Introduction to risk assessment and communication - probability

Ullrika Sahlin, Lund University

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It will rain tomorrow

The capital of Liberia is Monrovia

You will get a six next time you toss a dice a meter is a measure of a distance

a second is a measure of time

What is probability a measure of?

It will rain tomorrow

The capital of Liberia is Monrovia

You will get a six next time you toss a dice

The probability of an event is a measure of how likely it is to occur

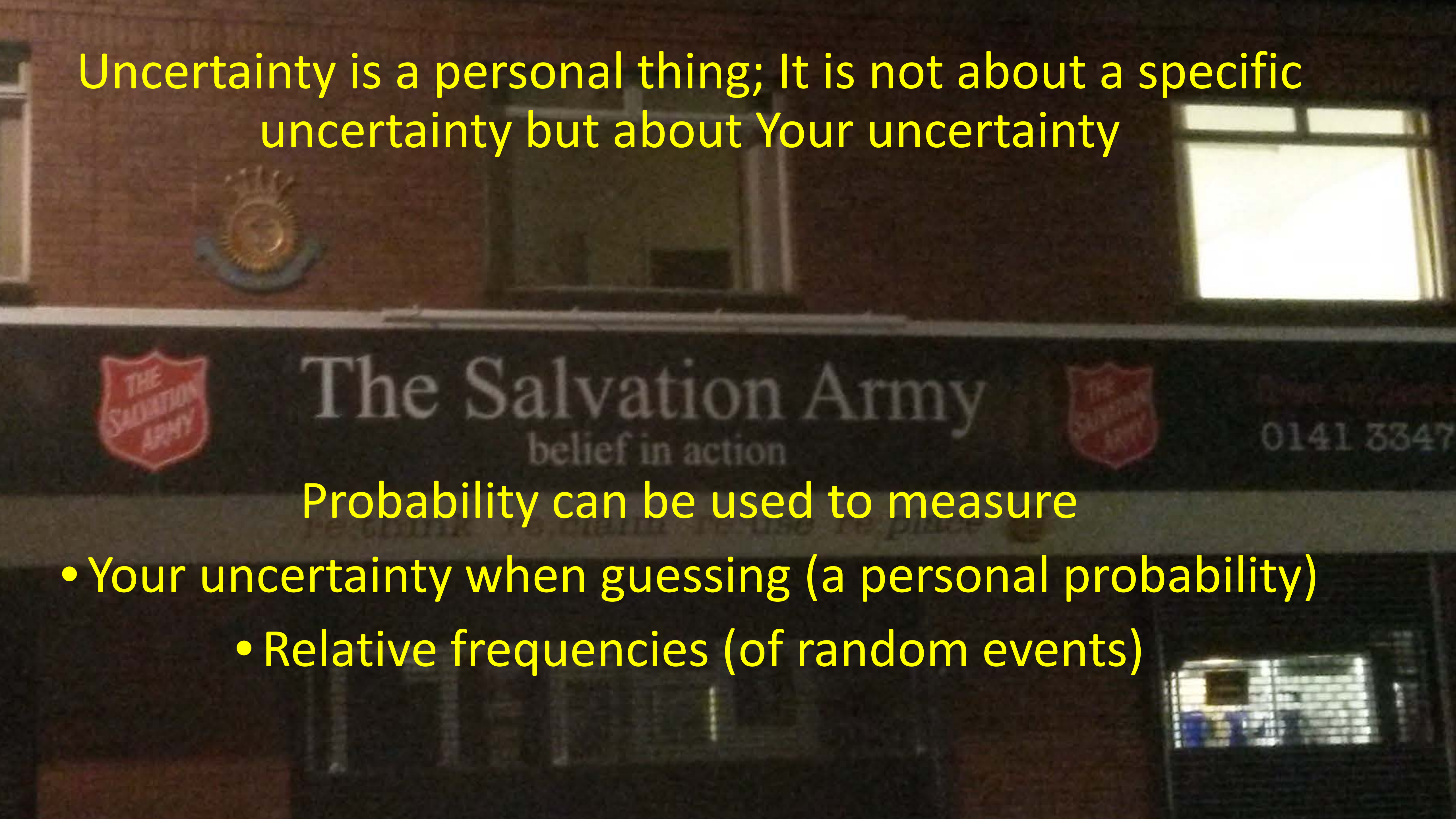
Probability 0 means that the event is certain not to occur

Probability 1 means that it is certain to occur

Uncertainty is a personal thing; It is not about a specific uncertainty but about Your uncertainty

