The Experiment 4 of Engineering Electromagnetics: Simulation and Analysis of Magnetic Focusing Phenomena

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This experiment use Matlab to simulate and analyze a magnetic phenomena: Magnetic focusing phenomena. For a beam of charged particles with small angle of divergence, given the same velocity component at the direction of the magnetic field B, their trajectory will have the same screw pitch. After a period, they will converge at another point. The phenomenon that the diverged charged particles focus at one point is similar to the phenomenon that lens can let the light beam focus at one point. Therefore, it is called as magnetic focusing. This problem can be solved by solving differential equations, but the purpose of this experiment is to use Matlab tools to analyze this dynamic process, so it is not limited to tedious mathematical derivation, but focus on understanding the physical nature of this dynamic process. Therefore, this experiment analyzes the velocity and position vector of each time segment through Matlab programming, so as to describe the trajectory of the charge in a period of time. In this experiment, the magnetic focusing phenomenon of 16 charges will be simulated, and its trajectory diagram will be drawn to get an intuitive feeling, and the phenomenon will be analyzed.

***Index Terms*—Magnetic focusing phenomena, Lorentz force, Matlab.**

I. Introduction

THIS is a experiment to use MATLAB to analyze the magnetic focusing phenomenon. For a beam of charged particles with small angle of divergence, given the same velocity component at the direction of the magnetic field B, their trajectory will have the same screw pitch. After a period, they will converge at another point. The phenomenon that the diverged charged particles focus at one point is similar to the phenomenon that lens can let the light beam focus at one point. Therefore, it is called as magnetic focusing. The magnetic focusing needs two conditions below:
(1) The charged particles have similar initial velocity v;
(2) The angle between v and B is sufficiently small so that each particle will do helical motion.

A. Theoretical analysis

The motion of charge in magnetic field is studied in this experiment, so the charge will experience Lorentz force in magnetic field. The Lorentz force can be expressed as follows:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

where $\vec{F}$ is the vector of Lorentz force, $\vec{B}$ is the vector of magnetic flux density, $\vec{v}$ is the vector of charge's velocity, q is the quantity of charge.

According to Newton's law of motion, charge will be accelerated under Lorentz force, hence the velocity and displacement will change. In the 3D Cartesian coordinate system, this process can be described by the following vector equations:

$$\vec{B} = B_x(t)\vec{a}_x + B_y(t)\vec{a}_y + B_z(t)\vec{a}_z \tag{2}$$

$$\vec{A} = \vec{F}/m \tag{3}$$

where $\vec{A}$ is the acceleration vector and m is the mass of the charge.

$$\vec{v}(t) = \vec{v}(1) + \int_0^t \vec{A}(t)dt \tag{4}$$

$$\vec{r}(t) = \vec{r}(1) + \int_0^t \vec{v}(t)dt \tag{5}$$

where $\vec{r}(t)$ is the position vector.

As can be seen from these quation above, this is a process that develops over time. In some cases, this process can be solved by solving the differential equation to obtain the analytical solution (i.e. mathematically exact solution) of the velocity vector and position vector at every moment. The purpose of this experiment is to use Matlab tools to analyze this dynamic process, so this experiment is not limited to tedious mathematical derivation, instead focus on understanding the physical nature of this dynamic process. Therefore, the time can be discretized by introducing a small time step Δt, and assuming that the acceleration vector remains unchanged within the time segment, thus, equations (4) and (5) can be written in the following discrete form:

$$\vec{v}(t + \Delta t) = \vec{v}(t) + \vec{A}(t)\Delta t \tag{6}$$

$$\vec{r}(t + \Delta t) = \vec{r}(t) + \vec{v}(t)\Delta t \tag{7}$$

Therefore the equation (6) and (7) can be used to calculate this phenomena with Matlab only needing to choose appropriate Δt.

B. Situation Declaration

There are 16 charges to be studied in this experiment, they have equal mass and m=0.02 kg and each carries q=0.016 C. The initial velocities are the same:$\vec{r}(1) = 0$ (at the origin of the coordinate). Electric field: $\vec{E} = 0$; magnetic flux density:$\vec{B} =$

$8\vec{a}_z Wb/m^2$. These 16 charges' initial velocities along z-axis are equal: $v_z(1) = 10m/s$. Their initial velocities along x-axis and y-axis can be expressed as: $v_x(1) = 0.1sin(k\pi/8)m/s$, $v_y(1) = 0.1cos(k\pi/8)m/s$, where $k = 0, 1, 2, \ldots\ldots, 15$.

