CÁLCULO I

Integral de Funções Elementares

Primitivas

Definição

Seja f uma função definida num intervalo I.

Uma *primitiva* de f em I é uma função F definida em I, tal que

$$F'(x) = f(x)$$

para todo x em I.

Primitivas

Exemplo

$$f(x) = x$$

$$F(x) = \frac{x^2}{2} \rightarrow F'(x) = \frac{2x}{2} = x$$

Primitivas

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$$F(x) = \frac{x^2}{2} \rightarrow F'(x) = \frac{2x}{2} = x$$

$$F_1(x) = \frac{x^2}{2} + 10, \quad F_2(x) = \frac{x^2}{2} - 1 \rightarrow F(x) = \frac{x^2}{2} + c$$

Primitivas

Corolário

Sejam f e g contínuas em I. Se $f'(x) = g'(x)$ em I, então:

$$g(x) = f(x) + c, c \in \mathbb{R}.$$

Primitivas

$$f(x) = x^3$$

$$F(x) = \frac{x^4}{4} \rightarrow F'(x) = \frac{4x^3}{4} = x^4$$

Notação: $\int f(x)dx = F(x) + c$

$$\int x^3 dx = \frac{x^4}{4} + c$$

Primitivas

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$$\int x^3 dx = \frac{x^4}{4} + c$$

Primitivas

Polinomiais

$$f(x) = x^n, n \neq -1$$

$$\int f(x)dx = \frac{x^{n+1}}{n+1} + c$$

Primitivas

Exemplo

$$\int \frac{1}{x^3}\, dx$$

$$\int \frac{1}{x^3}\, dx = \int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + c = \frac{x^{-2}}{-2} + c$$

$$\int \frac{1}{x^3}\, dx = -\frac{1}{2x^2} + c$$

Primitivas

Exemplo

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$$\int \frac{1}{x^3}\, dx = -\frac{1}{2x^2} + c$$

Primitivas

Exemplo

$$\int \sqrt{x}\ dx$$

$$\int \sqrt{x}\ dx = \int x^{\frac{1}{2}}\, dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

Primitivas

Exemplo

$$\int \sqrt{x}\ dx$$

$$\int \sqrt{x}\ dx = \int x^{\frac{1}{2}}\ dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\int \sqrt{x}\ dx = \frac{2\sqrt{x^3}}{3} + c$$

Primitivas

Exemplo

$$\int k \; dx = kx + c, k \in \mathbb{R}$$

$$\int 10 \; dx = 10x + c$$

$$\int -2 \; dx = -2x + c$$

Primitivas

Exemplo

$$\int \boldsymbol{k}\ \boldsymbol{dx} = \boldsymbol{kx} + \boldsymbol{c}, \boldsymbol{k} \in \mathbb{R}$$

$$\int \mathbf{10}\ \boldsymbol{dx} = \mathbf{10}\boldsymbol{x} + \boldsymbol{c}$$

$$\int -\mathbf{2}\ \boldsymbol{dx} = -\mathbf{2}\boldsymbol{x} + \boldsymbol{c}$$

Primitivas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \int x^{-1} dx = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \qquad a > 0, \quad a \neq 1$$

Primitivas

Exemplo

$$\int x\sqrt{x}\ dx$$

Primitivas

$$\int x\sqrt{x}\ dx = \int x^{1+\frac{1}{2}}\, dx = \int x^{\frac{3}{2}}\ dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= \frac{2\sqrt{x^5}}{5} + c$$

Primitivas

Exemplo

$$\int \frac{x^4 + 1}{x}\, dx$$

$$\int \frac{x^4 + 1}{x}\, dx = \frac{x^4}{4} + ln|x| + c$$

Primitivas

Exemplo

$$\int \frac{x^4+1}{x}\, dx = \int \left(\frac{x^4}{x} + \frac{1}{x}\right) dx = \int x^3 + \frac{1}{x} dx =$$

$$\int \frac{x^4+1}{x}\, dx = \frac{x^4}{4} + ln|x| + c$$

Primitivas

$$\int sen x\, dx = -\cos x + C$$

$$\int \cos x\, dx = sen x + C$$

$$\int sec\ x\, dx = \ln\left|tgx + secx\right| + C$$

$$\int cosec\ x\, dx = -\ln\left|cotgx + cosecx\right| + C$$

$$\int tg\ x\, dx = -\ln\left|cosx\right| + C$$

$$\int sec^2 x\, dx = tgx + C$$

$$\int cosec^2 x\, dx = -cotgx + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = arc\, senx + C$$

$$\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a} arc\, tg \frac{x}{a} + C$$

Primitivas

Exemplos

$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c$$

$$\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\operatorname{arctg}\frac{x}{2} + c$$

Primitivas

Seja $n \neq 0$ uma constante. Então:

$$\int e^{nx}\,dx = \frac{1}{n}e^{nx} + c$$

$$\int \cos nx\,dx = \frac{1}{n}\operatorname{sen} nx + c$$

$$\int \operatorname{sen} nx\,dx = -\frac{1}{n}\cos nx + c$$

Primitivas

Exemplos

$$\int e^{3x}\ dx = \frac{1}{3}e^{3x} + c$$

$$\int e^{-x}\ dx = -e^{-x} + c$$

Primitivas

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$$\int e^{3x}\ dx = \frac{1}{3}e^{3x} + c$$

$$\int e^{-x}\ dx = -e^{-x} + c$$

Primitivas

Exemplos

$$\int \cos 5x \; dx = \frac{1}{5} \operatorname{sen} 5x + c$$

$$\int \operatorname{sen} 2x \; dx = -\frac{1}{2} \cos 2x + c$$

Primitivas

Exemplos

$$\int \cos 5x \; dx = \frac{1}{5} \operatorname{sen} 5x + c$$

$$\int \operatorname{sen} 2x \; dx = -\frac{1}{2} \cos 2x + c$$

Primitivas

Exemplos

$$\int \cos^2 x \; dx = \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) dx$$

$$\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{4} \operatorname{sen} 2x + c$$

Primitivas

Exemplos

$$\int \cos^2 x \; dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x + c$$

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