Probability can be used to measure

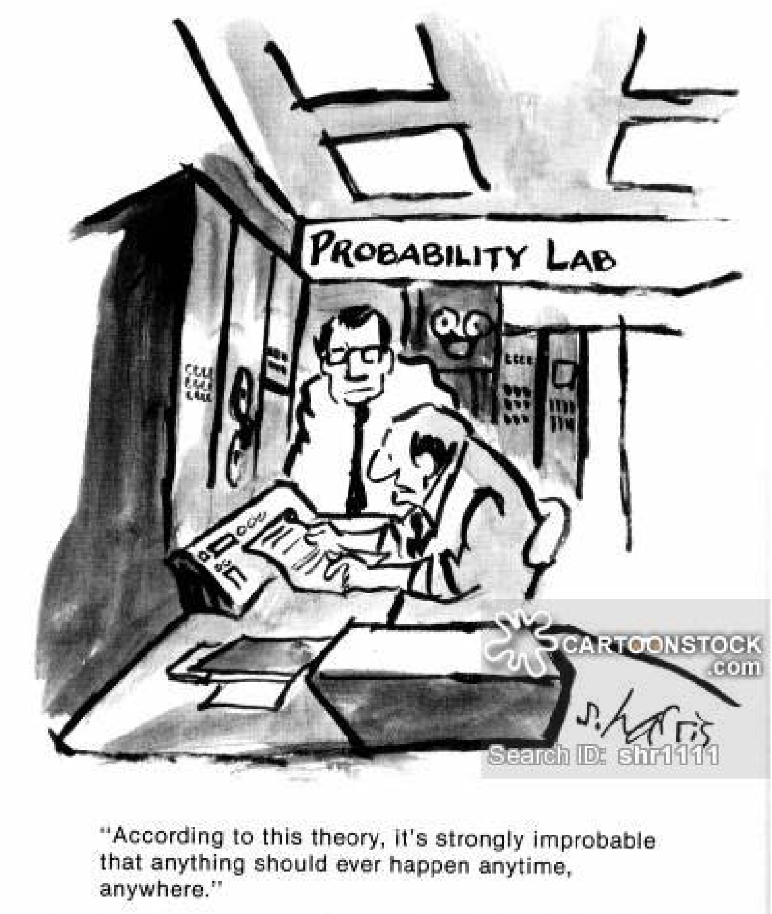
- Your uncertainty when guessing (a personal probability)
- Relative frequencies (of random events)



Probability as a tool to handle variation

- Probability distributions – from uncertainty about single events to uncertain quantities (random variables)

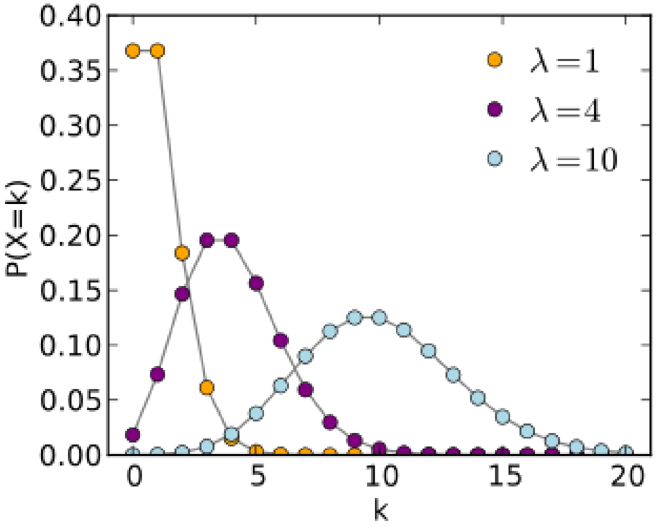
- Discrete distributions
- Continous distributions

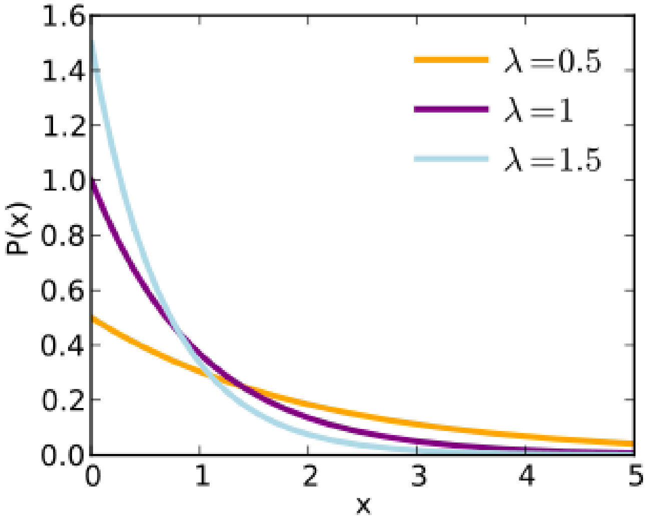


Poisson distribution (Discrete)

