Strain energy based fatigue life prediction

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Fatigue is a critical consideration in the design and maintenance of structures and mechanical components that are subjected to cyclic loading, such as those found in aircraft, automotive components, bridges, and industrial machinery. The repetitive nature of cyclic loading can lead to the progressive accumulation of damage within materials, potentially resulting in catastrophic failures. Therefore, understanding and managing fatigue is essential for ensuring the reliability, safety, and longevity of these systems. Fatigue in metallic materials refers specifically to the progressive and localized structural damage that occurs when a material undergoes repeated cyclic loading. This type of loading can cause failure at stress levels much lower than the material's ultimate tensile strength, making fatigue a subtle yet dangerous mode of failure that must be carefully managed during the design and operational phases of engineering components.

The generation of accurate fatigue properties typically requires extensive experimental testing, which is both expensive and time-consuming. These tests involve subjecting material samples to various cyclic loading scenarios to understand their fatigue behavior over time. Given the practical limitations of such testing, especially during the early design stages, there is a clear need for approximate methods that can predict the fatigue life of materials based on readily available data. Approximate fatigue life prediction models are valuable tools that allow designers to initially size structures according to their anticipated design fatigue life, thereby optimizing material usage and ensuring safety margins are met.

I. Introduction

Several predictive models for fatigue life have been proposed in the literature, each offering different approaches to capture the complex phenomena associated with fatigue damage. These models can be broadly categorized into

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linear damage rule (LDR) based models, multiaxial and variable amplitude loading models, stochastic-based models, continuum damage mechanics (CDM), and energy-based methods. LDR-based models, such as Miner's rule, are among the simplest and most widely used approaches, assuming that damage accumulates linearly with each cycle of loading. However, these models often fail to accurately represent the nonlinear nature of fatigue damage observed in many materials, especially under complex loading conditions [1, 2].

To address the limitations of LDR-based models, nonlinear cumulative damage theories and continuum damage mechanics approaches have been developed. These methods provide a more sophisticated framework for understanding fatigue by considering the progressive deterioration of material properties due to cyclic loading. Continuum damage mechanics, in particular, offers a detailed description of the damage process at a continuum level, enabling the modeling of the initiation and growth of micro-cracks and the accumulation of damage over time [3, 4]. Energy-based methods focus on the energy absorbed by the material during cyclic loading, using the strain energy (plastic energy, elastic energy, or the summation of both) as a key damage parameter. This approach accounts for load sequence and cumulative damage, making it a powerful tool for fatigue life prediction under variable amplitude loading conditions [5, 6].

The relationship between hysteresis energy and fatigue behavior was first highlighted by Inglis (1927), laying the foundation for energy-based approaches to fatigue life prediction. Since then, various energy-based methods have been developed to predict fatigue life using strain energy as the damage indicator. These methods provide a direct link between the energy dissipated in each loading cycle and the progression of fatigue damage, making them highly effective for capturing the effects of complex loading histories and material behaviors, such as cyclic hardening or softening [7, 8].

