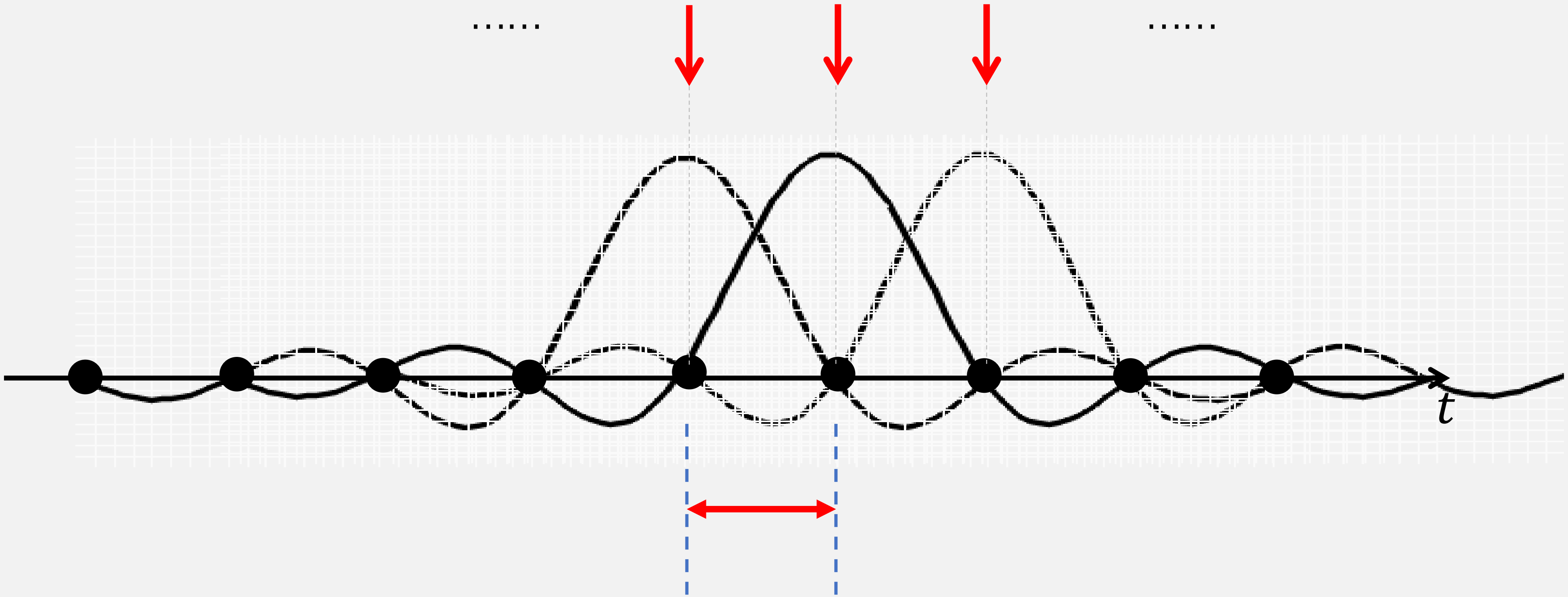
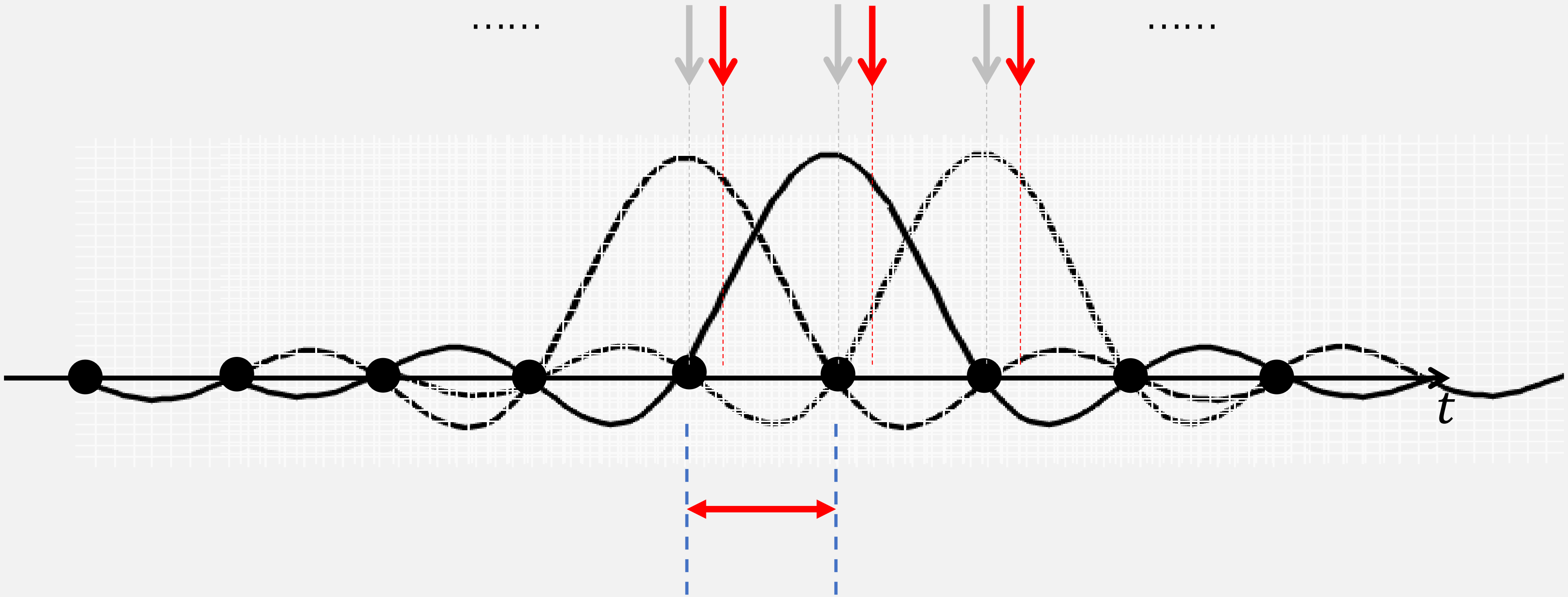
Sampling time



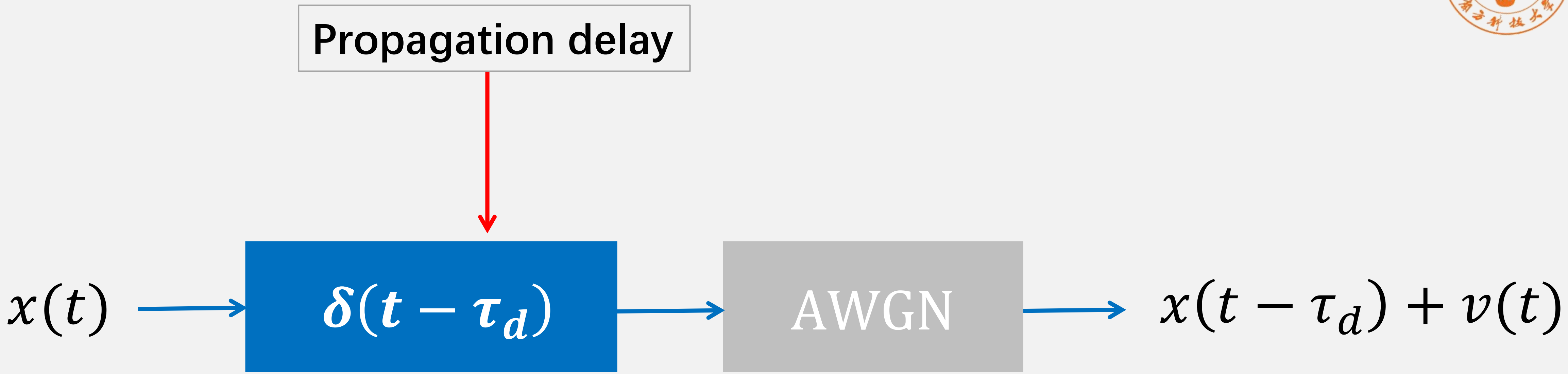
Sampling time



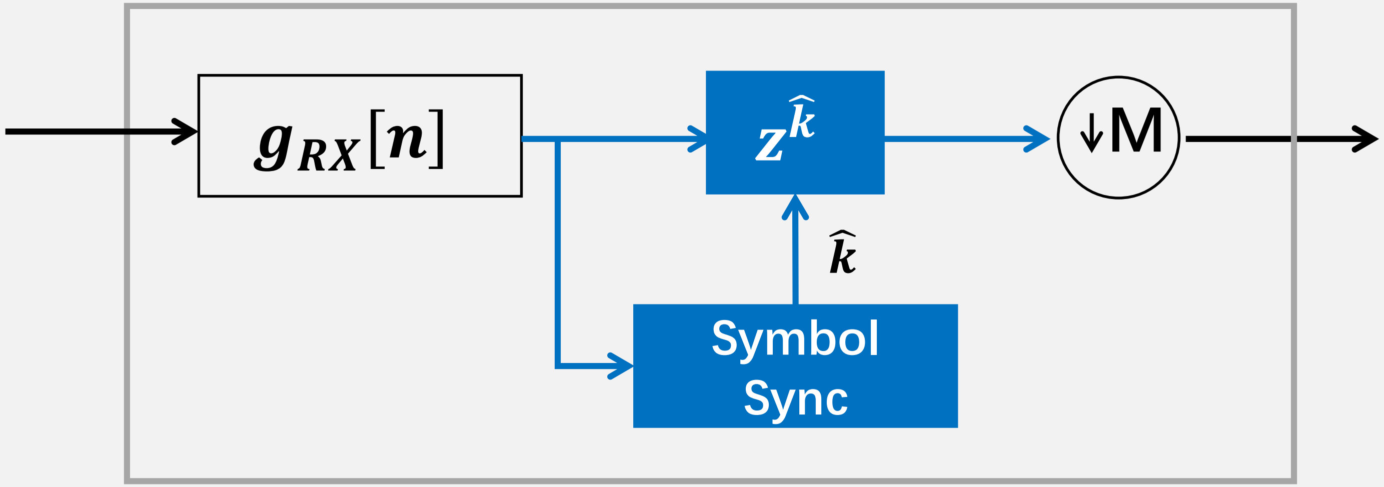
Lab 12 : Symbol Synchronization

主讲人：吴光 博士
Email: wug@sustech.edu.cn

Demo: Symbol synchronization

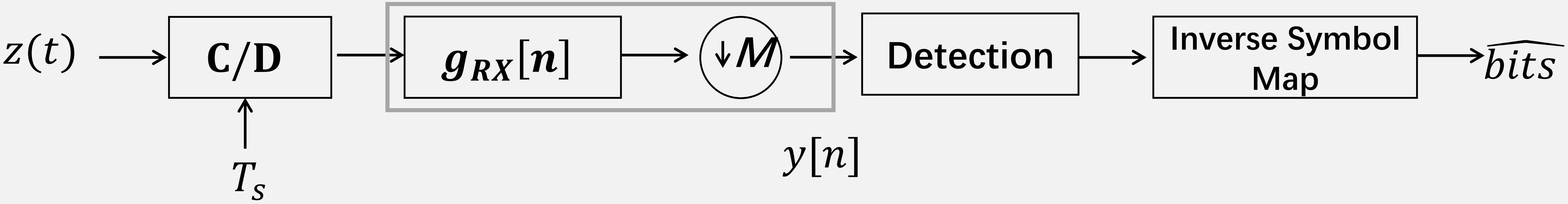


$$\boxed{z(t)=\alpha e^{j\varphi}x(t-\tau_d)+v(t)}$$



$g_{RX}[n]$
$z^{\hat{k}}$
$\downarrow M$
$\hat{k}$
Symbol Sync

Receiver



$z(t)$
C/D
T_s
$g_{RX}[n]$
$\downarrow M$
$y[n]$
Detection
Inverse Symbol Map
$\widehat{bits}$

- External synchronization method

- Self-synchronization method

I have an ability to

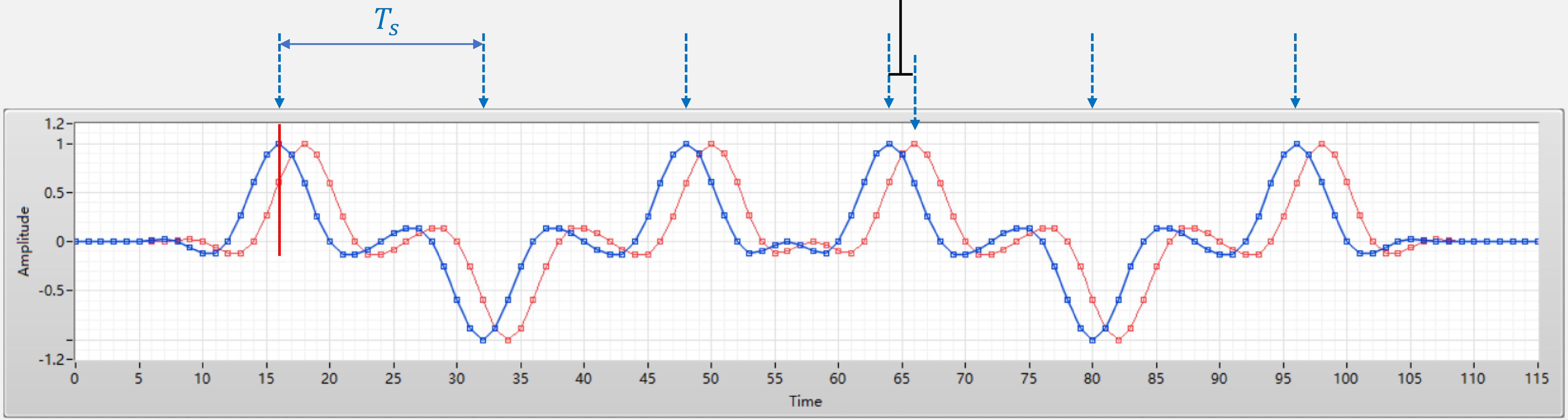
- Understand the symbol Synchronization
- Design and implement the Maximum Energy Algorithm
- Design and implement the Early-Late gate Algorithm

Algorithm Analysis and Verification

1. Maximum Energy Algorithm Analysis
2. Maximum Energy Algorithm Simulation
3. USRP Experiments
4. Early-Late Gate Algorithm Discussion

