INSTITUTO POLITECNICO NACIONAL

ESCUELA SUPERIOR DE CÓMPUTO (ESCOM)

ANÁLISIS DE ALGORITMOS

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EJERCICIO 05:

- DOMINIO ASINTÓTICO

FECHA DE ENTREGA:

- 01/04/2022

GRUPO:

- 3CM14

Dominio Asintótico

Objetivo: Demuestre para los dos primeros ejercicios el dominio asintótico de f(x) sobre g(x) y para los ejercicios del 3 al 7 demostrar que las funciones tienen una correcta cota asignada para las tres primeras funciones de complejidad tienen asignada correctamente la cota O “Cota superior ajustada” y que las últimas dos también tienen una correcta cota θ “exacta”).

Ejercicio 01:

$f(x) = x^2$

$g(x) = 2x^2 + 300x - 1000$

$\exists m \geq 0, k \geq 0 \mid |g(n)| \leq m|f(n)|, \forall x \geq k$

$|2x^2 + 300x - 1000| \leq m|x^2|$

$$\left|\frac{2x^2}{x^2}\right| + \left|\frac{300x}{x^2}\right| + \left|-\frac{1000}{x^2}\right| \leq m\frac{x^2}{x^2}$$

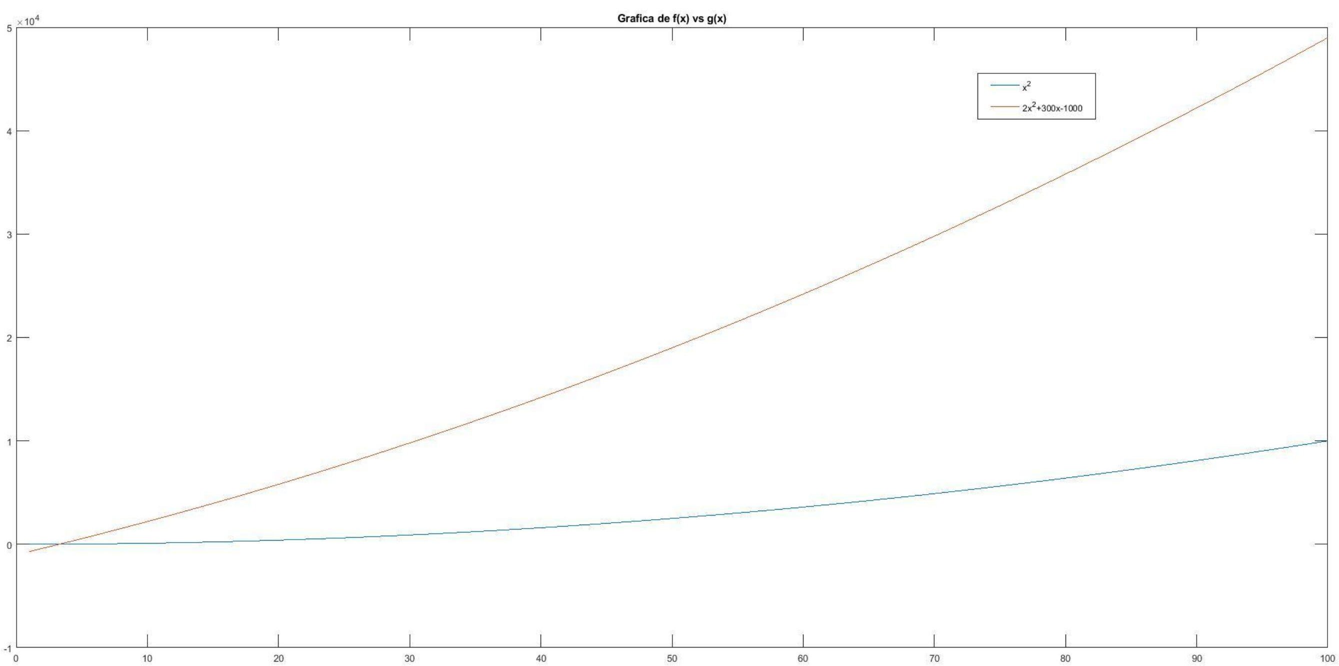
$$2 + \frac{300}{x} + \frac{1000}{x^2} \leq m \qquad si \quad x = 1 = k$$