The codes of the declaration is as follows:

```
clear all;
m = 0.02;
% Set charge mass;
q = 1.6e-2;
% Set charge quantity;
dt = 0.001;
% Set the time step to 0.001s;
t = 0:dt:10;
% Create an array of times;
k = 0:1:15;
vx = zeros(16,length(t));
vy = vx; vz = vx;
% Set the velocity vector component;
rx = zeros(16,length(t));
ry = rx; rz = rx;
% Set the position vector component;
Fx = zeros(16,length(t));
Fy = Fx; Fz = Fx;
% Set the force vector component;
ax = zeros(16,length(t));
ay = ax; az = ax;
% Set the acceleration vector component;
vz_o = 10;
vx_o = 0.1.*sin(k.*pi./8);
vy_o = 0.1.*cos(k.*pi./8);
vx(:,1) = vx_o;
vy(:,1) = vy_o;
vz(:,1) = vz_o;
% Set the initial value of velocity vector
    component;
Bz = 8; Bx = 0;By = 0;
% Set the initial value of magnetic field
    strength vector component;
```

II. Matlab Codes

Then, the formula derived in the theoretical part can be used to calculate the infinitesimal method. In order to write codes in Matlab more conveniently, we can establish a matrix for all the above amounts of exercise. Each row represents the data corresponding to a particle at a different time, so the matrix has 16 rows. After that, a for cycle is used to advance t each time to obtain all motion data. A three-dimensional trajectory diagram is drawn according to the position coordinates rx, ry and rz through which the motion passes. The codes are as follows:

```
for i=1:(length(t)-1)
 %calculate each point
 Fx(:,i)=q.*(vy(:,i).*Bz-vz(:,i).*By);
 Fy(:,i)=q.*(vz(:,i).*Bx-vx(:,i).*Bz);
 Fz(:,i)=q.*(vx(:,i).*By-vy(:,i).*Bx);
  %Calculate the space force at point i
 ax(:,i)=Fx(:,i)./m;
 ay(:,i)=Fy(:,i)./m;
 az(:,i)=Fz(:,i)./m;
 %Calculate the acceleration at point i
 vx(:,i+1)=vx(:,i)+ax(:,i).*dt;
 vy(:,i+1)=vy(:,i)+ay(:,i).*dt;
 vz(:,i+1)=vz(:,i)+az(:,i).*dt;
 %Calculate the i+1 point velocity
 rx(:,i+1)=rx(:,i)+vx(:,i).*dt;
 ry(:,i+1)=ry(:,i)+vy(:,i).*dt;
 rz(:,i+1)=rz(:,i)+vz(:,i).*dt;
 %Calculate the i+1 position
end
figure;
for j =1:16
    plot3(rx(j,:),ry(j,:),rz(j,:));
    hold on;
end
grid;
title(["The trajectory of a charged particle in
    an electromagnetic field";" 12113010"],"
    fontsize",12);
xlabel('X', 'fontsize', 12);
ylabel('Y', 'fontsize', 12);
zlabel('Z', 'fontsize', 12);
```

III. Result and Analysis

A. Basic Result and analysis

The result figure is as follows:

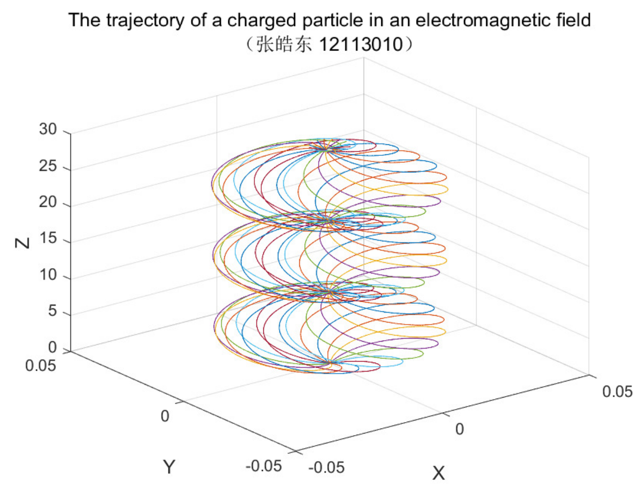

Fig. 1: The trajectory of 16 charged particle in an electromagnetic field

The projection of the trajectory diagram on the xy plane is as follows:

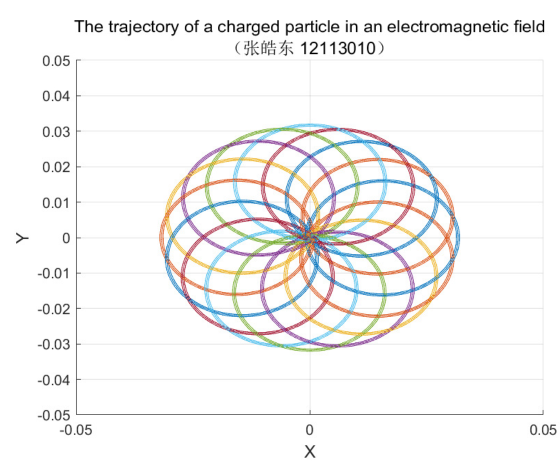

Fig. 2: The trajectory of 16 charged particle in an electromagnetic field in XY (planform)

In figure 1 and 2, it can be seen that each particle is in similar motion. In the xy direction, it moves in a circular motion with almost the same radius. Meanwhile, it moves in a straight line with uniform speed along the z axis with the same speed. Their paths diverge from each other because of their different initial position.

In order to observe the phenomenon of magnetic focusing more obviously and intuitively, we can adjust the angle of view of the picture and observe it from the angle parallel to the x-y plane. The effect picture is as follows:

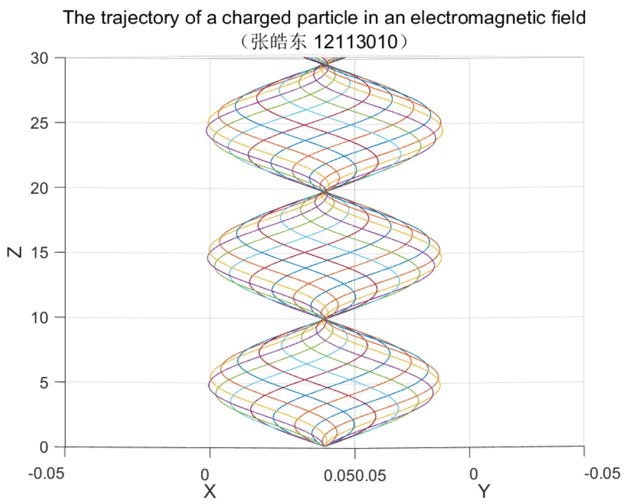

Fig. 3: The trajectory of 16 charged particle in an electromagnetic field

B. The effect of Δt

In this experiment, the method of microelement is used for calculation, and the change of velocity and displacement in Δt time is calculated each time. Therefore, different Δt will have a great impact on the results of this experiment. The following will show the results of different Δt renderings:

$\Delta t = 0.03$:

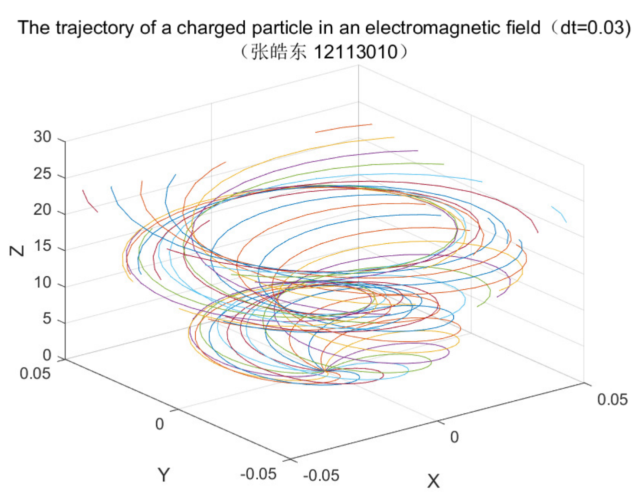

Fig. 4: The trajectory of 16 charged particle in an electromagnetic field ($\Delta t = 0.03$)

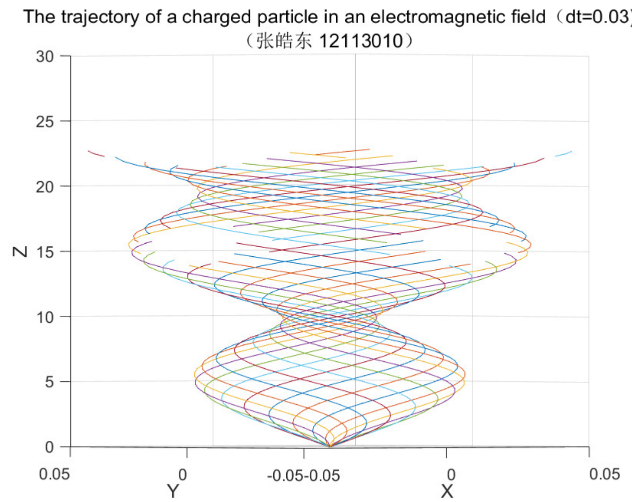

Fig. 5: The trajectory of 16 charged particle in an electromagnetic field ($\Delta t = 0.03$)