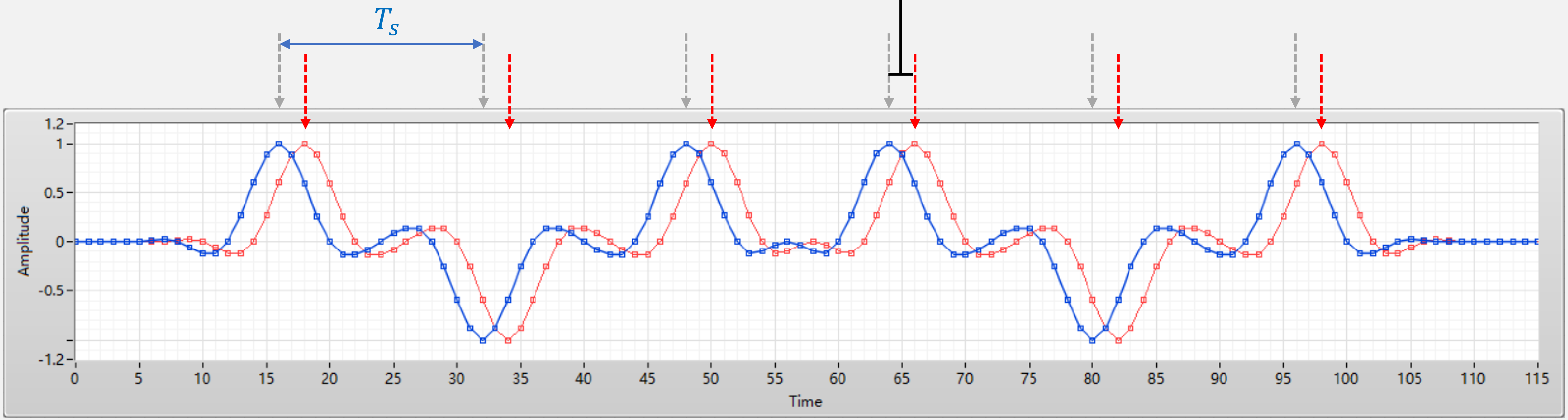
Maximum Energy Algorithm

$\tau_d = 0.125T_s$



The optimal sampling times are kT_s

$\tau_d = 0.125T_s$

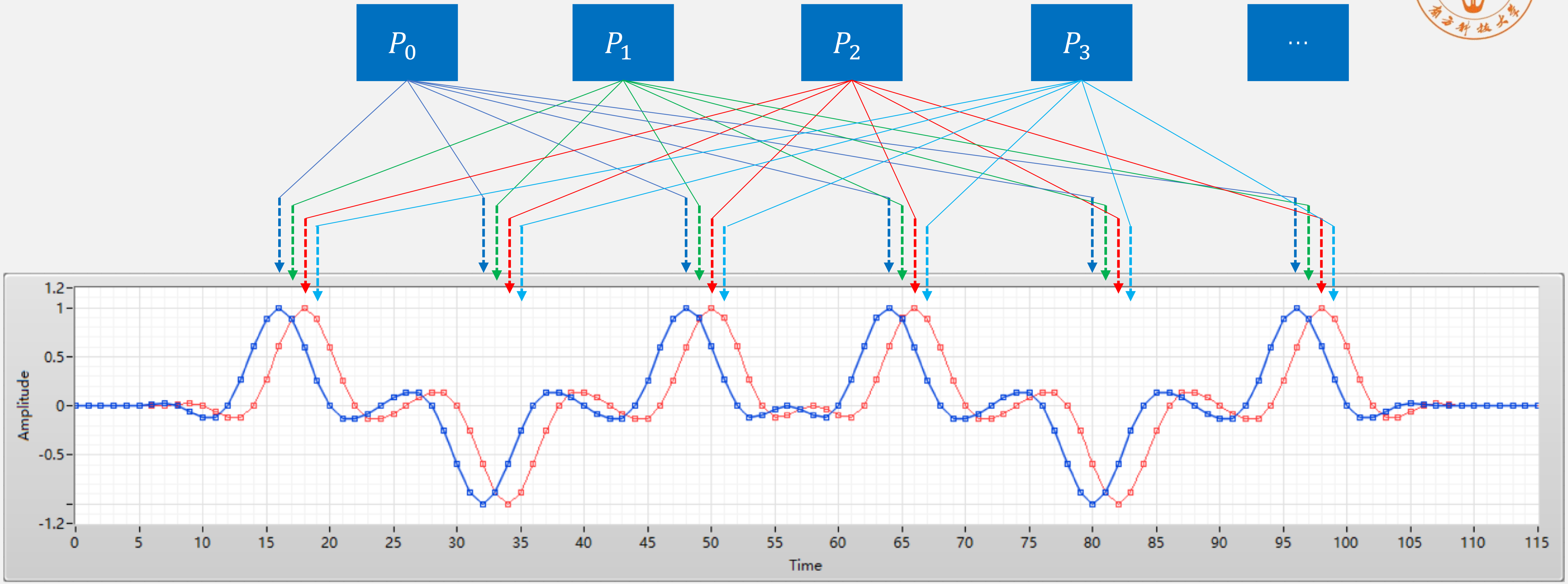


The optimal sampling times are $kT_s + \tau_d$

Maximum Energy Method

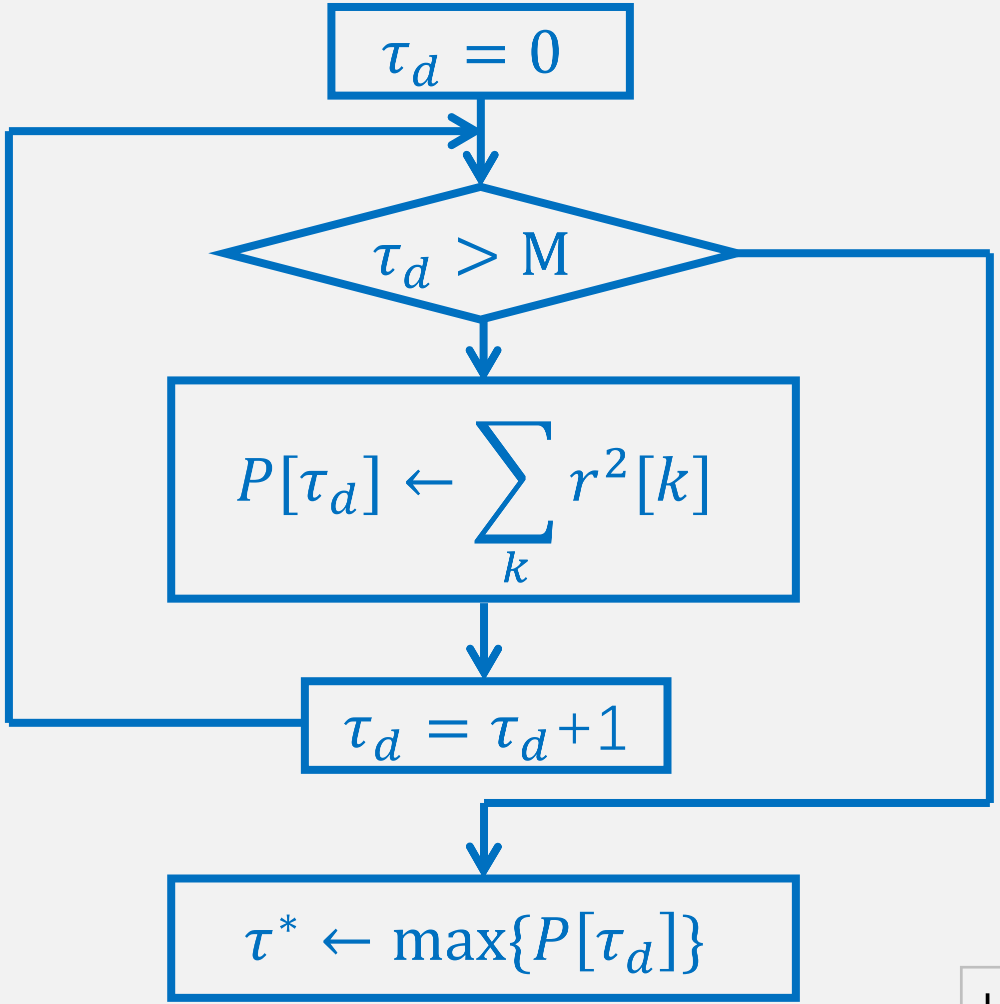
$$\hat{\tau}_d \approx arg\mathbf{max}\sum_k r^2(kT+\tau_d)$$

Attempt to find the sample point that maximizes the average received energy.



The optimal sampling times are $\boldsymbol{kT_s + \tau_d}$

Algorithm Analysis

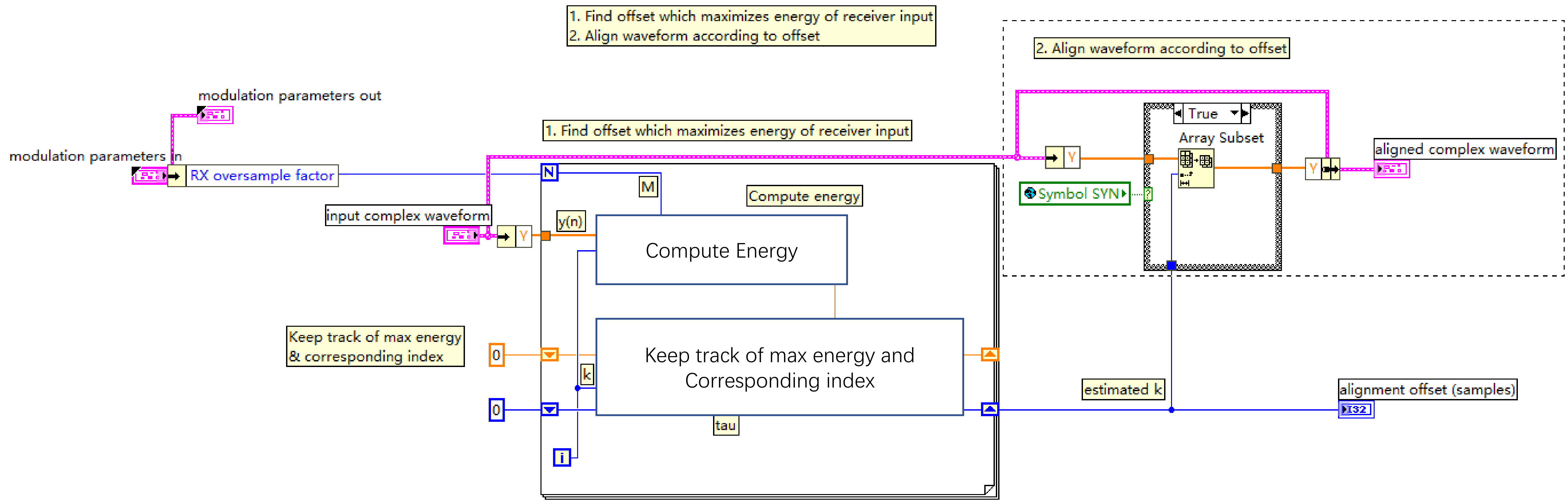


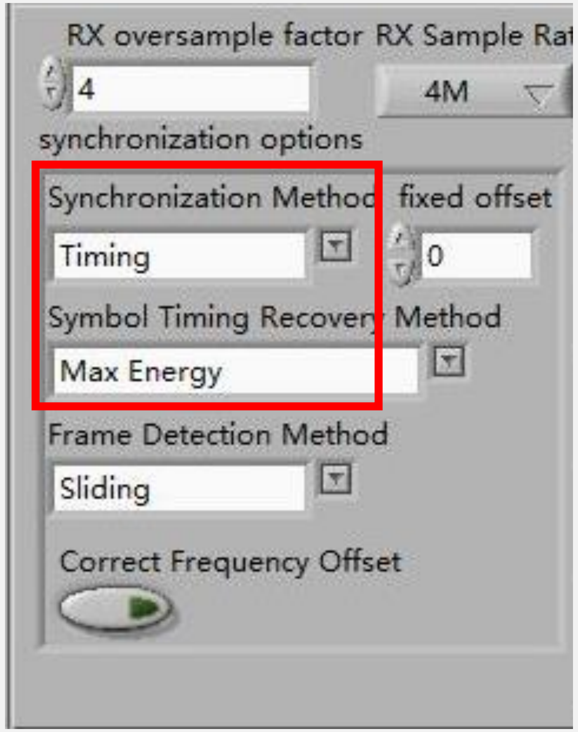
$$\hat{\tau}_d \approx arg\mathbf{max}\sum_i r^2(\tau_d + kT)$$

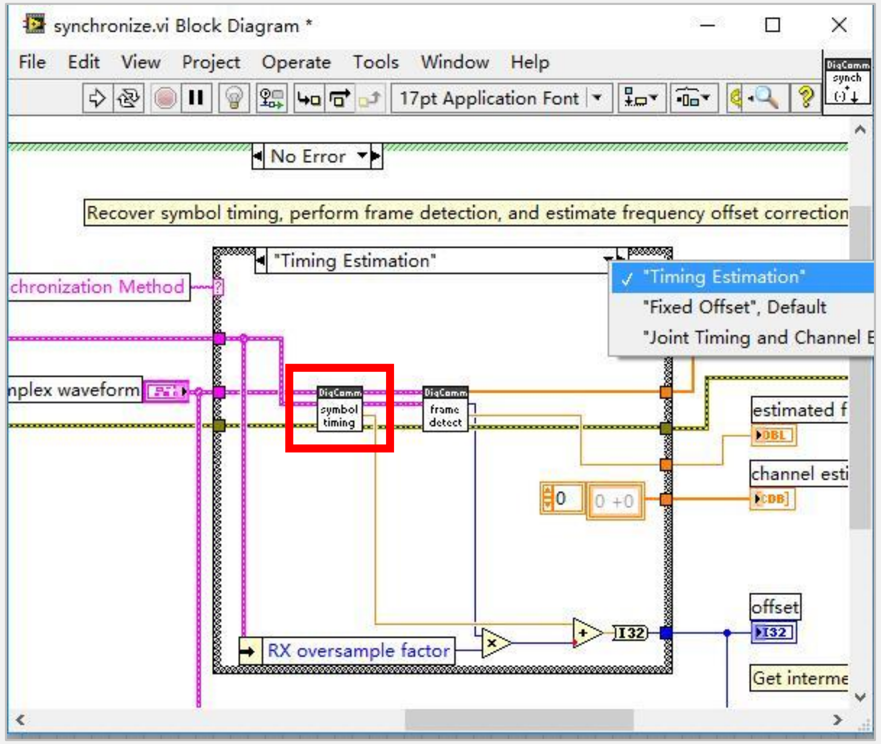
1. Find offset which maximizes energy of receiver input;
2. Align waveform according to offset;

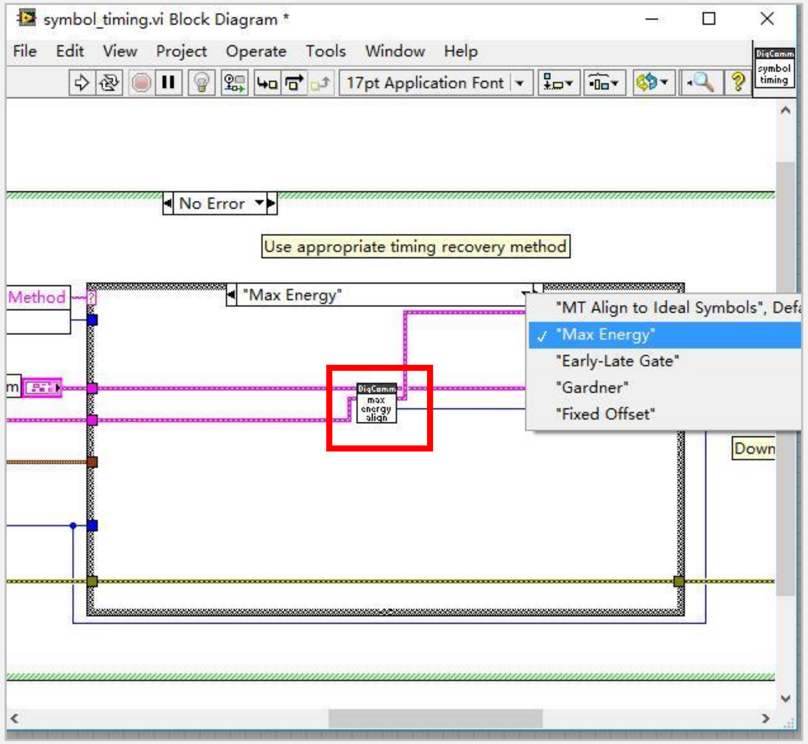
Hints： Use the shift register to save the array $P[\tau_d]$

