Homework 7

Due Nov 9th, 2022

Problem 1

Suppose $X_1, ..., X_n \overset{iid}{\sim} \Gamma(\alpha, \lambda)$, where α is known and $\lambda > 0$. Prove that $\bar{X}/\alpha$ is the best unbiased estimator of $1/\lambda$.

Problem 2

Suppose $X_1, ..., X_n \overset{iid}{\sim} N(0, \sigma^2)$.
(a) Find the UMVUE of σ.
(b) Find the UMVUE of σ^4.

Problem 3

Suppose $X_1, \ldots, X_n \overset{iid}{\sim} Geom(\theta)$, where the pmf of $Geom(\theta)$ is

$$P(X = i) = \theta(1-\theta)^{i-1}, \quad i = 1, 2, ..., \quad 0 < \theta < 1.$$

Geometric distribution can be interpreted as number of Bernoulli trials ($Ber(\theta)$) needed to get one success.
(a) Show that $T = \sum_{i=1}^{n} X_i$ is a sufficient statistic for θ, and has pmf

$$P_\theta(T = t) = \left(\begin{array}{c} t-1 \\ n-1 \end{array} \right) \theta^n (1-\theta)^{t-n}, t = n, n+1, n+2, \cdots.$$

(b) Compute $E_\theta(T)$ and use it to find the UMVUE of θ^{-1}.(Hint: $\sum_{i=0}^{\infty} z^i = \frac{1}{1-z}$ for $|z| < 1$.)
(c) Show that $\psi(X_1) = I(X_1 = 1)$ is an unbiased estimator of θ, use this fact to find the UMVUE of θ.

Problem 4

Suppose $X_1, \ldots, X_n \overset{iid}{\sim} f_a(x)$ where

$$f_a(x) = e^{-(x-a)} \mathbb{I}_{(a,\infty)}(x), \quad -\infty < a \infty.$$

Find the UMVUE of a.