$2 + 300 + 1000 \leq m$

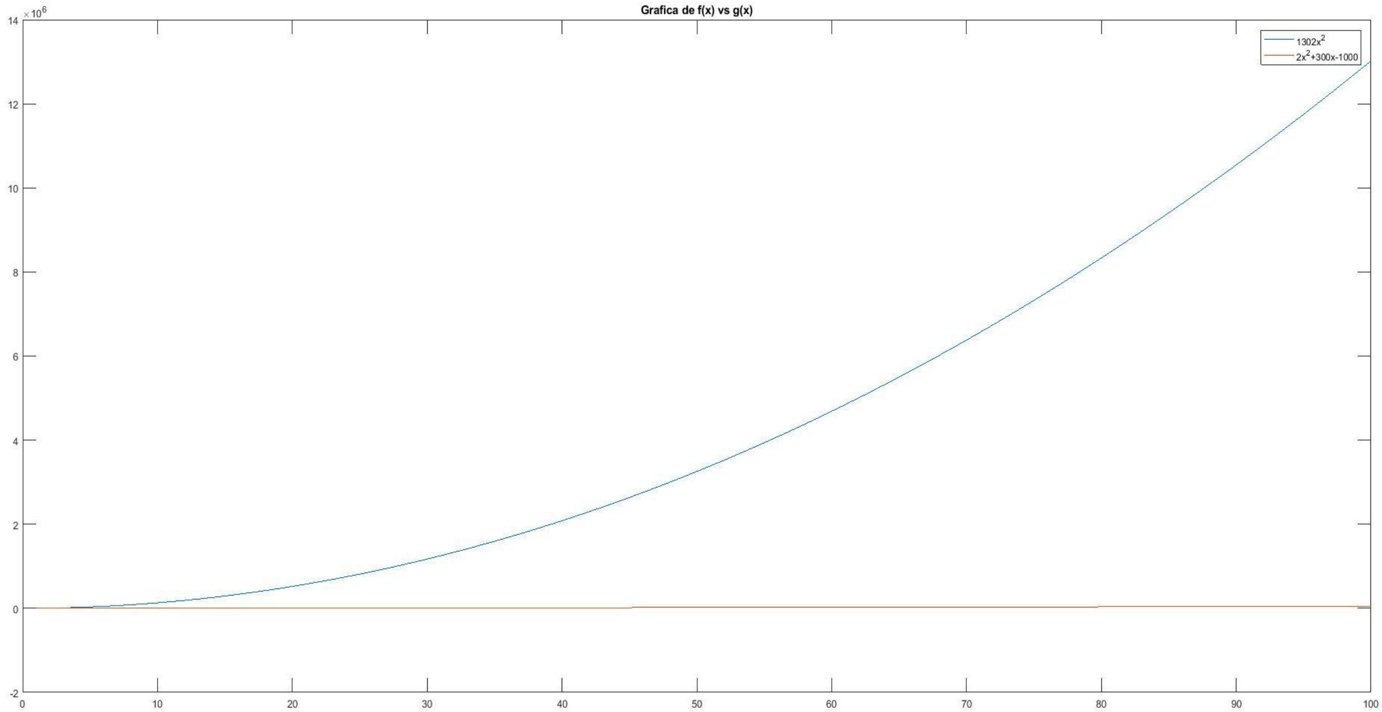
$m \geq 1302$

Con $m \geq 1302$ y $k \geq 1$ se demuestra que g(x) es dominada por f(x)

Gráfica de x^2 vs $2x^2 + 300x - 1000$



Gráfica de $1302x^2$ vs $2x^2 + 300x - 1000$



Ejercicio 02:

$f(x) = x^3$

$g(x) = 2x^3 - 30x + 500$

$\exists m \geq 0, k \geq 0 \mid |g(n)| \leq m|f(n)|, \forall x \geq k$

$|2x^3 - 30x + 500| \leq m|x^3|$

$$\left|\frac{2x^3}{x^3}\right| + \left|-\frac{30x}{x^3}\right| + \left|\frac{500}{x^3}\right| \leq m\frac{x^3}{x^3}$$