Programming for Maximum Energy Algorithm


1. Find offset which maximizes energy of receiver input
2. Align waveform according to offset
modulation parameters out
modulation parameters in
RX oversample factor
1. Find offset which maximizes energy of receiver input
input complex waveform
N
M
y(n)
Compute energy
Compute Energy
Keep track of max energy & corresponding index
0
0
k
Keep track of max energy and Corresponding index
tau
i
2. Align waveform according to offset
True
Array Subset
Symbol SYN
aligned complex waveform
estimated k
alignment offset (samples)
I32


RX oversample factor
RX Sample Rat
4
4M
synchronization options
Synchronization Method
fixed offset
Timing
0
Symbol Timing Recovery Method
Max Energy
Frame Detection Method
Sliding
Correct Frequency Offset


synchronize.vi Block Diagram *
File Edit View Project Operate Tools Window Help
17pt Application Font
No Error
Recover symbol timing, perform frame detection, and estimate frequency offset correction
"Timing Estimation"
chronization Method
✓ "Timing Estimation"
"Fixed Offset", Default
"Joint Timing and Channel E
nplex waveform
symbol timing
frame detect
estimated f
channel esti
offset
RX oversample factor
Get interme


symbol_timing.vi Block Diagram *
File Edit View Project Operate Tools Window Help
17pt Application Font
No Error
Use appropriate timing recovery method
Method
"Max Energy"
"MT Align to Ideal Symbols", Defa
✓ "Max Energy"
"Early-Late Gate"
"Gardner"
"Fixed Offset"
Down

Analysis for Maximum Energy Algorithm

$$z(t) = \alpha e^{j\phi}\sqrt{E_x}\sum_m s[m]g_{tx}(t - mT - \tau_d) + v(t)$$

$$y[n] = h\sum_m s[m]g\big((n - m)T - \tau_d\big) + v[n]$$

$$z(t) = \alpha e^{j\phi}\sqrt{E_x}\sum_{m} s[m]g_{tx}(t - mT - \tau_d) + v(t)$$

$$y[n] = h\sum_{m} s[m]g\big((n - m)T - \tau_d\big) + v[n]$$

$$y[n] = \underbrace{hs[n]g(\tau_d)}_{\text{理想值}} + \underbrace{h\sum_{m\neq n} s[m]g\big((n-m)T-\tau_d\big)}_{\text{ISI}} + \underbrace{v[n]}_{\text{噪声}}$$

$$y[n] = h\sum_{m} s[m]g\big((n-m)T-\tau_d\big) + v[n]$$

$$y(t) = h \sum_{m=-\infty}^{+\infty} s[m]g(t - mT - \tau_d) + v(t)$$

$$J(\tau) = E[|y(nT + \tau)|^2] = |h|^2 \sum_{m=-\infty}^{+\infty} |g(mT + \tau - \tau_d)|^2 + N_o$$

$$y(t) = h \sum_{m=-\infty}^{+\infty} s[m]g(t - mT - \tau_d) + v(t)$$

$$J(\tau) = E[|y(nT + \tau)|^2] = |h|^2 \sum_{m=-\infty}^{+\infty} |g(mT + \tau - \tau_d)|^2 + N_o$$

$$E[|y(nT+\tau)|^2] = |h|^2 \sum_{m=-\infty}^{+\infty} \left|g\left(mT+\tau_{frac}-\hat{\tau}_{frac}\right)\right|^2 + N_o$$

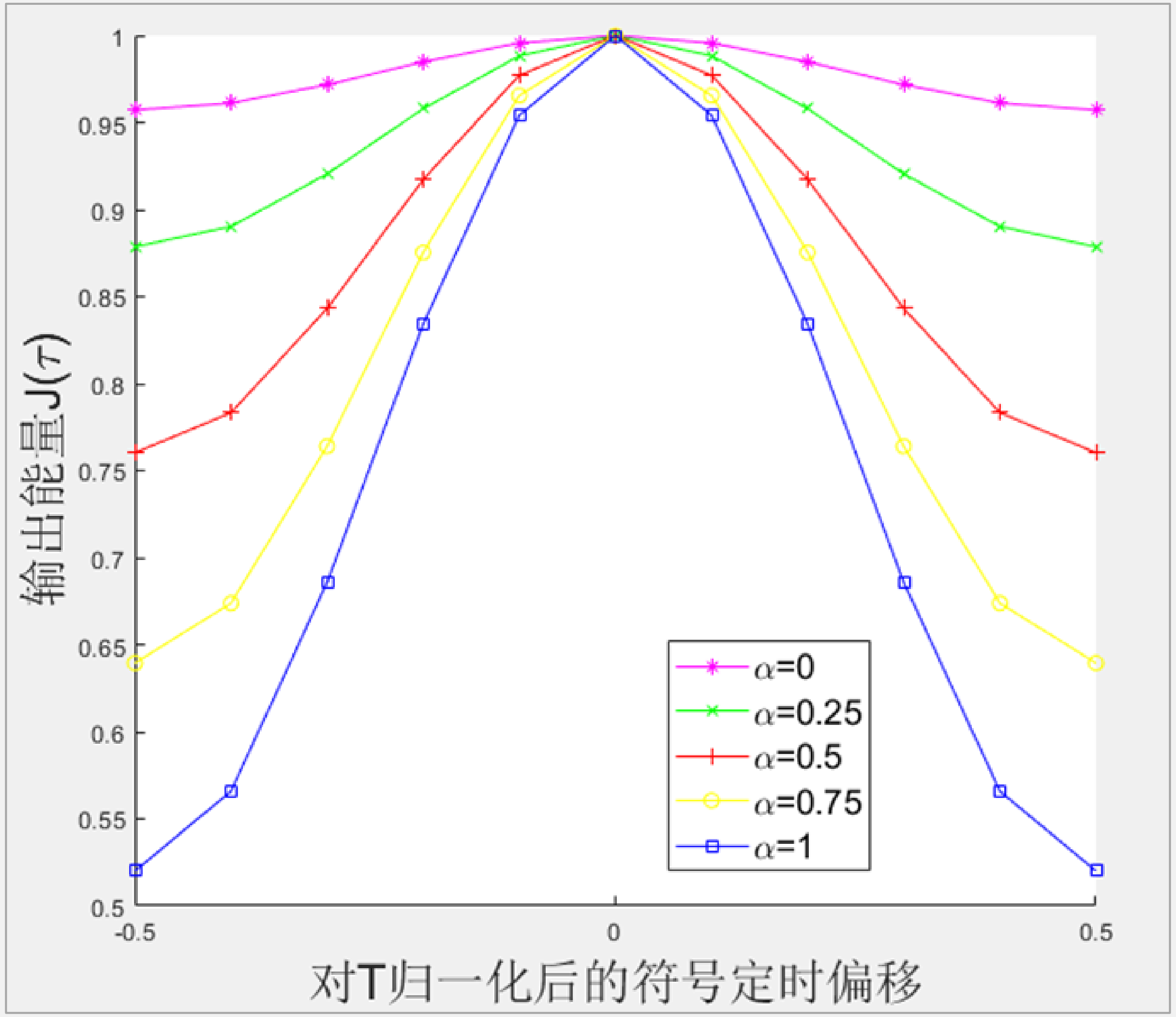
$dT+\tau_{frac}$ $\hat{d}T+\hat{\tau}_{frac}$

$$J(\tau) = E[|y(nT+\tau)|^2] = |h|^2 \sum_{m=-\infty}^{+\infty} |g(mT+\tau-\tau_d)|^2 + N_o$$

$$E[|y(nT+\tau)|^2] = |h|^2 \sum_{m=-\infty}^{+\infty} \left|g\left(mT + \tau_{frac} - \hat{\tau}_{frac}\right)\right|^2 + N_o$$

$$\hat{\tau}_d = \arg \max_{\tau \in [0,T)} J(\tau)$$

$$E[|y(nT+\tau)|^2] \leq |h|^2 |g(0)|^2 + N_o$$

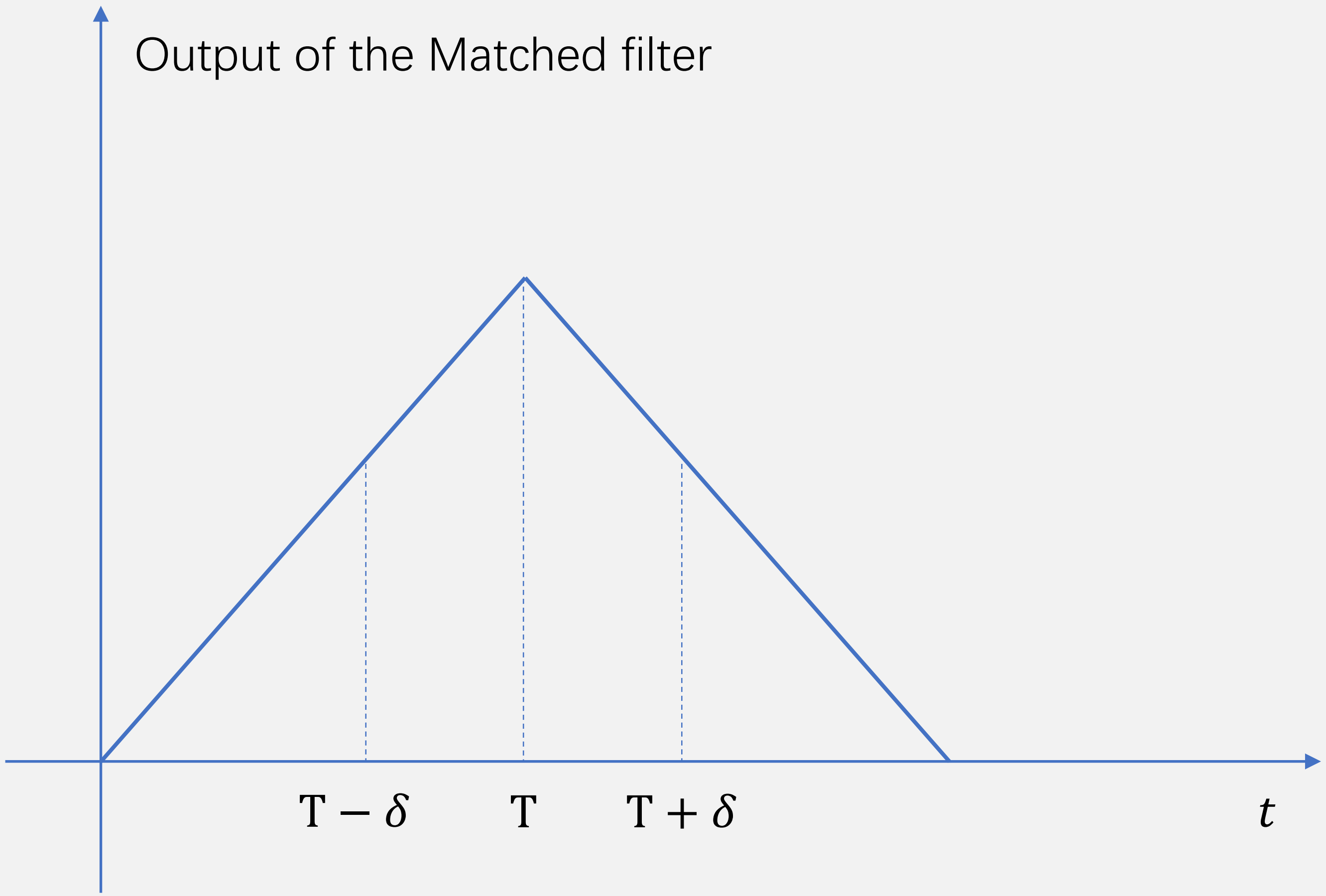


1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
-0.5
0
0.5
输出能量J(τ)
对T归一化后的符号定时偏移
α=0
α=0.25
α=0.5
α=0.75
α=1

Algorithm Analysis and Verification

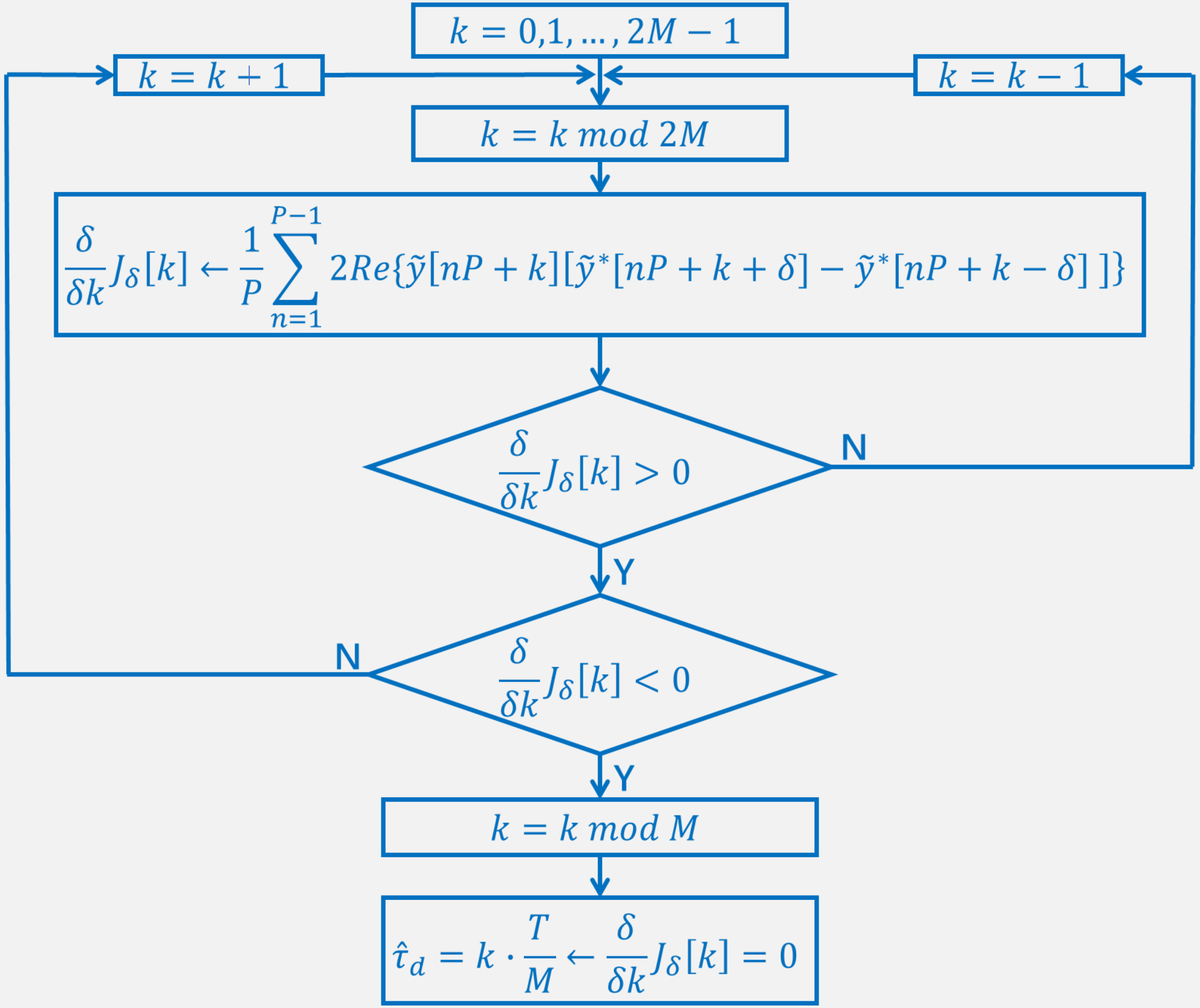
1. Maximum Energy Algorithm Analysis
2. Maximum Energy Algorithm Simulation
3. USRP Experiments
4. Early-Late Gate Algorithm Discussion

