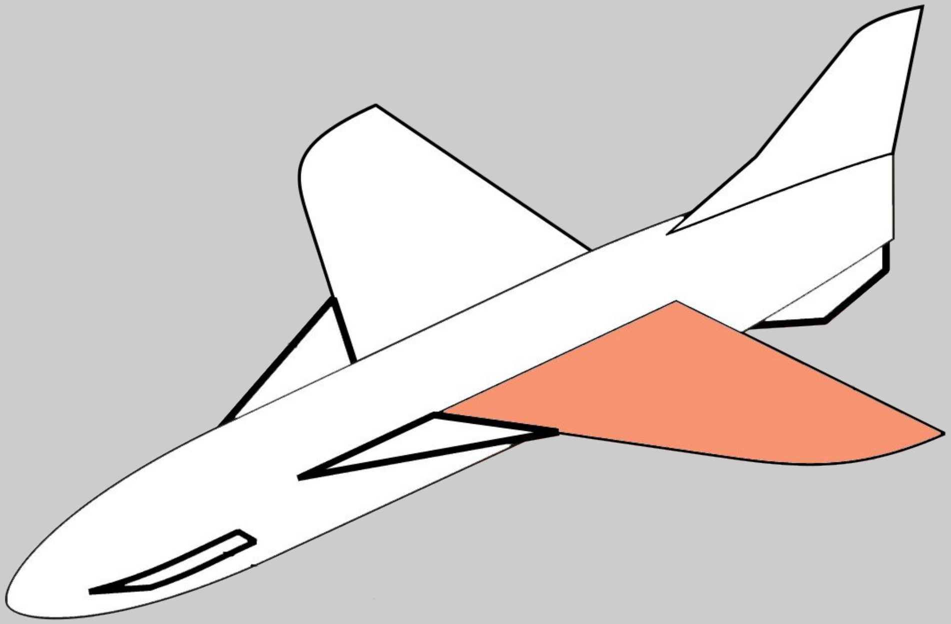
CÁLCULO I

Integral de Riemann

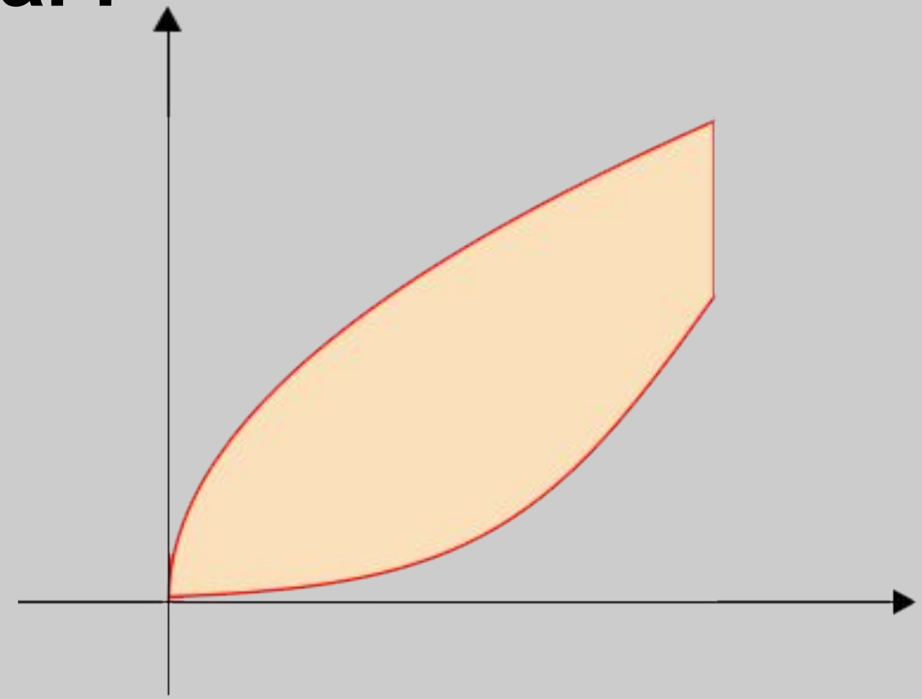
Cálculo Integral

- **Como calcular a área de uma região não regular?**