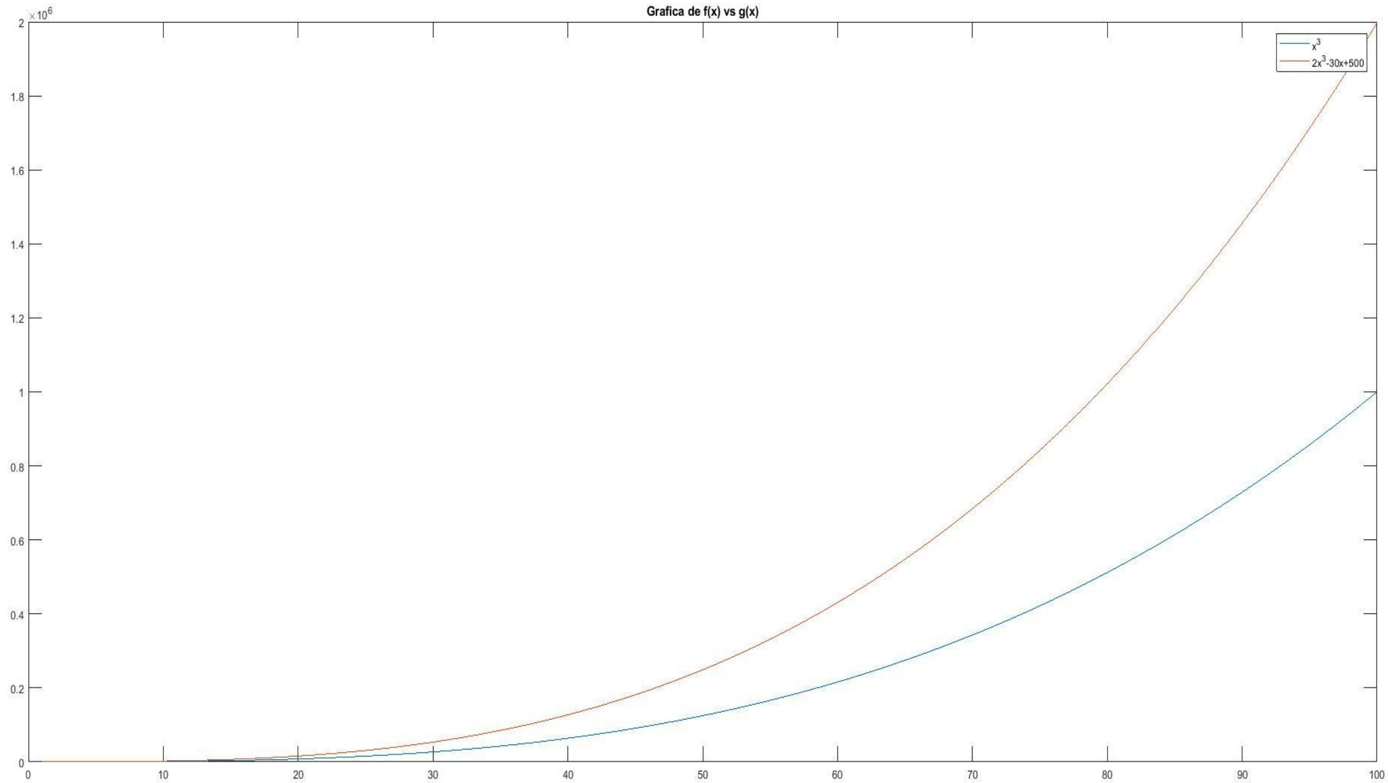
$$2 + \frac{30}{x^2} + \frac{500}{x^3} \leq m \qquad si \quad x = 1 = k$$

