# Homework 1

Due Sept 22nd, 2022

Problem 1

Let $X_1, \ldots, X_n \overset{iid}{\sim} U(0, a)$. Find the joint pdf of R and V, where $R = X_{(n)} - X_{(1)}$ and $V = \frac{1}{2}(X_{(n)} + X_{(1)})$.

Problem 2

Let X and Y be iid $N(0, 1)$ random variables. Define $Z = \min(X, Y)$. What distribution does Z follow?

Problem 3

A random right triangle can be constructed in the following manner. Let X be a random angle whose distribution is uniform on $(0, \pi/2)$. For each X, construct a triange as pictured below. Here Y =height of the random triangle. For a fixed constant d, find the distribution of Y and $E(Y)$.
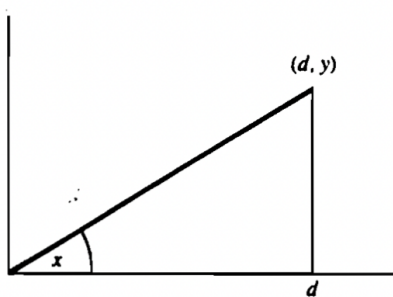
Problem 4

X_1 and X_2 are independent $N(0, \sigma^2)$.
(a) Find the joint distribution of Y_1 and Y_2, where

$$Y_1 = X_1^2 + X_2^2, \quad Y_2 = \frac{X_1}{\sqrt{Y_1}}.$$

(b) Show that Y_1 and Y_2 are independent. Interpret the result geometrically.

Problem 5

Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \chi^2_m$. Find the distribution of $\frac{1}{n} \sum_{i=1}^n X_i$. Hint: use the characteristic function.