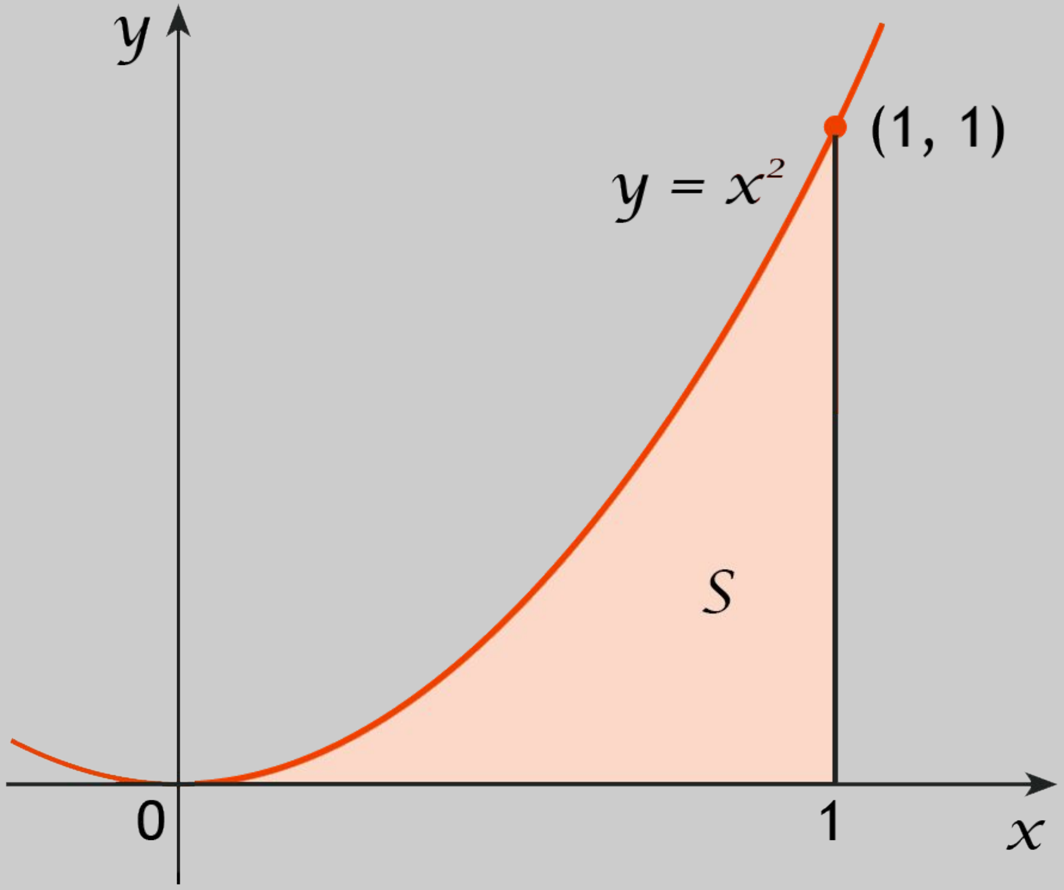


Cálculo Integral

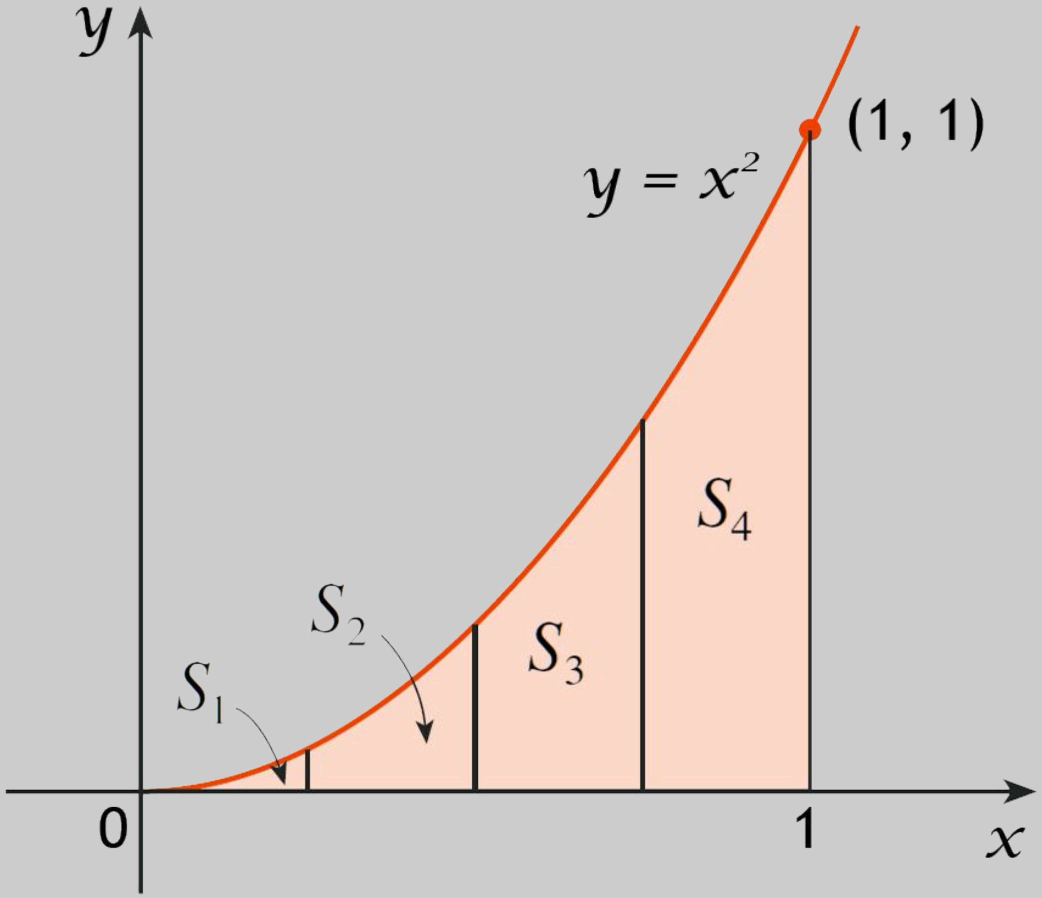
- **Como calcular a área de uma região não regular?**



