**Lab 2：Pulse Shaping and Matched Filtering**

|  |  |
| --- | --- |
| **Author** | Name：宋宜航 张皓东  Student ID:12112717 12113010 |
| **Introduction**  **The pulse shaping**  Pulse shaping is a fundamental process in communication systems. The purpose of this is to transform the 0- and 1-bit streams that need to be transmitted into the form of a waveform by a specific function, thus facilitating the subsequent transmission of signals during the communication process. The above-mentioned process of converting the bit stream into a waveform for transmission is usually achieved through the convolution process. The 0- and 1-bit stream can be regarded as a combination of a set of unit impulse functions, and then certain functions can be used to convolve with it. And the waveform is moved to the position of the corresponding unit impulse function to form a set of waveforms. This process is called as pulse shaping. The process can be represented by the following schematic diagram:    The above-mentioned convolution process can be visualized from the process diagram below, that is, the convolution process of the combination of the pulse shaping function and the unit impulse function.    There are many considerations in pulse forming. In communication systems, data is transmitted in the form of pulses, often in the time domain. To improve signal integrity and minimize interference, it is essential to shape these pulses so that they conform to specific criteria, such as reducing bandwidth, minimizing inter-symbol interference (ISI), and improving signal-to-noise ratio.  Therefore, there are various certain functions to do pulse shaping such as: 1. Raised Cosine Filter: The raised cosine filter is one of the most commonly used pulse shaping techniques. It has a time-domain response resembling a raised cosine function. The filter's characteristics are defined by a parameter, often denoted as "alpha" (α), which controls the roll-off factor and bandwidth. 2. Gaussian Filter: Gaussian pulse shaping employs Gaussian functions as the shaping waveforms. Gaussian pulses are characterized by their bell-shaped response in both time and frequency domains. They offer advantageous characteristics, such as minimal ISI and efficient bandwidth usage. 3. Root Raised Cosine Filter: The root raised cosine filter is used to mitigate ISI and control spectral efficiency. It has a frequency response that resembles a raised cosine filter, which is beneficial for reducing sidelobes in the frequency domain. The root raised cosine function is the main pulse shaping function that this experiment mainly focuses on. Its specific content will be introduced in detail later.  **Nyquist's first criterion**  The Nyquist first criterion, also known as the Nyquist sampling theorem, is a fundamental principle in digital signal processing and communication systems. It outlines the requirements for accurately sampling analog signals and how to prevent distortion during the digitization process. According to the Nyquist first criterion, for precise signal reconstruction, it is essential to ensure that the frequency content of the analog signal is restricted. This means that the highest frequency component of the analog signal must be less than or equal to half of the sampling frequency. The sampling frequency refers to the rate at which samples are taken from the analog signal, typically measured in samples per second (Hertz). According to the Nyquist theorem, the sampling frequency must be at least twice the frequency of the highest component in the signal, known as the Nyquist frequency. When a signal is sampled at a rate equal to or higher than the Nyquist frequency, it can be precisely reconstructed from the discrete samples without loss of information or distortion. The Nyquist First Criterion is widely applied in digital signal processing, communication systems, audio processing, image acquisition, and various fields to ensure effective and accurate digitization of analog signals.  The Nyquist first criterion should also be followed during pulse shaping. Specifically, it is how to design the sample interval in the pulse shaping process to ensure that there is less overlap and interference between the frequencies corresponding to different waveforms. For example, the corresponding relationship in the frequency domain between different waveform combinations of pulse shaping is as follows:    The corresponding relationship between the specific shaping function in the time domain and frequency domain is as follows:    To connect the above two, it is necessary to ensure that the spectrum interval of the two signals is greater than twice the bandwidth so that the spectrum of the two signals has less overlapping interference, which means that the Nyquist first criterion must be obeyed.  **Root raised cosine function**  The Root Raised Cosine function is a widely used waveform in digital communication systems. It plays a crucial role in pulse shaping. The Root Raised Cosine function is a mathematical function that is characterized by its time-domain and frequency-domain properties. The mathematic expression of the root raised cosine function is as follows:    As we all known, the expression of traditional raised cosine function is that:    Therefore, it is a variation of the traditional Raised Cosine function and is used for pulse shaping in digital communication systems. The main features of the Root Raised Cosine function are as follows:  1. The Root Raised Cosine function has a time-domain response that resembles the square root of a Raised Cosine function. This response is designed to minimize inter-symbol interference (ISI) by providing zero crossings at the symbol boundaries, which helps in symbol synchronization.  2. In the frequency domain, the Root Raised Cosine function exhibits desirable spectral characteristics. It has a roll-off factor, often denoted as "α," that controls the shape of the function's frequency response. A smaller α results in a wider bandwidth, while a larger α results in a narrower bandwidth.  **Inter-symbol interference (ISI)**  Inter-Symbol Interference (ISI) is a phenomenon in digital communication systems where symbols (or bits) in a transmitted signal overlap or interfere with adjacent symbols, making it challenging to accurately detect and decode the received data. ISI occurs when the transmitted signal's pulse extends into the time periods assigned for other symbols. This can lead to misinterpretation of the received signal and errors in data recovery.  In the context of pulse shaping, ISI is a critical concern. ISI can degrade the effectiveness of pulse shaping in the following ways: ISI can cause spectral overlap, which may lead to increased bandwidth, reducing the effectiveness of bandwidth-efficient modulation schemes. ISI can introduce distortion into the received signal, affecting the quality of data transmission. Pulse shaping is designed to reduce ISI by ensuring that the signal pulses do not interfere with adjacent symbols. High levels of ISI can result in increased error rates during symbol detection. This can be particularly problematic in high-speed communication systems, where accurate symbol recovery is essential.  To mitigate ISI and optimize the performance of pulse shaping, various techniques are employed. One common approach is to use pulse shaping waveforms, such as the Root Raised Cosine function as mentioned above, which minimizes ISI by providing zero crossings at symbol boundaries. Additionally, equalization techniques, such as linear or decision feedback equalizers, can be used at the receiver to compensate for ISI and improve signal recovery. It is worth noting that ISI is inevitable because the bandwidth of the signal is always limited, so the time span of the signal on the corresponding time domain is infinite, so there must be interference between the signals. This is a contradiction.  **Matched filter**  Marched filter is used to receive the signal, the overall procedure including the pulse shaping and matched filtering is that:    Therefore, a pulse shaping function corresponds to a matched filter function. The purpose of matched filtering is to make the SINR of the received signal as large as possible. Through the following theoretical derivation, it can be found that the optimal receiver function should be the pulse shaping function. of conjugate symmetry. The SINR can be expressed as:    The g(0) can be expressed as:    So |g(0)| is that:    Therefore, we can conclude that the optimal matched filter is that:    As mentioned earlier, different pulse shaping functions correspond to different matched filter functions. For example, if the pulse shaping function is the Raised-cosine function, then the matched filter function is unit impulse function. The schematic diagram is as follows:    And if the pulse shaping function is the Root Raised-cosine function, then the matched filter function is also the Root Raised-cosine function. The schematic diagram is as follows:    **The pulse shaping**  Pulse shaping is a fundamental process in communication systems. The purpose of this is to transform the 0- and 1-bit streams that need to be transmitted into the form of a waveform by a specific function, thus facilitating the subsequent transmission of signals during the communication process. The above-mentioned process of converting the bit stream into a waveform for transmission is usually achieved through the convolution process. The 0- and 1-bit stream can be regarded as a combination of a set of unit impulse functions, and then certain functions can be used to convolve with it. And the waveform is moved to the position of the corresponding unit impulse function to form a set of waveforms. This process is called as pulse shaping. The process can be represented by the following schematic diagram:    The above-mentioned convolution process can be visualized from the process diagram below, that is, the convolution process of the combination of the pulse shaping function and the unit impulse function.    There are many considerations in pulse forming. In communication systems, data is transmitted in the form of pulses, often in the time domain. To improve signal integrity and minimize interference, it is essential to shape these pulses so that they conform to specific criteria, such as reducing bandwidth, minimizing inter-symbol interference (ISI), and improving signal-to-noise ratio.  