CÁLCULO I

Teorema Fundamental do Cálculo

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Se f for integrável em $[a, b]$ e se F for uma primitiva de f em $[a, b]$, então

$$\int_a^b f(x)dx = F(b) - F(a)$$

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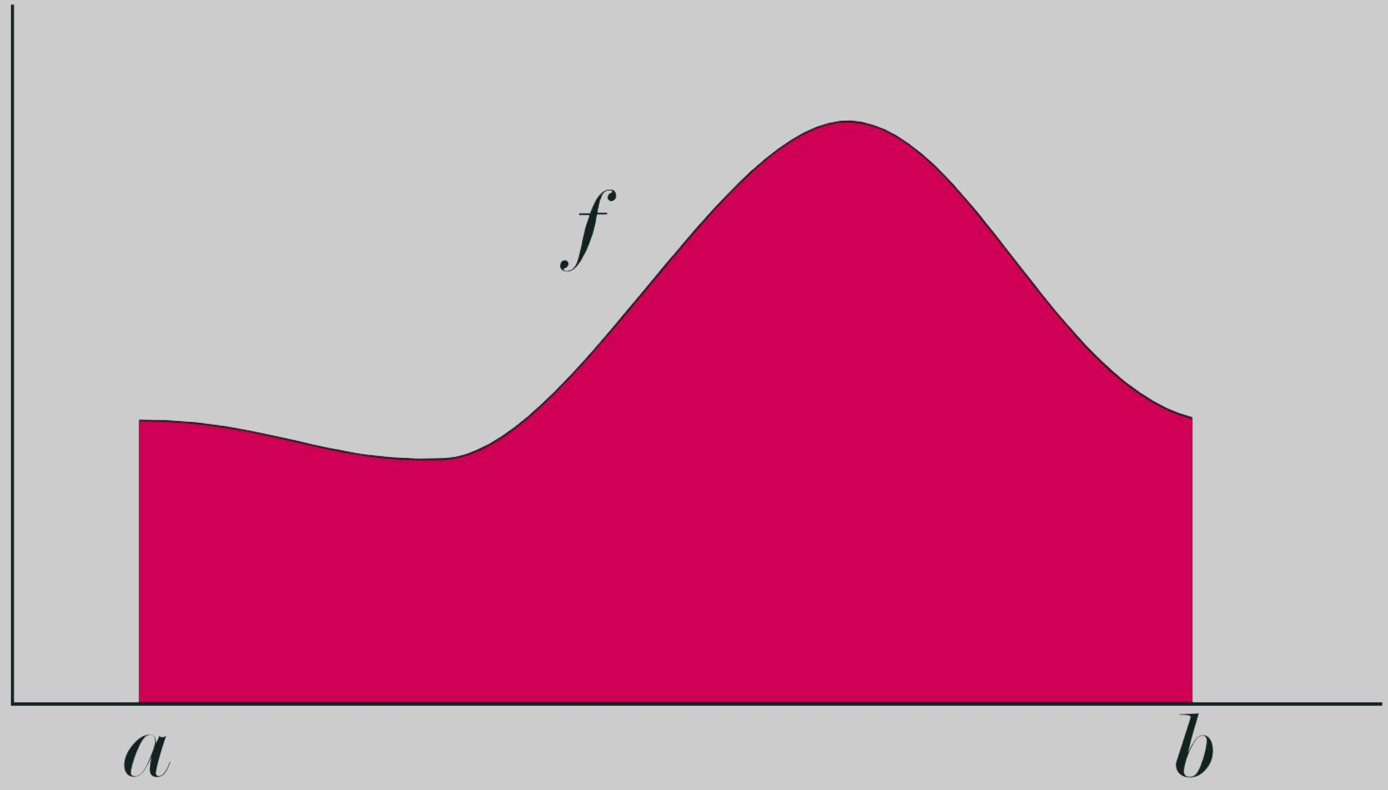

Teorema Fundamental do Cálculo

Exemplo

$$\int_{-1}^{2} 2\,dx = [2x]_{-1}^{2} = (2 \cdot 2) - \big(2 \cdot (-1)\big) = 4 + 2 = 6$$

