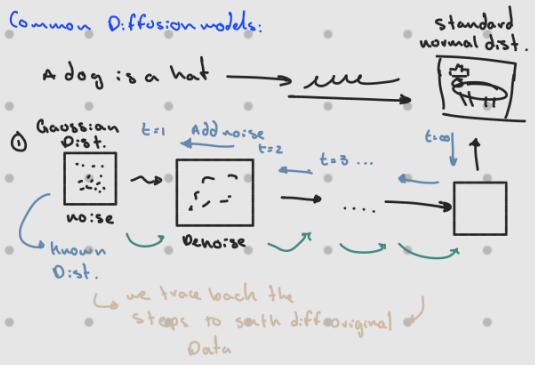
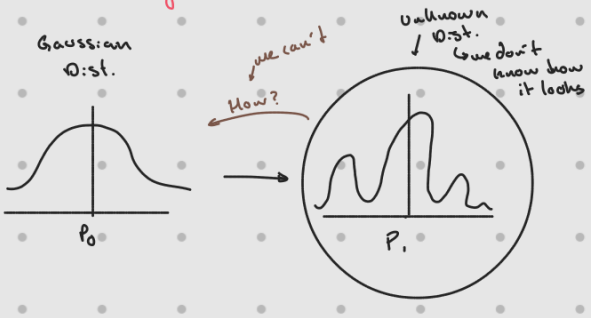
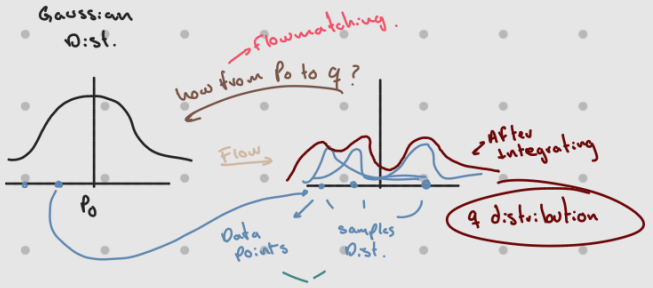
Flow matching.

Common Diffusion models:



Flow-matching:





but, how we make Data-Samples? CONDITIONAL FLOWS.

Continuos Normalizing Flows

1) Probability Density Path:

$p : [0,1] \times \mathbb{R}^d \to \mathbb{R}_{>0}$

time variable · Data dist (Input) · Time-Dependent Prob. Density

Time-Dependent function.

→ we give a time, and place in data space tells ↳ Prob. Density at that place/time.

2) Time-Dependent vector field.

$v : [0,1] \times \mathbb{R}^d \to \mathbb{R}^d$ → same last goes to Data Dist.

3) Time-Dependent diffeomorphic map [Flow]:

$\phi : [0,1] \times \mathbb{R}^d \to \mathbb{R}^d$

↳ Flow connected to vector field by:

$$\frac{d}{dt}\phi_t(x) = v_t(\phi_t(x))$$

change flow · vector field · time-Dependent · $\phi_0(x) = x$ · Time-Dependent

↳ Flow at time 0, at any point is that point

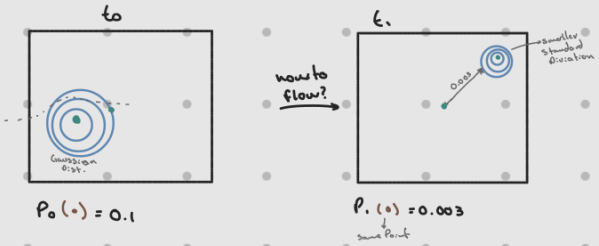
but how do we get from t_0 to t_1 dist.

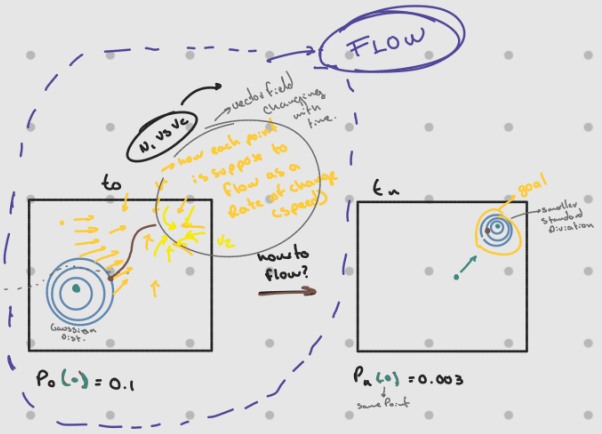
↳ using ODE solver to solve flows until we get to $t = x$

$$\therefore \quad p_t = [\phi_t] * p_0$$

↳ Goal Dist. = Flow * Original Dist.

2D-space Data





HOW TO GET VECTOR FIELD?

↳ what Algorithm must learn

Flow-matching:

$$\mathcal{L}_{FM}(\theta) = \mathbb{E}_{t,\ p_t(x)} \left\| v_t(x) - u_t(x) \right\|^2$$

- how to get? → $p_t(x)$: Prob. Dens. Path
- match for each pos. and time → $v_t(x)$: neural network
- $u_t(x)$: vector field (generates Prob. Dens. Path)
- How to get?

; we can't know, for now, the Prob density of target dist.

↳ we can Approximate by Samples with Gaussian models.

$$p_1(x) \approx q(x)$$

Constructing p_t, u_t from Conditional Probability Paths and vector fields: