Homework 6

Due Nov 3rd, 2022

Problem 1

Suppose $X_1, ..., X_n \overset{iid}{\sim} Ber(p)$.
(a) Show that the variance of the MLE of p attains the Cramer-Rao lower bound.
(b) For $n \geq 4$, show that the product $X_1X_2X_3X_4$ is an unbiased estimator of p^4, and use this fact to find the best unbiased estimator of p^4.

Problem 2

Let $X_1, ..., X_n \overset{iid}{\sim} Poi(\lambda)$, and let $\bar{X}$ and S^2 denote the sample mean and variance respectively.
(a) Prove that $\bar{X}$ is the best unbiased estimator of λ without using the Cramer-Rao theorem.
(b) Prove that $E(S^2|\bar{X}) = \bar{X}$ and use it to show that $Var(S^2) > Var(\bar{X})$.

Problem 3

Suppose $X_1, ..., X_n \overset{iid}{\sim} Ber(p)$. Find the UMVUE of $p(1-p)$. Make sure to prove that the estimator is indeed a UMVUE of $p(1-p)$.

Problem 4

Prove the following statement:
Let T be a complete sufficient statistic for θ and let $\phi(T)$ be any estimator based on T. Then $\phi(T)$ is the unique unbiased estimator of its expected value.