Early-Late Gate Algorithm

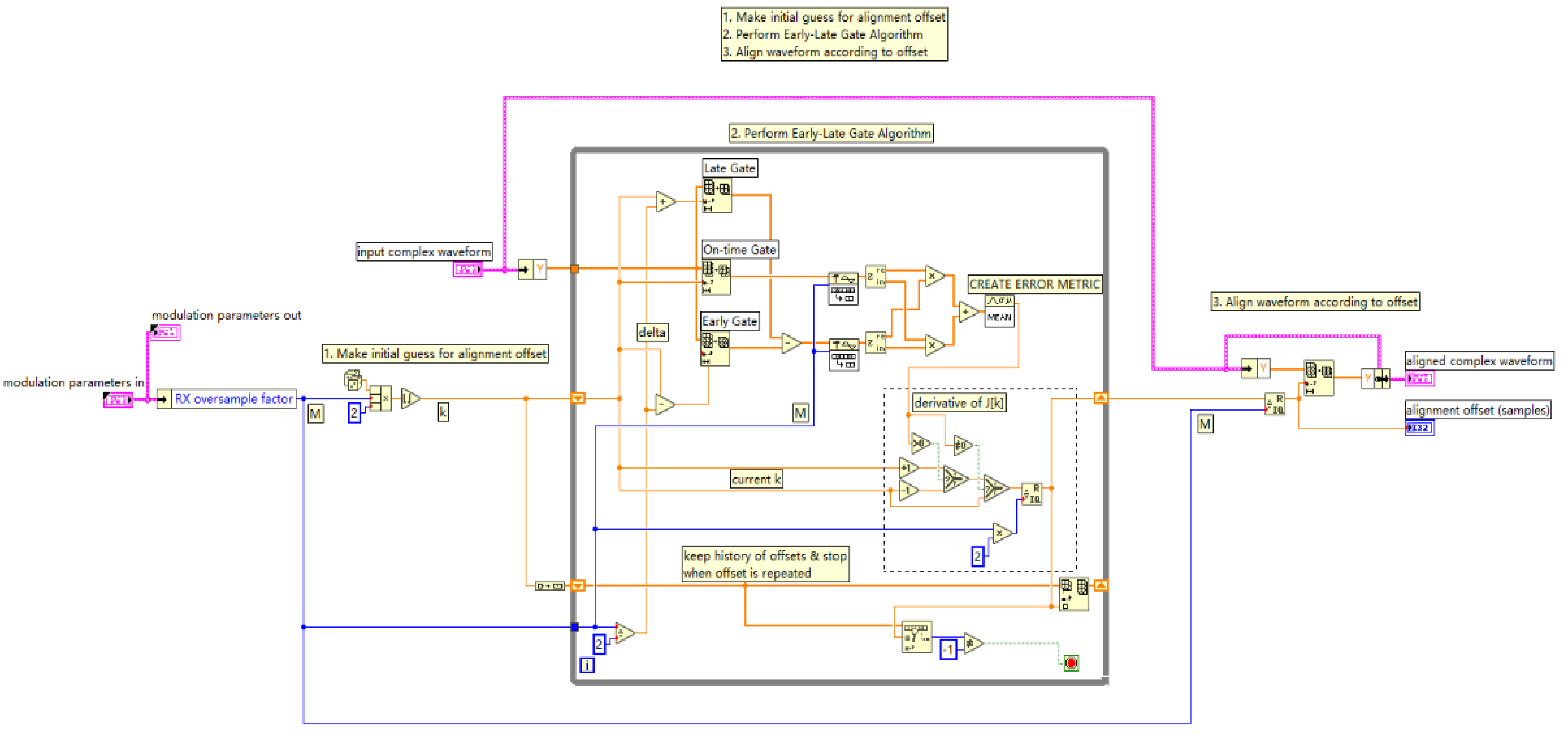

Output of the Matched filter
T − δ
T
T + δ
t

$$\frac{\delta}{\delta k} J_{\delta}[k] \cong \frac{1}{P} \sum_{n=1}^{P-1} 2Re\{\tilde{y}[nP+k][\tilde{y}^{*}[nP+k+\delta] - \tilde{y}^{*}[nP+k-\delta]\,]\}$$

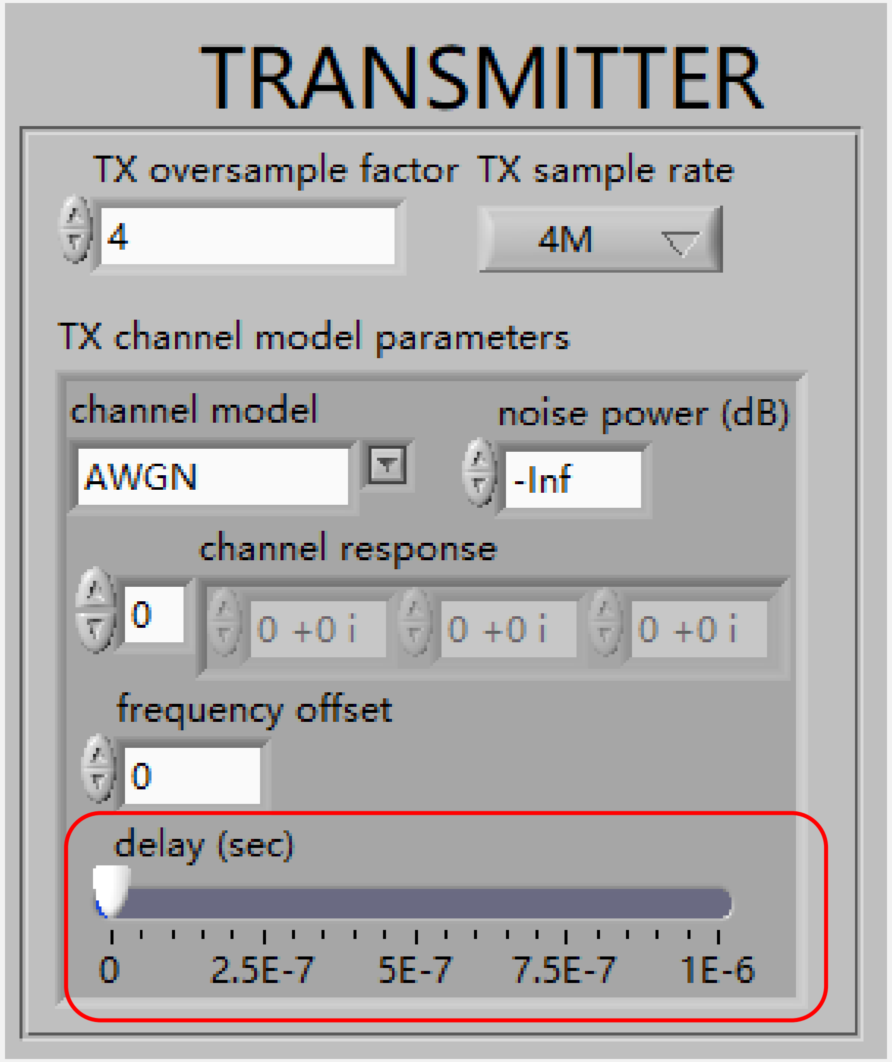
$$\frac{\delta}{\delta k} J_{\delta}[k] = 0$$

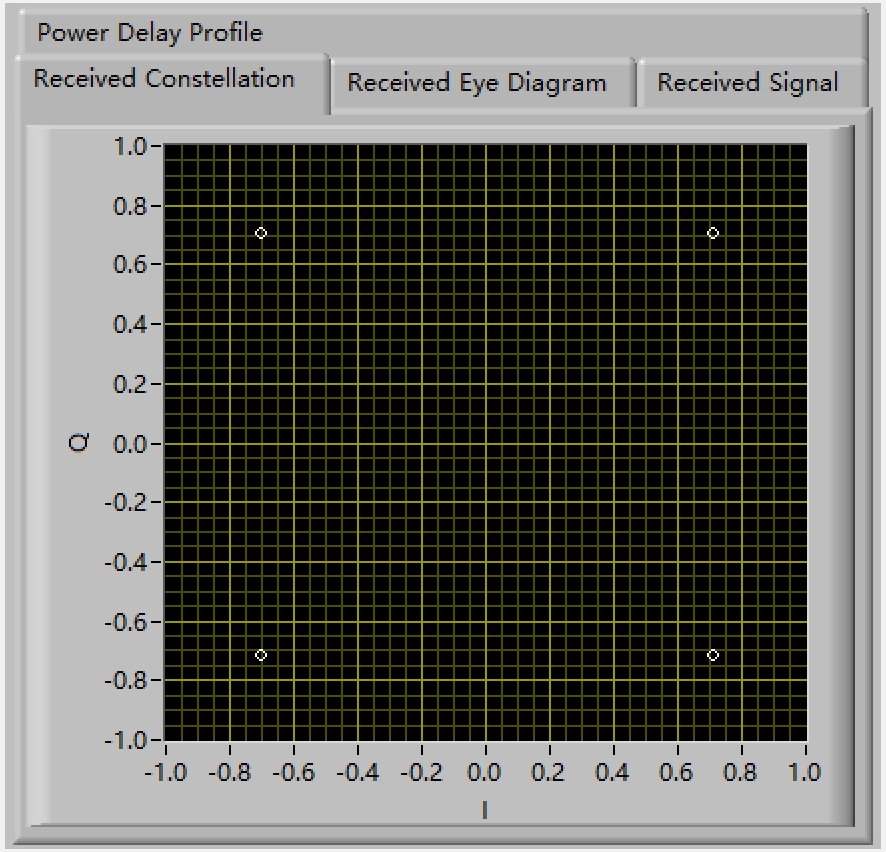

$k = 0,1,\dots,2M-1$
$k = k+1$
$k = k-1$
$k = k \bmod 2M$
$\frac{\delta}{\delta k} J_\delta[k] \leftarrow \frac{1}{P}\sum_{n=1}^{P-1} 2Re\{\tilde{y}[nP+k][\tilde{y}^*[nP+k+\delta] - \tilde{y}^*[nP+k-\delta]\,]\}$
$\frac{\delta}{\delta k} J_\delta[k] > 0$
N
Y
$\frac{\delta}{\delta k} J_\delta[k] < 0$
N
Y
$k = k \bmod M$
$\hat{\tau}_d = k \cdot \frac{T}{M} \leftarrow \frac{\delta}{\delta k} J_\delta[k] = 0$

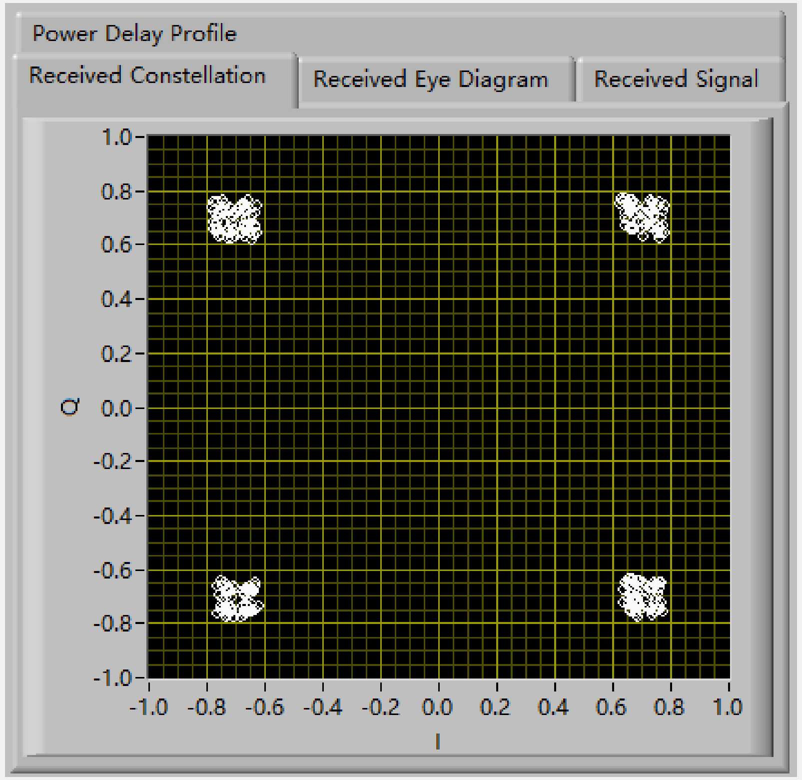
Programming for Early-Late Gate Algorithm

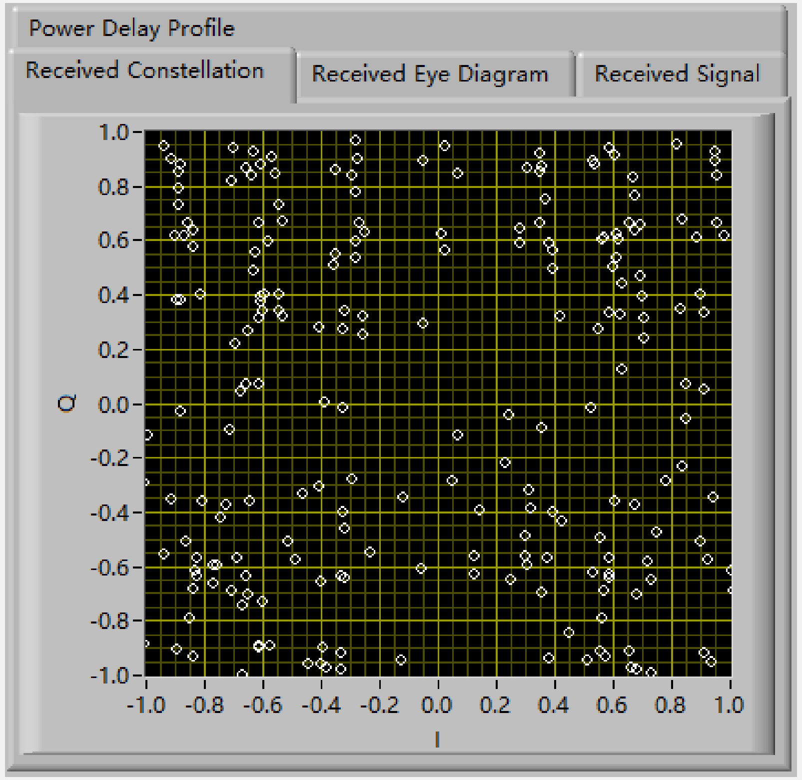

1. Make initial guess for alignment offset
2. Perform Early-Late Gate Algorithm
3. Align waveform according to offset
2. Perform Early-Late Gate Algorithm
Late Gate
On-time Gate
Early Gate
input complex waveform
delta
CREATE ERROR METRIC
MEAN
3. Align waveform according to offset
aligned complex waveform
alignment offset (samples)
modulation parameters out
modulation parameters in
RX oversample factor
1. Make initial guess for alignment offset
M
2
k
M
derivative of J[k]
current k
M
2
keep history of offsets & stop
when offset is repeated
2
i
-1
I32