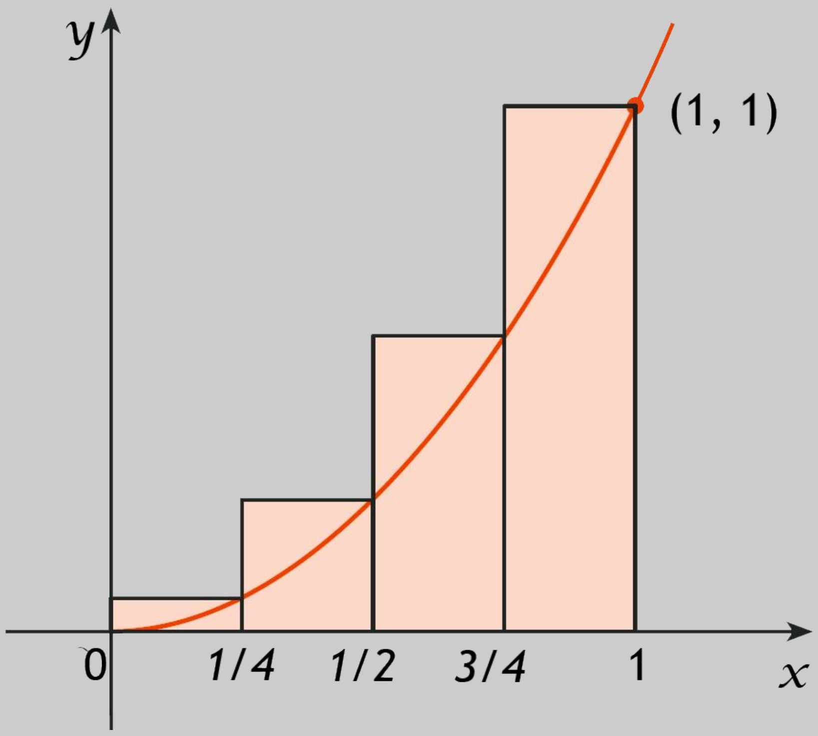
Integral de Riemann

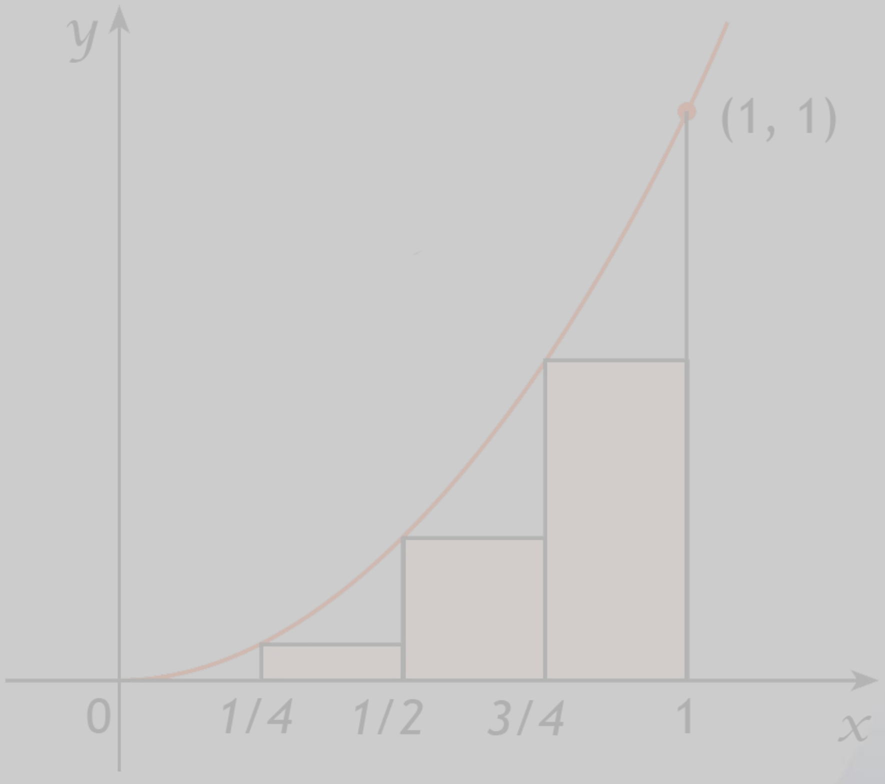


Integral de Riemann

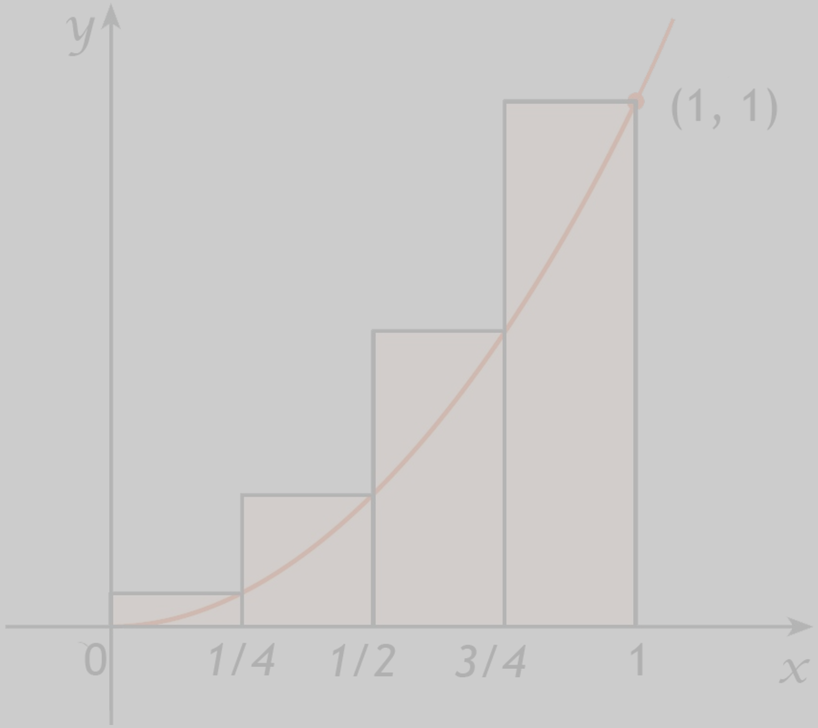


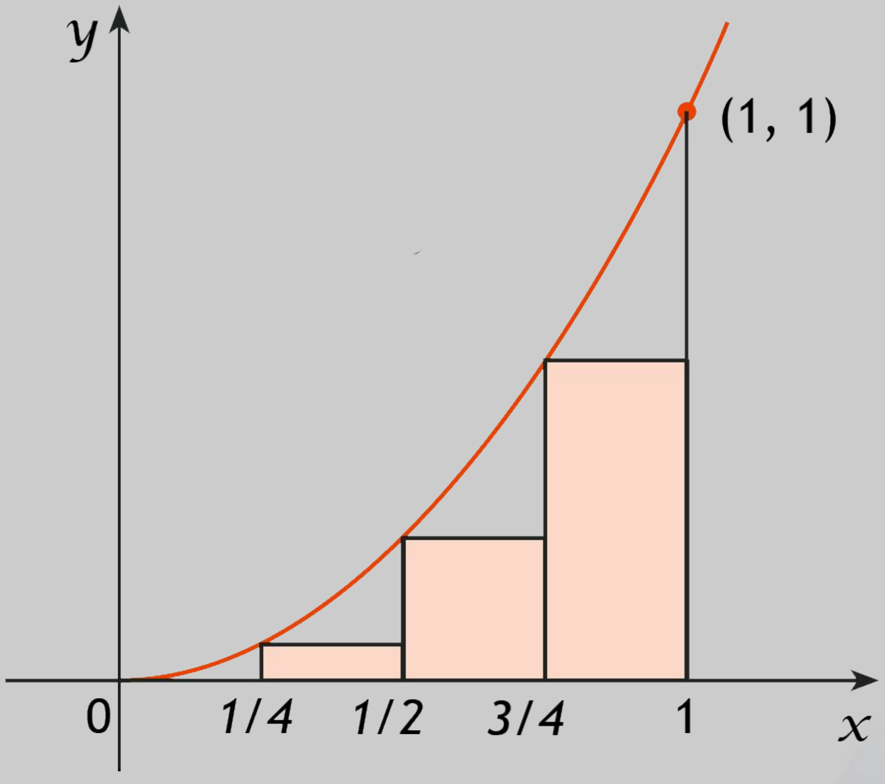