Therefore, there are various certain functions to do pulse shaping such as: 1. Raised Cosine Filter: The raised cosine filter is one of the most commonly used pulse shaping techniques. It has a time-domain response resembling a raised cosine function. The filter's characteristics are defined by a parameter, often denoted as "alpha" (α), which controls the roll-off factor and bandwidth. 2. Gaussian Filter: Gaussian pulse shaping employs Gaussian functions as the shaping waveforms. Gaussian pulses are characterized by their bell-shaped response in both time and frequency domains. They offer advantageous characteristics, such as minimal ISI and efficient bandwidth usage. 3. Root Raised Cosine Filter: The root raised cosine filter is used to mitigate ISI and control spectral efficiency. It has a frequency response that resembles a raised cosine filter, which is beneficial for reducing sidelobes in the frequency domain. The root raised cosine function is the main pulse shaping function that this experiment mainly focuses on. Its specific content will be introduced in detail later.  **Nyquist's first criterion**  The Nyquist first criterion, also known as the Nyquist sampling theorem, is a fundamental principle in digital signal processing and communication systems. It outlines the requirements for accurately sampling analog signals and how to prevent distortion during the digitization process. According to the Nyquist first criterion, for precise signal reconstruction, it is essential to ensure that the frequency content of the analog signal is restricted. This means that the highest frequency component of the analog signal must be less than or equal to half of the sampling frequency. The sampling frequency refers to the rate at which samples are taken from the analog signal, typically measured in samples per second (Hertz). According to the Nyquist theorem, the sampling frequency must be at least twice the frequency of the highest component in the signal, known as the Nyquist frequency. When a signal is sampled at a rate equal to or higher than the Nyquist frequency, it can be precisely reconstructed from the discrete samples without loss of information or distortion. The Nyquist First Criterion is widely applied in digital signal processing, communication systems, audio processing, image acquisition, and various fields to ensure effective and accurate digitization of analog signals.  The Nyquist first criterion should also be followed during pulse shaping. Specifically, it is how to design the interval in the up-sampling process to ensure that there is less overlap and interference between the frequencies corresponding to different waveforms. For example, the corresponding relationship in the frequency domain between different waveform combinations of pulse shaping is as follows:    The corresponding relationship between the specific shaping function in the time domain and frequency domain is as follows:    To connect the above two, it is necessary to ensure that the spectrum interval of the two signals is greater than twice the bandwidth so that the spectrum of the two signals has less overlapping interference, which means that the Nyquist first criterion must be obeyed.  **Root raised cosine function**  The Root Raised Cosine function is a widely used waveform in digital communication systems. It plays a crucial role in pulse shaping. The Root Raised Cosine function is a mathematical function that is characterized by its time-domain and frequency-domain properties. The mathematic expression of the root raised cosine function is as follows:    As we all known, the expression of traditional raised cosine function is that:    Therefore, it is a variation of the traditional Raised Cosine function and is used for pulse shaping in digital communication systems. The main features of the Root Raised Cosine function are as follows:  1. The Root Raised Cosine function has a time-domain response that resembles the square root of a Raised Cosine function. This response is designed to minimize inter-symbol interference (ISI) by providing zero crossings at the symbol boundaries, which helps in symbol synchronization.  2. In the frequency domain, the Root Raised Cosine function exhibits desirable spectral characteristics. It has a roll-off factor, often denoted as "α," that controls the shape of the function's frequency response. A smaller α results in a wider bandwidth, while a larger α results in a narrower bandwidth.  **Inter-symbol interference (ISI)**  Inter-Symbol Interference (ISI) is a phenomenon in digital communication systems where symbols (or bits) in a transmitted signal overlap or interfere with adjacent symbols, making it challenging to accurately detect and decode the received data. ISI occurs when the transmitted signal's pulse extends into the time periods assigned for other symbols. This can lead to misinterpretation of the received signal and errors in data recovery.  In the context of pulse shaping, ISI is a critical concern. ISI can degrade the effectiveness of pulse shaping in the following ways: ISI can cause spectral overlap, which may lead to increased bandwidth, reducing the effectiveness of bandwidth-efficient modulation schemes. ISI can introduce distortion into the received signal, affecting the quality of data transmission. Pulse shaping is designed to reduce ISI by ensuring that the signal pulses do not interfere with adjacent symbols. High levels of ISI can result in increased error rates during symbol detection. This can be particularly problematic in high-speed communication systems, where accurate symbol recovery is essential.  