Exponential distribution (Continous)

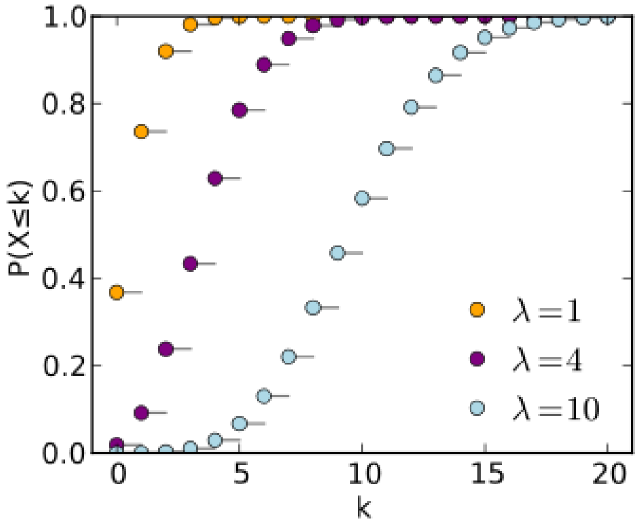
Probability mass function

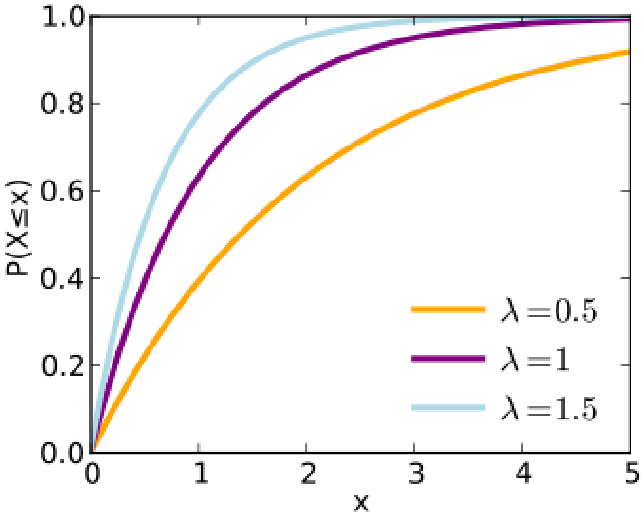




Probability density function

Cumulative distribution function





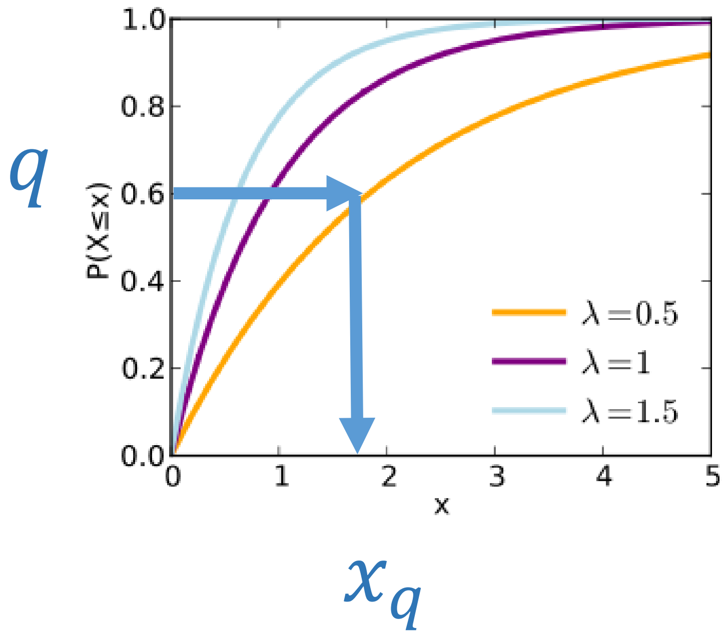
Cumulative distribution function

Expectation

- For any distribution of an unknown quantity, the result of taking the probability of any value of the quantity, multiplying it by the value, and adding all the products, is called the expectation of the uncertain quantity
- When the expectation is describing a location it is sometimes referred to as the mean
- When the quantity is important for decisions its expectation is sometimes called prevision – which do not refeer to any underlying probability distribution
- Many decision rules only care about previsions, e.g. to Maximize Expected Utility

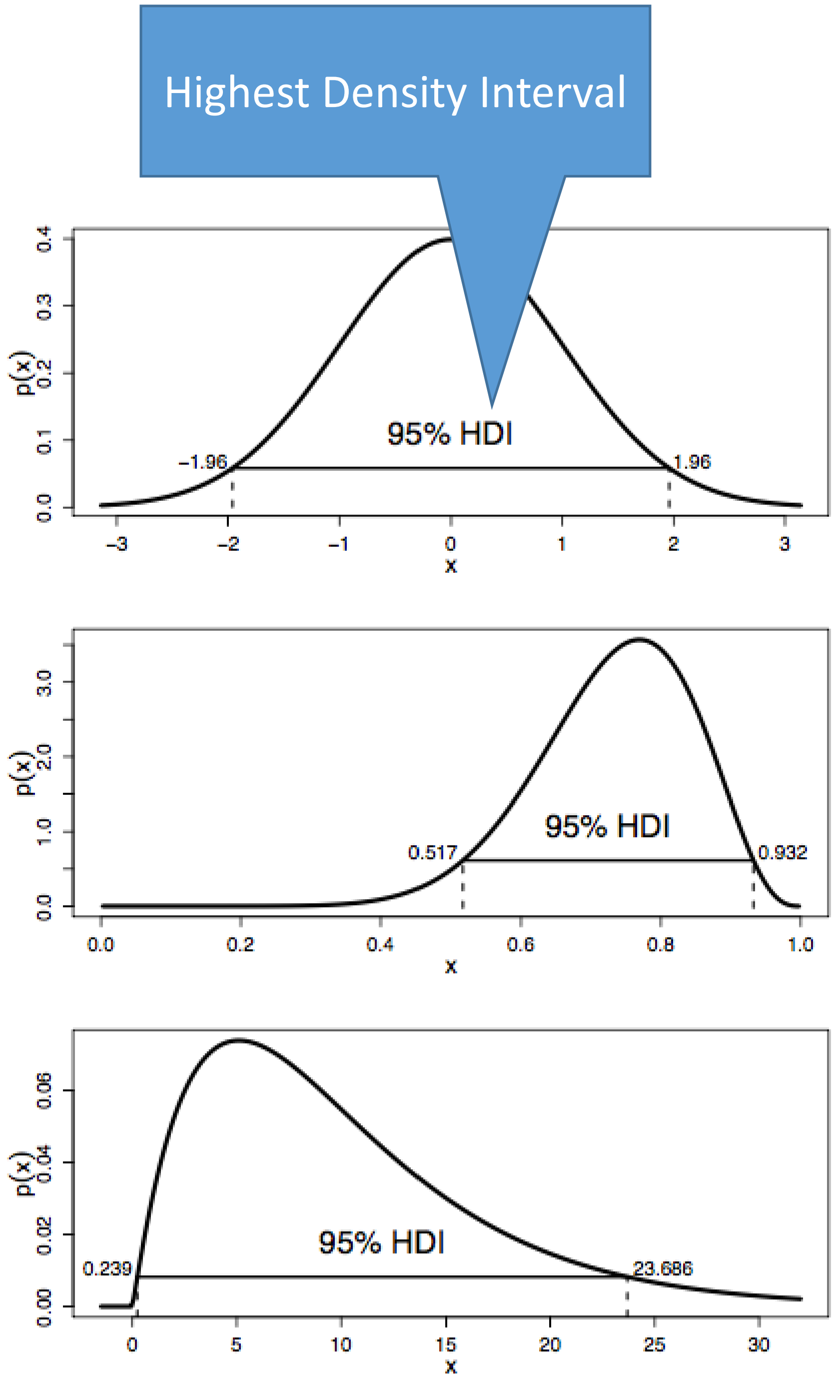
Summaries of distributions

- Probabilities: P(X = 3) or P(X > 5)
- Quantiles: the quantity x_q $such\ that$ $P(X \leq x_q) = q$
- Intervals
- Location measures
- Measures of scale or dispersion
- Measures of shape
- Measures of correlation



Summaries of distributions

- Probabilities
- Quantiles
- Intervals: a 100s% probability interval for X contains the true value of X with probabity s
- Location measures
- Measures of scale or dispersion
- Measures of shape
- Measures of correlation



Summaries of distributions

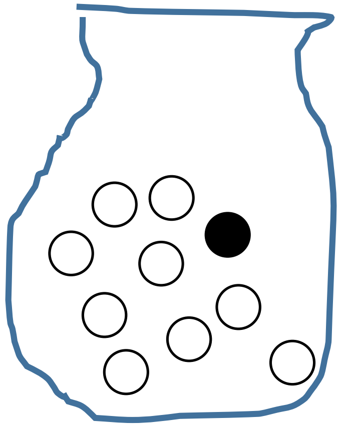
- Probabilities
- Quantiles
- Intervals
- Location measures: most probable (mode), median, mean
- Measures of scale or dispersion: variance, standard deviation, range
- Measures of shape: skewness
- Measures of correlation

