Screening Task 1

September 27, 2020

0.1 Task instructions

Here, we are asked to build a 4 qubit circuit formed by L layers of even are odd blocks of parametrized gates, where the odd blocks are R_x gates each with a parameter $\theta_{i,n} \in (0, 2\pi)$, where $i \in 1, 2, 3, 4$ and $n \in$ odd; and the even block are formed by R_z gates with different $\theta_{i,n}$ parameters with even n followed by c_z gates between each pair of qubits. The resultant 4 qubit state $|\psi(\theta) >$ is used to calculate the minimum distance

$$\epsilon = \min_\theta |||\psi(\theta) > -|\phi(\theta) > ||,$$

where $|\phi(\theta) >$ is a randomly generated 4 qubit vector. Therefore, we need to write a rutine that finds the optimal $\theta_{i,n}$ parameters that results in the minimum ϵ for each L.

[1]:
```
from qiskit import QuantumCircuit, execute, Aer
from qiskit.tools.jupyter import *
from qiskit.visualization import *
import numpy as np
from qiskit.quantum_info import random_statevector
from qiskit_textbook.tools import array_to_latex
import random
from qiskit.aqua.components.optimizers import COBYLA, AQGD
import matplotlib.pyplot as plt
```

0.1.1 Solution

First, we create the random 4 qubit vector $|\phi(\theta) >$ that will be used as the reference vector, and the maximum number of layers L_{max} for the circuit

[2]:
```
Phi =np.fromiter(random_statevector(16).to_dict().values(), dtype = "complex_")␣
 ↪  #phi random vector

L_max = 8   #maximum number of layers
```

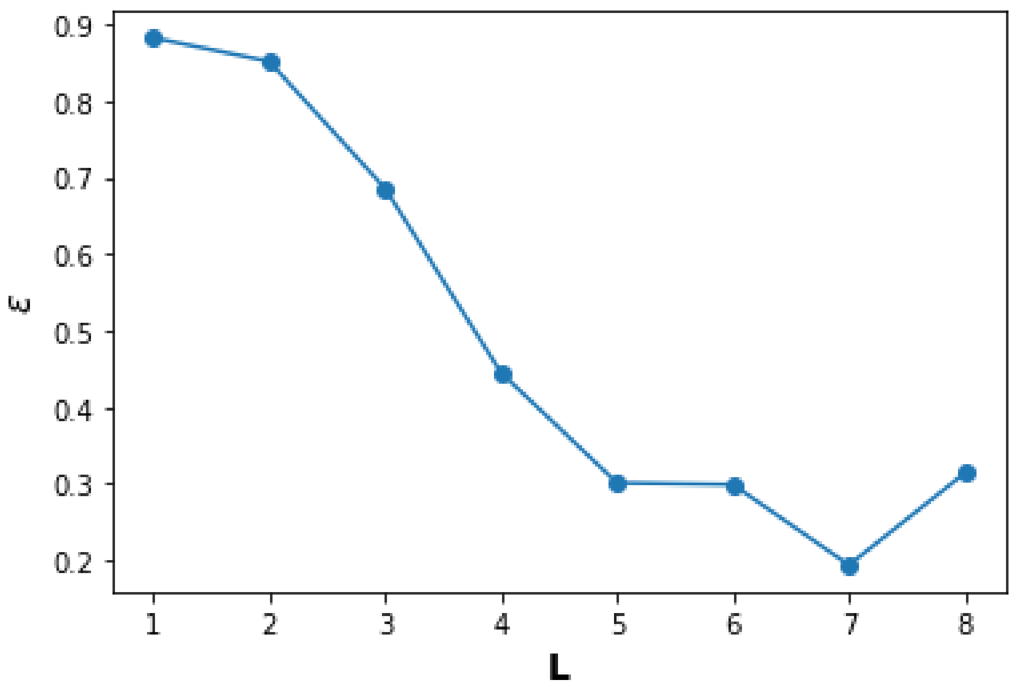
And the rutine to get the optimal $\theta_{i,n}$ parameters for each circuit with the number of layers $L \in (1, L_{max})$ is presented here. We use the Constrained Optimization By Linear Approximation (COBYLA) method for the optimization.