Integral de Riemann





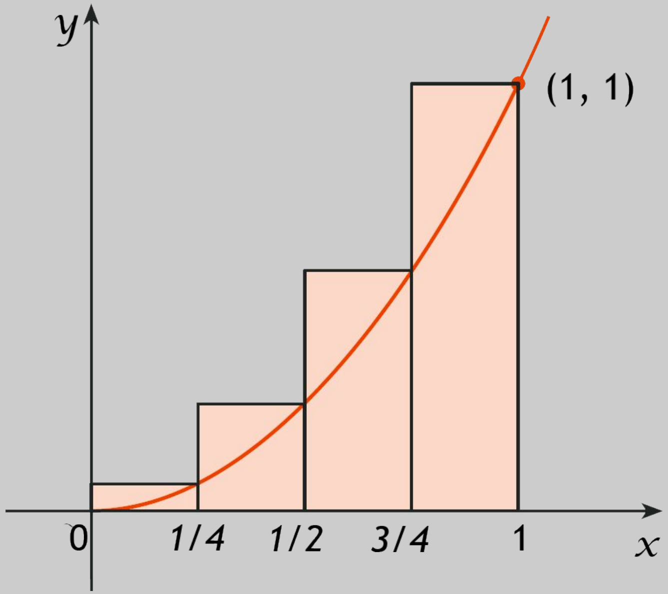
Integral de Riemann





Integral de Riemann

$$S_4 = \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 + \frac{1}{4}(1)^2 = \frac{15}{32} = 0.46875$$