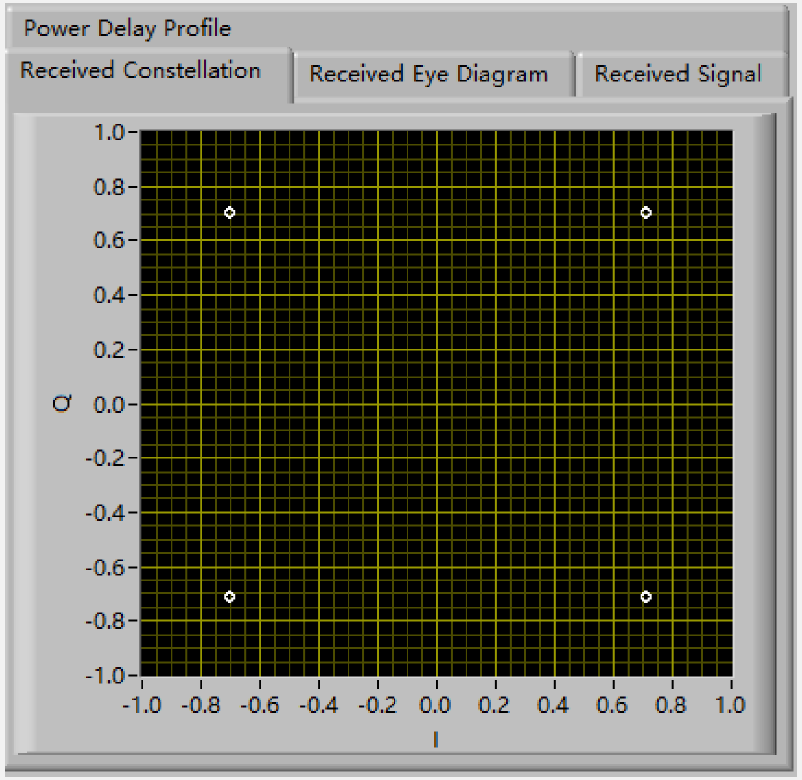
System Testing

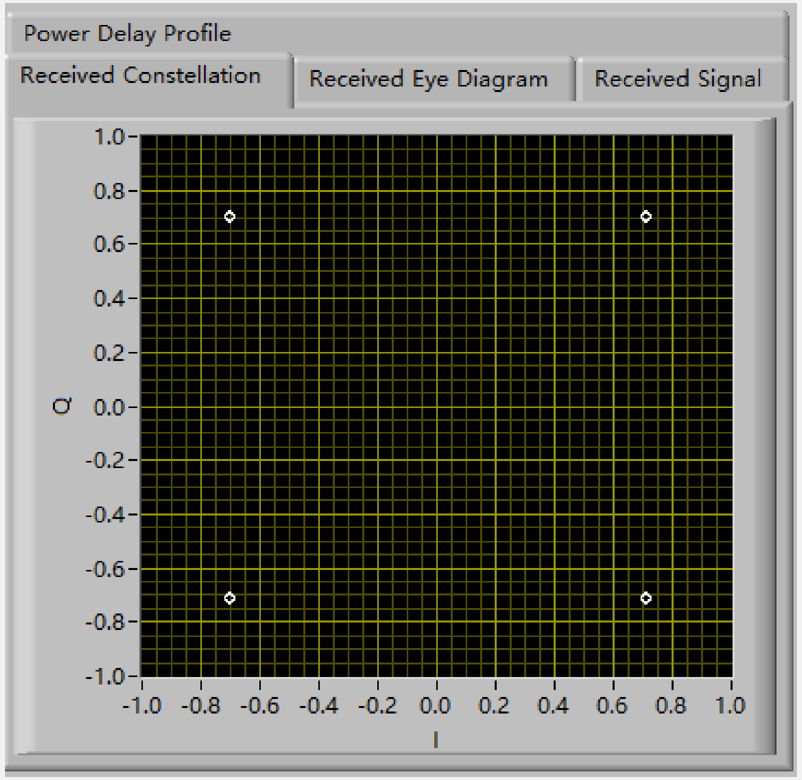

TRANSMITTER
TX oversample factor
4
TX sample rate
4M
TX channel model parameters
channel model
AWGN
noise power (dB)
-Inf
channel response
0
0 +0 i
0 +0 i
0 +0 i
frequency offset
0
delay (sec)
0
2.5E-7
5E-7
7.5E-7
1E-6

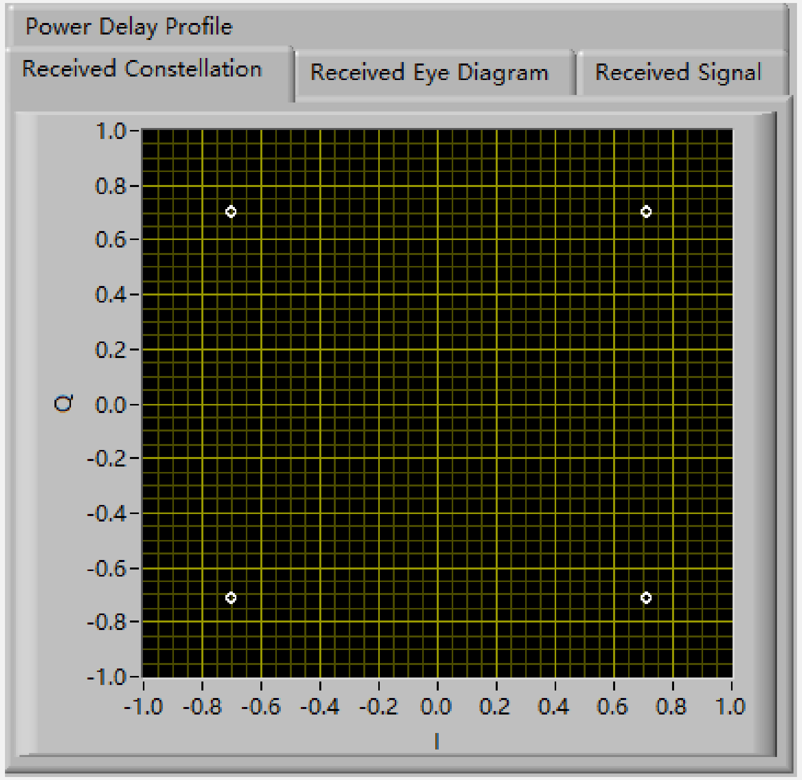

Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
Q
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
I

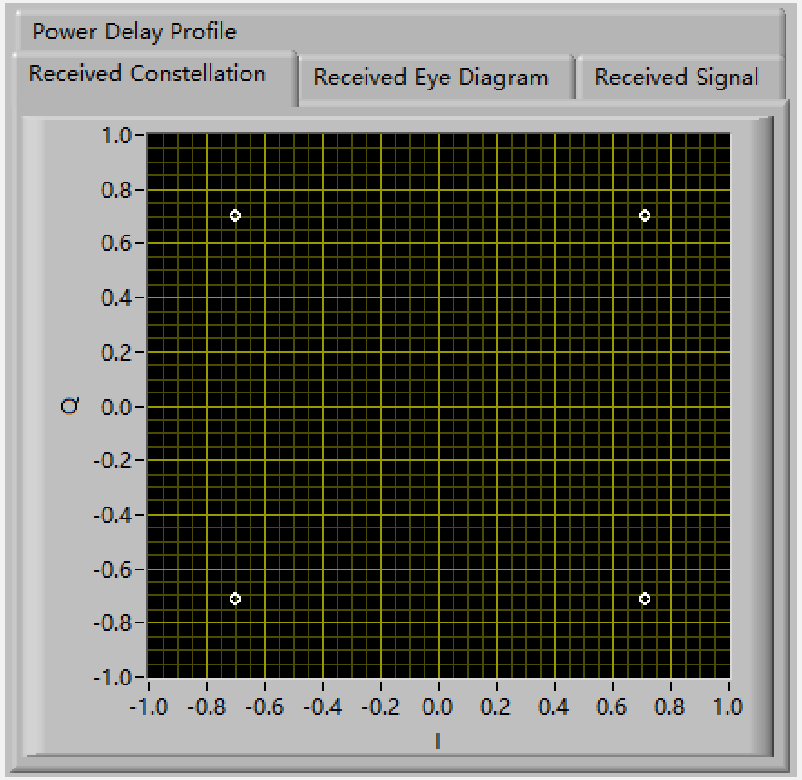

Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
Q
I
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0


Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
Q
I
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0

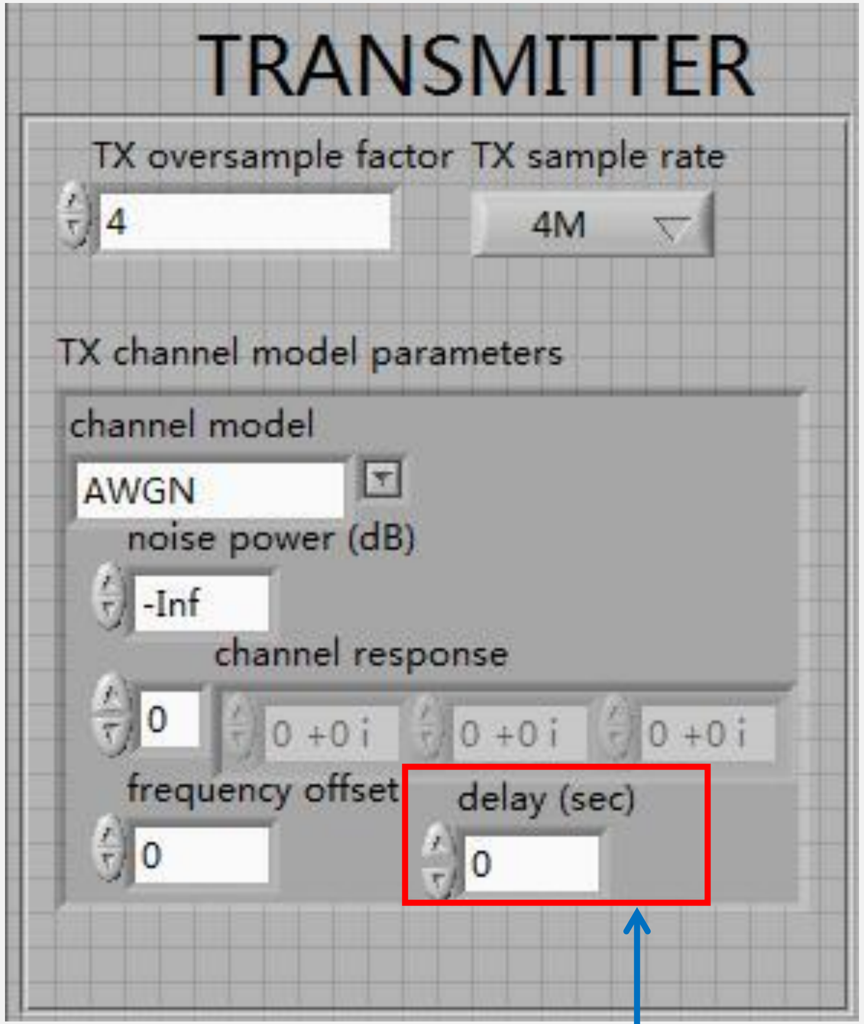

Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
Q
I

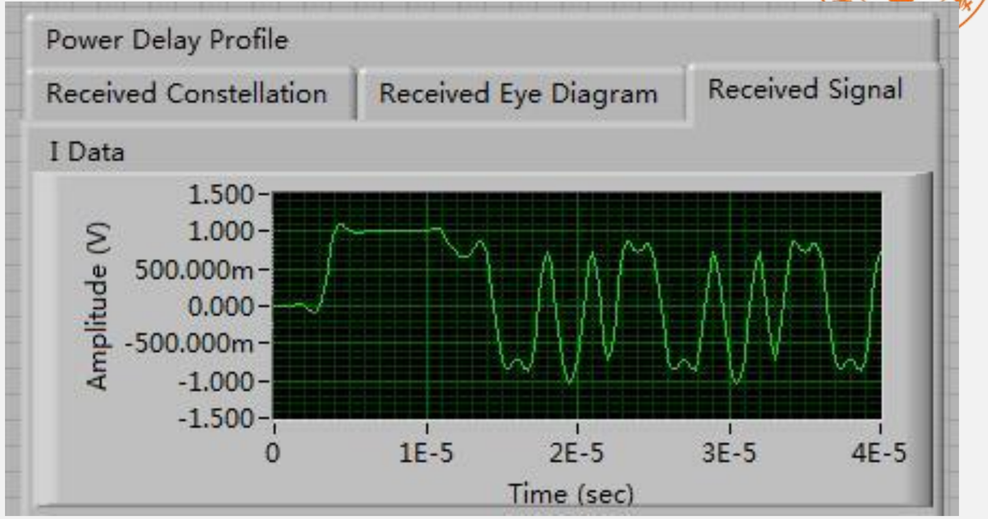

Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
Q
I

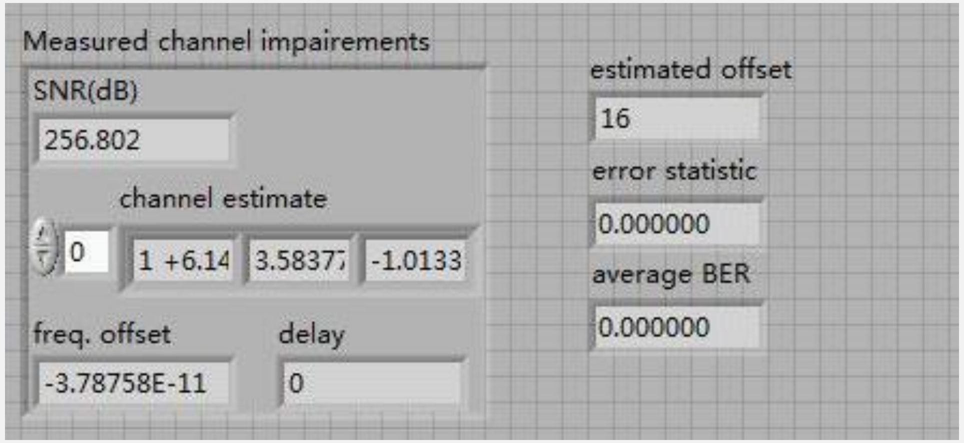

Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
Q
I


Power Delay Profile
Received Constellation
Received Eye Diagram
Received Signal
Q
I