Glinka [9] introduced the energy density approach under multiaxial loading, offering a scalar parameter to estimate damage and providing an early foundation for energy-based fatigue evaluation in metals subjected to non-uniform stress states. This work demonstrated the potential of energy methods for fatigue modeling in high-strain regions. Golos and Ellyin [10] developed a total strain energy density model that combined both elastic and plastic energy components, offering a cumulative approach to damage characterization. Their formulation captured cyclic energy dissipation effectively and paved the way for energy-based damage quantification. Xiao et al. [11] proposed using hysteresis energy per cycle as a predictor of fatigue failure, showing strong correlation between energy loss per cycle and fatigue life under uniaxial loading. Their model highlighted the direct link between internal energy dissipation and fatigue failure mechanisms. Ellyin and Kujawski [12] extended the strain energy concept to variable amplitude loading, developing an approach to predict fatigue life under realistic loading histories. Their contribution emphasized the importance of integrating energy concepts with loading spectrum analysis. Jiang and Sehitoglu [13] proposed a detailed constitutive model based on cyclic plasticity and strain energy accumulation, capable of capturing ratcheting behavior and multiaxial hardening effects. Their framework contributed significantly to high-fidelity fatigue simulations under complex loading. Lemaitre [14] introduced a continuum damage mechanics (CDM) model based on energy dissipation, offering a theoretical approach to represent material degradation and fatigue crack initiation in a thermodynamically consistent manner. This model formed a basis for energy-based internal variable theories. Papadopoulos [15] integrated strain energy with critical plane methods for multiaxial fatigue life prediction, improving the precision of failure assessment under out-of-phase and non-proportional loading. This method enhanced multiaxial fatigue life estimation through orientation-specific energy evaluation. Jinso and Nelson [16] presented a comparative analysis of critical plane and energy-based models, highlighting the robustness of energy approaches in predicting fatigue life under low-cycle conditions. Their work provided clarity on model selection based on fatigue regimes. Morel et al. [17] experimentally validated energy-based models under biaxial loading conditions, demonstrating their effectiveness in capturing material responses in pressure-dominated and shear-dominated states. This experimental work supported theoretical developments with practical test data. Klingbeil [18] introduced a total dissipated energy theory for crack growth prediction, linking energy absorption to fatigue crack propagation rates. The formulation provided a physics-based criterion for modeling damage evolution at the crack tip. Lagoda et al. [19] analyzed the applicability of energy parameters under random loading, confirming that energy-based methods can accurately represent load sequence effects and damage accumulation. Their study addressed fatigue under service-like irregular loads. Scott-Emuakpor et al. [20] implemented energy-based fatigue criteria in multiaxial laboratory experiments and demonstrated good correlation between accumulated energy and fatigue damage. Their experimental results supported the use of energy metrics in real-world multiaxial fatigue scenarios. Li et al. [21] formulated theoretical relationships between strain energy and crack initiation life, enabling estimation of early fatigue damage using strain-based energy expressions. Their analytical approach provided tools for early-stage damage assessment. Letcher et al. [22] introduced an energy partition-based critical lifetime model that delineated fatigue life phases using elastic and plastic energy components. Their work contributed to clearer physical interpretations of fatigue progression. Goswami et al. [23] investigated plastic strain energy-based life estimation in low-carbon steels under low-cycle fatigue. Their study reinforced the role of plastic dissipation in determining fatigue resistance in ductile materials. Zhu et al. [24] enhanced the energy-based framework by integrating mean stress correction factors, enabling improved predictions across different stress ratios. This refinement made energy methods more adaptable to realistic loading environments. Wang et al. [25] developed a hybrid strain energy and plastic strain model for fatigue–creep life prediction in AISI H13 steel under thermomechanical conditions. Their model captured temperature-dependent energy dissipation mechanisms effectively. Huffman et al. [26] applied strain energy density principles to assess fatigue crack growth in pressure vessel steels, correlating energy release rates with crack propagation behavior. Their work linked traditional fracture parameters with modern energy-based interpretations. Seiler et al. [27] proposed a phase-field model for ductile fatigue fracture incorporating strain energy accumulation and cyclic plasticity. The model enabled detailed simulation of crack path evolution driven by local energy distributions. Kalina and Morel [28] introduced a simplified phase-field approach using strain energy as the damage-driving factor, reducing computational time while maintaining predictive capability for ductile fracture. Their work bridged the gap between computational efficiency and physical fidelity. Strain energy-based fatigue life prediction has progressed from early scalar energy density models for proportional loading to more advanced multiaxial and variable amplitude loading frameworks. Early models, such as those by Glinka and Golos & Ellyin, captured basic damage mechanisms but lacked accuracy under complex loading or mean stress conditions. Subsequent developments incorporated cyclic plasticity, critical plane analysis, and continuum damage mechanics, offering improved prediction fidelity at the cost of computational effort. More recent studies have integrated phase-field methods and thermal effects to address crack initiation and propagation, though challenges remain in generalizing these models across materials and loading conditions. Overall, while energy-based methods provide a robust foundation for fatigue modeling, their limitations in high-cycle fatigue, load history handling, and computational demands continue to motivate ongoing research.

II. Proposed methodology

The prediction of fatigue life in metallic materials has long relied on the availability of detailed cyclic stress-strain data, often obtained through extensive experimental procedures. These procedures, while accurate, are time-consuming, costly, and not always feasible, particularly during the early stages of product development or when resources are limited. In response to these challenges, the present study proposes a method that predicts fatigue life using only static mechanical properties—such as ultimate tensile strength (UTS), yield strength, and elongation—thus addressing a well-documented gap between static and fatigue properties in material characterization.

Recent research has indicated the potential of correlating static mechanical properties to fatigue behavior, thereby simplifying life prediction without sacrificing significant accuracy [29, 30]. These approaches offer promising alternatives, especially when experimental data is scarce. The proposed method builds on this idea by formulating a predictive model that utilizes readily available static properties to estimate fatigue life, thereby providing a cost-effective and accessible tool for engineers.

Here, the proposed method was validated under constant amplitude loading conditions, a commonly adopted scenario in fatigue research due to its simplicity and well-understood mechanics [31]. Constant amplitude loading serves as a baseline for developing and validating fatigue models and is often used to calibrate predictive tools before extending them to more complex loading histories. Despite its controlled nature, constant amplitude loading does not reflect the stochastic and multi-scale loading environments encountered in real-world applications. Therefore, while the proposed method shows efficacy under constant loading, its broader utility lies in its potential for extension to variable amplitude scenarios.To address real-world loading conditions, where stress amplitudes fluctuate over time, the method can be expanded into a cycle-by-cycle cumulative damage model. This approach aligns with the widely accepted Palmgren-Miner linear damage hypothesis (Miner's rule), which posits that damage accumulates linearly with each cycle, and failure occurs when the total damage exceeds a critical threshold. While Miner's rule is often criticized for its simplification—such as ignoring load sequence effects—it remains a cornerstone in fatigue analysis due to its ease of implementation and reasonable accuracy in many practical cases [29].

Integrating the static-property-based fatigue model into a Miner-type framework enables predictions under variable loading without direct cyclic testing. Moreover, this extension makes the method applicable to more realistic service conditions, such as those found in transportation or structural components subject to fluctuating loads.

Fatigue behavior is further influenced by microstructural changes during cyclic loading, particularly cyclic hardening and softening, which alter the material's resistance to damage. For instance, materials that undergo cyclic hardening exhibit an increase in dislocation density and yield strength, potentially extending fatigue life, while cyclic softening often correlates with microstructural degradation, such as persistent slip band formation, reducing fatigue resistance [32, 33].

