The Experiment 2 of Engineering Electromagnetics: The Analysis about Electric Distribution of A Line Charge with Integration and Infinitesimal Method

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***Abstract*—This experiment is to calculate the electric field distribution of the continuous line charge and plot the relevant figures with MATLAB. In this experiment, two method are used, one is to use integration method to calculate the distribution of electric field, and another is to use infinitesimal method to obtain the approximate calculation of the distribution. These two method are compared and analyzed at last. The infinitesimal method is to divide the continuous line charge into several point charges and add up the contribution of each charge to the electric field distribution. There must be some error between this method and the integral method which is There must be some error between this method and the integral method which is considered an accurate method of calculation. This error is related to the number of segments that are divided from the line charge. And the more segments divided are, the smaller the error is.**

***Index Terms*—Electric field, Line charge, Integration method, Infinitesimal method, MATLAB**

I. Introduction

This is a report for the second experiment of Engineering Electromagentics theory. This experiment is to calculate the electric field distribution including potential distribution, contours distribution and electric field line of a continuous line charge using integration method and infinitesimal separately method with MATLAB and analyze and compare them at last.

In this experiment, a continuous line charge with charge density of $\rho = 1 \times 10^{-9} C/m$ is distributed on the line segment between points A(-1,0) and B(1,0).

There are many symbols and notations that are used in this experiment, so the following table lists all the important symbols and notations that will be used.

TABLE I: List of symbols and notations

Symbol	Meaning	Unit
k	Electromagnetic constant of 1×10^9	F/m
E	Electric field intensity	V/m
V	Electric potential	V
ρ	The charge density of 1×10^{-9}	C/m
Q	The electric charge	C
ΔQ	The charge amount of each segment	C
V_i	The potential of the i^{th} segment	V

The basic electrostatic knowledge and relevant formulas used in this experiment are as follows:

The electric field intensity E generated by point charge in vacuum is:

$$\boldsymbol{E} = k\frac{Q}{R^2}\boldsymbol{a_R} \tag{1}$$

If infinity is the zero potential point, the potential generated by point charge in vacuum is:

$$V = k\frac{Q}{R} \tag{2}$$

The strength of the electric field can be obtained by using the negative gradient of the electric potential, i.e.

$$\boldsymbol{E} = -\nabla V \tag{3}$$

II. INTEGRATION METHOD

The purpose of this experiment is to obtain the electric distribution of a continuous line charge. Firstly we use the integration method to calculate the result that is accurate.

A. Theoretical Analysis

First of all, we should deduce the expression of the potential of the line charge. Let the coordinates of the point on the plane is (X_0, Y_0). And then divide the line charge into infinitesimal pieces, each of which is small enough so that the length is dx. So, the charge of each pieces is $Q = \rho dx$. Because the potential is scalar so that the total potential in this field is

$$V = k\int_{-1}^{1}\frac{\rho dx}{R} \tag{4}$$

where R is the distance between the point in the field and the object point being studied.

$$R = \sqrt{(X_0 - x)^2 + Y_0^2} \tag{5}$$

So the equation (4) can be deduced to the expression as follows:

$$V = k\rho\ln\left(\frac{1 - X_0 + \sqrt{(1 - X_0)^2 + Y_0^2}}{-1 - X_0 + \sqrt{(-1 - X_0)^2 + Y_0^2}}\right) \tag{6}$$

Therefore, we can use this equation to calculate the potential. So that we can use equation(3) to calculate the electric strength **E**. Then the electric field distribution including potential distribution, contours distribution and electric field line can be presented.

B. MATLAB Codes

Firstly, we give the declaration of the field. The codes are as follows:

```
clear;
k = 9e9;
p = 1e-9;
% Set the linear density of linear
    charge
x1 = -1;
y1 = 0;
x2 = 1;
y2 = 0;
xm = 2.5;
% Set the range in the x direction of
    the field
ym = 2.5;
% Set the range in the y direction of
    the field
x=linspace(-xm,xm,150);
% Divide the X-axis into 150 equal
    parts
y=linspace(-ym,ym,150);
% Divide the Y-axis into 150 equal
    parts
[X,Y]=meshgrid(x,y);
% Form the coordinates of each point
    in the field
```

Then we can calculate the potential distribution using the equation(6). The codes are as follows:

```
figure;
V = k*p*log((1-X+sqrt((1-X).^2+Y.^2))
    ./(-1-X+sqrt((-1-X).^2+Y.^2)));
% Calculate the potential at each
    point in the field
mesh(X,Y,V);
% Plot the distribution of potential
hold on;
title(["Potential distribution of
    linear charge in a vacuum";"
    12113010"],"fontsize",13);
xlabel("X(m)","fontsize",13);
ylabel("Y(m)","fontsize",13);
```

We also can choose appropriate scale of the field to plot the graph of the contours distribution which means that the potential in each line is same. The codes are as follows:

```
figure;
Vmin=0;
% Set the minimum potential value of
    the equipotential line
Vmax=150;
% Set the maximum potential value of
    the equipotential line
Veq=linspace(Vmin,Vmax,35);
% Set the potential values of 35
    alleles
contour(X,Y,V,Veq);
% Plot the contours
grid on
hold on
plot([x1,x2],[y1,y2],'b','LineWidth'
    ,1);
title(["Equipotential line of the line
    charge in a vacuum";" 12113010"],
    "fontsize", 13);
xlabel("X(m)", "fontsize", 13);
ylabel("Y(m)", "fontsize", 13);
```

The electric fluxline is used to represent the direction of the electric field intensity, because the direction of the electric field intensity is the tangential direction of the electric fluxline. The electric field intensity can be obtained by calculating the gradient of the potential in MATLAB, namely the equation(3). And then we can use streamline() function to generate the clusters of electric fluxline respectively with each charges as the starting point. The codes are as follows:

```
figure;
[Ex,Ey]=gradient(-V);
% Calculate the two components of the
    electric field intensity at each
    point in the field
del_theta=120;
% Set the angle difference between
    adjacent power lines
theta=(0:del_theta:360).*pi/180;
% Generating the radian value of the
    power line
for xs=x1:0.25:x2
xs_1=xs+0.001*cos(theta);
ys=0.001*sin(theta);
streamline(X,Y,Ex,Ey,xs_1,ys)
end
grid on
hold on
plot([x1,x2],[y1,y2],'b','LineWidth'
    ,1);
contour(X,Y,V,Veq);
title(["Equipotential line and
    electric fluxline of the line
    charge in vacuum" ; ...
    "smooth continuous curves"+"
        12113010"], "fontsize", 13);
```

```
xlabel("X(m)", "fontsize", 13);
ylabel("Y(m)", "fontsize", 13);
```

C. Simulation Result and Analysis

The result can be shown as follows. These figures can be considered as the correct distribution of the line charge for subsequent reference and comparison.

Figure 1 is the potential distribution of the line charge. It can be seen from the figure that the electric potential will have a peak in the area of the line segment and decrease rapidly in the edge area.

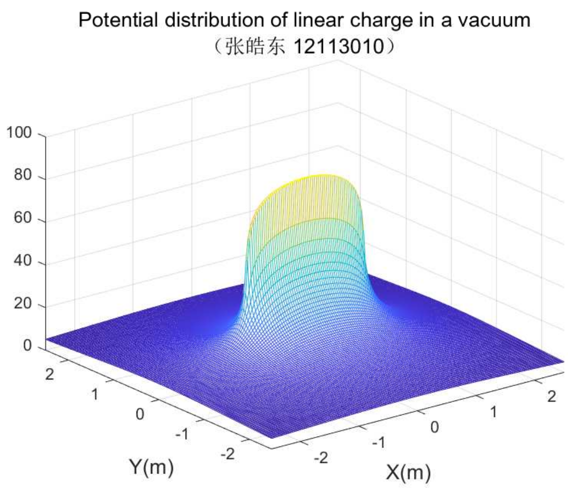


Fig. 1: The Potential Distribution of Line Charge

Figure 2 is the contours distribution of the line charge. As can be seen from the figure, the equipotential charge is elliptically distributed around the line, and the line charge has been highlighted in the figure.