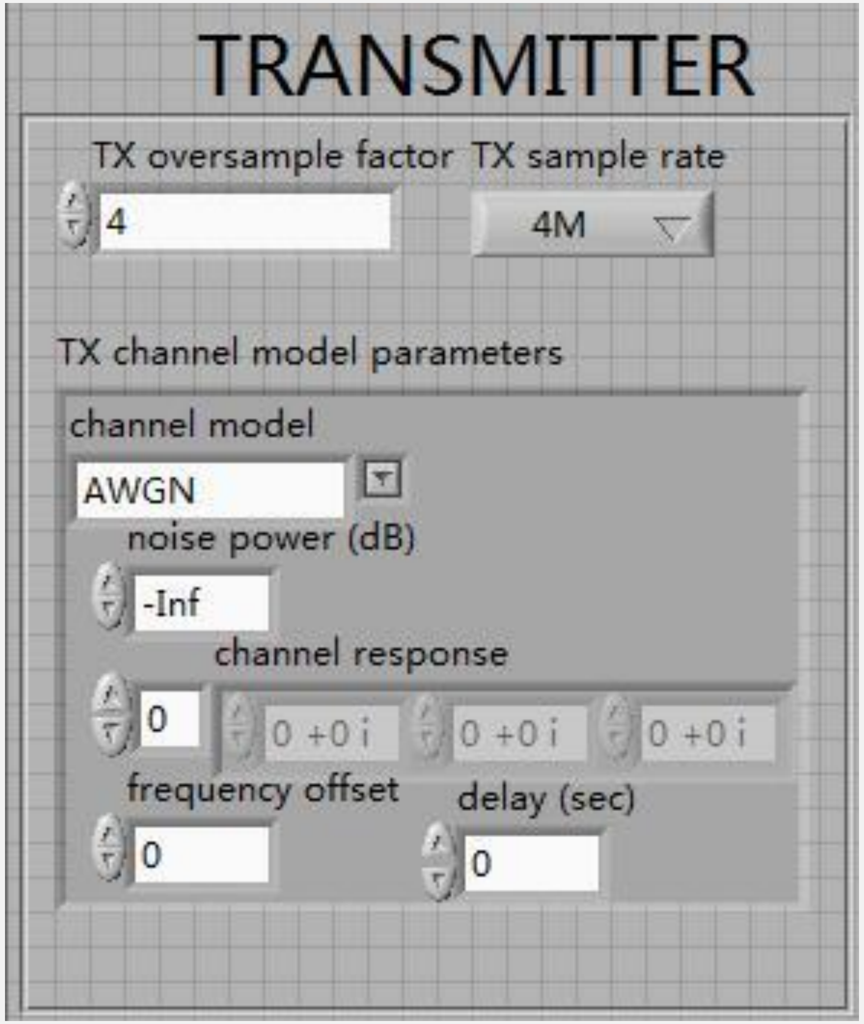
Accuracy Analysis

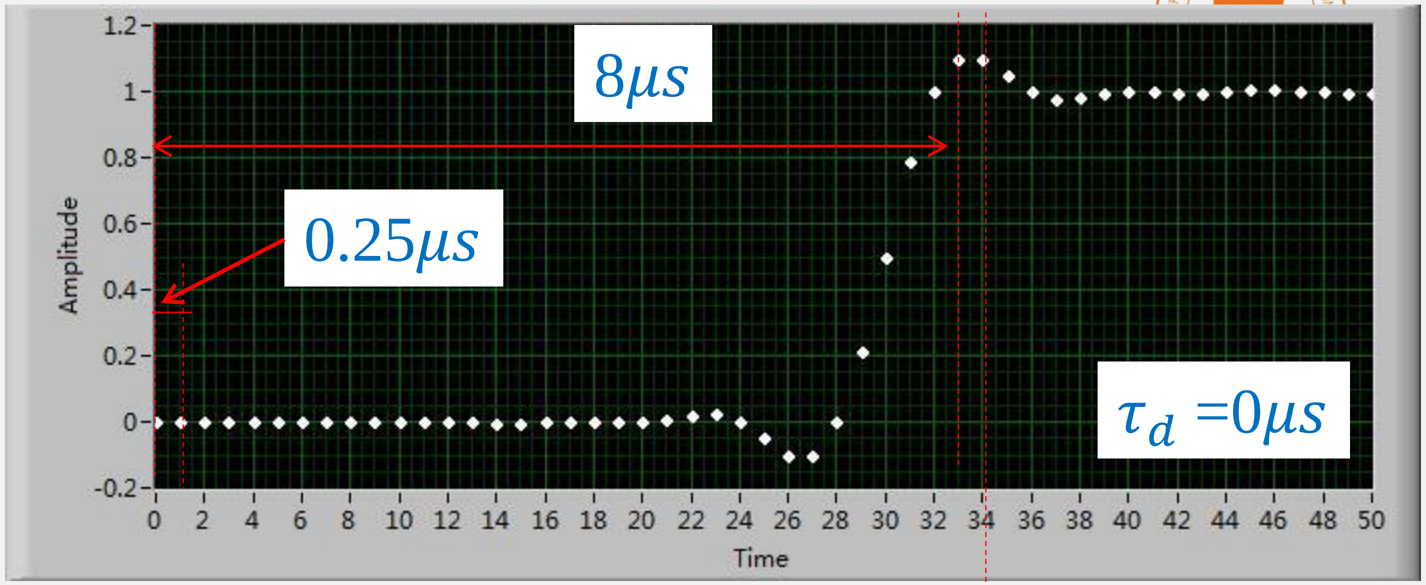


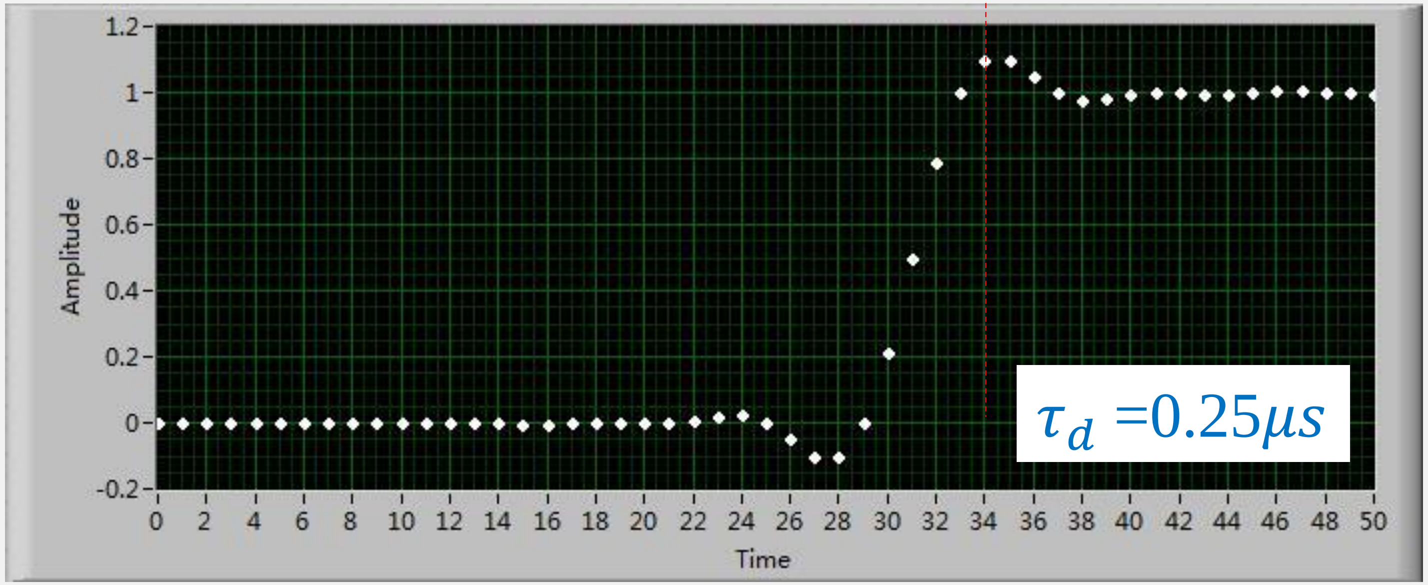


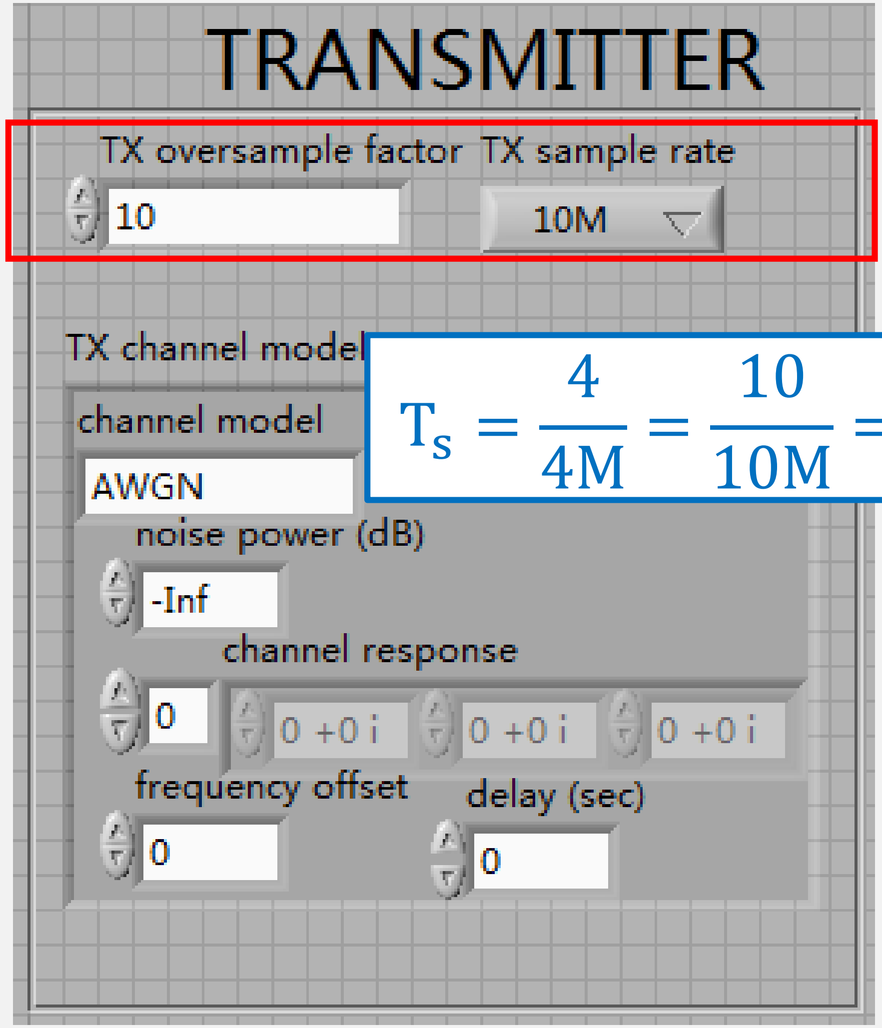


Set the propagation delay

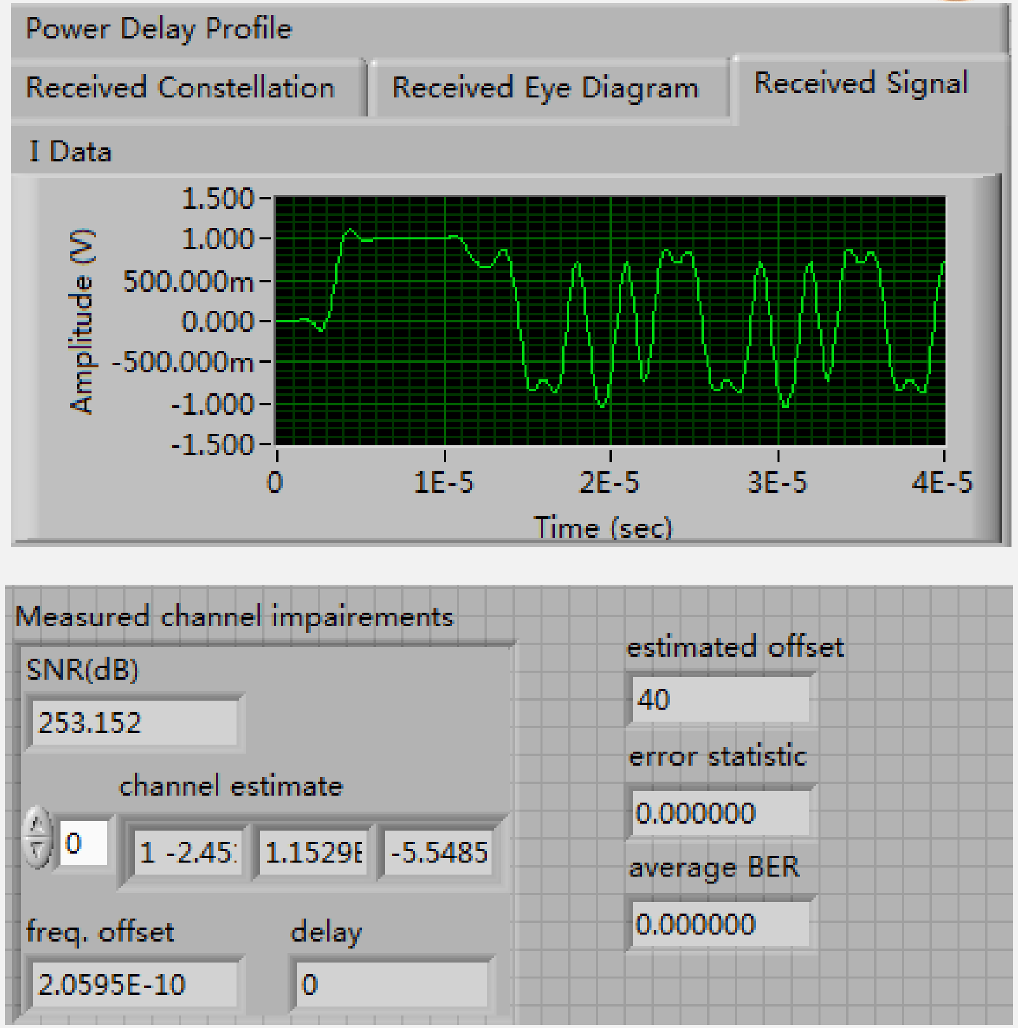

TRANSMITTER
TX oversample factor
4
TX sample rate
4M
TX channel model parameters
channel model
AWGN
noise power (dB)
-Inf
channel response
0
0 +0 i
0 +0 i
0 +0 i
frequency offset
0
delay (sec)
0