To mitigate ISI and optimize the performance of pulse shaping, various techniques are employed. One common approach is to use pulse shaping waveforms, such as the Root Raised Cosine function as mentioned above, which minimizes ISI by providing zero crossings at symbol boundaries. Additionally, equalization techniques, such as linear or decision feedback equalizers, can be used at the receiver to compensate for ISI and improve signal recovery. It is worth noting that ISI is inevitable because the bandwidth of the signal is always limited, so the time span of the signal on the corresponding time domain is infinite, so there must be interference between the signals. This is a contradiction.  **Matched filter**  Marched filter is used to receive the signal, the overall procedure including the pulse shaping and matched filtering is that:    Therefore, a pulse shaping function corresponds to a matched filter function. The purpose of matched filtering is to make the SINR of the received signal as large as possible. Through the following theoretical derivation, it can be found that the optimal receiver function should be the pulse shaping function. of conjugate symmetry. The SINR can be expressed as:    The g(0) can be expressed as:    So |g(0)| is that:    Therefore, we can conclude that the optimal matched filter is that:    As mentioned earlier, different pulse shaping functions correspond to different matched filter functions. For example, if the pulse shaping function is the Raised-cosine function, then the matched filter function is unit impulse function. The schematic diagram is as follows:    And if the pulse shaping function is the Root Raised-cosine function, then the matched filter function is also the Root Raised-cosine function. The schematic diagram is as follows:    **Lab results & Analysis**：  **2.1 Pulse Shaping and Matched Filtering**    Fig2.1.1 Pulse Shaping block diagram  **Fig 2.1.2 Matched Filtering block diagram**  **2.1.2 Program Process**  **Pulse Shaping:**  We mainly use three blocks in LabView. They are MT Generate Filter Coefficient, Upsample and Convolution. The basic idea is that we use the input parameter to generate the corresponding pulse shaping and then after we upsample the input signal, we convolution the input signal and the pulse, then we can get the final result. Also, the reason to use upsampling block is that the symbol rate may be less than the sample rate required for pulse shapes with excess bandwidth.  To prove the above process, we will use mathematical expressions and equations below:  Let Tx be some sampling period such that 1/Tx is greater than twice the maximum frequency of gtx(t). For simplicity we take Tx = T/L. Other choices of Tx would require a resampling operation. The continuous-time complex baseband signal is:  Snipaste_2022-10-16_18-02-30  Since x(t) is band limited by virtue of the bandlimited pulse shape gtx(t), there exists a sequence {c[n]} such that:  Snipaste_2022-10-16_18-06-04  From these two equations, we can get that:  Snipaste_2022-10-16_18-07-21  And this equation is just what we get after we up-sample the input signal and then convolution with the pulse.  **Matched Filtering:**  This block consists of convolution and also MT generate filter coefficient. As can be seen in the block diagram, we just need to unbundle the waveform signal, and then convolution it with the existing matched filtering array.  The mathematical is shown below: Let z(t) denote the complex base band input to the continuous-to-discrete converter. Assume that z(t) has already been band limited by the RF in the analog front end. Let Tz = T/M for some integer M such that 1/Tz is greater than the Nyquist rate of the signal. This is known as oversampling. The result is shown below:  Snipaste_2022-10-16_18-20-49  That is what exactly using the input signal to do a convolution with the generated matched filtering parameter.  **2.1.2 Simulation Result**  **Pulse Shaping:**  Snipaste_2022-10-16_18-24-04  Fig 2.1.1 block diagram in transmitter  Snipaste_2022-10-16_18-25-25  Fig2.1.2 simulation result of constellation graph  Snipaste_2022-10-16_18-26-25  Fig 2.1.3 simulation result in eye diagram  From the first transmitter diagram, we can see that the pulse shaping block diagram’s icon is different from others, indicating that I use my own block. The next two simulation results show that the bit error rate=0, and the eye diagram is good.  **Matched Filtering:**  Snipaste_2022-10-16_18-30-49  Fig 2.1.4 block diagram in receiver  Snipaste_2022-10-16_18-25-25  Fig2.1.5 simulation result of constellation graph  Snipaste_2022-10-16_18-26-25  Fig2.1.5 simulation result of eye diagram  Also, we can see that in the receiver’s block diagram, I have substitute the original block with my own. And the simulation results shown in the front panel shows that the program is correct.  **2.1 Variation curve of SNR with receiver raised cosine roll-off factor when transmitter raised cosine roll-off factor is constant**    Fig2.1.5 SNR to Rx’s  Analysis:  As can be seen from the image, when the roll-off factor of the transmitter is determined, the SNR of the receiver changes with the change of the roll-off factor of the receiver. And when receiver’s is closer to transmitter , SNR is maximum. Theoretically, SNR reaches the maximum value when the receiver and transmitter roll-off factors are the same.  According to the theoretical analysis of 1.2 in the theoretical introduction, when the root raised cosine pulse shaping function is used, the best matched filter should be the same as the root raised cosine function. Therefore, the transmitter and receiver should use the same roll-off factor to achieve the maximum SNR. The relationship between SNR and a measured in practice is consistent with the theoretical analysis.  At the same time, it can be observed that when the transmitter is larger, the maximum SNR it can achieve is also larger.  **2.3 USRP Constellation and Bandwidth in Different Raised Cosine Roll-off Factor**  To run the program on USRP, we will use top\_tx.vi and top\_rx.vi two blocks. The files top tx.vi and top rx.vi are the top level of the transmitter and receiver respectively. This level connects the digital communications blocks of the transmitter and receiver in transmitter.vi and receiver.vi with the VIs needed to control the NI-USRP. Just as the picture depicts.  Snipaste_2022-10-16_18-38-47  Fig2.3.1 USRP hierarchy diagram  In this lab, we change the value of alpha to 0, 0.5 and 1. And see the eye diagram, the constellation diagram and the frequency diagram. To measure the frequency component of the I/Q signal, we design the blocks below:  程序框图  Fig 2.3.2 frequency component measurement block diagram  **Here are the results:**  When alpha=0;    Fig 2.3.3 constellation diagram when alpha=0    Fig 2.3.4 eye diagram when alpha=0  F3_1  Fig 2.3.5 received signal when alpha=0  f0  Fig 2.3.6 frequency component when alpha=0  From the received signal and constellation front panel, we can see that the bit error rate is 0 while the constellation has a tend to diverge. And we can observer that the bandwidth is nearly 500000 when alpha=0.  When alpha=0.5    Fig 2.3.7 constellation diagram when alpha=0.5    Fig 2.3.8 eye diagram when alpha=0.5    Fig 2.3.9 received signal when alpha=0.5  f0.5  Fig 2.3.10 frequency component when alpha=0.5  From the received signal and constellation front panel, we can see that the bit error rate is 0 while the constellation is more converged than alpha=0. And we can observer that the bandwidth is nearly 700000 when alpha=0.5.  When alpha =1    Fig 2.3.11 constellation diagram when alpha=1    Fig 2.3.12 eye diagram when alpha=1  F3_2  Fig 2.3.13 received signal when alpha=1  f1dk  Fig 2.3.14 frequency component when alpha=1  From the received signal and constellation front panel, we can see that the bit error rate is 0 while the constellation is the most converged. And we can observer that the bandwidth is nearly 900000 when alpha=1.  In conclusion, the above experiment shows that if we increases the value of alpha, the performance of the constellation graph will be better, because the points are stay closer. However, if we look at the frequency component, we know that the power of the noise is usually a constant, so we just need to find a point from which the curve becomes flat. Finally, we find that the bandwidth is 50000,70000 and 90000 when alpha =0,0.5 and 1 respectively. These results can be perfectly explained by the theorem we mentioned before. When the alpha increases, the performance is better but the more bandwidth resources are occupied. This is consistent with the trade-off between high performance and the resources. | |
| **Experience**   1. **Screenshot on course**   Pulse shaping simulation results, alpha = 0.5, constellation diagram and eye diagram  Alpha=0.5       1. **Problems we meet and the experience**   In this experiment we learned how to use pulse shaping functions and matched filters. Not only from the theoretical derivation to understand how to achieve the best receiver, but also using the root rise cosine function, programming verification on LabVIEW. The influence of roll off factor on ISI and SNR is also explored. Understand that SNR maximizes when roll factor of receiver and transmitter is the same.  I am not familiar with many vi in LabVIEW simulation, and I am not familiar with input and output parameters, which makes it easy for me to use them incorrectly.  However, in the hardware experiment, we meet some trouble. As the directivity of the antenna, we had to find an angle of them to receive the signal. We also tried to increase the gain. However, once we did that, the distortion and the noise will also be amplified.  3**. Respective contributions**  We two finish the whole LabVIEW program together. Song Yihang complete procedure design including pulse shaping and matched filter simulation, SINR graph usrp realization and performance analysis. The introduction of basic principle in pulse shaping and matched filter were completed, and the conception of Nyquist principle, Root Raised Cosine function and ISI were elaborated by Zhang Haodong. | |
| **Score** | 98 |