Incorporating these effects into the cumulative damage model could improve its predictive fidelity. Several researchers have shown that accounting for cyclic deformation behavior significantly enhances fatigue life predictions, especially for materials like steels and aluminum alloys that exhibit pronounced hardening or softening under certain loading regimes [34]. By embedding cyclic hardening/softening factors, the proposed method may be calibrated to better reflect the evolving material state, thereby enabling more robust life predictions over extended service durations.

The method introduced in this study contributes to the ongoing evolution of fatigue prediction tools by offering a static-property-based alternative that is practical, scalable, and adaptable. It reduces dependency on exhaustive cyclic testing, facilitating faster material selection and component design—particularly valuable in industries requiring rapid prototyping and cost control.

In summary, the present work provides a methodologically sound and practically viable approach for fatigue life prediction using static properties. The proposed method is schematically shown in Figure 1. Its adaptability to variable amplitude loading and potential incorporation of cyclic material behavior underscore its relevance for both academic research and industrial application. Future work could focus on integrating machine learning techniques to optimize parameter calibration and further enhance prediction accuracy, thereby expanding the utility of the proposed framework.

III. Approach for generation of stress-strain curve

The stress-strain relationship of the material is generally obtained through experimental results. However, there are methods to generated the same using material constants obtained from monotonic testing like yield stress, elastic modulus etc. Hooke's law states that below the yield point, the stress is linearly proportional to the strain. However, for strain energy computation inclusion of plastic behaviour is important. Therefore, in this formulation, Ramberg-Osgood law is utilized. It defines a power-law relationship of stress-strain describing elastic and plastic strains completely.

The Ramberg–Osgood model is widely used to describe the nonlinear relationship between stress and strain in metallic materials, especially in the elastic–plastic transition region. It enables estimation of strain hardening behavior using a few measurable properties such as Young's modulus, yield strength, and ultimate tensile strength. The total

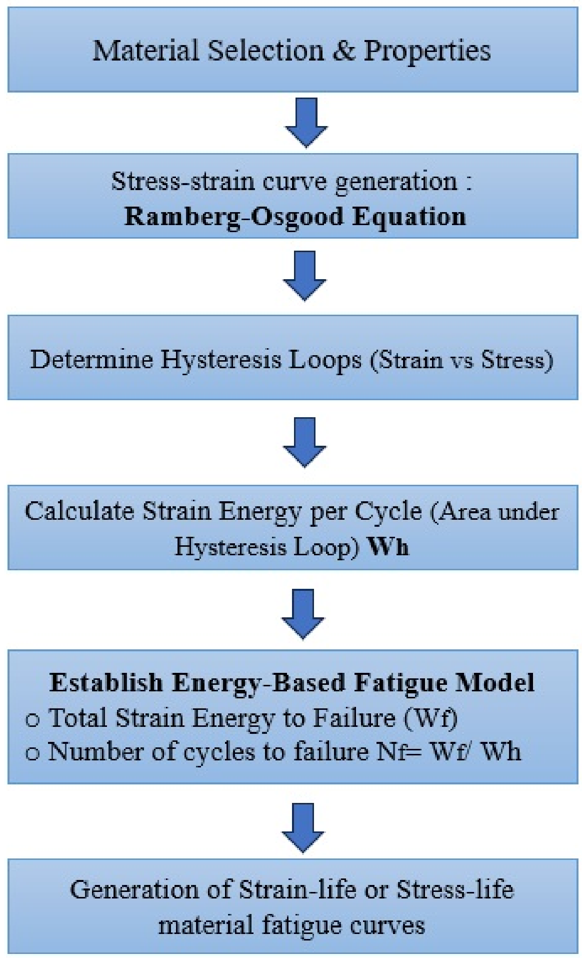


Fig. 1 Methodology for Strain-Energy based fatigue life prediction

strain ϵ is split into elastic and plastic parts: The stress strain relationship is given by Equation 1

$$\begin{aligned} \varepsilon &= \varepsilon_{elastic} + \varepsilon_{plastic} \\ \varepsilon &= \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^n \end{aligned} \tag{1}$$

where:

ε is the total strain,

σ is the applied stress,

E is the Young's modulus,

K and n are material constants defining plastic behavior.

An alternative form using the yield strength σ_0 and an offset constant α is:

$$\varepsilon = \frac{\sigma}{E} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \tag{2}$$

This form is particularly useful when aligning the curve to a known offset yield, such as 0.2%.

Determination of the constants K and n theoretically

Let S_{ty} be the yield strength, S_{tu} be the ultimate tensile strength and ε_f be the total strain at failure. These parameters can be obtained from experimental tensile test data. The plastic strain at yield stress is commonly taken as:

$$\varepsilon_{py} = 0.002 \tag{3}$$

The plastic strain at ultimate stress can be estimated as:

$$\varepsilon_{pu} = \varepsilon_f - \frac{S_{tu}}{E} \tag{4}$$

The strain-hardening exponent n is then given by:

$$n = \frac{\ln\left(\frac{S_{tu}}{S_{ty}}\right)}{\ln\left(\frac{\varepsilon_{p,u}}{0.002}\right)} \tag{5}$$

The strength coefficient K is calculated as:

$$K = \frac{S_{ty}}{(0.002)^n} \tag{6}$$

The constant n has significance such that higher n indicates sharper transition from elastic to plastic and lower n yields a smoother transition and softer hardening. The model captures strain hardening well before necking. Strain softening and post-necking require additional models. The Ramberg–Osgood model provides a simple yet powerful method to construct nonlinear stress–strain curves with minimal experimental data. Its flexibility makes it suitable for metals exhibiting gradual yielding and hardening. For finite element and other analyses, it offers a good balance between accuracy and simplicity.

