Homework 9

Due Dec 8th, 2022

Problem 1

Suppose $X_1, ..., X_{20} \overset{iid}{\sim} Ber(p)$. We want to test the hypothesis

$$H_0 : p = 0.2 \quad \text{vs} \quad H_1 : p \neq 0.2.$$

Suppose the decision rule is $\phi(\boldsymbol{x})$ and $\phi(\boldsymbol{x}) = 1$ if and only if $\sum_{i=1}^{20} x_i \geq 7$ or $\sum_{i=1}^{20} x_i \leq 1$.
(a) Find the power of the decision rule when $p = 0,\ 0.1,\ 0.2,\ ...\ 0.9$. Draw a graph of the power as a function of p.
(b) Find the type 1 error probability of the rule ϕ. What is the type 2 error probability when $p = 0.05$?

Problem 2

Suppose $X_1, \ldots, X_{10} \overset{iid}{\sim} U(0, \theta)$. Construct a test for testing

$$H_0 : \theta \leq 1, \quad \text{vs}, \quad H_a : \theta \geq 1,$$

at level α (Construct a test means finding a rejection region).

Problem 3

Suppose $X_1, \ldots, X_n \overset{iid}{\sim} U(0, \theta)$. Use $\max X_i$ to find a $(1 - \beta, 1 - \gamma)$ tolerance interval.