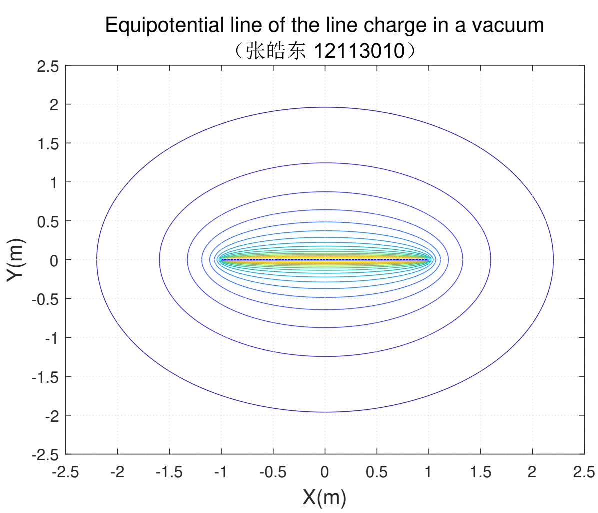


Fig. 2: The Contours Distribution of Line Charge

Figure 3 is the electric fluxline distribution and contours distribution of the line charge.

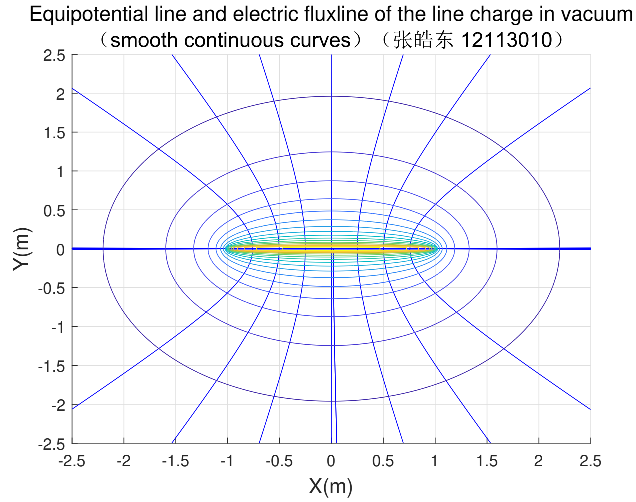


Fig. 3: The Contours and Electric Fluxline Distribution of Line Charge (With Smooth Continuous Curves)

III. INFINITESIMAL METHOD

We can think of this line charge as a series of point charges, each of which is spaced the same distance apart, and we can think of it as an arrangement of N point charges, using this distribution to approximate the line charge distribution. We need to calculate the distribution of each point charge and add up all the point charge distributions. Since this method is only an approximation, it must be different from the correct method, that is, the integral method. We will take different values of N, plot the distribution corresponding to each N, and compare and analyze the difference.

A. Theoretical Analysis

We can think of this line charge as a series of N point charges, so the amount of charge in each of these point charge is:

$$\Delta Q = \rho \frac{|AB|}{N} \tag{7}$$

So the potential of each segments is:

$$V_i = k \frac{\Delta Q}{R_i} \tag{8}$$

where R_i is the distance between the point in the field and the each object point being studied.

$$R_i = \sqrt{(X_0 - x_i)^2 + Y_0^2} \tag{9}$$

Then the total potential can be obtained from:

$$V = \sum_{i=0}^{N-1} V_i = \sum_{i=0}^{N-1} k \frac{\Delta Q}{R_i} \tag{10}$$

The calculation method of electric field intensity is the same as above, that is using negative gradient to calculate it namely equation(3).

B. MATLAB Codes

The codes to plot the distribution are similar to the codes above. We choose N=10 as an example to give the codes here, the other codes of N=20,50,100 are very similar to this, only the value of n is different, the method is the same. So, those codes will not be shown here.

Firstly we give the declaration of the field:

```
n = 10;
k = 9e9;
p = 1e-9;
% Set the linear density of linear
    charge
x1 = -1; y1 = 0;
x2 = 1; y2 = 0;
xm = 2.5;
ym = 2.5;
x=linspace(-xm,xm,150);
y=linspace(-ym,ym,150);
[X,Y]=meshgrid(x,y);
del_x = (x2-x1)/n;
% Divide the line charge to N segments
del_Q = p.*(x2-x1)./n;
% The charge amount of each point
    charge
```

Then we can calculate the potential distribution using the equation(10) with for loop. The codes are as follows:

```
figure;
V_sum = 0;
for x_n = x1+del_x/2:del_x:x2-del_x/2
    V_sum = V_sum + k*del_Q./sqrt((X-
        x_n).^2+Y.^2);
end
mesh(X,Y,V_sum);
hold on;
title(["Potential distribution of
    linear charge in a vacuum";"with
    infiitesimal method (N=10)";"
    12113010"],"fontsize",13);
xlabel("X(m)","fontsize",13);
ylabel("Y(m)","fontsize",13);
```