$2 + 30 + 500 \leq m$

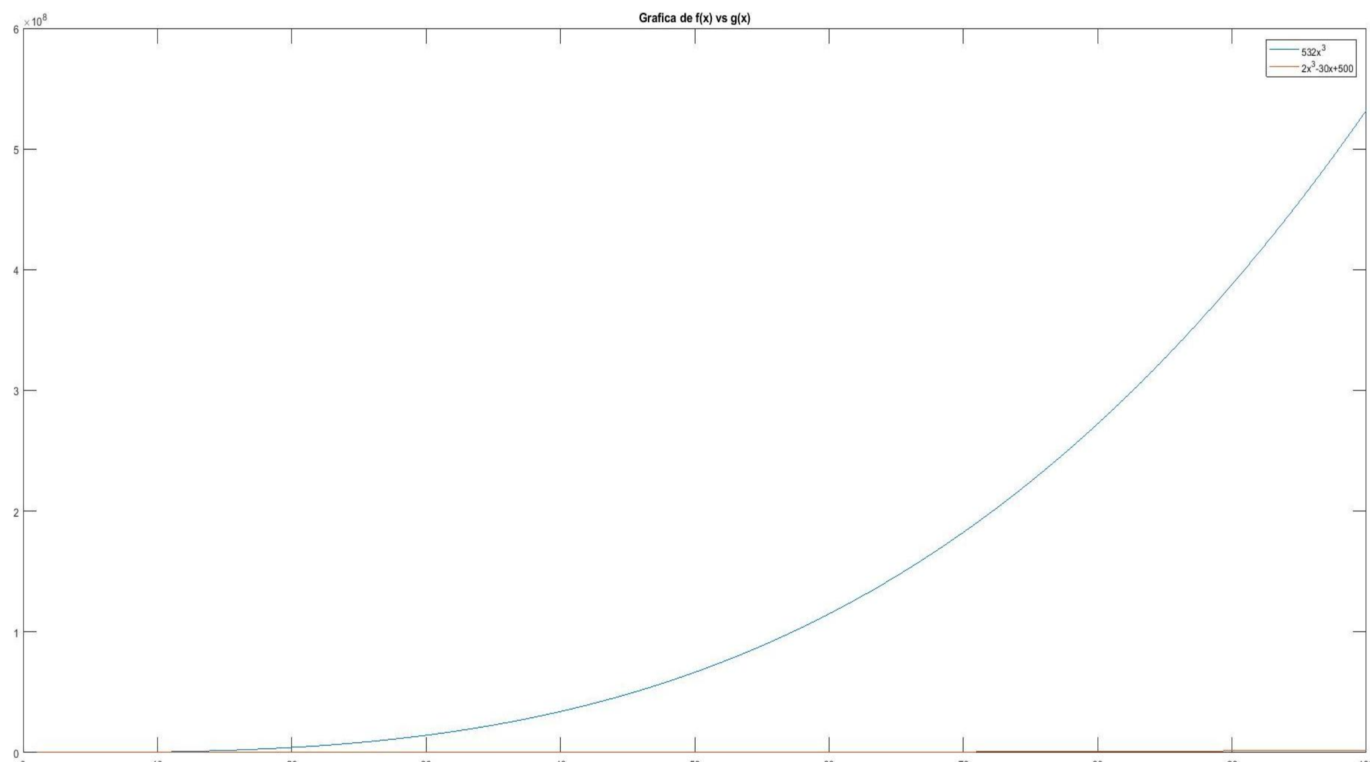
$m \geq 532$

Con $m \geq 532$ y $k \geq 1$ se demuestra que g(x) es dominada por f(x)

Gráfica de x^3 vs $2x^3 - 30x + 500$



Gráfica de $532x^3$ vs $2x^3 - 30x + 500$



Ejercicio 03:

$f_t(n) = 3n^2 + 9n + 12 \in O(2n^2)$

$f(x) = O\big(g(x)\big) \leftrightarrow \exists c > 0, x_0 > 0 \mid \forall n > x_0, |f(n)| \leq c|g(n)|$

