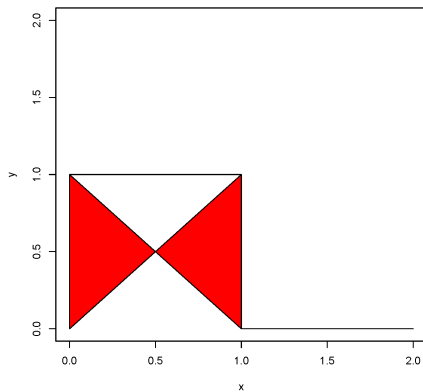


14.310x: Data Analysis for Social Scientists
Describing Data, Joint and Conditional Distributions – Part II

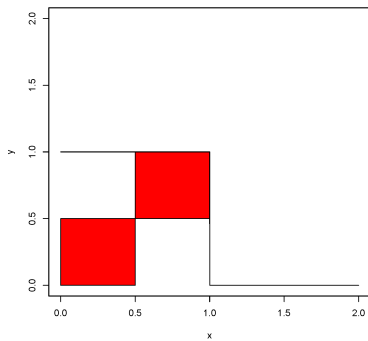
Suppose two sisters, Caroline and Anna, sleep in adjoining rooms. Each has a speaker with which she plays music, and each speaker has a volume dial going from 0 to 1. The joint distribution of the volumes of the two speakers is $f_{XY}(x, y) = c(x + y^2)$ over the unit square, 0 otherwise. Caroline's volume is denoted by X , Anna's by Y .

Question 1

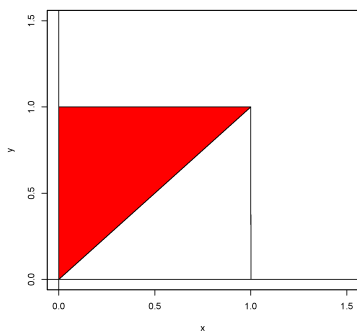
Which of the following figures represent the domain (in red) in which the density function is defined as $f_{XY}(x, y) = c(x + y^2)$?



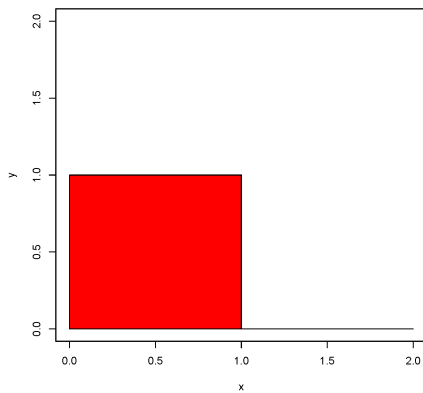
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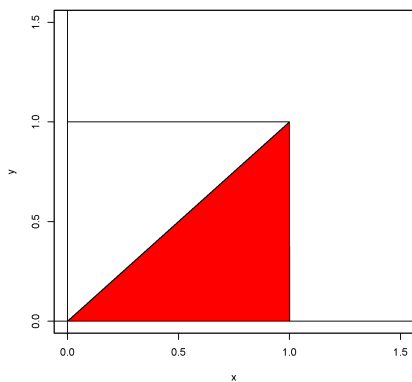
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Question 2

What does the constant c represent? (Select all that apply)

- ☐ The constant c is a parameter whose value assures that the joint PDF integrates to 1.
- ☐ The constant c represents a parameter that changes both the joint PDF and the joint CDF of the random variables X and Y .
- ☐ The constant c is an irrelevant parameter in the shape of the joint CDF of the random variables X and Y .
- ☐ The constant c is a parameter that helps to infer whether the random variables X and Y are independent.

Question 3

What is the value of the constant c in this case?

