

Homework 3

Due Oct 10th, 2022

Problem 1

Let X_1, \dots, X_n be independent random variables with densities

$$f_{X_i}(x|\theta) = e^{i\theta - x} \mathbb{I}(x \geq i\theta).$$

Prove that $T = \min_i (X_i/i)$ is a sufficient statistic for θ .

Problem 2

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$, show that \bar{X} and $X_{(n)} - X_{(1)}$ are independent.

Problem 3

Let X_1, \dots, X_n be a random sample from a population with location pdf $f(x - \theta)$. Show that the order statistics, $T(X_1, \dots, X_n) = (X_{(1)}, \dots, X_{(n)})$, are a sufficient statistic for θ and no further reduction is possible.

Problem 4

Suppose X follows a discrete distribution with the following pmf:

$$P(X = 0) = p, \quad P(X = 1) = 3p, \quad P(X = 2) = 1 - 4p, \quad 0 < p < 1/4.$$

Is the family of distributions of X complete? What about the following family:

$$P(X = 0) = p, \quad P(X = 1) = p^2, \quad P(X = 2) = 1 - p - p^2, \quad 0 < p < 1/2.$$