Similarly we also can choose appropriate scale of the field to plot the graph of the contours distribution. And the point charges are also plotted in this figure. The codes are as follows:

```
figure;
Vmin=0;
% Set the minimum potential value of
    the equipotential line
Vmax=150;
% Set the maximum potential value of
    the equipotential line
Veq=linspace(Vmin,Vmax,35);
% Set the potential values of 35
    alleles
contour(X,Y,V_sum,Veq);
% Plot the contours
grid on
hold on
plot([x1,x2],[y1,y2],'b','LineWidth'
    ,1);
for x_n = x1+del_x/2:del_x:x2-del_x/2
    plot(x_n,0,'o','Color','0,0,0');
end
title(["Equipotential line of the line
     charge in a vacuum";"with
    infiitesimal method (N=10)";"
    12113010"], "fontsize", 13);
xlabel("X(m)", "fontsize", 13);
ylabel("Y(m)", "fontsize", 13);
```

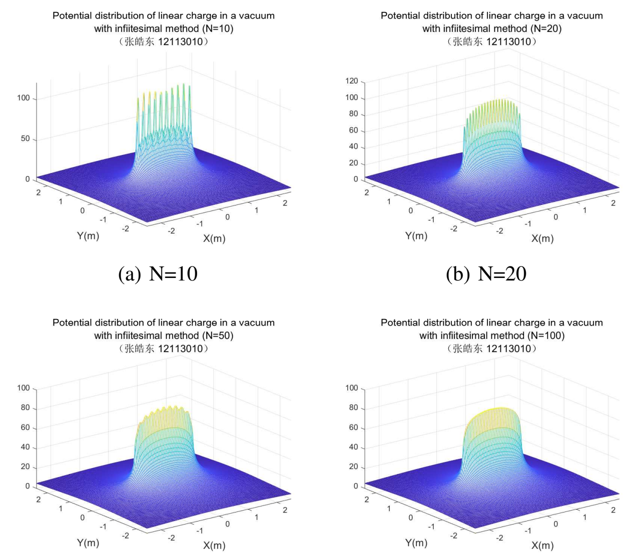
And similarly then we can use equation(3) and streamline() function to generate the clusters of electric fluxline respectively with each charges as the starting point. The codes are as follows:

```
figure;
[Ex,Ey]=gradient(-V_sum);
% Calculate the two components of the
    electric field intensity at each
    point in the field
del_theta=120;
theta=(0:del_theta:360).*pi/180;
% Generating the radian value of the
    power line
ys=0.001*sin(theta);
for x_n = x1+del_x/2:del_x:x2-del_x/2
streamline(X,Y,Ex,Ey,x_n+0.001*cos(
    theta),ys)
end
grid on
hold on
plot([x1,x2],[y1,y2],'b','LineWidth'
    ,1);
contour(X,Y,V_sum,Veq);
hold on
for x_n = x1+del_x/2:del_x:x2-del_x/2
    plot(x_n,0,'o','Color','0,0,0');
end
title(["Equipotential line and
    electric fluxline of the line
    charge in vacuum" ;"with
    infiitesimal method (N=10)"; ...
     "smooth continuous curves"+"
        12113010"], "fontsize", 13);
xlabel("X(m)", "fontsize", 13);
ylabel("Y(m)", "fontsize", 13);
```

C. Simulation Result and Analysis

We choose the number of segments N=10,20,50,100 separately and plot the each distribution. The results of the four different segmentation methods are listed below according to the different classifications.