The stress-strain curve is generated using the above discussed method for steel alloy AISI 4130, plotted and compared with experimental data in Figure 2. The tensile test results is carried out with dog bone shape specimen using 0.063-in.-thick sheet of AISI 4130 at different temperatures at Jet Propulsion Laboratory, USA. [35]

IV. Methodology for fatigue life prediction

The methodology proposed in this work for predicting the fatigue life of a material, specifically metals is based on the strain energy. The first step towards it is generating the stress strain curve as described in previous section.

Following that, one needs to find the hysteresis cycle during various types of loading. The work done during cyclic loading leads to deformation of the material and is transformed into strain energy.

The hysteresis curve formed during loading and unloading are explained in Figures 3 and 4. In the first case, the loading is cycling between 0 - Max - 0 in which stress ratio R = Min/Max stress becomes 0. The trace points are 0-1-2-3-4 in which loading is 0 - 1 and unloading 1 - 2, the area enclosed as 0 - 1 - 2 - 0 is the strain energy absorbed in the material during such loading and unloading. Similarly, the next loading cycle follows 2 - 1 - 3(loading) and 3 - 4 (unloading). In this cycle the area enclosed as 2 -1 3 -4-2 is the strain energy absorbed. In the second case, where the loading is acted as fully reversible, the stress ratio R = Min/Max stress = -1. The area enclosed during such hysteresis loop formed is equivalent approximately to 4 times the area enclosed by the previous case (R=0).
Now, the task is to find the area enclosed in the hysteresis cycle in stress-strain plane. A representative cyclic loading for stress ratio R = 0 is shown in Figure 5. The area enclosed in the hysteresis loop is found out geometrically. The total rectangular area enclosed till ε_c and σ is $A0 = \Delta\sigma * \Delta\epsilon$. The area enclosed by stress-strain curve is given as $A1 = \int_{\epsilon_1}^{\epsilon_1} \sigma \, d\epsilon$. Therefore, the area enlosed inside the loop can be found out by subtracting the upper and lower area from total rectangular area, $E_h = A0 - 2A1$.

The enclosed area can be computed using Equation 7 based on generalized coordinate system.

$$\begin{aligned} W_h &= 2 * \int_{\epsilon_1}^{\epsilon_1} \sigma \, d\epsilon - \Delta\sigma * \Delta\epsilon \\ &= 2 * \int_{0}^{\epsilon_c} \sigma \, d\epsilon - \Delta\sigma * \Delta\epsilon \\ &= \Delta\sigma * \Delta\epsilon - 2 * \int_{0}^{\sigma_c} \epsilon \, d\sigma \end{aligned} \tag{7}$$

The concept which is utilized in this work is that the total strain energy which a material can sustain is the strain energy obtained from the monotonic stress strain curve. The area enclosed inside it provides a estimate of the total

