Protocol Analysis Modulo Combination of Theories: A Case Study in Maude-NPA[⋆]

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Abstract. There is a growing interest in formal methods and tools to analyze cryptographic protocols *modulo* algebraic properties of their underlying cryptographic functions. It is well-known that an intruder who uses algebraic equivalences of such functions can mount attacks that would be impossible if the cryptographic functions did not satisfy such equivalences. In practice, however, protocols use a collection of well-known functions, whose algebraic properties can naturally be grouped together as a union of theories $E_1 \cup \ldots \cup E_n$. Reasoning symbolically modulo the algebraic properties $E_1 \cup \ldots \cup E_n$ requires performing $(E_1 \cup \ldots \cup E_n)$-unification. However, even if a unification algorithm for each individual E_i is available, this requires combining the existing algorithms by methods that are highly non-deterministic and have high computational cost. In this work we present an alternative method to obtain unification algorithms for combined theories based on *variant narrowing*. Although variant narrowing is less efficient at the level of a single theory E_i, it does not use any costly combination method. Furthermore, it does not require that each E_i has a dedicated unification algorithm in a tool implementation. We illustrate the use of this method in the Maude-NPA tool by means of a well-known protocol requiring the combination of three distinct equational theories.

Keywords: Cryptographic protocol verification, equational unification, variants, exclusive or, narrowing

1 Introduction

In recent years there has been growing interest in the formal analysis of protocols in which the crypto-algorithms satisfy different algebraic properties [10,13,30,17]. Applications such as electronic voting, digital cash, anonymous communication,

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and even key distribution, all can profit from the use of such cryptosystems. Thus, a number of tools and algorithms have been developed that can analyze protocols that make use of these specialized cryptosystems [30,29,6,2,14].

Less attention has been paid to combinations of algebraic properties. However, protocols often make use of more than one type of cryptosystem. For example, the Internet Key Exchange protocol [24] makes use of Diffie-Hellman exponentiation (for exchange of master keys), public and private key cryptography (for authentication of master keys), shared key cryptography (for exchange of session keys), and exclusive-or (used in the generation of master keys). All of these functions satisfy different equational theories. Thus it is important to understand the behavior of algebraic properties in concert as well as separately. This is especially the case for protocol analysis systems based on unification, where the problem of combining unification algorithms [3,36] for different theories is known to be highly non-deterministic and complex, even when efficient unification algorithms exist for the individual theories, and even when the theories are disjoint (that is, share no symbols in common).

The Maude-NPA protocol analysis tool, which relies on unification to perform backwards reachability analysis from insecure states, makes use of two different techniques to handle the combination problem. One is to use a general-purpose approach to unification called *variant narrowing* [21], which, although not as efficient as special purpose unification algorithms, can be applied to a broad class of theories that satisfy a condition known as the *finite variant property* [12]. A second technique applicable to special purpose algorithms, or theories that do not satisfy the finite variant property, uses a more general framework for combining unification algorithms; this is described in [15].

One advantage of using variant narrowing is that there are well-known methods and tools for checking that a combination of theories has the finite variant property, including checking its local confluence and termination, and also its satisfaction of the finite variant property itself [18]. Furthermore, under appropriate assumptions some of these checks can be made modularly (see, e.g., [34] for a survey of modular confluence and termination proof methods). This makes variant narrowing easily applicable for unification combination and very suitable for experimentation with different theories. Later on, when the theory is better understood, it may be worth the effort to invest the time to apply the framework described in [15] to integrate more efficient special purpose algorithms.

In this paper we describe a case study involving the use of variant narrowing to apply Maude-NPA to the analysis of a protocol that involves three theories: (i) an associative-commutative theory satisfied by symbols used in state construction, (ii) a cancellation theory for public key encryption and decryption, and (iii) the equational theory of the exclusive-or operator. This theory combination is illustrated in the analysis of a version of the Needham-Schroeder-Lowe protocol [29], denoted NSL⊕, in which one of the concatenation operators is replaced by an exclusive-or [8].

The rest of this paper is organized as follows. In Section 2 we give some necessary background. In Section 3 we give an overview of Maude-NPA. In Sections 4 and 5 we describe variant narrowing and how it is used in Maude-NPA. In Section 6 we describe our use of Maude-NPA on the NSL⊕ protocol. In Section 7 we discuss related work, and Section 8 concludes the paper.