$0 \leq |f(n)| \leq c|g(n)|$

$0 \leq |3n^2 + 9n + 12| \leq c|2n^2|$

$$\frac{0}{2n^2} \leq \frac{3n^2}{2n^2} + \frac{9n}{2n^2} + \frac{12}{2n^2} \leq c\frac{2n^2}{2n^2}$$

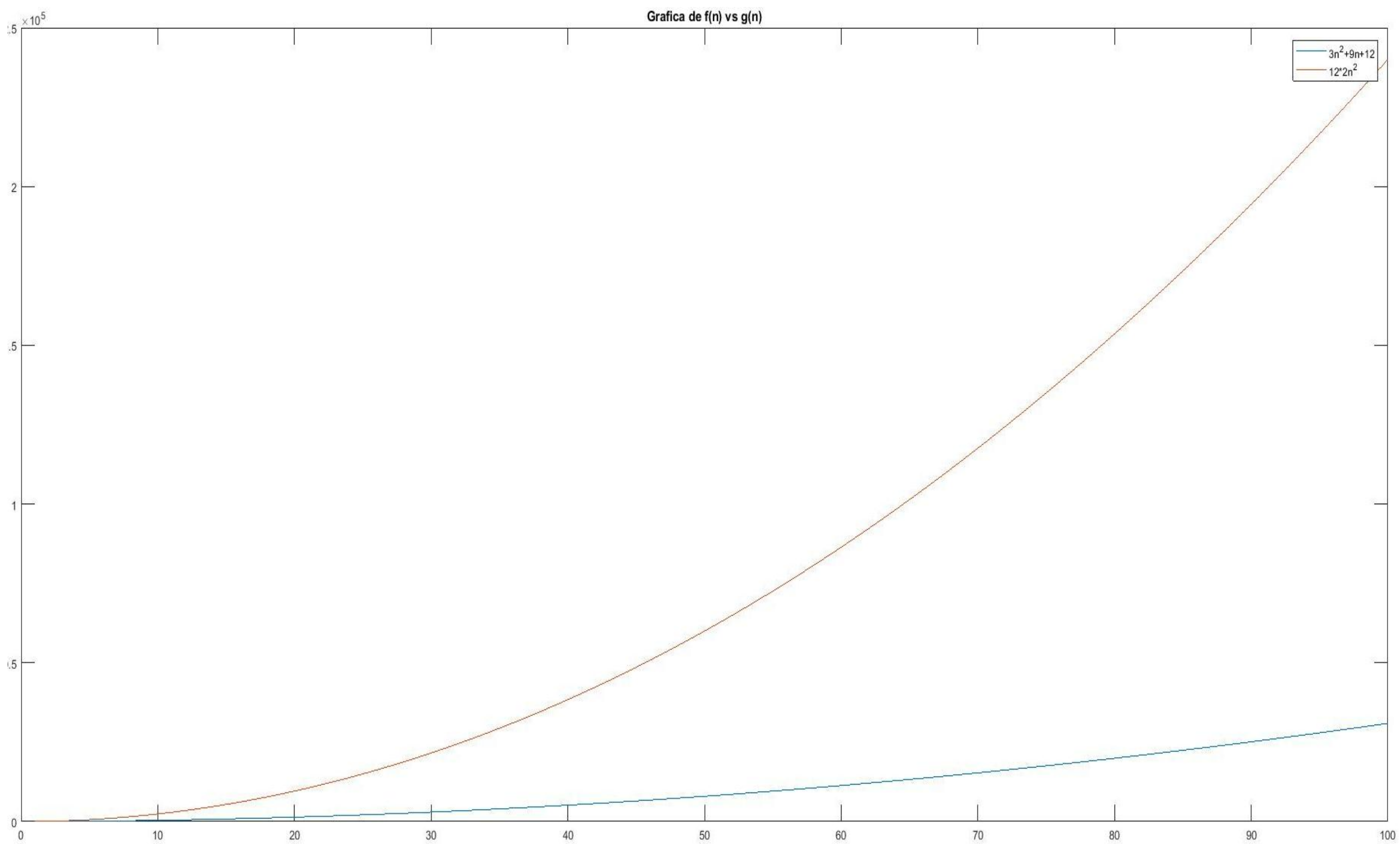
$$0 \leq \frac{3}{2} + \frac{9}{2n} + \frac{6}{n^2} \leq c \qquad si\ \ x_0 = n = 1$$