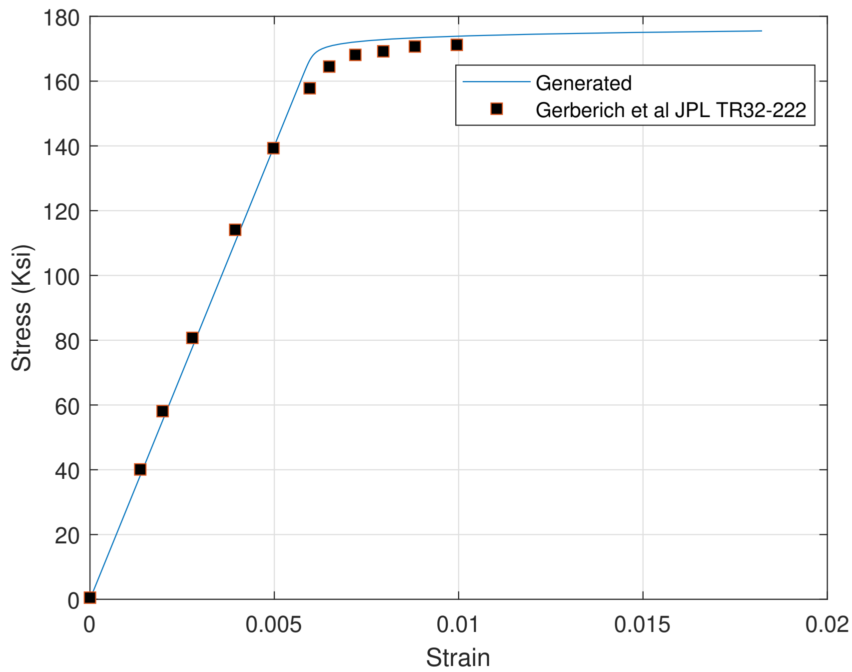


Fig. 2 Generated Stress-Strain curve for steel alloy AISI 4130

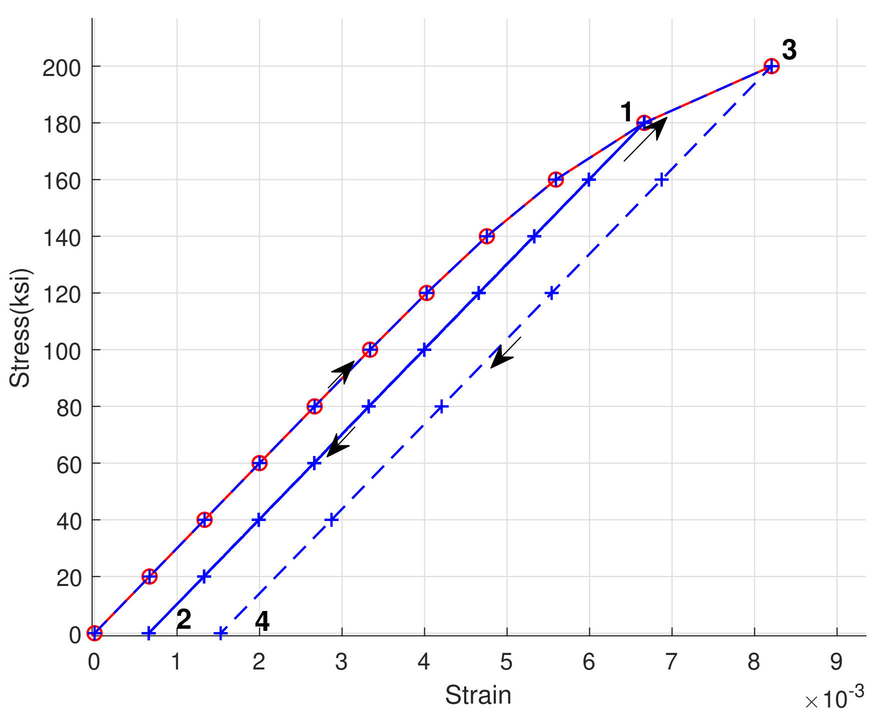


Fig. 3 Stress-Strain Hysteresis loop for R = 0 loading

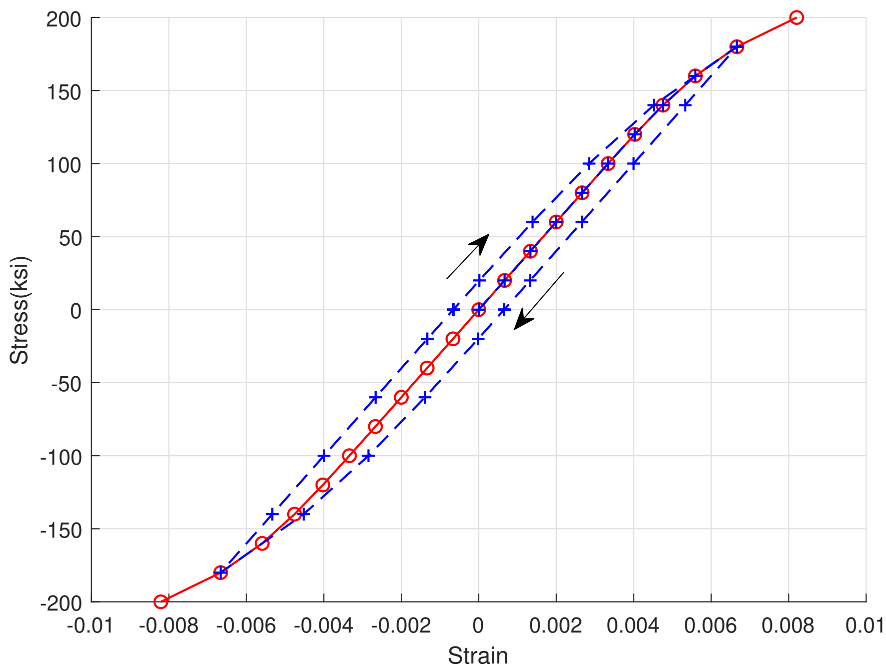
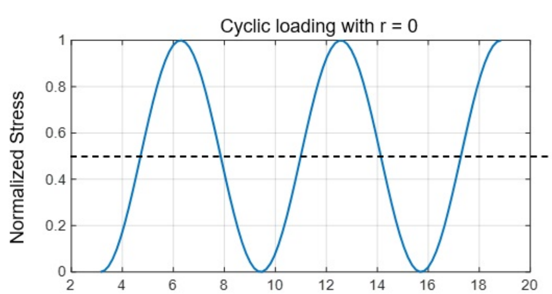


Fig. 4 Stress-Strain Hysteresis loop for R = -1 loading

[] 

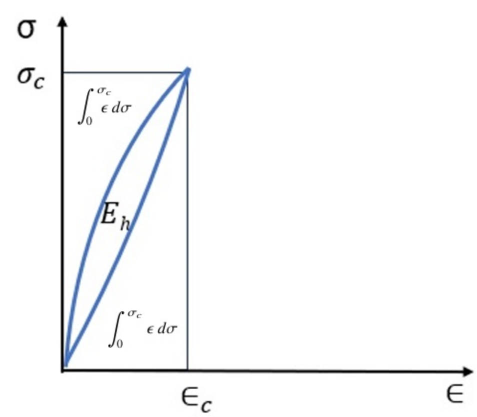
[] 

Fig. 5 Cyclic hysteresis loop formation

strain energy which a material can absorb when work would be done on the same.