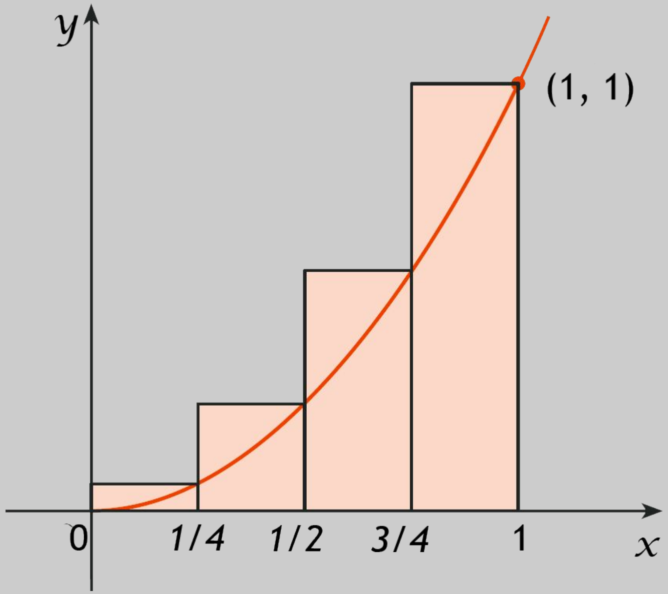


Áreas Superiores (S)

$A < 0.46875$

Integral de Riemann

$$S_4 = \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 + \frac{1}{4}(1)^2 = \frac{15}{32} = 0.46875$$

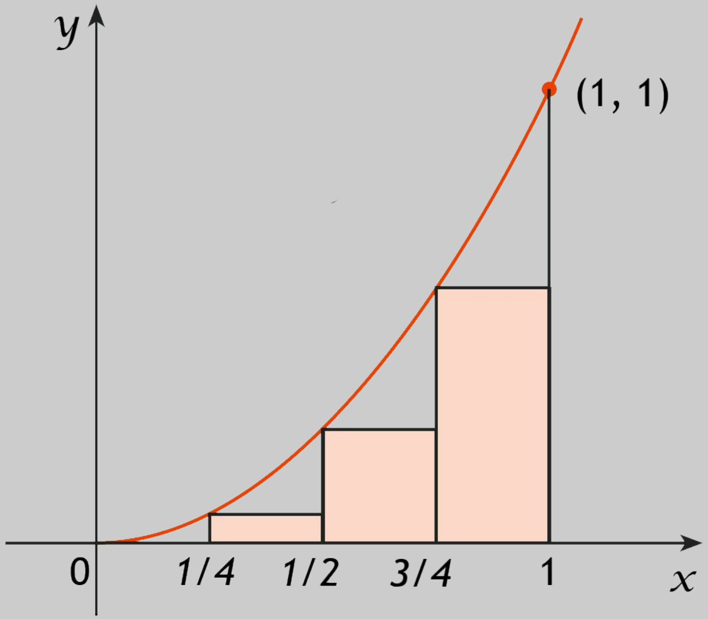


Áreas Superiores (S)

$$A < 0.46875$$

Integral de Riemann

$$I_4 = \frac{1}{4}(0)^2 + \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 = \frac{7}{32} = 0.21875$$

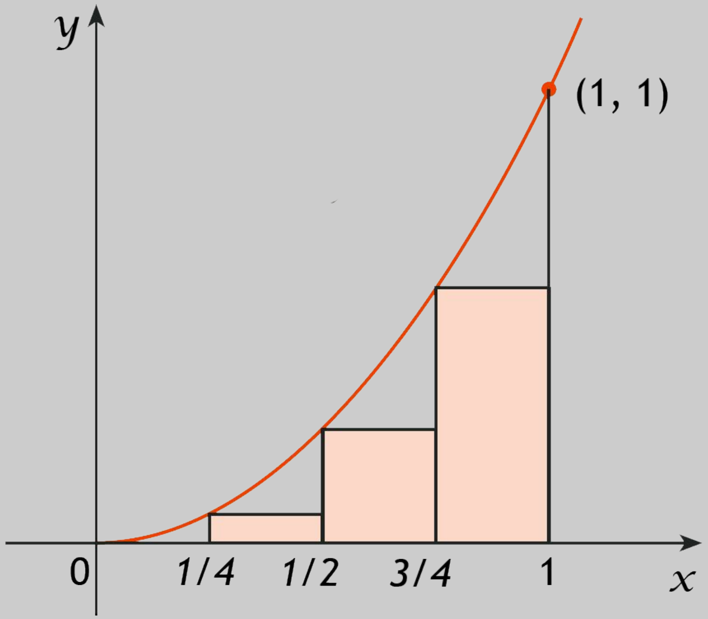


Áreas Inferiores (I)