[3]:
```python
Epsilon = []

for L in range(1, L_max+1):      #loop for each L case

    def qc_gates(theta):         #function to create the circuit
        theta_d = []

        for i_d in range (0,L):
            theta_r = [theta[8*i_d:8*i_d+4],theta[8*i_d+4:8*(i_d+1)]]
            theta_d.append(theta_r)

        theta = np.asarray(theta_d)

        for i_l in range(0, L):
            for i in range (0, 4):
                circ.rx(theta[i_l][0][i], i)     #rx gates

            for i in range (0, 4):
                circ.rz(theta[i_l][0][i], i)     #rz gates

            circ.cz(0, 1)                        #cz gates
            circ.cz(0, 2)
            circ.cz(0, 3)
            circ.cz(1, 2)
            circ.cz(1, 3)
            circ.cz(2, 3)

    def cost(theta_c):                #function to calculate epsilon
        global circ

        circ = QuantumCircuit(4)      #initialization of quantum circuit

        qc_gates(theta_c)             #evaluate gate function

        backend = Aer.get_backend('statevector_simulator')

        job = execute(circ, backend)

        result = job.result()

        st_vec = result.get_statevector()

        return np.linalg.norm(st_vec-Phi)

    theta_0 = np.random.rand(8*L)*2*np.pi   #initial random parameters theta

    bound = []
```

```
    for i_b in range(0, 8*L):
        bound.append([0, 2*np.pi])

    optimizer = COBYLA(maxiter=2000)        #optimization

    theta_opt, eps, _ = optimizer.optimize(len(theta_0), cost,␣
↪initial_point=theta_0, variable_bounds=bound)

    Epsilon.append(eps)                    #store result
```

Finally, we plot the resultant minimum distance ϵ for each layer L

[4]:
```
plt.plot(np.arange(1,L_max+1), Epsilon, marker='o')
plt.xlabel('L',fontsize=14, fontweight='bold')
plt.ylabel('$\epsilon$', fontsize=14, fontweight='bold')
plt.show()
```



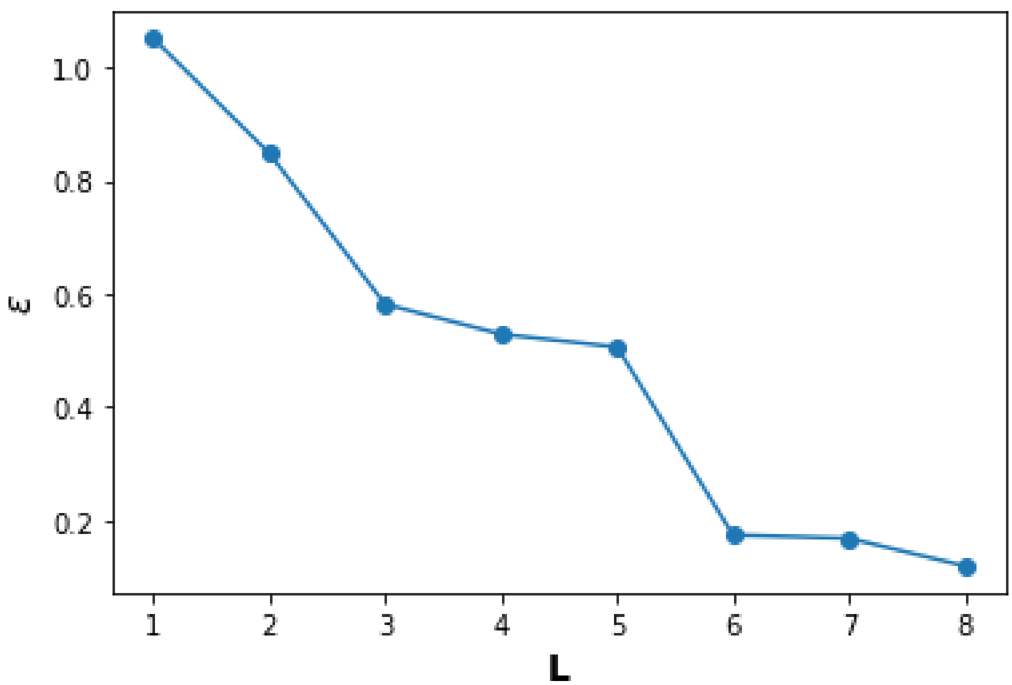
As we can see, the minimum distance ϵ decreases as the number of layers L in the circuit increases until it reaches a minmum value $\epsilon \approx 0.2$ for $L = 7$. Further increasing the number L of layers produces an increase of the resultant parameter to $\epsilon \approx 0.3$