The area enclosed by the stress–strain curve of a material under loading conditions is widely recognized as a measure of the total strain energy density absorbed by the material during deformation. This strain energy corresponds to the mechanical work performed on the material per unit volume and serves as an indicator of the material's capacity to absorb energy before failure. It is particularly significant in the context of fatigue and fracture mechanics, where energy-based approaches are employed to assess damage accumulation and failure thresholds. The area under the elastic portion of the curve represents the recoverable (elastic) strain energy, while the area under the plastic portion corresponds to the dissipated (plastic) strain energy [36–38]. Thus, evaluating this area provides insight into both the ductility and toughness of materials, making it a fundamental parameter in energy-based fatigue models and continuum damage mechanics frameworks [29, 39, 40]. Mathematically, it can be found by Equation 8

$$W_f = \int_0^{\epsilon_u} \sigma \, d\epsilon = \sigma\epsilon - \int_0^{\sigma_u} \epsilon \, d\sigma \tag{8}$$

Now, the fatigue life is essentially proportional to the ratio of the total strain energy to the energy absorbed during a single cyclic loading. This relationship also depends on the loading ratio which is manifested in a constant K. The proposed fatigue life model is given in Equation 9

$$N_f = K * \frac{W_f}{W_h} = K * \frac{\int_0^{\epsilon_u} \sigma \, d\epsilon}{2 * \int_0^{\epsilon_c} \sigma \, d\epsilon - \Delta\sigma * \Delta\epsilon} \tag{9}$$

$$N_f = K * \frac{\sigma_u \epsilon_u - \frac{\sigma_u^2}{2E} - \frac{0.002\sigma_u^{n+1}}{(n+1)\sigma_y^n}}{\sigma_c \epsilon_c - \frac{\sigma_c^2}{2E} - \frac{0.002\sigma_c^{n+1}}{(n+1)\sigma_y^n}} \tag{10}$$

V. Results and discussions

The proposed method is implemented for metallic materials choosing steel and Aluminium alloys commonly used in Aerospace and other applications. The static material properties are obtained from the NASA TN D1574 and Military handbook MIL-HDBK - 5J [41, 42] are mentioned in Table 1. The input data required for the proposed fatigue prediction model are yield strength (σ_y), ultimate strength (σ_u), fracture strength (σ_f), elongation (e_f), Young's modulus (E). These properties are basically used to form the stress-strain curve of the given material using the Eqn. 2-5. The fatigue life value at each given stress is computed using

The experimental fatigue data is obtained from [41–43] and compared with the results from the proposed method.

S.No	Material	σ_y ksi	σ_u ksi	σ_f ksi	e_f	E ksi	Data Source
1	AISI 4130 (soft)	113	130	245	1.12	30000	NASA TN D1574
2	2014-T6	67	74	91	0.29	10000	NASA TN D1574
3	7050-T7451	64	76	81	0.08	10300	MIL-HDBK-5J

Table 1 Input data for the model: Static strength properties of materials

A. Case 1 : Steel Alloy 4130 (Soft)

The material considered for Case 1 is steel alloy AISI 4130 which is a high strength steel alloy. The static properties have been obtained from [41, 43] as mentioned in Table 1. At first, it is utilized to form the stress-strain curve of the material which forms the basis for further computation. Thereafter, the fatigue life is computed using Equations 9 and 10 at different stress or strain ranges, incremented in step of 50 MPa and corresponding strain conversion as per Ramberg-Osgood relationship. The E-N (Strain-Life) curve is formed using the derived data which is compared with the experimental data extracted from [41, 43]. In the work by Smite et. al [41], the test data was obtained by axial testing of hourglass shaped specimen till failure at different stress and strain ranges. The result obtained is compared and plotted in Figure 6. It can be observed that predicted values are closely matching with experimental data.

B. Case 2 : Aluminum Alloy 2014-T6

The material considered for Case 2 is an Aluminum alloy 2014-T6. This alloy is widely used in aerospace applications having composition Aluminum - Copper (4.4%). The static properties are again obtained from tested data NASA TN D 1574 [41, 43] as mentioned in Table 1. The fatigue life prediction is carried out for different stress or strain ranges.

The comparison is made with the experimental data which are plotted in Figure 10. It can be observed that predicted values are closely matching with experimental data at low and high cycle fatigue region, however there ar some mismatch at knee location of the E-N curve. The difference in the prediction could be due to the cyclic hardening or softening of the material. In case of cyclic hardening there is the increase in cyclic yield stress for given strain range as shown in Figure 7. And in case of cyclic softening there is observed to be decrease in cyclic yield stress for given strain range as shown in Figure 8. This phenomenon happens due to various reasons like material composition, microstructure, loading conditions, loading rate, environmental factors etc. [31, 44] This leads to change in the total energy for the failure W_f. The comparison between monotonic and cyclic stress-strain curve is shown in Figure 9. In case of Aluminum alloy like 2024-T4, the cyclic stress strain curve is higher than monotonic, therefore area under the curve will be higher and hence, $W_f^c > W_f^m$. Therefore, if cyclic stress-strain curve is considered the predicted strain-life curve will correct and shift towards right which will be more closer to the experimental data in literature.