$A > 0.21875$

Integral de Riemann

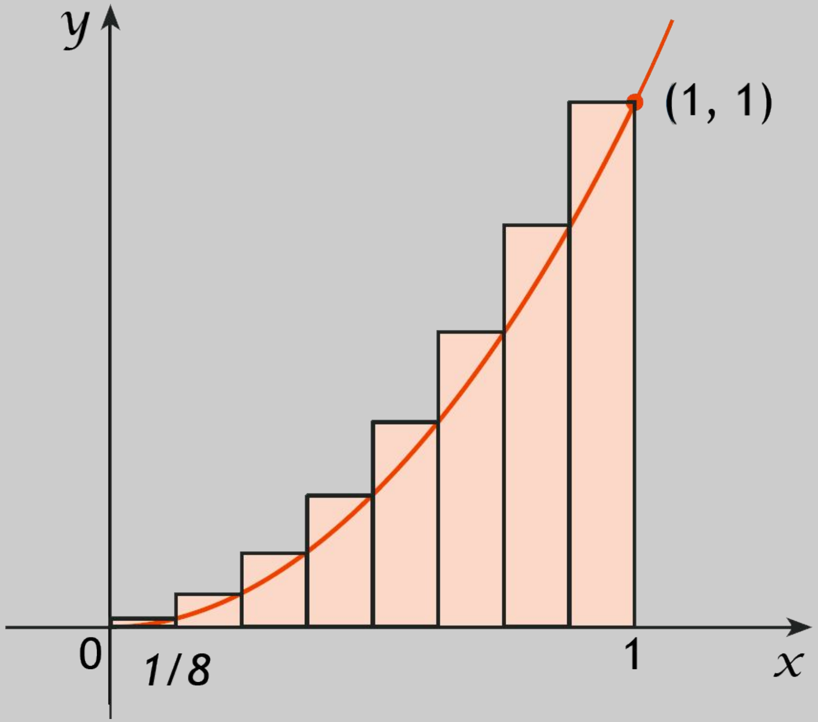
$$I_4 = \frac{1}{4}(0)^2 + \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 = \frac{7}{32} = 0.21875$$

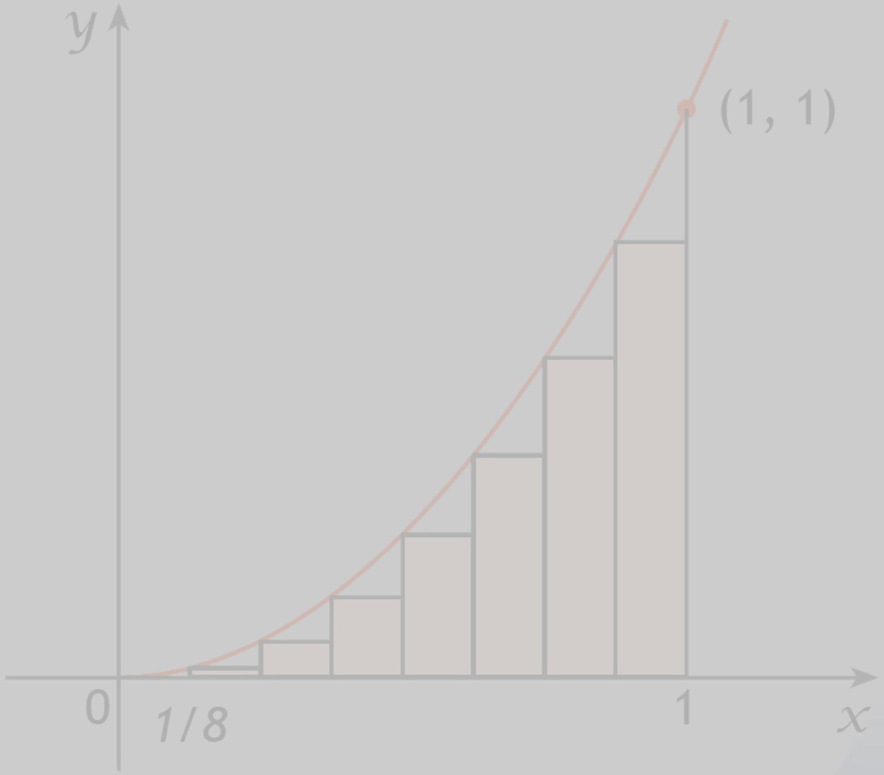


Áreas Inferiores (I)