$0 \leq 12 \leq c \quad \therefore \quad c \geq 12$

$3n^2 + 9n + 12 \leq 12|2n^2|$

Entonces $3n^2 + 9n + 12$ = O$(2n^2)$

Gráfica de $3n^2 + 9n + 12$ vs $12|2n^2|$



Ejercicio 04:

$f_t(n) = 2n + 8 \ \in O(n)$

$f(x) = O\big(g(x)\big) \leftrightarrow \ \exists c > 0, x_0 > 0 \mid \forall n > x_0, |f(n)| \leq \ c|g(n)|$

$0 \leq |f(n)| \leq c|g(n)|$

$0 \leq |2n + 8| \leq c|n|$

$\frac{0}{n} \leq \frac{2n}{n} + \frac{8}{n} \leq c\frac{n}{n}$

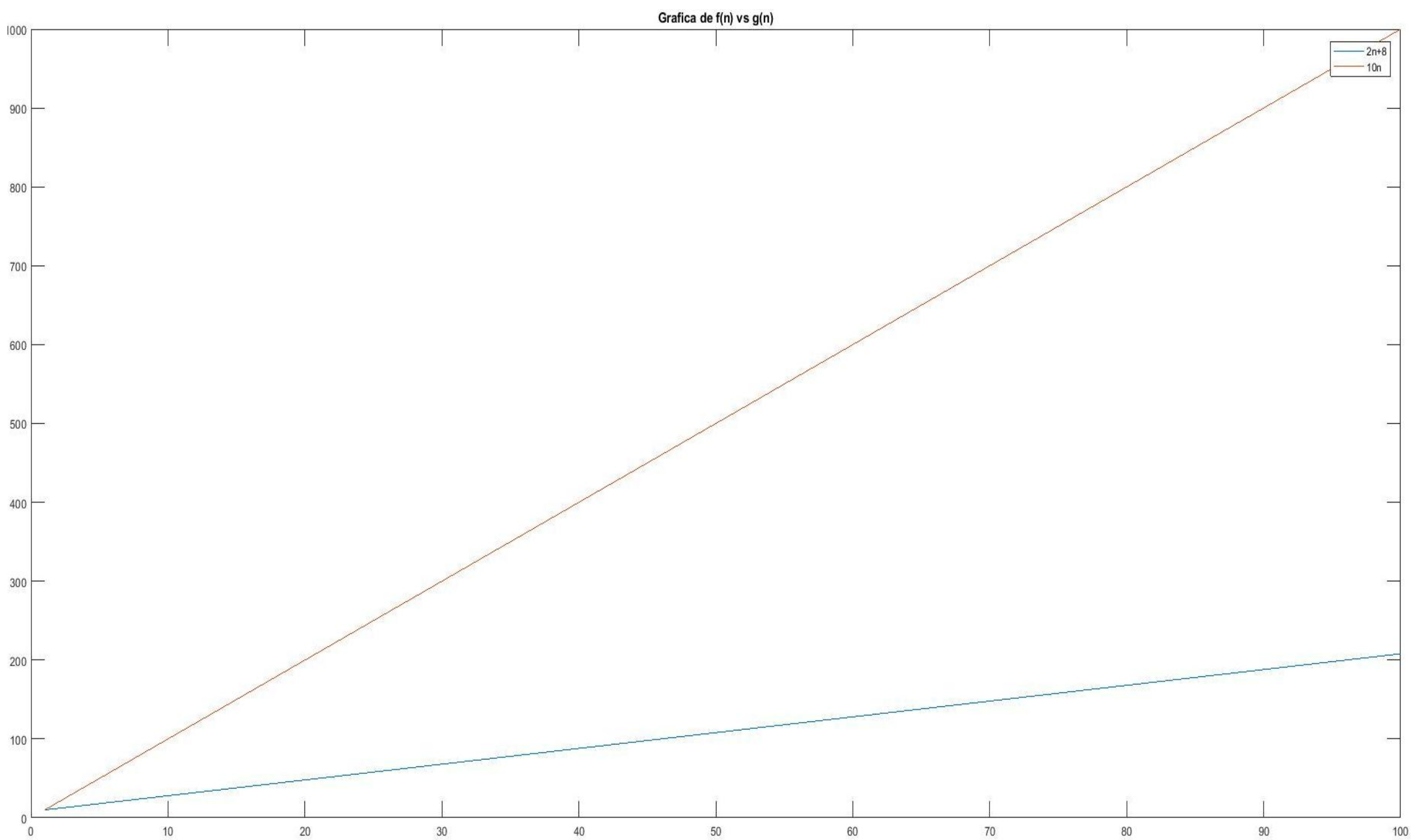
$0 \leq 2 + \frac{8}{n} \leq c \quad Si \ \ x_0 = n = 1$