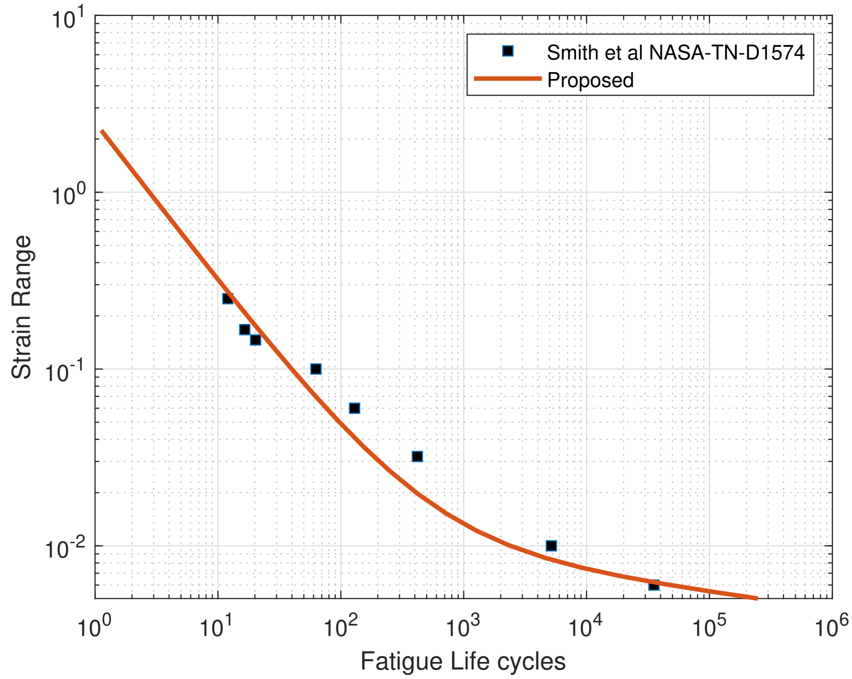


Fig. 6 Predicted Strain-Life curve for 4130 Steel (Soft)

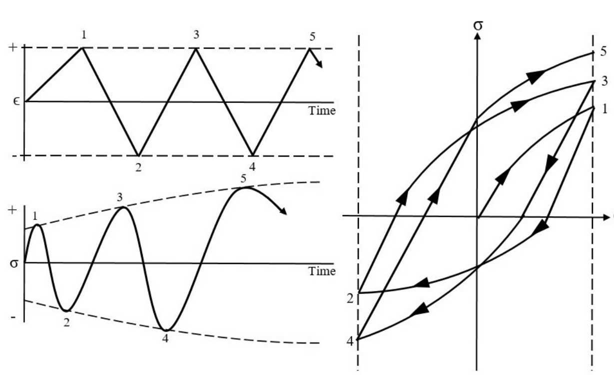


Fig. 7 Cyclic Hardening [31]

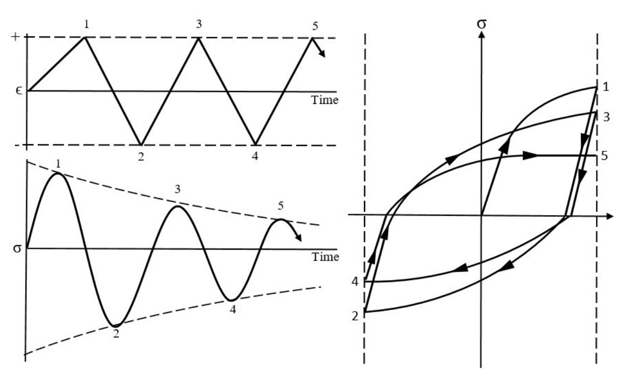


Fig. 8 Cyclic softening [31]

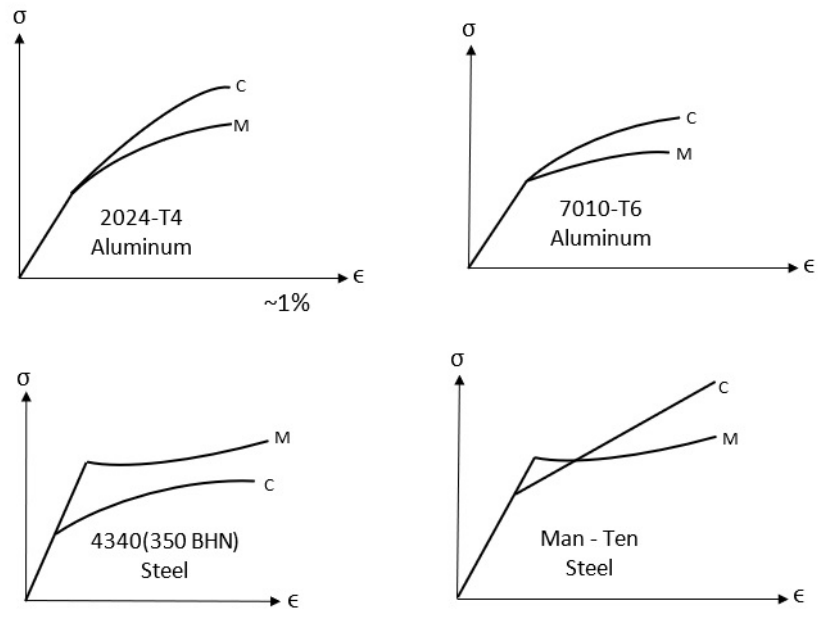


Fig. 9 Stress-strain curve Comparison for Monotonic and Cyclic [31]