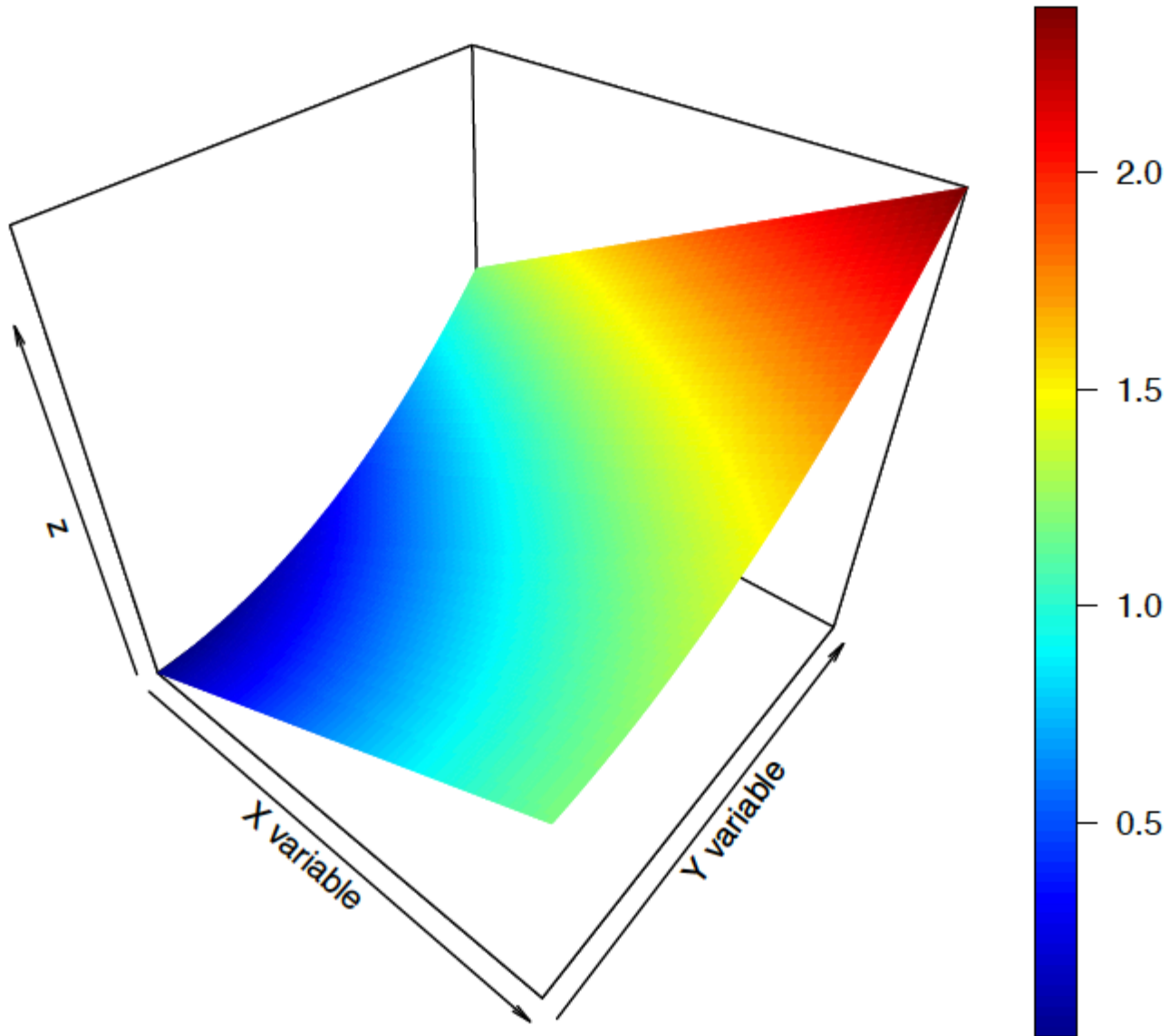
Note: Please review our guidelines on precision regarding rounding answers [here](#).

Now we are going to work in R to plot the bivariate PDF. Download the code [here](#) and take a look at the following code in order to create a grid and a 3-dimensional plot of the PDF. Please note that you might need to install the package `plot3D`.

Question 4

The following plot was created by running the code. A student is claiming that this plot is wrong since there are certain regions in which the PDF shows values larger than 1. Is this student correct that there is a mistake and therefore the plot does not correspond to the information given in the problem?

Plotting joint pdf



- ☐ Yes
- ☐ No

Question 5

Are the volumes of the two speakers independent random variables?

- ☐ Yes

- No

Question 6

Recall that Caroline's speaker volume is denoted by X and Anna's speaker volume is denoted by Y . What is the formula for the marginal distribution of Anna's speaker volume?

- $\frac{5}{6}\left(\frac{1}{2} + y^2\right)$
- $\frac{6}{5}\left(\frac{1}{2} + y^2\right)$
- $\frac{6}{5}\left(\frac{1}{2} + \sqrt{y}\right)$
- $\frac{5}{6}\left(\frac{1}{2} + \sqrt{y}\right)$

Question 7

Recall that Caroline's volume is denoted by X and Anna's volume is denoted by Y . What is the conditional distribution of Caroline's volume as a function of Anna's?

- $\frac{x+y^2}{\frac{1}{2}+y^2}$
- $\frac{\frac{5}{6}(x+y^2)}{\frac{6}{5}\left(\frac{1}{2}+y^2\right)}$
- $\frac{x+\sqrt{y}}{\frac{1}{2}+y^2}$
- $\frac{\frac{6}{5}(x+y^2)}{\frac{1}{2}+y^2}$

Question 8

From this conditional distribution can you infer whether Caroline likes Anna's music or not?

Hint: Think about the probability that Caroline's volume is high when the volume of Anna's music increases.

- Caroline does like Anna's music
- Caroline does not like Anna's music

Question 9

What is the probability that Caroline's volume is less than $\frac{1}{2}$ if Anna's volume is $\frac{1}{2}$?

Note: Please review our guidelines on precision regarding rounding answers [here](#).

Question 10

Recall that Caroline's speaker volume is denoted by X and Anna's speaker volume is denoted by Y . What is the marginal distribution of Caroline's speaker volume?

- ☐ $\frac{5}{6}\left(x + \frac{2}{3}\right)$
- ☐ $\frac{5}{6}\left(x + \frac{1}{3}\right)$
- ☐ $\frac{6}{5}\left(x + \frac{2}{3}\right)$
- ☐ $\frac{6}{5}\left(x + \frac{1}{3}\right)$

Question 11

Is there a First Order Stochastic Dominance (FOSD) relationship between the random variables X and Y ? (We suggest you compute the CDF's of both variables and plot them in R.)

- ☐ The distribution of X FOSD the distribution of Y
- ☐ The distribution of Y FOSD the distribution of X
- ☐ There is no clear relationship

Question 12

From this information, does Anna or Caroline prefer higher volumes?

- ☐ Anna
- ☐ Caroline
- ☐ We can't say