$0 \leq 2 + 8 \leq c$

$0 \leq 10 \leq c \quad \therefore \quad c \geq 10$

Entonces $2n + 8$ = O(n)

Gráfica de $2n + 8$ vs $10n$



Ejercicio 05:

$f_t(n) = 2n^3 - 3n^2 + 9n + 120 \quad \in O(n^3)$

$f(x) = O\big(g(x)\big) \leftrightarrow\ \exists c > 0, x_0 > 0 \mid \forall n > x_0, |f(n)| \leq\ c|g(n)|$

$0 \leq |f(n)| \leq c|g(n)|$

$0 \leq |2n^3 - 3n^2 + 9n + 120\ \ | \leq c|n^3|$

$\frac{0}{n^3} \leq \frac{2n^3}{n^3} - \frac{3n^2}{n^3} + \frac{9n}{n^3} + \frac{120}{n^3}\ \leq \frac{cn^3}{n^3}$

$0 \leq 2 - \frac{3}{n} + \frac{9}{n^2} + \frac{120}{n^3}\ \leq c \qquad Si\ n = x_0 = 1$

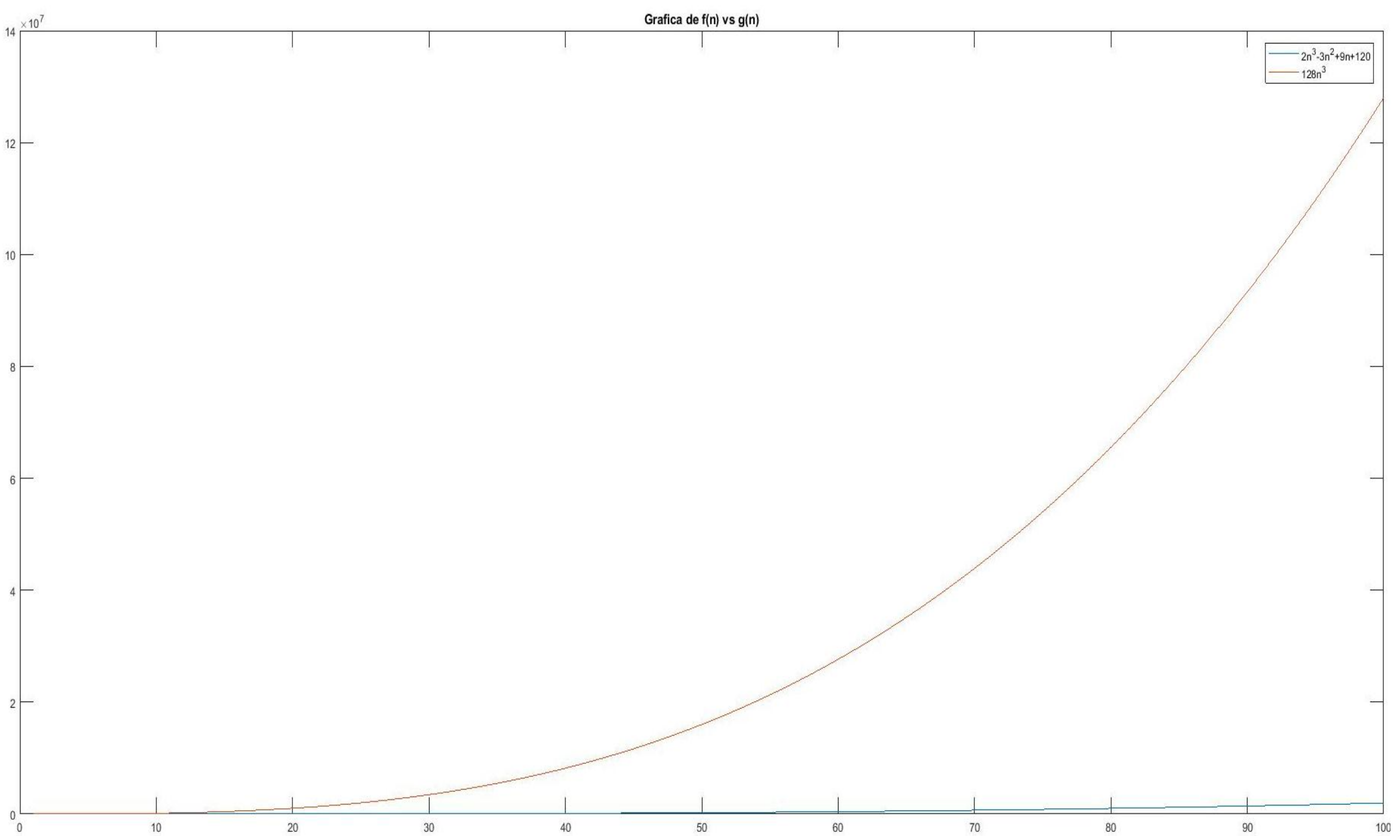
$0 \leq 2 - 3 + 9 + 120\ \ \leq c$

$0 \leq 128 \leq c \quad \therefore\ \ c \geq 128$

$2n^3 - 3n^2 + 9n + 120 \leq 128n^3$

Entonces $2n^3 - 3n^2 + 9n + 120$ = O(n^3)

Gráfica de $2n^3 - 3n^2 + 9n + 120$ vs $128n^3$



Ejercicio 06:

$f_t(n) = 2n^3 + 3n^2 + 9n + 120 \quad \in \theta(n^3 + n^2)$

$f(x) = \theta\big(g(x)\big) \leftrightarrow \exists c_1 > 0, c_2 > 0, x_0 > 0 \mid \forall x > x_0, c_1|g(x)| \leq |f(x)| \leq c_2|g(x)|$

$c_1|n^3 + n^2| \leq |2n^3 + 3n^2 + 9n + 120| \leq c_2|n^3 + n^2|$

$c_1(n^3 + n^2) \leq 2n^3 + 3n^2 + 9n + 120 \leq c_2(n^3 + n^2)$

$$c_1\frac{n^3 + n^2}{n^3 + n^2} \leq \frac{2n^3}{n^3 + n^2} + \frac{3n^2}{n^3 + n^2} + \frac{9n}{n^3 + n^2} + \frac{120}{n^3 + n^2} \leq c_2\frac{n^3 + n^2}{n^3 + n^2}$$

$$c_1 \leq \frac{2n + 3}{n + 1} + \frac{9}{n^2 + n} + \frac{120}{n^3 + n^2} \leq c_2 \quad Si\ n = 1$$

$c_2 \geq 67$

Entonces $2n^3 + 3n^2 + 9n + 120 = O(n^3 + n^2)$

$f(n) = \Omega\big(g(n)\big) \leftrightarrow \exists c > 0, x_0 > 0 \mid \forall n > x_0, c|g(n)| \leq c|f(n)|$

$0 \leq c(n^3 + n^2) \leq 2n^3 + 3n^2 + 9n + 120$

$$\frac{0}{n^3 + n^2} \leq c\frac{n^3 + n^2}{n^3 + n^2} \leq \frac{2n^3 + 3n^2 + 9n + 120}{n^3 + n^2}$$

$$0 \leq c \leq \frac{2n + 3}{n + 1} + \frac{9}{n^2 + n} + \frac{120}{n^3 + n^2}$$

$$\lim_{n\to\infty}\frac{2n + 3}{n + 1} + \lim_{n\to\infty}\frac{9}{n^2 + n} + \lim_{n\to\infty}\frac{120}{n^3 + n^2}$$