Uncertainty and decisions

- A gamble is an act having an element of uncertainty
- Making decisions means to choose between gambles
- Risk is the integration of uncertainty in outcomes and their associated loss
- Uncertainty in the knowledge bases adds uncertainty to our understanding of the risk

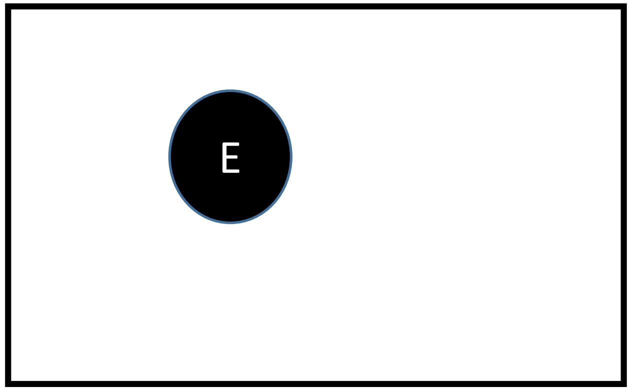
A standard for probability

- The urn analogy



$P(E) = 1/10$

- Venn diagram



E: ”the ball is black”
not E: ”the ball is white”

not E is the complementary event

$\Omega = \{E, not\ E\}$

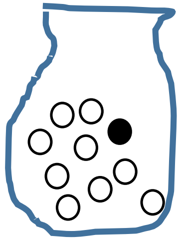
$P(\Omega) = 1$

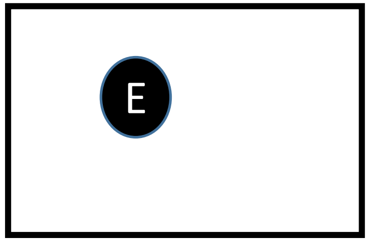
$P(E) = 1 - P(not\ E)$

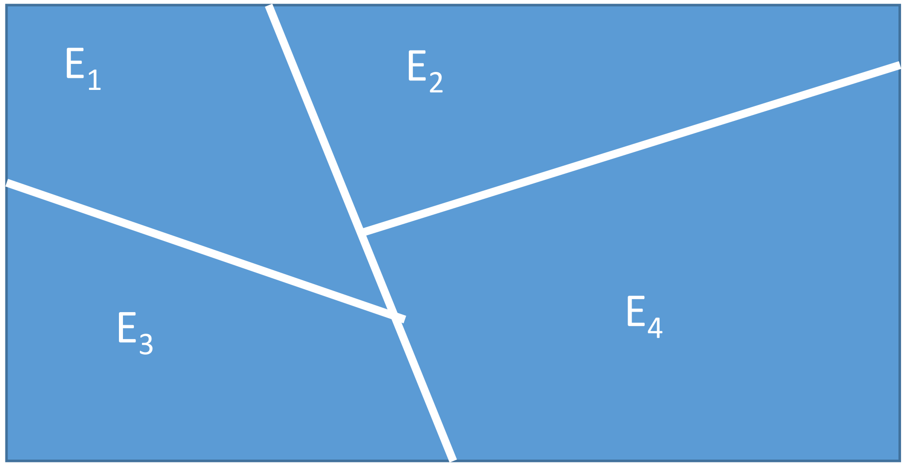
The probability of all events must sum up to one!