$\Delta t = 0.01$:

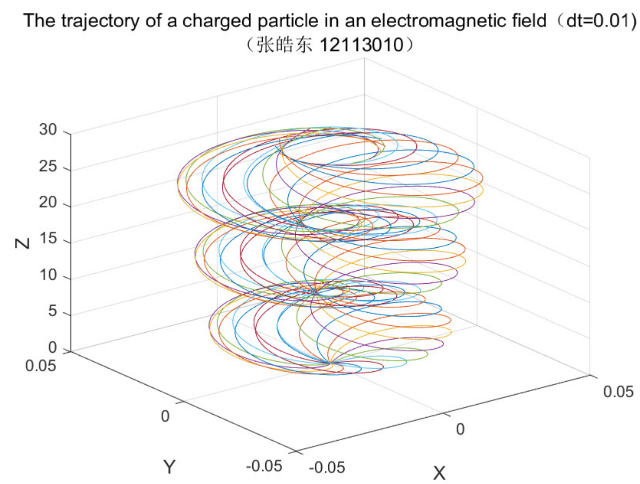

Fig. 6: The trajectory of 16 charged particle in an electromagnetic field ($\Delta t = 0.01$)

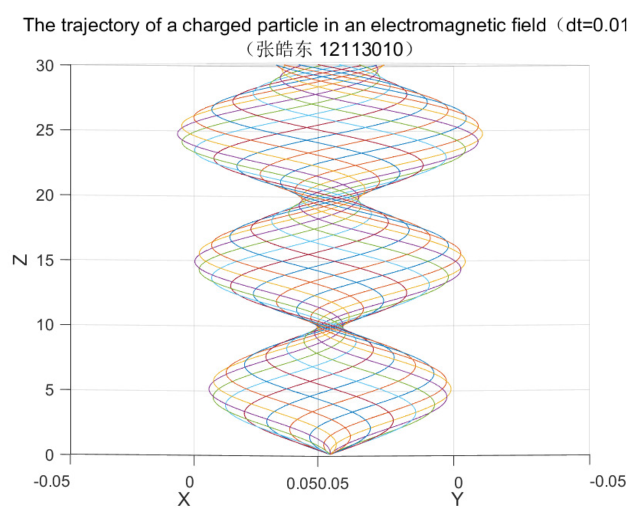

Fig. 7: The trajectory of 16 charged particle in an electromagnetic field ($\Delta t = 0.01$)

$\Delta t = 0.0001$:

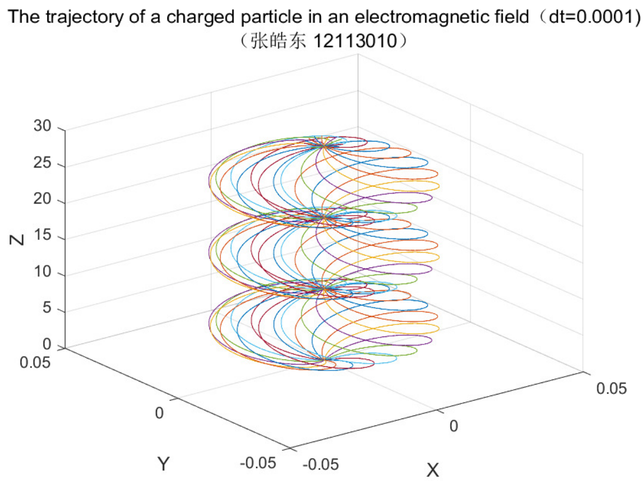

Fig. 8: The trajectory of 16 charged particle in an electromagnetic field ($\Delta t = 0.0001$)