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Exemplo

$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \left(\frac{2^3}{3}\right) - \left(\frac{0^3}{3}\right) = \frac{8}{3}$$

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$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \left(\frac{2^3}{3}\right) - \left(\frac{0^3}{3}\right) = \frac{8}{3}$$

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Exemplo

$$\int_1^2 (x^3 + 2x + 1)dx = \left[\frac{x^4}{4} + x^2 + x\right]_1^2 = \left(\frac{2^4}{4} + 2^2 + 2\right) - \left(\frac{1^4}{4} + 1^2 + 1\right) = 10 - \frac{10}{4} = \frac{31}{4}$$

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Exemplo

$$\int_1^2 (x^3 + 2x + 1)dx = \left[\frac{x^4}{4} + x^2 + x\right]_1^2 = \left(\frac{2^4}{4} + 2^2 + 2\right) -$$

$$\left(\frac{1^4}{4} + 1^2 + 1\right) = 10 - \frac{9}{4} = \frac{31}{4}$$

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Exemplo

$$\int_1^3 \frac{1}{x^3}\,dx = \int_1^3 x^{-3}\,dx = \left[-\frac{1}{2x^2}\right]_1^3 = \left(-\frac{1}{18}\right) - \left(-\frac{1}{2}\right)$$

$$= \frac{4}{9}$$

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$$\int_1^3 \frac{1}{x^3}\,dx = \int_1^3 x^{-3}\,dx = \left[-\frac{1}{2x^2}\right]_1^3 = \left(-\frac{1}{18}\right) - \left(-\frac{1}{2}\right) = \frac{4}{9}$$

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Exemplo

$$\int_1^2 \left(\frac{1}{x} + \frac{1}{x^2}\right) dx = \left[\ln x - x^{-1}\right]_1^2 = \left(\ln 2 - \frac{1}{2}\right) - (\ln 1 - 1)$$

$$= \ln 2 + \frac{1}{2}$$

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$$\int_1^2 \left(\frac{1}{x} + \frac{1}{x^2}\right) dx = \left[\ln x - x^{-1}\right]_1^2 = \left(\ln 2 \ - \frac{1}{2}\right) - (\ln 1 - 1)$$

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Exemplo

$$\int_0^{\frac{\pi}{4}} \cos 2x \, dx = \left[\frac{1}{2}\operatorname{sen} 2x\right]_0^{\frac{\pi}{4}} = \left(\frac{1}{2}\operatorname{sen}\frac{\pi}{2}\right) - \left(\frac{1}{2}\operatorname{sen} 0\right) =$$

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Exemplo

$$\int_0^{\frac{\pi}{4}} \cos 2x \, dx = \left[\frac{1}{2} \operatorname{sen} 2x\right]_0^{\frac{\pi}{4}} = \left(\frac{1}{2} \operatorname{sen} \frac{\pi}{2}\right) - \left(\frac{1}{2} \operatorname{sen} 0\right) = \frac{1}{2}$$

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Exemplo

$$\int_0^{\frac{\pi}{3}} (3 + \operatorname{sen} 3x)\,dx = \left[3x - \frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{3}} =$$

$$\left(\pi - \frac{1}{3}\cos \pi\right) - \left(0 - \frac{1}{3}\cos 0\right) = \pi + \frac{2}{3}$$

Teorema Fundamental do Cálculo

Exemplo

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Exemplo

$$\int_0^1 e^{-x}\,dx = \left[-e^{-x}\right]_0^1 = \left(-e^{-1}\right) - \left(-e^{0}\right) = 1 - \frac{1}{e}$$

Teorema Fundamental do Cálculo

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Exemplo

$$\int_1^4 \frac{1+x}{\sqrt{x}}dx = \int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\right)dx = \int_1^4 x^{-\frac{1}{2}} + x^{\frac{1}{2}}dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 =$$

$$= \left[2\sqrt{x} + \frac{2\sqrt{x^3}}{3}\right]_1^4 = \left(4 + \frac{16}{3}\right) - \left(2 + \frac{2}{3}\right) = \frac{20}{3}$$

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