We may explore what happens when we use different parametrized gates in our circuit. For instance, if we change the R_x gates in the odd blocks for R_y gates, we get:

[5]:
```python
Epsilon = []

for L in range(1, L_max+1):     #loop for each L case

    def qc_gates(theta):        #function to create the circuit
        theta_d = []

        for i_d in range (0,L):
            theta_r = [theta[8*i_d:8*i_d+4],theta[8*i_d+4:8*(i_d+1)]]
            theta_d.append(theta_r)

        theta = np.asarray(theta_d)

        for i_l in range(0, L):
            for i in range (0, 4):
                circ.ry(theta[i_l][0][i], i)     #ry gates

            for i in range (0, 4):
                circ.rz(theta[i_l][0][i], i)     #rz gates

            circ.cz(0, 1)                        #cz gates
            circ.cz(0, 2)
            circ.cz(0, 3)
            circ.cz(1, 2)
            circ.cz(1, 3)
            circ.cz(2, 3)

    def cost(theta_c):                 #function to calculate epsilon
        global circ

        circ = QuantumCircuit(4)     #initialization of quantum circuit

        qc_gates(theta_c)            #evaluate gate function

        backend = Aer.get_backend('statevector_simulator')

        job = execute(circ, backend)

        result = job.result()

        st_vec = result.get_statevector()

        return np.linalg.norm(st_vec-Phi)

    theta_0 = np.random.rand(8*L)*2*np.pi   #initial random parameters theta

    bound = []
```

```
    for i_b in range(0, 8*L):
        bound.append([0, 2*np.pi])

    optimizer = COBYLA(maxiter=2000)        #optimization

    theta_opt, eps, _ = optimizer.optimize(len(theta_0), cost,␣
↪initial_point=theta_0, variable_bounds=bound)

    Epsilon.append(eps)                     #store result
```

[6]:
```
plt.plot(np.arange(1,L_max+1), Epsilon, marker='o')
plt.xlabel('L',fontsize=14, fontweight='bold')
plt.ylabel('$\epsilon$', fontsize=14, fontweight='bold')
plt.show()
```



We can note that the minimum ϵ in this case is lower than the optimal in the previous circuit, reaching a value of $\epsilon = 0.12$ for $L = 8$.