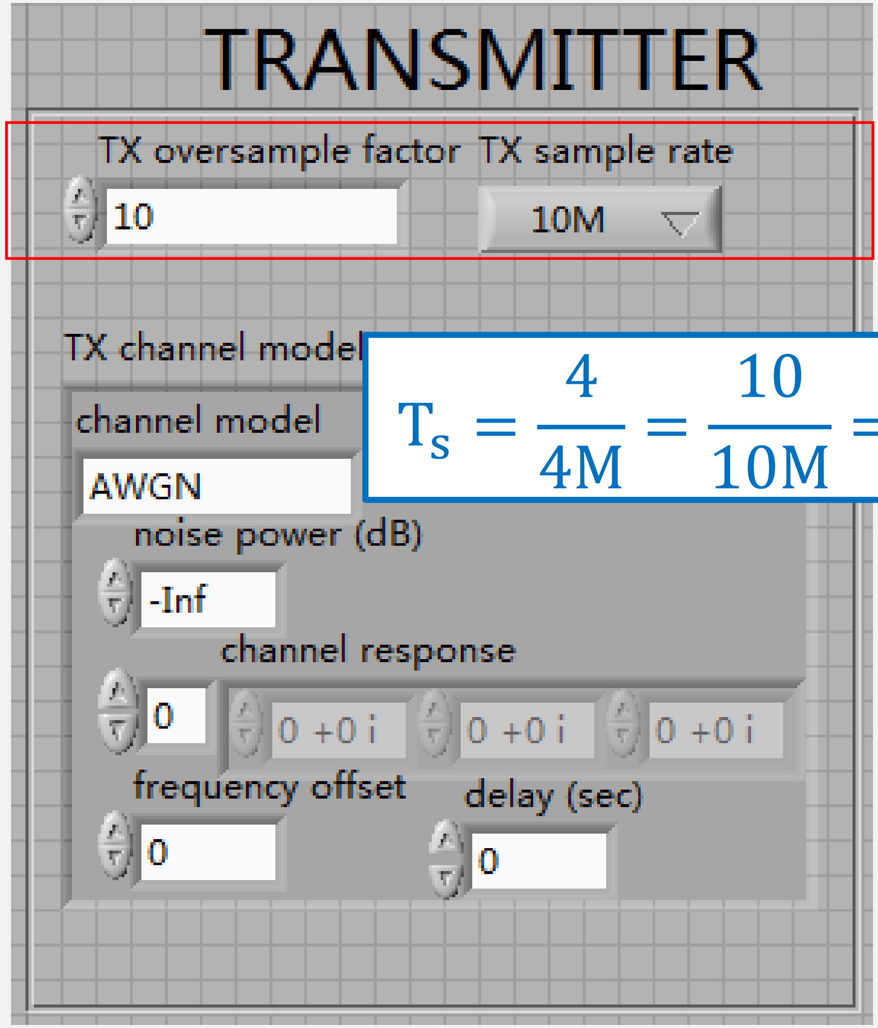

8μs
0.25μs
τd =0μs
Amplitude
Time


τd =0.25μs
Amplitude
Time

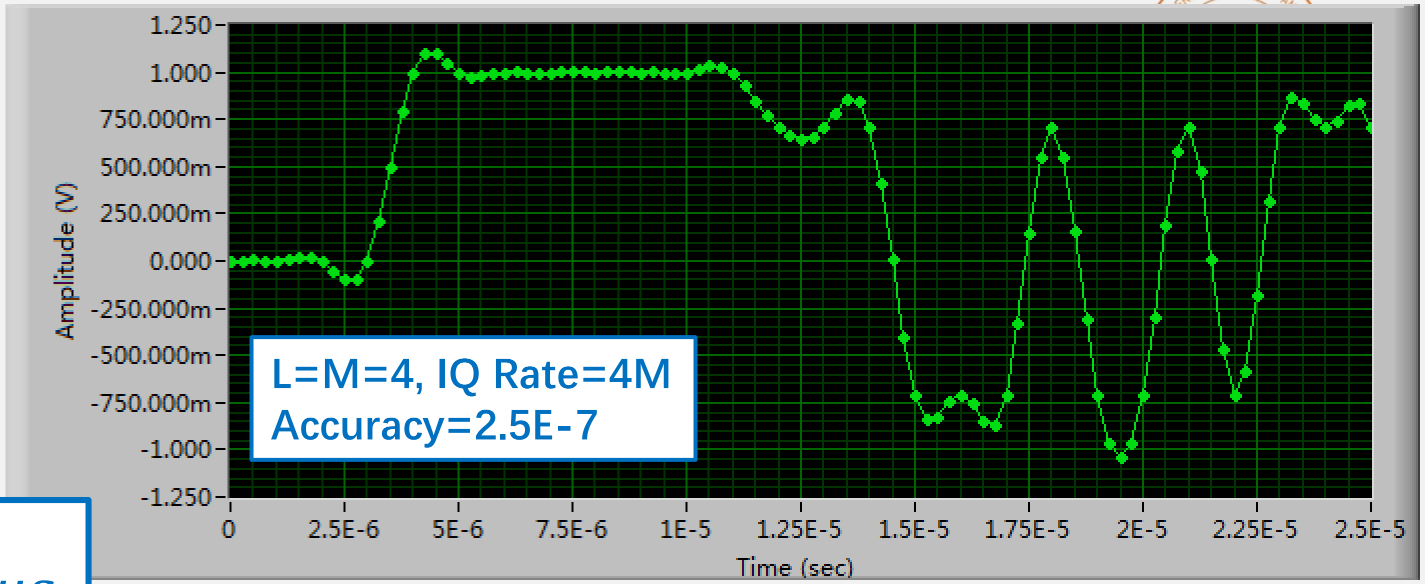


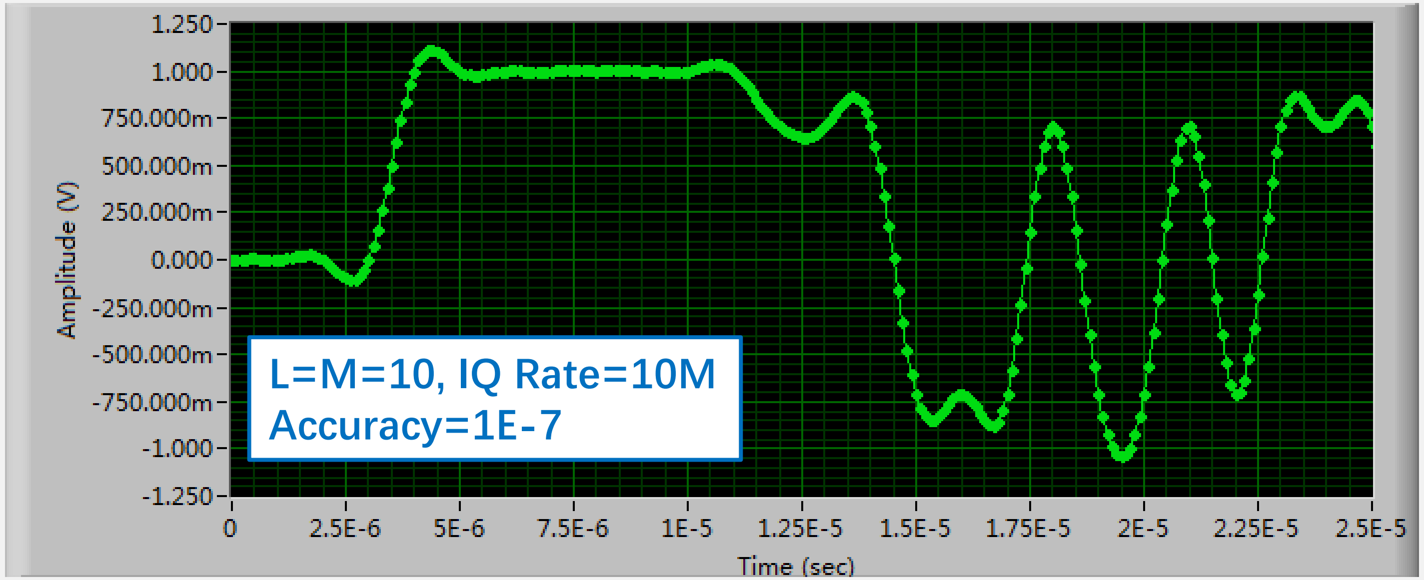
$$T_s = \frac{4}{4M} = \frac{10}{10M} = 1\mu s$$