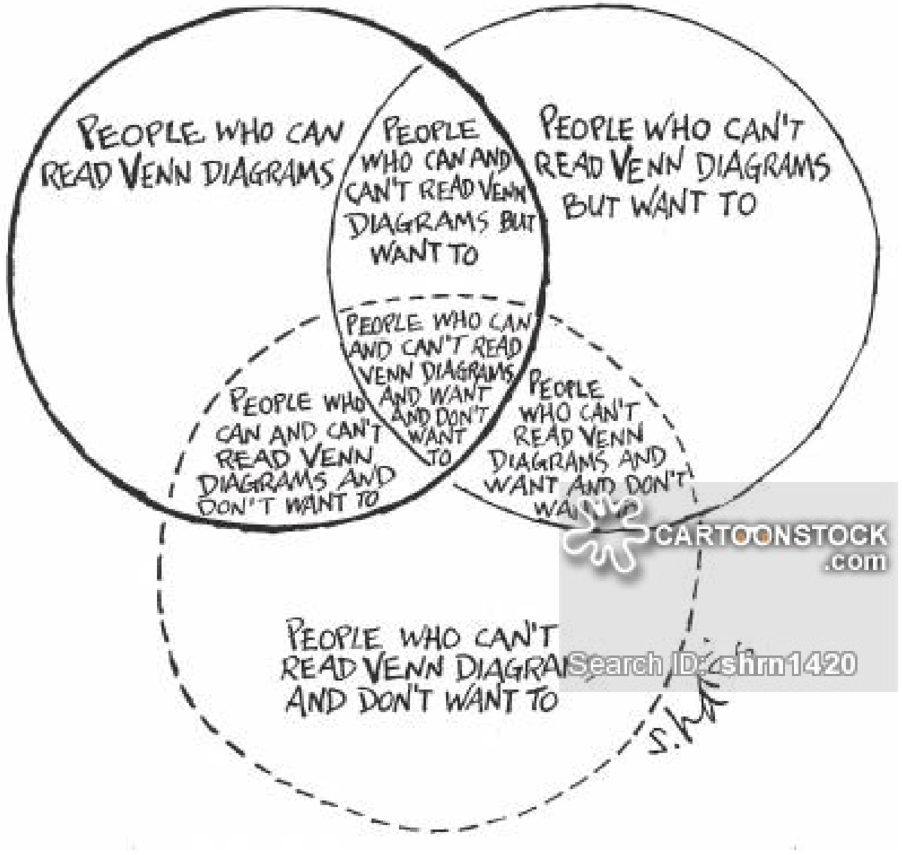
What to put in the urn

- Events in outcome space









Conditioning

- K is our knowledge bases right now
- $P(E|K)$
- F is a forecast
- $P(E|F \& K)$

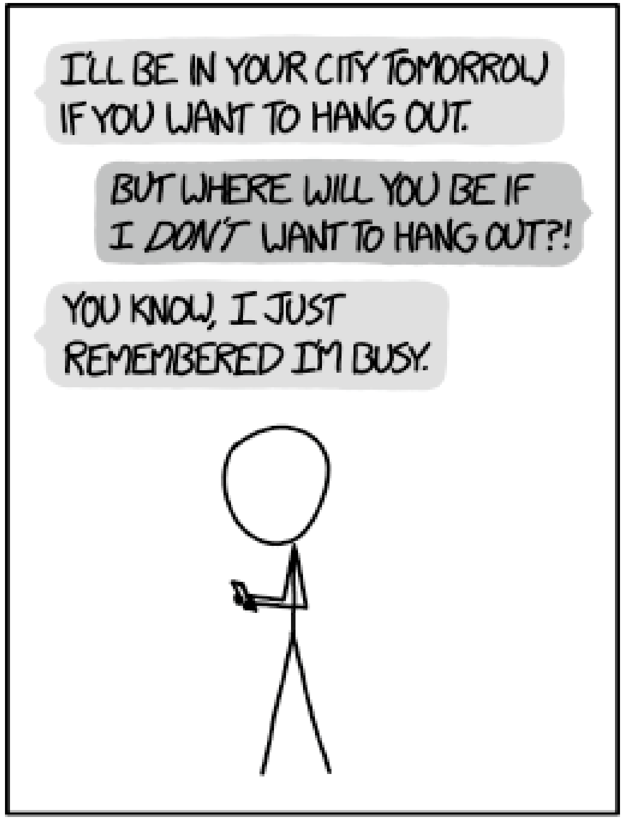
Conditional versus unconditional probability

P(rain tomorrow)

P(rain tomorrw|it has rained today)

P(rain|K_1)

P(rain|K_2)



WHY I TRY NOT TO BE PEDANTIC ABOUT CONDITIONALS.

Independence

Event E is independent of F given K if

$$P(E|K) = P(E|F \,\&\, K)$$

or (dropping the condition on knowledge bases)

$$P(E) = P(E|F)$$

By symmetry

$$P(F) = P(F|E)$$

and

$$P(E \,\&\, F) = P(E)P(F)$$

Positive association given current knowledge bases

$P(E|\ F\ \&\ K) > P(E|K)$

or (dropping K)

$P(E|F) > P(E)$

Implies that

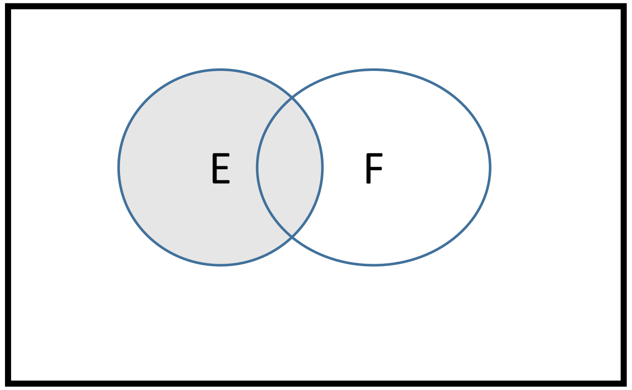
$P(F|E) > P(F)$

- A senior policeman said ”The proportion of members of an ethnic minority amongst those convicted of mugging was higher than the proportion of the general population.”
- E: person belong to ethnic minority
- F: convicted
- Due to symmetry: ”The members of the ethnic minority are more likely to be convicted of mugging than is a random member of the population”

More probability rules

Addition rule

P(E or F) = P(E) + P(F) – P(E & F)



Multiplication rule

P(E & F) = P(E|F)P(F)

P(E|F) -------------P(F|E)

Transposed conditionals

How to go from one to the other?

