Homework 5

Due Oct 27th, 2022

Problem 1

Let W be a statistic, show that $\mathbb{E}_\theta(W-\theta)^2 = Var(W) + (\mathbb{E}_\theta(W)-\theta)^2$.

Problem 2

$X_1, \ldots, X_n \overset{iid}{\sim} f(x|\mu)$ where

$$f(x|\mu) = e^{-(x-\mu)} \cdot \mathbb{I}(x > \mu), \quad \mu \in (-\infty, \infty).$$

(a) Find $\hat{\mu}_{mle}$.
(b) Use method of moments to find an unbiased estimator for μ.
(c) Compare the estimators from (a) and (b), which one has a smaller MSE?

Problem 3

Let $F(x)$ and $f(x)$ be the distritbution and density functions for iid random variables $X_1, \ldots, X_n$. Show that

$$\int \cdots \int_{a<x_1<\ldots<x_n<b} f(x_1)\cdots f(x_n) dx_1 \cdots dx_n = \frac{1}{n!}[F(b)-F(a)]^n.$$

Problem 4

If $f(x|\theta)$ satisfies

$$\frac{d}{d\theta}\mathbb{E}_\theta\left(\frac{\partial}{\partial\theta}\log f(X|\theta)\right) = \int \frac{\partial}{\partial\theta}\left[\left(\frac{\partial}{\partial\theta}\log f(x|\theta)\right) f(x|\theta)\right] dx$$

(true for an exponential family), show that

$$\mathbb{E}_\theta\left[\left(\frac{\partial}{\partial\theta}\log f(X|\theta)\right)^2\right] = -\mathbb{E}_\theta\left(\frac{\partial^2}{\partial\theta^2}\log f(X|\theta)\right).$$