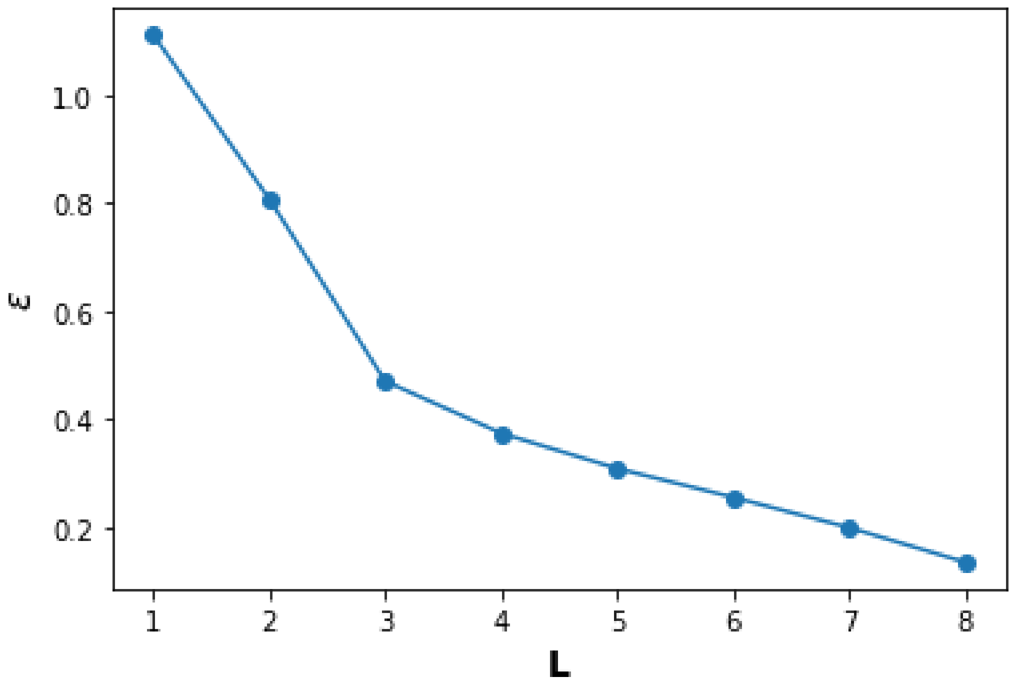
Now, replacing the R_z gates in the even blocks for R_y gates, we get:

[8]:
```
Epsilon = []

for L in range(1, L_max+1):      #loop for each L case
```

```
def qc_gates(theta):        #function to create the circuit
    theta_d = []

    for i_d in range (0,L):
        theta_r = [theta[8*i_d:8*i_d+4],theta[8*i_d+4:8*(i_d+1)]]
        theta_d.append(theta_r)

    theta = np.asarray(theta_d)

    for i_l in range(0, L):
        for i in range (0, 4):
            circ.rx(theta[i_l][0][i], i)     #rx gates

        for i in range (0, 4):
            circ.ry(theta[i_l][0][i], i)     #ry gates

        circ.cz(0, 1)                        #cz gates
        circ.cz(0, 2)
        circ.cz(0, 3)
        circ.cz(1, 2)
        circ.cz(1, 3)
        circ.cz(2, 3)

def cost(theta_c):              #function to calculate epsilon
    global circ

    circ = QuantumCircuit(4)    #initialization of quantum circuit

    qc_gates(theta_c)           #evaluate gate function

    backend = Aer.get_backend('statevector_simulator')

    job = execute(circ, backend)

    result = job.result()

    st_vec = result.get_statevector()

    return np.linalg.norm(st_vec-Phi)

theta_0 = np.random.rand(8*L)*2*np.pi   #initial random parameters theta

bound = []

for i_b in range(0, 8*L):
    bound.append([0, 2*np.pi])
```

```
    optimizer = COBYLA(maxiter=2000)        #optimization

    theta_opt, eps, _ = optimizer.optimize(len(theta_0), cost,␣
↪initial_point=theta_0, variable_bounds=bound)

    Epsilon.append(eps)                     #store result
```

[9]:
```
plt.plot(np.arange(1,L_max+1), Epsilon, marker='o')
plt.xlabel('L',fontsize=14, fontweight='bold')
plt.ylabel('$\epsilon$', fontsize=14, fontweight='bold')
plt.show()
```



In this case, we can see that ϵ quickly drops to a value of 0.47 for $L = 3$, followed by a less steep linear decrease for $L \geq 4$ achiving a minimum of $\epsilon = 0.135$ for $L = 8$