

Understanding Analysis Solutions

Ulisse Mini

Contents

1	The Real Numbers	1
1.2	Some Preliminaries	1

Chapter 1

The Real Numbers

1.2 Some Preliminaries

Exercise 1.2.1

- (a) Prove that $\sqrt{3}$ is irrational. Does a similar similar argument work to show $\sqrt{6}$ is irrational?
- (b) Where does the proof break down if we try to prove $\sqrt{4}$ is irrational?

Solution

- (a) Suppose for contradiction that p/q is a fraction in lowest terms, and that $(p/q)^2 = 3$. Then $p^2 = 3q^2$ implying p is a multiple of 3 since 3 is not a perfect square. Therefor we can write p as $3r$ for some r , substituting we get $(3r)^2 = 3q^2$ and $3r^2 = q^2$ implying q is also a multiple of 3 contradicting the assumption that p/q is in lowest terms.
For $\sqrt{6}$ the same argument applies, since 6 is not a perfect square.
- (b) 4 is a perfect square, meaning $p^2 = 4q^2$ does not imply that p is a multiple of four as p could be 2.