The potential distribution of each situation are displayed in Fig.4:

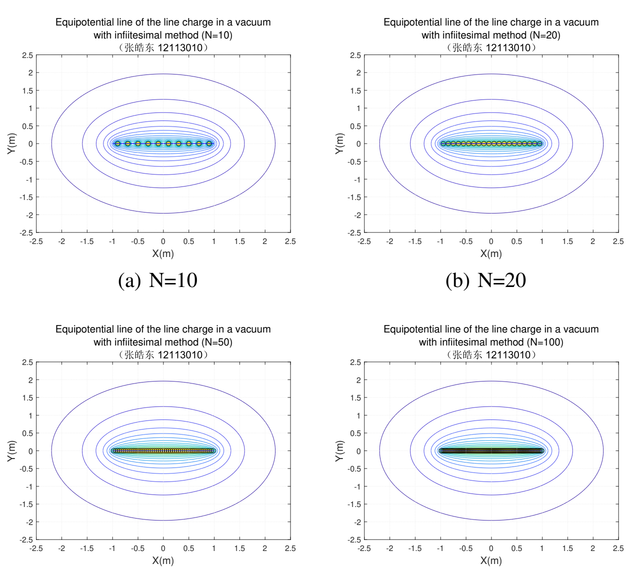


(a) N=10 (b) N=20

(c) N=50 (d) N=100

Fig. 4: The Potential Distribution of Line Charge Using Infinitesimal Method With N=10,20,50,100 Separately

The contours distribution of each situation are displayed in Fig.5:

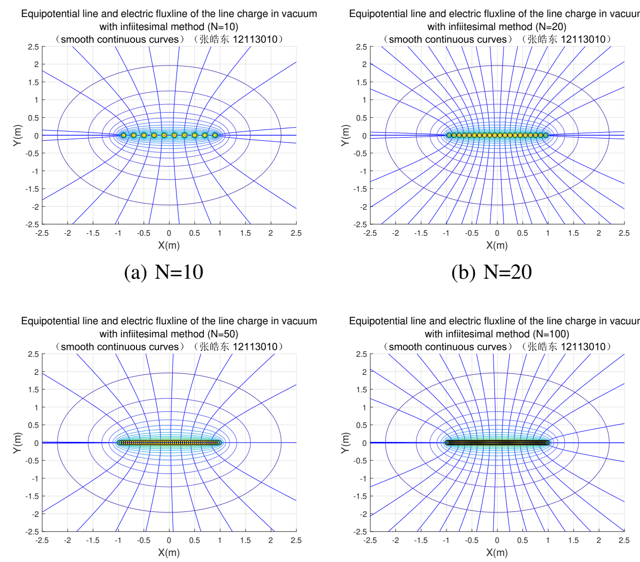


(a) N=10 (b) N=20

(c) N=50 (d) N=100

Fig. 5: The Contours Distribution of Line Charge Using Infinitesimal Method With N=10,20,50,100 Separately

The electric fluxline distribution and contours distribution are displayed in Fig.6:



(a) N=10 (b) N=20

(c) N=50 (d) N=100

Fig. 6: The Electric Fluxline and Contours Distribution of Line Charge Using Infinitesimal Method With N=10,20,50,100 Separately

D. Comparative Analysis

First of all, we can intuitively and qualitatively find the gap between using the infinitesimal method and directly using the integral to calculate accurately from Figure 4. It can be seen that when N is small, the potential distribution presents a discrete state, and when N=10 it can be clearly seen. However, as N increases, this discrete state gradually disappears, and it is getting closer to the exact image we got in Figure 1. Especially when N=100, it is difficult for us to distinguish it from the exact image with the naked eye. The gap between the images is gone. Therefore, through qualitative analysis and observation, we can draw a conclusion: with the increase of the number of segments, the results obtained by using the infinitesimal method are getting closer to the accurate results obtained by using the integral method.

However, the above are all qualitative analysis. Next, we will use MATLAB to conduct quantitative analysis to study the relationship between the error and the number of segments.

We will use the root mean square error(RMSE) to evaluate the err between the result using infinitesimal method with different values of segments and the accurate value. RMSE is used to indicate the degree of deviation between the data and the true value. Its mathematical expression is:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(X_i - X_{real})^2}{N}} \tag{11}$$