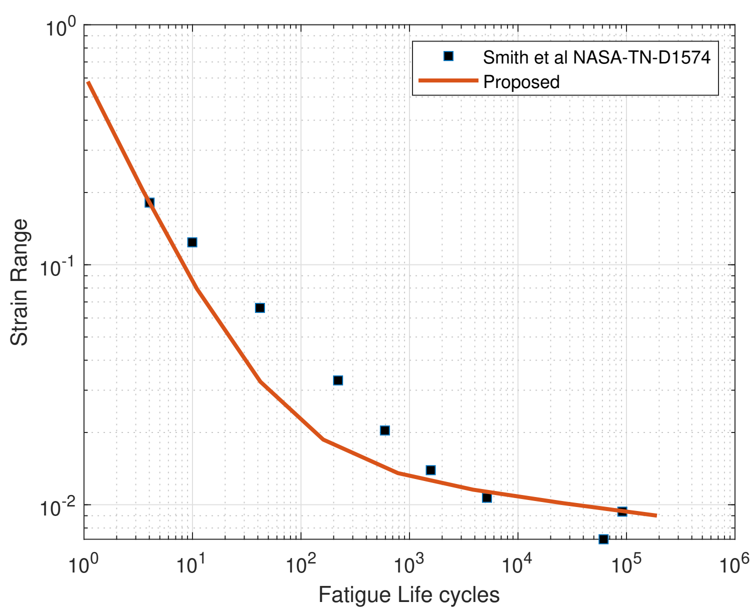


Fig. 10 Predicted Stress-Life curve for 2014-T6 Aluminum alloy

C. Case 3 : Aluminum Alloy 7050-T7451

The material considered for this case is a high strength Aluminum alloy 7050-T7451 consisting of Zn (6%) also in addition to Al-Cu. The static properties have been obtained from Military Handbook (MIL-HDBK-5J) [42] and further fatigue life prediction are compared with the experimental data also provided in the same handbook. The result obtained is compared and plotted in Figure 11. It can be observed that predicted values are closely matching with experimental data.

VI. Conclusions

The proposed method effectively predicts the fatigue life of metallic materials using only static properties, addressing a significant gap between static and fatigue properties that has been observed in the existing literature. Traditionally, fatigue life predictions have required detailed cyclic stress-strain data, which can be time-consuming and costly to obtain. By leveraging static properties, this method provides a more accessible and efficient alternative for predicting fatigue life, making it especially useful in early design phases or when testing resources are limited.

In this study, the method was applied under constant amplitude loading conditions, which is a commonly studied scenario in fatigue research. Constant amplitude loading simplifies the analysis and serves as a foundational approach for understanding fatigue behavior. However, real-world applications often involve variable amplitude loading, where the stress levels fluctuate over time. To address this, the proposed method can be extended to a cycle-by-cycle approach, forming a cumulative damage model that accounts for varying stress amplitudes. This extension aligns with established theories such as Miner's rule, which is widely used for cumulative damage assessment under variable loading conditions.

Additionally, incorporating the effects of cyclic hardening or softening into the cumulative damage model could further enhance its accuracy. Cyclic hardening or softening occurs as a material is repeatedly loaded, which can alter its fatigue response. Literature indicates that accounting for these changes can significantly impact fatigue life predictions, as materials that harden under cyclic loading may exhibit longer fatigue lives, whereas those that soften may experience reduced fatigue resistance. By including these effects, the proposed method could provide a more comprehensive and realistic prediction of material behavior under varying load conditions, thereby offering a robust tool for engineers and designers to evaluate the fatigue life of metallic components.

Overall, this study contributes to the field by providing a practical and scalable method for fatigue life prediction, potentially reducing the reliance on extensive cyclic testing and offering a pathway for more reliable predictions in complex loading scenarios.

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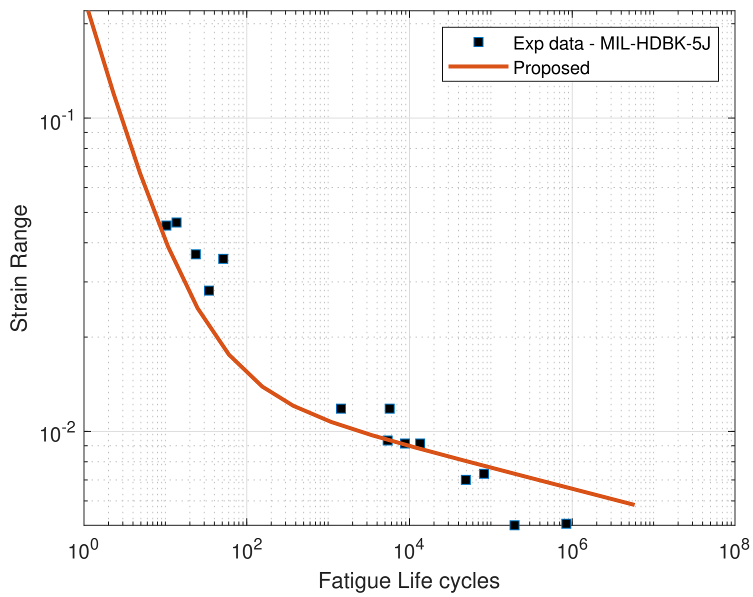

Fig. 11 Predicted Stress-Life curve for 7050-T7451 Aluminum alloy