Project 2 in FYS3150

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October 23, 2018

https://github.com/UlrikSeip/Projects/tree/master/prosjekt3

1 ABSTRACT

In this project we simulate the orbits of all the 8 planets in the solar system, and Pluto. Comparing the Forward Euler and the Velocity Verlet methods we find the Velocity Verlet method to be preferable due to its conservation of energy. We then test the Velocity Verlet method against the analytically derived escape velocity and perihelion of Mercury.

2 INTRODUCTION

Our solar system is littered with asteroids, planets and moons. This plethora objects floating around in space makes for a perfect exercise in solving multi body differential equations in 3 dimensions.

When simulating orbits for several celestial bodies with high accuracy, the computation can be expensive, and so it is paramount to strike a balance between efficiency and accuracy. To explore this balance, we will run simulations using the Velocity-Verlet integration method, and comparing with the Forward-Euler method. Having found the optimal way to simulate the orbits, we move on to test the effect of the gravitational pull between planets, and we also look at the (something about the perhelion of Mercury).

3 METHOD

3.a Newtons law of gravitation

One of the most common representations of newtons law of gravitation is

$$F_G = \frac{M_{\text{Earth}} v^2}{r}\hat{\boldsymbol{r}} = \frac{G M_{\odot} M_{\text{Earth}}}{r^2}\hat{\boldsymbol{r}} \tag{1}$$

Where $G = 6.67 \cdot 10^-11 \frac{m^3}{kgs^2}$ is the gravitational constant, m_1 and m_2 are the masses of the bodies exerting a force upon each other, F_G is said force, and r is the distance between the bodies. Using Keplers laws this can be further simplified to

$$F_G = \frac{M_{\odot} M_{\text{Earth}} 4\pi^2}{r^2}\frac{\text{AU}^3}{\text{yr}^2}\hat{\boldsymbol{r}} \tag{2}$$

We can then rewrite this for a point mass and acceleration as

$$a = \frac{M_s 4\pi^2}{r^2}\frac{\text{AU}^3}{\text{yr}^2}\hat{\boldsymbol{r}} \tag{3}$$

3.b The Forward Euler method

To use equation () for the Forward Euler method we need an expression for $\Delta \boldsymbol{v}$. We therefore introduce a time step dt. We also define $\hat{\boldsymbol{r}} = \cos(\theta)\hat{\boldsymbol{\imath}} + \sin(\theta)\hat{\boldsymbol{\jmath}} + \cos(\phi)\hat{\boldsymbol{k}}$. This gives us

$$\frac{d\boldsymbol{v}}{dt} = \frac{v_{i+1} - v_i}{dt} = -4\pi^2 \frac{M_s}{r^2}\hat{\boldsymbol{r}}$$

$$\boldsymbol{v_{i+1}} = -4\pi^2 \frac{M_s}{r^2} dt\hat{\boldsymbol{r}} + \boldsymbol{v_i} \tag{4}$$

stuff about x_{i+1}

3.c The Velocity Verlet method

3.d Testing the algorithms

4 RESULTS

4.a The Forward Euler method

4.b The Velocity Verlet method

4.c Testing the algorithms

5 CONCLUSIONS

6 APENDICES

7 REFERENCES

References

[1] Computational Physics, Lecture Notes Fall 2015, Morten Hjort-Jensen p.215-220