A principle for learning

$$P(\theta) + data \rightarrow P(\theta|data)$$

$$P(data|\theta)$$

$$P(\theta|data) \cdot P(data) = P(data|\theta) \cdot P(\theta)$$

Bayes Rule: $$P(\theta|data) = \frac{P(data|\theta) \cdot P(\theta)}{P(data)}$$

Cromwell's rule

"Think it possible you may be mistaken"

What happens if $P(F) = 0$

Bayes rule:

$P(F|E) = P(E|F)P(F) / P(E)$

You should not have probability 1 (or 0) for any event, other than one demonstrated by logic!

i.e. $0 < P(E|K) < 1$

but $P(E|K) = 1$ if and only if K logically implies the truth of E

Bayesian inference can be used for any data analysis

Purpose

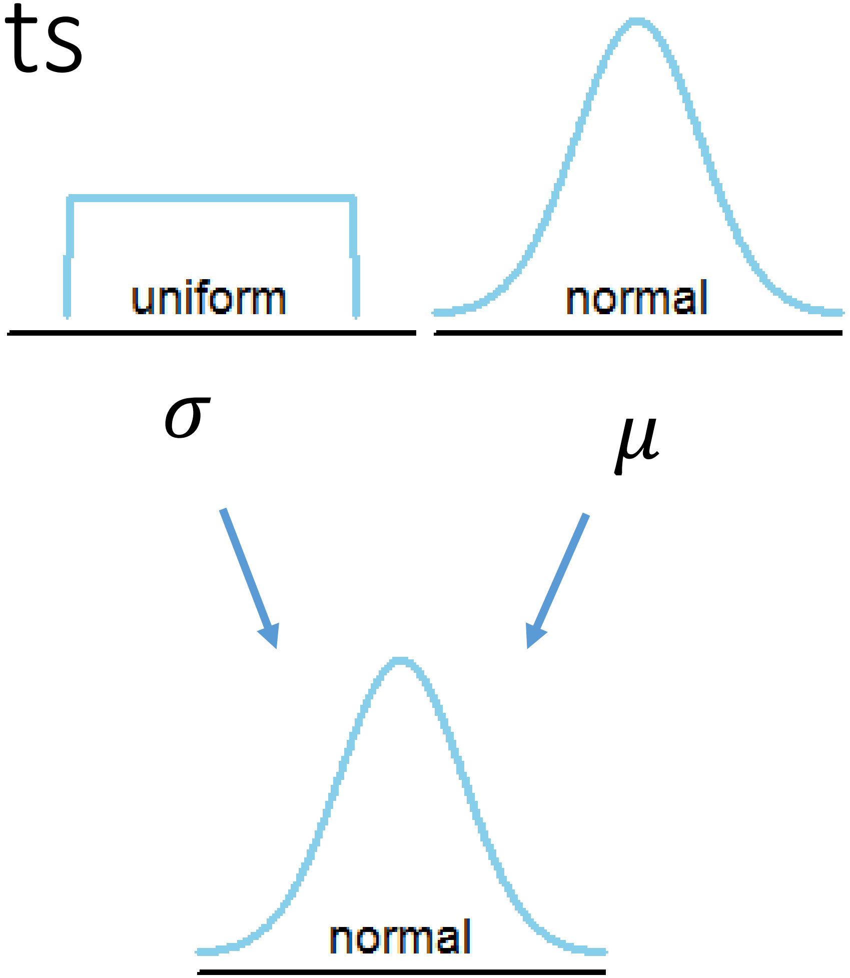
- Hypothesis testing
- Estimation
- Assessment
- Quantification of uncertainty
- Decision analysis

So

- It gives you what you want
- At the curse of too high degree of freedom
- Hower, you must understand what you are doing (see also the Folk Theorem)
- You can integrate data and expert knowledge
- Running can sometimes time consuming
- It is fun

http://www.sumsar.net/blog/2015/12/bayes-js-a-small-library-for-doing-mcmc-in-the-browser/

The height of US presidents



$$h \sim N(\mu, \sigma)$$

Check out Rasmus Bååths blog on Bayesian stuff
http://www.sumsar.net/

http://www.sumsar.net/blog/2015/12/bayes-js-a-small-library-for-doing-mcmc-in-the-browser/

