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Ejemplo 14: Sea $X_1, X_2, \dots, X_n$ una m.a de una densidad Normal con media μ y varianza σ^2 encuentra el estimador máximo verosímil de $\theta = (\mu, \sigma^2)$.

Recordamos la f.d.p de la Normal

$$f_x(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{si } x>0$$

1) Escribimos la función de verosimilitud $L(\theta)$

$$L(\mu,\sigma^2) = \prod_{i=1}^{n} f(x_i, \mu, \sigma^2)$$

$$= \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

$$= (\sigma\sqrt{2\pi})^{-n} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

2) Sacamos el logaritmo natural $\ln(L(\theta))$

$$\ln L(\mu,\sigma^2) = \ln{-n}(\sigma\sqrt{2\pi})\, e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\ln L(\mu,\sigma^2) = -n\ln\sigma - n\ln\sqrt{2\pi} - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

3) Derivar parcialmente respecto al parámetro o parámetros de interés, igualar a cero y despejar el estimador

$$\frac{\partial}{\partial\mu}\ln L(\mu,\sigma^2) = \cancel{\frac{\partial}{\partial\mu}-n\ln\sigma}^{0} - \cancel{\frac{\partial}{\partial\mu}n\ln\sqrt{2\pi}}^{0} - \frac{\partial}{\partial\mu}\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\mu}\ln L(\mu,\sigma^2) = -\frac{1}{2\sigma^2}2(x_i-\mu)(-1) = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$

$$\frac{\partial}{\partial\sigma^2}\ln L(\mu,\sigma^2) = \frac{\partial}{\partial\sigma^2}\,-n\ln\sigma - \cancelto{0}{\frac{\partial}{\partial\sigma^2}\,n\ln\sqrt{2\pi}} - \frac{\partial}{\partial\sigma^2}\frac{1}{2\sigma^2}$$

$$\sum_{i=1}^{n}(x_i-\mu)^2 = -n\frac{\partial}{\partial\sigma^2}\ln\sigma - \frac{\partial}{\partial\sigma^2}\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\sigma^2}\ln L(\mu,\sigma^2) = -n\left(\frac{1}{2\sigma^2}\right) - \frac{1}{2}\frac{\partial}{\partial\sigma^2}\left(\sigma^{-2}\right)\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\sigma^2}\ln L(\mu,\sigma^2) = -\frac{n}{2\sigma^2} + \frac{1}{2}\left(\frac{1}{\sigma^4}\right)\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$= \frac{1}{\sigma^2}\left[\sum_{i=1}^{n}x_i - n\hat{\mu}\right] \Rightarrow \hat{\mu} = \bar{x}$$

$$-\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\frac{1}{2\hat\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = \frac{n}{2\hat\sigma^2}$$

$$\frac{\cancel{2}\hat\sigma^2}{\cancel{2}\hat\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = n$$

$$\frac{1}{\hat\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 = n$$

$$\hat\sigma^2 n = \sum_{i=1}^{n}(x_i-\mu)^2$$

$$\hat\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$

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