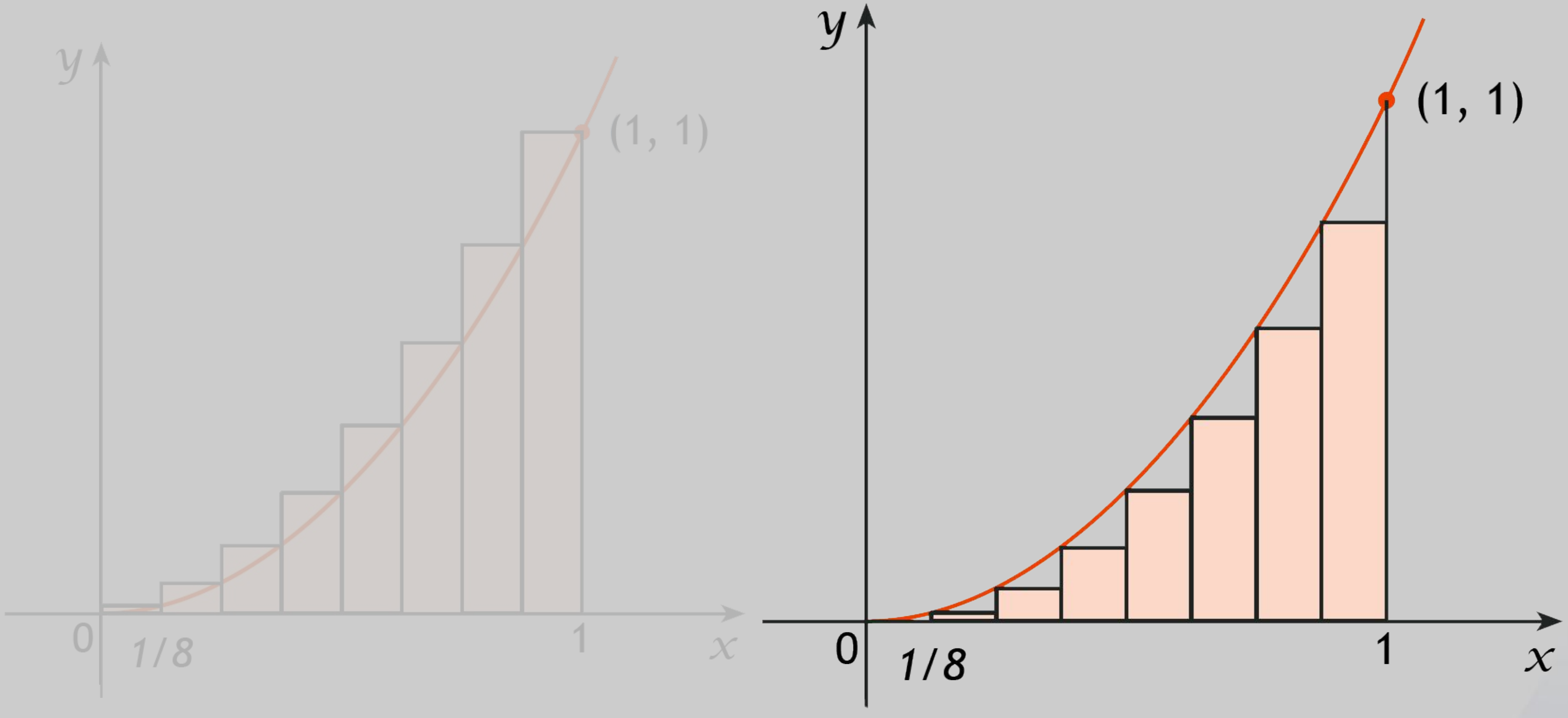
$$A > 0.21875$$

Integral de Riemann





Integral de Riemann



Integral de Riemann

Áreas Superiores

$$S_8 = 0.3984375$$

Áreas Inferiores

$$I_8 = 0.2734375$$

$$0.2734375 < A < 0.3984375$$

Integral de Riemann

Áreas Superiores

$$S_8 = 0.3984375$$

Áreas Inferiores

$$I_8 = 0.2734375$$

$$0.2734375 < A < 0.3984375$$

Integral de Riemann

n	I_n	S_n
10	0,2850000	0,3850000
20	0,3087500	0,3587500
30	0,3168519	0,3501852
50	0,3234000	0,3434000
100	0,3283500	0,3383500
1000	0,3328335	0,3338335

Integral de Riemann

n	I_n	S_n
10	0,2850000	0,3850000
20	0,3087500	0,3587500
30	0,3168519	0,3501852
50	0,3234000	0,3434000
100	0,3283500	0,3383500
1000	0,3328335	0,3338335

$0.3328335 < A < 0.3338335$

Integral de Riemann

$$\lim_{n\to\infty} R_n = \lim_{\Delta x\to 0}\sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx = L$$

- $P: a = x_0 < x_1 < \cdots < x_n = b$
- L, que quando existe é único, denomina-se integral (de Riemann) de f em $[a, b]$.