The codes with MATLAB are as follows:

```
clc;
clear;
```

```
figure;
grid on;
k = 9e9;
p = 1e-9;
x1 = -1; y1 = 0;
x2 = 1; y2 = 0;
xm = 2.5;
ym = 2.5;
x=linspace(-xm,xm,150);
y=linspace(-ym,ym,150);
[X,Y]=meshgrid(x,y);
V_real = k*p*log((1-X+sqrt((1-X).^2+Y
    .^2))./(-1-X+sqrt((-1-X).^2+Y.^2)))
    ;
% Use the integral method to get the
    real potential value
N = zeros(120);
RMSE = zeros(120);
for n = 1:1:120
% Choose a different number of
    segments
del_x = (x2-x1)/n;
% the length of each segment
del_Q = p.*(x2-x1)./n;
% The amount of charge in each segment
V = 0;
for x_n = x1+del_x/2:del_x:x2-del_x/2
   V = V + k*del_Q./sqrt((X-x_n).^2+Y
       .^2);
end
% Use the method of accumulation to
    obtain the corresponding potential
    value under each segment number
V_e = (V-V_real).^2;
rmse = sqrt(sum(V_e(:))./(150*150));
% calculate the RMSE
N(n) = n;
RMSE(n) = rmse;
end
axis equal;
plot(N, RMSE,'-x', 'LineWidth',1);
xlim([0,120]);
title(["The RMSE(Root Mean Square
    Error) with segments";"
    12113010"],"fontsize", 13)
xlabel("Segments N");
ylabel("RMSE");
```

Therefore, the graph about the relationship between RMSE and the numbers of segments can be obtained and the result is shown in Fig.7. Here we take N from 1 to 120, and take the step as 1. It can be seen from the figure that as the number of segments increases, the difference between the potential distribution and the real value which is represented by RMSE here will appear rapid attenuation, it can be seen that this curve is very steep, that is, the attenuation speed is very large, and it can also be seen that when N=100, the value of RMSE has approached 0, so we will see that when N=100, the obtained potential distribution is not different from that obtained in Fig.1.

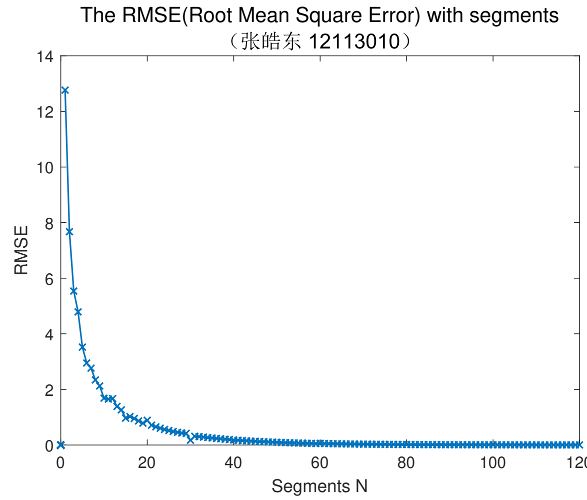


Fig. 7: The relationship between RMSE and numbers of segments

IV. CONCLUSION

In this experiment, we use the integral method and the infinitesimal method to calculate and draw the electric field distribution of a line charge in vacuum, including potential distribution, contours distribution and electric field line distribution. The result calculated by using the integral method is considered to be the correct result. Taking it as a reference, we compared the effect of using the infinitesimal method, analyzed the size of the error generated by it, and qualitatively analyzed it through visual observation and calculated RMSE. Quantitative analysis is used to study the relationship between the size of the error and the number of segments. By calculating the RMSE and drawing its relationship with the number of segments, we can draw a conclusion: as the number of segments increases, the error of the calculation results by the infinitesimal method will increase. It is rapidly reduced to 0, and it can be observed that when N¿50, the error can already be approximately 0.

This experiment tells us that the micro-element method is indeed a very effective method. When the number of micro-elements is enough, we can use the accumulation of these micro-elements to approximate and estimate the real result, which inspires us to solve complex problems. During the analysis, the idea and method of the micro-element can be effectively used for calculation, so as to effectively solve the problem. In fact, the idea of calculus is derived from the idea of microelement method.

In the operation of this experiment, I also learned a lot. First of all, I have a deep memory and intuitive feeling for the electric field distribution of line charges. Then I also felt the power of the infinitesimal method. I saw that with the increase of the number of segments, the speed of error reduction was beyond my imagination. Therefore, the idea of Weiyuan left a deep impression on me. There are many inspirations for my study and work. Finally, this experiment also further understands the role of RMSE in statistics.

ACKNOWLEDGMENT

Thanks to the teacher of the course of engineering electromagnetic field theory, Professor Jia, who taught us the theoretical knowledge, proposed the purpose and requirements of the experiment, provided the template and some codes of the experiment, and made a contribution to the theoretical derivation in this article.