Energy-Based Fatigue Life Prediction for AISI 4340 Steel

1 Material Properties (Inputs)

- Elastic modulus: $E = 210{,}000\,\text{MPa}$
- Yield strength: $\sigma_y = 470\,\text{MPa}$
- Ultimate tensile strength: $\sigma_u = 745\,\text{MPa}$
- Engineering elongation at fracture: $\varepsilon_f = 0.22\,(22\%)$

2 Derived Material Constants

$$\varepsilon_{\text{true},f} = \ln(1+\varepsilon_f) = \ln(1.22) \approx 0.1989$$

$$\varepsilon_{p,y} = 0.002$$

$$\varepsilon_{p,u} = \varepsilon_{\text{true},f} - \frac{\sigma_u}{E} \approx 0.1963$$

$$n = \frac{\ln(\sigma_u/\sigma_y)}{\ln(\varepsilon_{p,u}/\varepsilon_{p,y})} \approx 9.95$$

$$K = \frac{\sigma_y}{(\varepsilon_{p,y})^{1/n}} \approx 878\,\text{MPa}$$

3 Energy-Based Model Parameters

$$W_f = \sigma_u \varepsilon_{\text{true},f} - \frac{\sigma_u^2}{2E} - \frac{(n+1)\sigma_y^n}{0.002 \cdot \sigma_u^{n+1}}$$

$$W_f \approx 133.53$$

4 Calibration Point

Given data:

$$\varepsilon_a = 0.012, \quad N_f = 1030\,\text{cycles}$$

Step 1: Cyclic stress from Ramberg–Osgood

$$\sigma_c \approx 549.05\,\text{MPa}$$

Step 2: Hysteresis energy at calibration strain

$$W_{h,cal} = \sigma_c \varepsilon_a - \frac{\sigma_c^2}{2E} - \frac{(n+1)\sigma_y^n}{0.002 \cdot \sigma_c^{n+1}}$$

$$W_{h,cal} \approx 5.40$$

Step 3: Life scaling constant

$$K_{\text{life}} = \frac{N_{f,cal} \cdot W_f}{W_{h,cal}} = \frac{1030 \cdot 133.53}{5.40} \approx 41.65$$

5 Predicted Fatigue Life

Stress σ_c (MPa)	Strain ε_c	Hysteresis Energy W_h	Predicted Life N_f (cycles)
100	4.76×10^{-4}	0.0238	233,604
150	7.14×10^{-4}	0.0536	103,818
200	9.53×10^{-4}	0.0953	58,356
250	1.19×10^{-3}	0.1497	37,164
300	1.45×10^{-3}	0.2206	25,218
350	1.77×10^{-3}	0.3256	17,084
400	2.31×10^{-3}	0.5271	10,551
450	3.44×10^{-3}	1.0128	5,492
500	6.08×10^{-3}	2.2765	2,443
550	1.22×10^{-2}	5.4924	1,013 (Calibration Point)
600	2.55×10^{-2}	13.2342	416
650	5.34×10^{-2}	30.7118	181
700	1.08×10^{-1}	68.0282	82