Integral de Riemann

$$\lim_{n\to\infty} \boldsymbol{R_n} = \lim_{\Delta x\to 0} \sum_{i=1}^{n} \boldsymbol{f(x_i)\Delta x} = \int_a^b \boldsymbol{f(x)dx = L}$$

- $\boldsymbol{P: a = x_0 < x_1 < \cdots < x_n = b}$
- ***L*, que quando existe é único, denomina-se integral (de Riemann) de $\boldsymbol{f}$ em $[\boldsymbol{a, b}]$.**

Integral de Riemann

$$\lim_{n\to\infty} \boldsymbol{R_n} = \lim_{n\to\infty} \frac{\mathbf{1}}{\boldsymbol{n}}\left(\frac{\mathbf{1}}{\boldsymbol{n}}\right)^{\mathbf{2}} + \frac{\mathbf{1}}{\boldsymbol{n}}\left(\frac{\mathbf{2}}{\boldsymbol{n}}\right)^{\mathbf{2}} + \cdots + \frac{\mathbf{1}}{\boldsymbol{n}}\left(\frac{\boldsymbol{n}}{\boldsymbol{n}}\right)^{\mathbf{2}} =$$

$$\lim_{n\to\infty} \frac{\mathbf{1}}{\boldsymbol{n}^{\mathbf{3}}}\left(\mathbf{1}^{\mathbf{2}} + \mathbf{2}^{\mathbf{2}} + \cdots + \boldsymbol{n}^{\mathbf{2}}\right)$$

Integral de Riemann

$$\lim_{n\to\infty} \boldsymbol{R_n} = \lim_{n\to\infty} \frac{\mathbf{1}}{\boldsymbol{n}}\left(\frac{\mathbf{1}}{\boldsymbol{n}}\right)^{\mathbf{2}} + \frac{\mathbf{1}}{\boldsymbol{n}}\left(\frac{\mathbf{2}}{\boldsymbol{n}}\right)^{\mathbf{2}} + \cdots + \frac{\mathbf{1}}{\boldsymbol{n}}\left(\frac{\boldsymbol{n}}{\boldsymbol{n}}\right)^{\mathbf{2}} =$$

$$\lim_{n\to\infty} \frac{\mathbf{1}}{\boldsymbol{n}^3}\left(\mathbf{1}^2 + \mathbf{2}^2 + \cdots + \boldsymbol{n}^2\right)$$

$$\boxed{\mathbf{1}^2 + \mathbf{2}^2 + \cdots + \boldsymbol{n}^2 = \frac{\boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\mathbf{2}\boldsymbol{n}+\mathbf{1})}{\mathbf{6}}}$$

Integral de Riemann

$$\lim_{n\to\infty} \frac{\mathbf{1}}{\boldsymbol{n}^{\mathbf{3}}} \frac{\boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\mathbf{2}\boldsymbol{n}+\mathbf{1})}{\mathbf{6}} = \lim_{n\to\infty} \frac{\mathbf{2}\boldsymbol{n}^{\mathbf{3}}+\mathbf{3}\boldsymbol{n}^{\mathbf{2}}+\boldsymbol{n}}{\mathbf{6}\boldsymbol{n}^{\mathbf{3}}} =$$

$$\lim_{n\to\infty} \frac{n^3\left(2+\frac{3}{n}+\frac{1}{n^2}\right)}{n^3(6)} = \lim_{n\to\infty} \frac{2}{6} = \frac{1}{3}$$

Integral de Riemann

$$\lim_{n\to\infty} \frac{1}{\boldsymbol{n^3}} \frac{\boldsymbol{n(n+1)(2n+1)}}{\boldsymbol{6}} = \lim_{n\to\infty} \frac{\boldsymbol{2n^3+3n^2+n}}{\boldsymbol{6n^3}} =$$

$$\lim_{n\to\infty} \frac{\boldsymbol{n^3\left(2+\frac{3}{n}+\frac{1}{n^2}\right)}}{\boldsymbol{n^3(6)}} = \lim_{n\to\infty} \frac{\boldsymbol{2}}{\boldsymbol{6}} = \frac{\boldsymbol{1}}{\boldsymbol{3}}$$

Integral de Riemann

Propriedades

Sejam f e g integráveis em $[a, b]$ e c uma constante. Então:

i. $f + g$ **é integrável em** $[a, b]$ **e**

$$\int_a^b f(x) + g(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx.$$

Integral de Riemann

Propriedades

Sejam f e g integráveis em $[a,b]$ e c uma constante. Então:

ii. **cf é integrável em $[a,b]$ e**

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx.$$

Integral de Riemann

Propriedades

Sejam f e g integráveis em $[a,b]$ e c uma constante. Então:

iii. Se $f(x) \geq 0$ em $[a,b]$, então $\int_a^b f(x)dx \geq 0$.

Integral de Riemann

Propriedades

Sejam f e g integráveis em $[a,b]$ e c uma constante. Então:

iv. Se $c \in \,]a,b[$ então

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

CÁLCULO I

Integral de Riemann