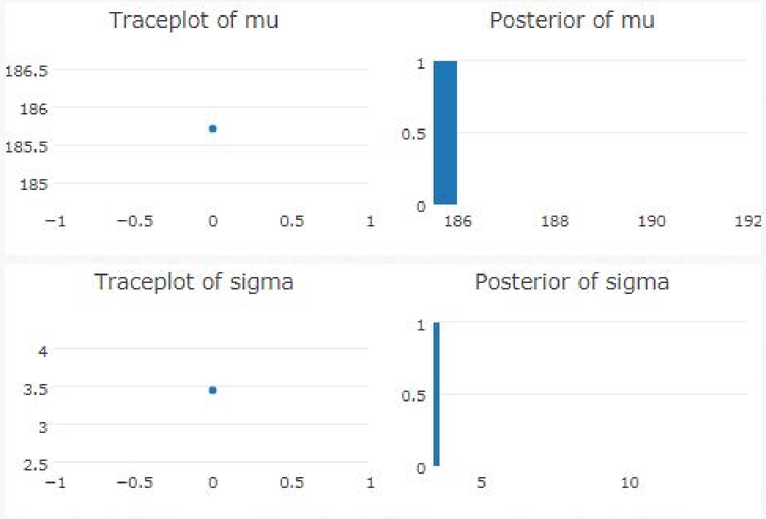
Sampling from the posterior of the height model

Here is a motivating example: Say that you have the heights of the last ten American presidents…

```
// The heights of the last ten American presidents in cm, from Kennedy to Obama
var heights = [183, 192, 182, 183, 177, 185, 188, 188, 182, 185];
```

… and that you would like to fit a Bayesian model assuming a Normal distribution to this data. Well, you can do that **right now** by clicking "Start sampling" below! This will run an MCMC sampler in your browser implemented in JavaScript.

Start sampling | Stop sampling and plot histograms | Clear samples



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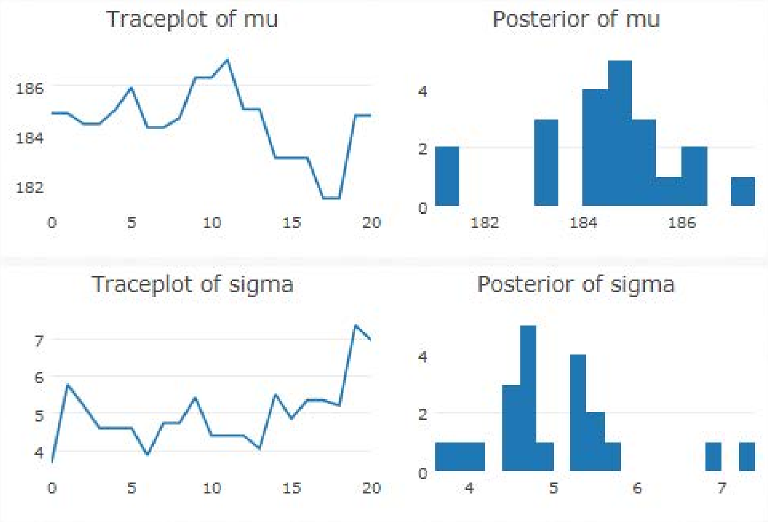

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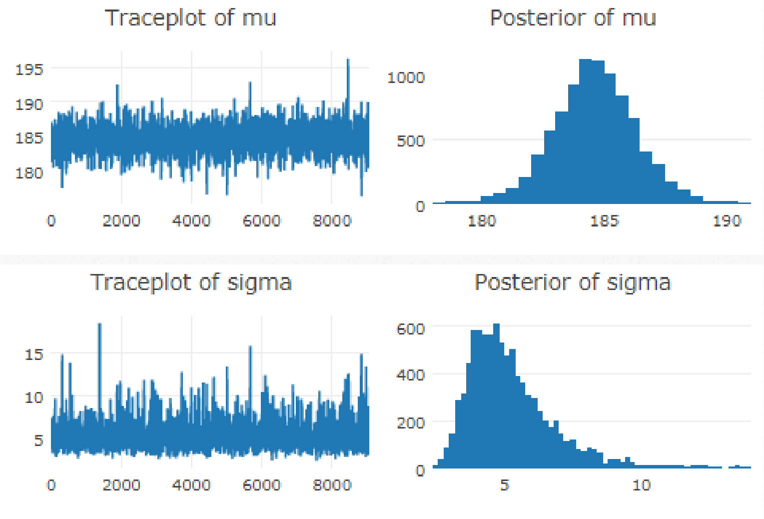

Distributions a la Kruschke style