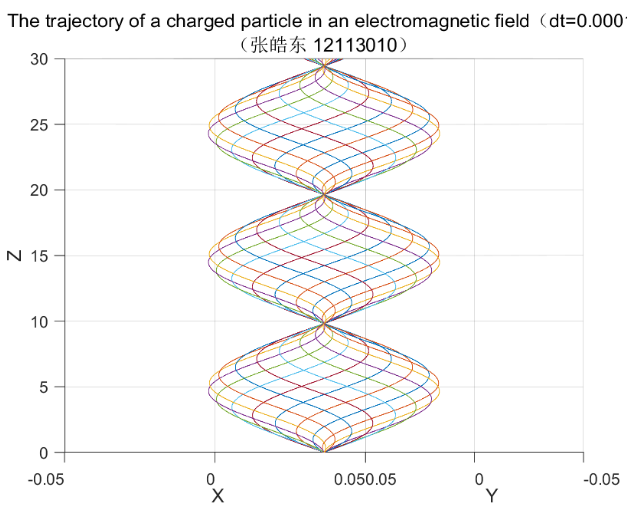

Fig. 9: The trajectory of 16 charged particle in an electromagnetic field ($\Delta t = 0.0001$)

As can be seen from the above renderings of different Δt, the smaller the Δt, the trajectory is the more accurate, the effect is more in line with the real situation. When Δt is lesser than 0.001, there are no much difference of the effect.Because the larger the Δt is, the larger the running time cost is. Therefore, Δt=0.001 is enough.

C. The effect of T

T is the total time of particle motion. Obviously, the larger T is, the further the particle moves on the z-axis, so the number of tracks shown in the figure will increase. As above, we have selected three envelopes and focal points, but if we do not limit them, we can see all the tracks, which is very messy, as shown in the figure below, which is not conducive to our analysis. So in the figure above we all took three envelopes and display them. For example, when T=20s:

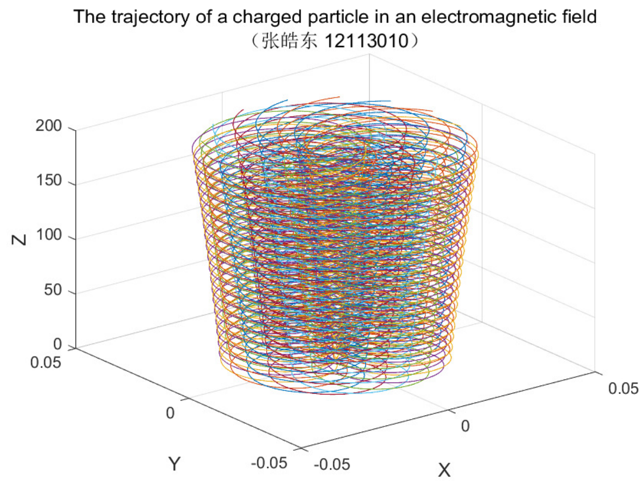

Fig. 10: The trajectory of 16 charged particle in an electromagnetic field (T=20s).

It is worth noting that as T increase, requirements for dt also increase, because the errors exists actually in using of dt calculate, the greater the T, such an error will gradually accumulate to produce a greater impact, then the impact of dt will be more obvious, with T=20 as an example, if we choose dt=0.01, the results are as follows, we can see that the error is quite larger in the later part of the trajectory, and it diverges quite a bit.

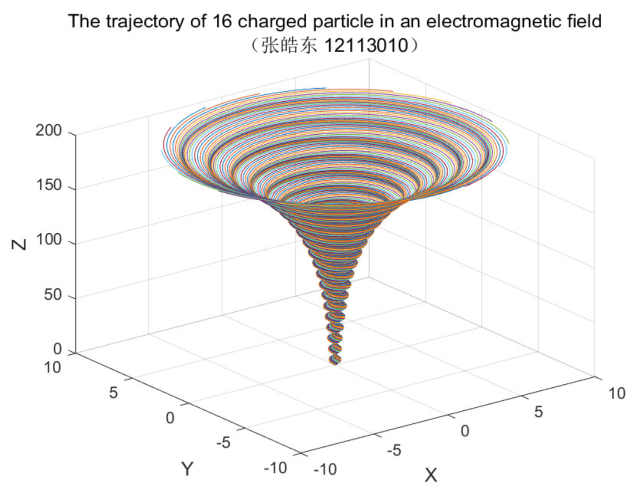

Fig. 11: The trajectory of 16 charged particle in an electromagnetic field (T=20s, dt=0.01s).

IV. Conclusion

In this experiment, the motion of 16 particles in the magnetic field was simulated on Matlab by using the method of microelement, and the influence of parameters such as dt and T on the simulation effect was studied. This physical phenomenon is called magnetic focusing, we have a profound and intuitive understanding of magnetic focusing phenomenon through this experiment, and clear the physical nature of this phenomenon. In the analysis of dt, we have a further understanding and perception of the error generated by the infinitesimal method.

Acknowledgment

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