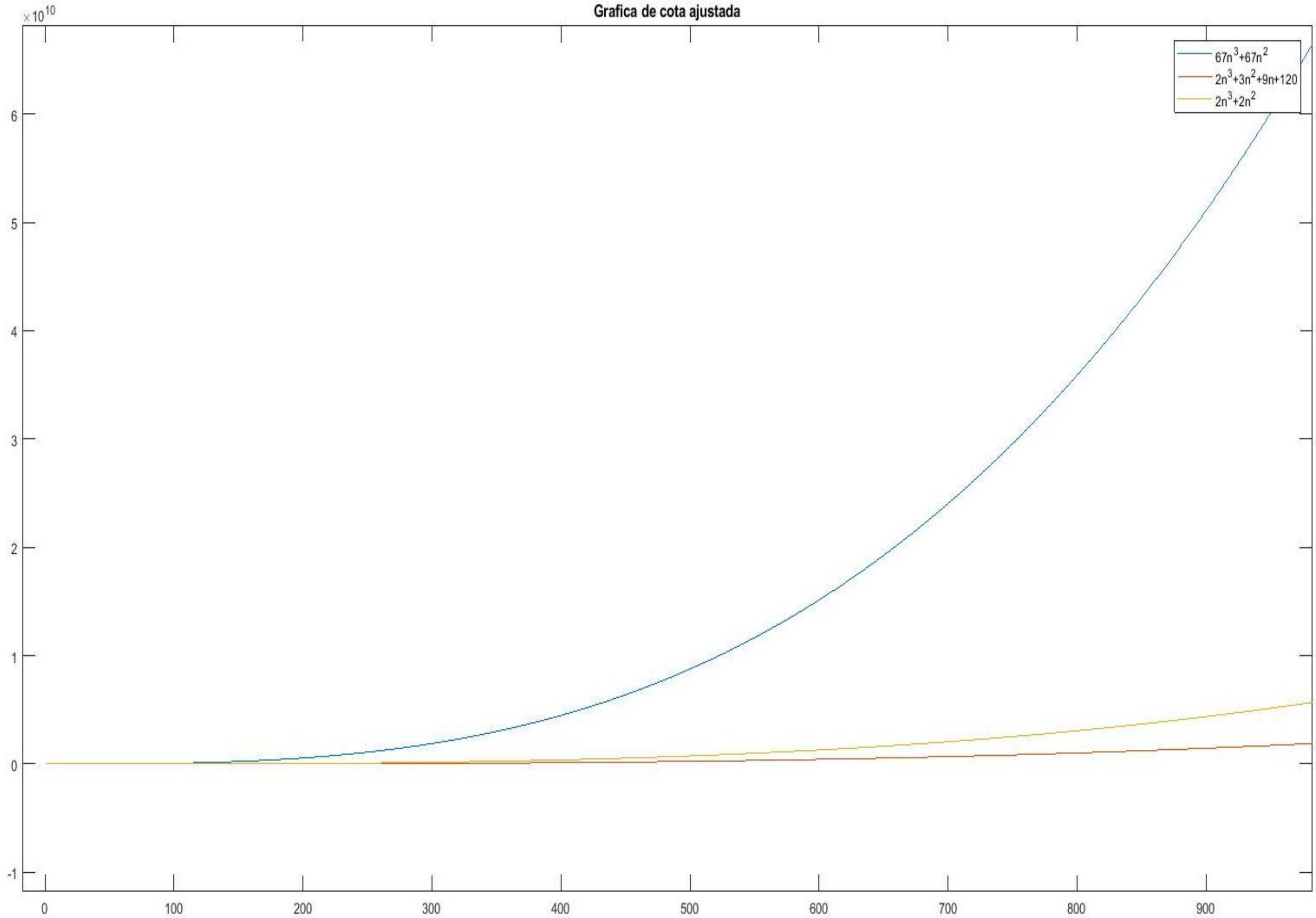
$2 + 0 + 0$

$c_1 \geq 2$

$2(n^3 + n^2) \leq 2n^3 + 3n^2 + 9n + 120 \leq 67(n^3 + n^2) \quad con\ c_1 = 2\ c_2 = 67 \quad con\ n \geq 1$

$\therefore\ 2n^3 + 3n^2 + 9n + 120 = \theta(n^3 + n^2)$

Gráfica de $2(n^3 + n^2) \leq 2n^3 + 3n^2 + 9n + 120 \leq 67(n^3 + n^2)$



Ejercicio 07:

$f_t(n) = 2n^2 + 9n \ \ \in \theta(n^2)$

$f(x) = \theta\big(g(x)\big) \leftrightarrow \exists c_1 > 0, c_2 > 0, x_0 > 0 \ | \ \forall x > x_0, c_1|g(x)| \leq |f(x)| \leq c_2|g(x)|$

Probamos para O

$\exists m \geq 0, k \geq 0 \ | \ |g(n)| \leq m|f(n)|, \forall x \geq k$

$|2n^2 + 9n \ | \leq c|n^2|$

$\frac{2n^2}{n^2} + \frac{9n}{n^2} \leq c\frac{n^2}{n^2}$

$2 + \frac{9}{n} \leq c \quad Si\ n = 1$

$2 + 9 \geq c$

$11 \geq c = c_2$

Entonces $2n^2 + 9n = O(n^2)$
Probamos para Ω

$f(n) = \Omega\big(g(n)\big) \leftrightarrow \exists c > 0, x_0 > 0 \mid \forall n > x_0, c|g(n)| \leq c|f(n)|$

$0 \leq c(n^2) \leq 2n^2 + 9n$

$$\frac{0}{n^2} \leq c\frac{n^2}{n^2} \leq 2\frac{n^2}{n^2} + \frac{9}{n^2}$$

$$0 \leq c \leq 2 + \frac{9}{n}$$

$$\lim_{n\to\infty}(2 + \frac{9}{n}) = 2 \qquad \therefore \quad c_1 \geq 2$$

$2|n^2| \leq |2n^2 + 9n| \leq 11|n^2|$

$\therefore\ 2n^2 + 9n = \theta(n^2)$

Gráfica de $2(n^2) \leq 2n^2 + 9n \leq 11(n^2)$