# Homework 4

Due Oct 21, 2022

## Problem 1

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\theta, a\theta^2)$, where $a$ is a known positive constant and $\theta > 0$.
(a) Show that the parameter space does not contain a two-dimensional open set.
(b) Show that the statistic $T = (\bar{X}, S^2)$ is a sufficient statistic for $\theta$, but the family of distributions is not complete.

## Problem 2

Let $X_1, \ldots, X_n$ be a random sample from the following population:

$$f(x, \theta) = \theta x^{\theta - 1}, \quad 0 < x < 1, \quad \theta > 0.$$

(a) Is $\sum_{i=1}^{n} X_i$ sufficient for $\theta$?
(b) Find a complete sufficient statistic for $\theta$.

## Problem 3

Suppose $X_1, ..., X_n$ are independently sampled from the following pmf

$$P(X = k) = -\frac{1}{\ln(1-p)} \frac{p^k}{k}, \quad 0 < p < 1, k = 1, 2, \ldots$$

Use the method of moment to find an estimator for $p$.

## Problem 4

Suppose $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Find a method of moment estimator for $P(X > 1)$.