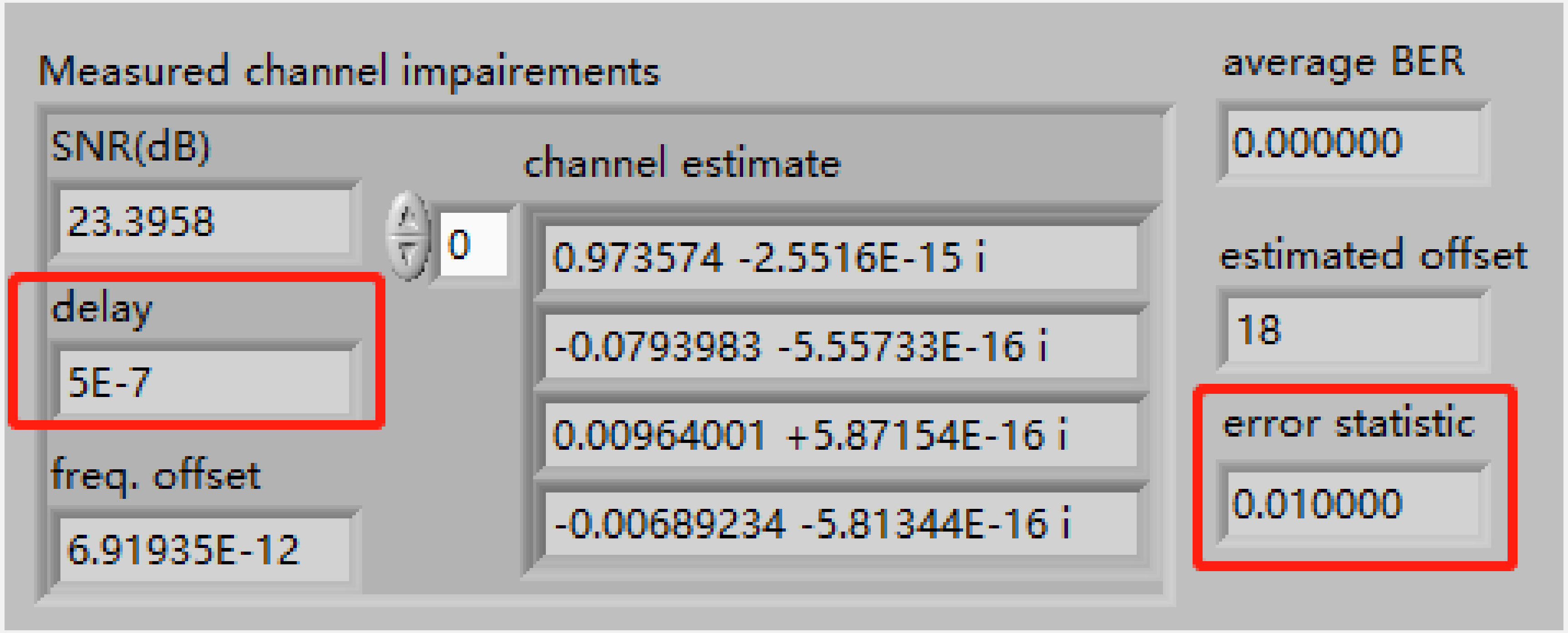


$$T_s = \frac{4}{4M} = \frac{10}{10M} = 1\mu s$$

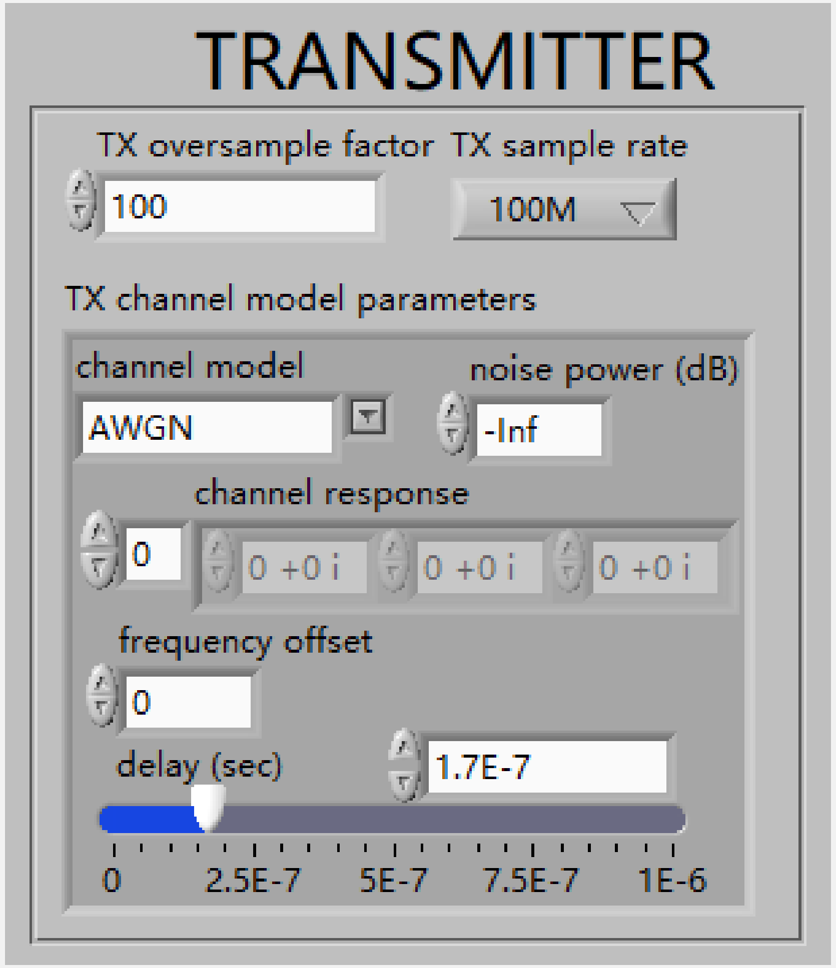


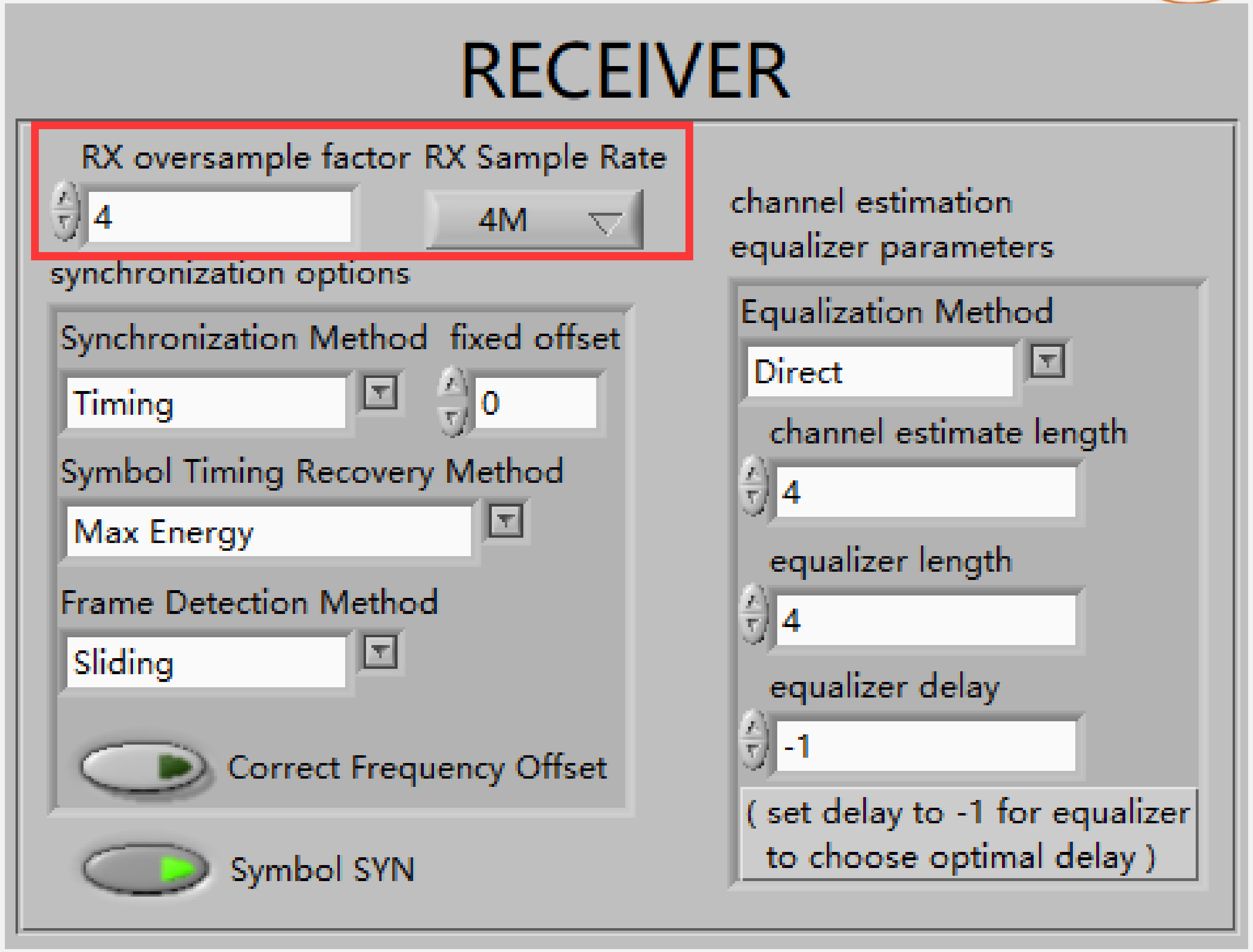


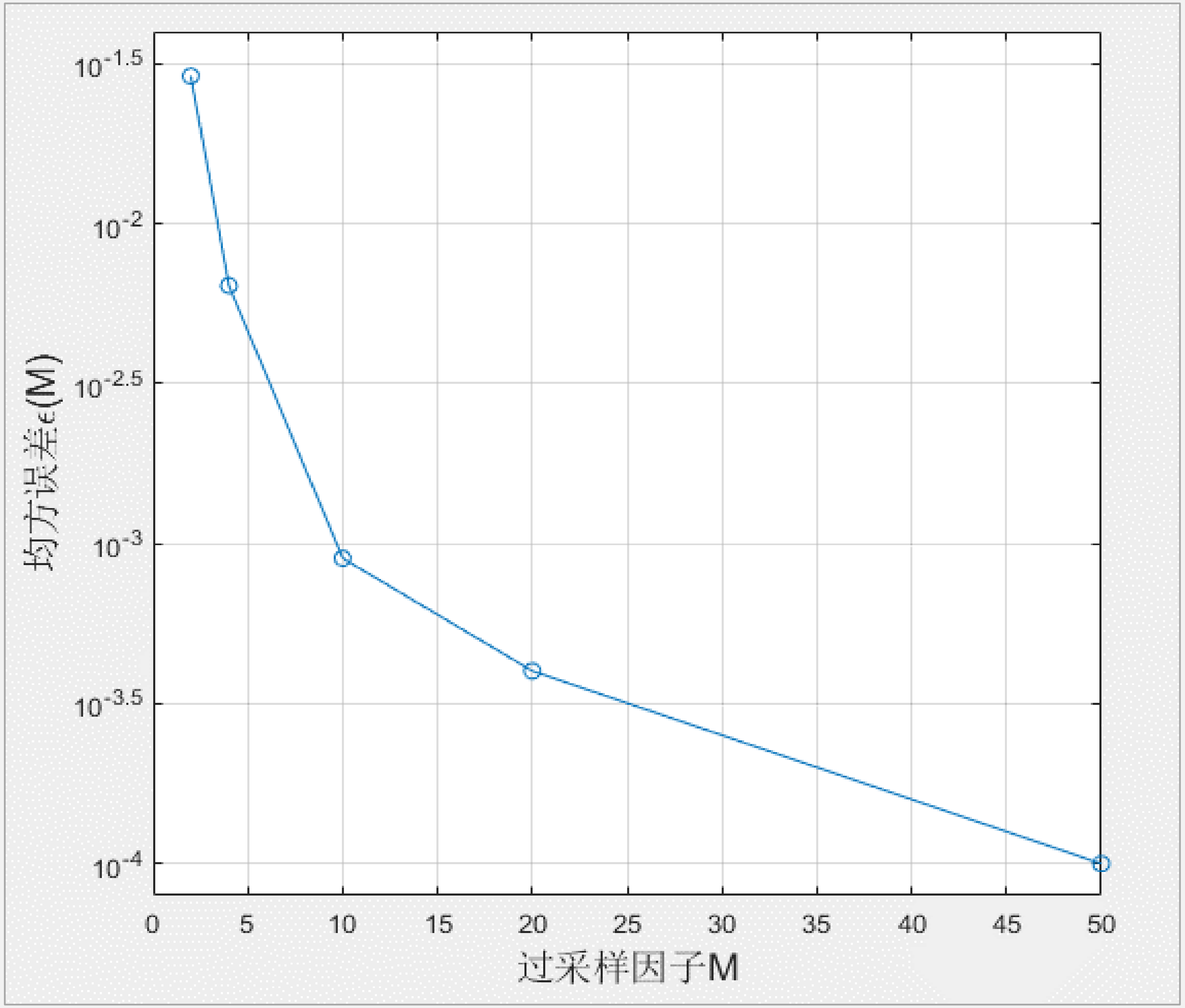
$$\epsilon[M] = \boldsymbol{E}\left\{\left\|\frac{\hat{\tau}(M) - \tau_d}{T_s}\right\|^2\right\}$$


Measured channel impairements
SNR(dB)
23.3958
delay
5E-7
freq. offset
6.91935E-12
channel estimate
0
0.973574 -2.5516E-15 i
-0.0793983 -5.55733E-16 i
0.00964001 +5.87154E-16 i
-0.00689234 -5.81344E-16 i
average BER
0.000000
estimated offset
18
error statistic
0.010000

参数名	参数值
信道延迟τ_d	$0.17\mu s$
符号速率f_s	$1MHz$
发射端过采样因子M_{tx}	100
接收端过采样因子$M(M_{rx})$	$2,4,10,20,50$
传输信道	AWGN
信道信噪比(dB)	$-\infty$
符号定时同步方法	Max Energy


TRANSMITTER
TX oversample factor
TX sample rate
100
100M
TX channel model parameters
channel model
noise power (dB)
AWGN
-Inf
channel response
0
0 +0 i
0 +0 i
0 +0 i
frequency offset
0
delay (sec)
1.7E-7
0
2.5E-7
5E-7
7.5E-7
1E-6


RECEIVER
RX oversample factor
RX Sample Rate
4
4M
synchronization options
Synchronization Method
fixed offset
Timing
0
Symbol Timing Recovery Method
Max Energy
Frame Detection Method
Sliding
Correct Frequency Offset
Symbol SYN
channel estimation
equalizer parameters
Equalization Method
Direct
channel estimate length
4
equalizer length
4
equalizer delay
-1
(set delay to -1 for equalizer to choose optimal delay)

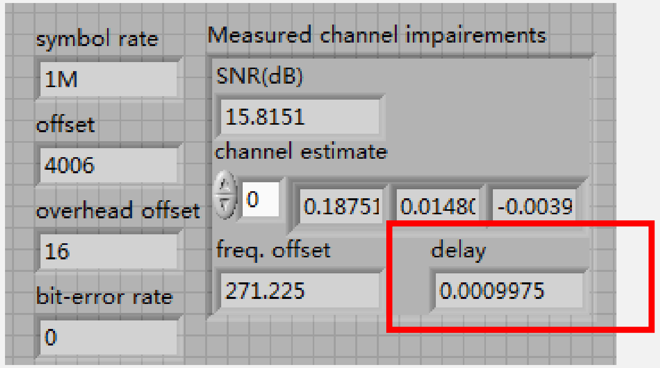


$10^{-1.5}$
10^{-2}
$10^{-2.5}$
10^{-3}
$10^{-3.5}$
10^{-4}
均方误差ε(M)
0
5
10
15
20
25
30
35
40
45
50
过采样因子M

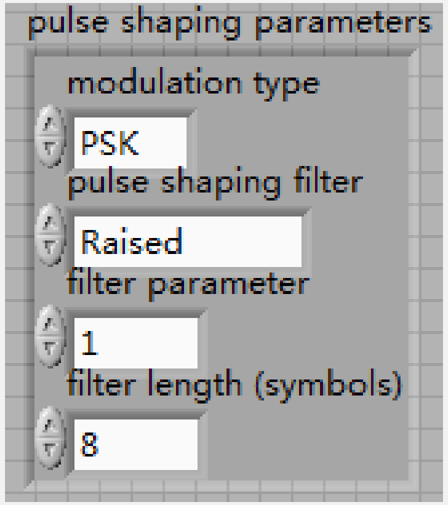
Algorithm Analysis and Verification

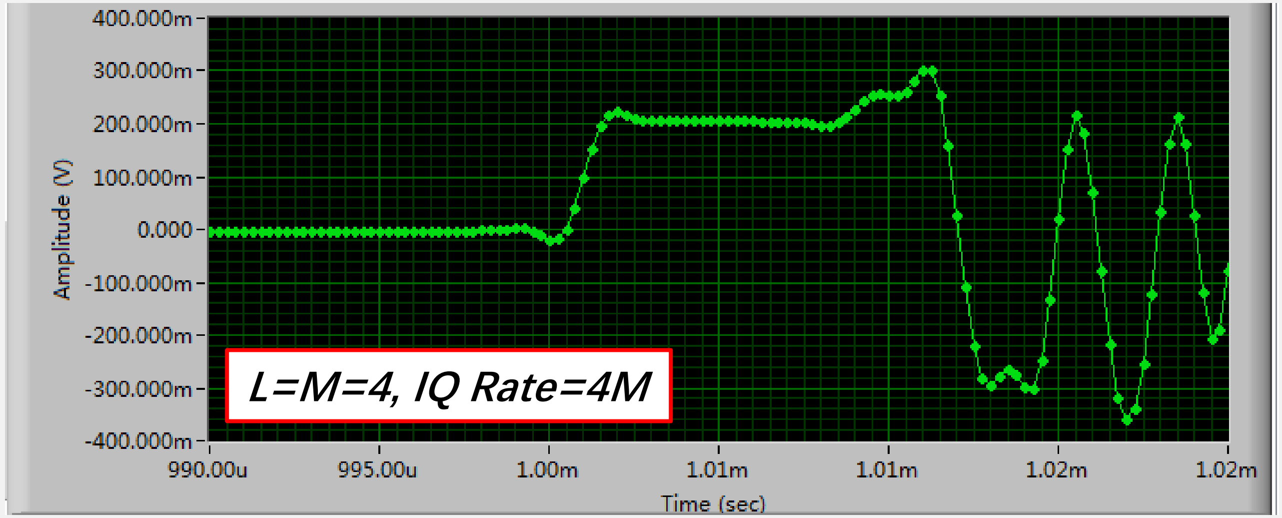
1. Maximum Energy Algorithm Analysis
2. Maximum Energy Algorithm Simulation
3. USRP Experiments
4. Early-Late Gate Algorithm Discussion

USRP Experiment

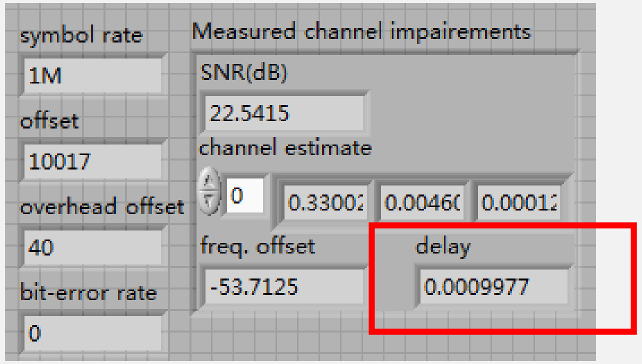


How to calculate the delay here ?

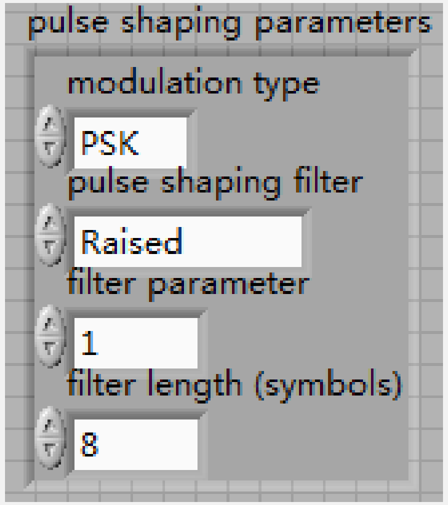


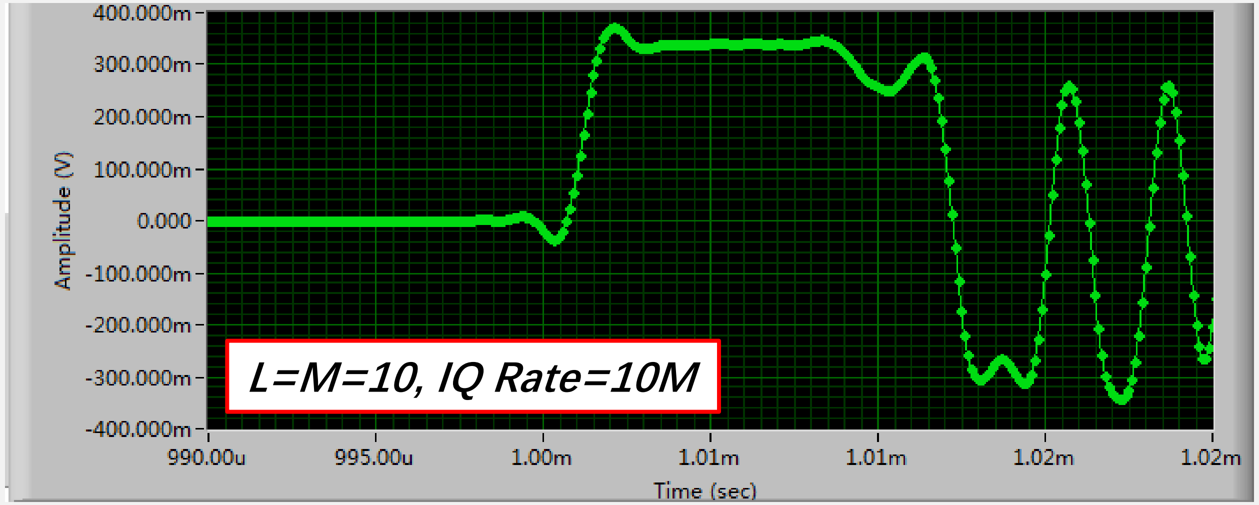


USRP Experiment



How to calculate the delay here ?





Algorithm Analysis and Verification

1. Maximum Energy Algorithm Analysis
2. Maximum Energy Algorithm Simulation
3. USRP Experiments
4. Early-Late Gate Algorithm Discussion

- **Question ?**