2 Background on Term Rewriting

We follow the classical notation and terminology from [37] for term rewriting and from [31,32] for rewriting logic and order-sorted notions. We assume an *order-sorted signature* Σ with a finite poset of sorts $(\mathsf{S}, \leq)$ (such that each connected component of $(\mathsf{S}, \leq)$ has a top sort) and a finite number of function symbols. We assume an S-sorted family $\mathcal{X} = \{\mathcal{X}_{\mathsf{s}}\}_{\mathsf{s} \in \mathsf{S}}$ of disjoint variable sets with each $\mathcal{X}_{\mathsf{s}}$ countably infinite. $\mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$ denotes the set of terms of sort s, and $\mathcal{T}_{\Sigma,\mathsf{s}}$ the set of ground terms of sort s. We write $\mathcal{T}_{\Sigma}(\mathcal{X})$ and $\mathcal{T}_{\Sigma}$ for the corresponding term algebras. We write $Var(t)$ for the set of variables present in a term t. The set of positions of a term t is written $Pos(t)$, and the set of non-variable positions $Pos_{\Sigma}(t)$. The subterm of t at position p is $t|_p$, and $t[u]_p$ is the result of replacing $t|_p$ by u in t. A *substitution* σ is a sort-preserving mapping from a finite subset of $\mathcal{X}$ to $\mathcal{T}_{\Sigma}(\mathcal{X})$.

A Σ*-equation* is an unoriented pair $t = t'$, where $t \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$, $t' \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}'}$, and s and s' are sorts in the same connected component of the poset $(\mathsf{S}, \leq)$. For a set E of Σ-equations, an E*-unifier* for a Σ-equation $t = t'$ is a substitution σ s.t. $\sigma(t) =_E \sigma(t')$. A *complete* set of E-unifiers of an equation $t = t'$ is written $CSU_E(t = t')$. We say that $CSU_E(t = t')$ is *finitary* if it contains a finite number of E-unifiers. A *rewrite rule* is an oriented pair $l \to r$, where $l \notin \mathcal{X}$ and $l, r \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$ for some sort $\mathsf{s} \in \mathsf{S}$. An *(unconditional) order-sorted rewrite theory* is a triple $\mathcal{R} = (\Sigma, E, R)$ with Σ an order-sorted signature, E a set of Σ-equations, and R a set of rewrite rules. The rewriting relation $\to_{R,E}$ on $\mathcal{T}_{\Sigma}(\mathcal{X})$ is $t \xrightarrow{p}_{R,E} t'$ (or $\to_{R,E}$) if $p \in Pos_{\Sigma}(t)$, $l \to r \in R$, $t|_p =_E \sigma(l)$, and $t' = t[\sigma(r)]_p$ for some σ. Assuming that E has a finitary and complete unification algorithm, the narrowing relation $\rightsquigarrow_{R,E}$ on $\mathcal{T}_{\Sigma}(\mathcal{X})$ is $t \overset{p}{\rightsquigarrow}_{\sigma,R,E} t'$ (or $\rightsquigarrow_{\sigma,R,E}$, $\rightsquigarrow_{R,E}$) if $p \in Pos_{\Sigma}(t)$, $l \to r \in R$, $\sigma \in CSU_E(t|_p = l)$, and $t' = \sigma(t[r]_p)$.

We say that the relation $\to_{R,E}$ is *terminating* if there is no infinite sequence $t_1 \to_{R,E} t_2 \to_{R,E} \cdots t_n \to_{R,E} t_{n+1} \cdots$. We say that the relation $\to_{R,E}$ is *confluent* if whenever $t \to^*_{R,E} t'$ and $t \to^*_{R,E} t''$, there exists a term t''' such that $t' \to^*_{R,E} t'''$ and $t'' \to^*_{R,E} t'''$. An order-sorted rewrite theory (Σ, E, R) is confluent (resp. terminating) if the relation $\to_{R,E}$ is confluent (resp. terminating). In a confluent, terminating, order-sorted rewrite theory, for each term $t \in \mathcal{T}_{\Sigma}(\mathcal{X})$, there is a unique (up to E-equivalence) R, E-irreducible term t' obtained from t by rewriting to canonical form, which is denoted by $t \to^!_{R,E} t'$ or $t{\downarrow}_{R,E}$ (when t' is not relevant). The relation $\to_{R,E}$ is E*-coherent* [25] if $\forall t_1, t_2, t_3$ we have $t_1 \to_{R,E} t_2$ and $t_1 =_E t_3$ implies $\exists t_4, t_5$ such that $t_2 \to^*_{R,E} t_4$, $t_3 \to^+_{R,E} t_5$, and $t_4 =_E t_5$.

3 Protocol Specification and Analysis in Maude-NPA

Given a protocol $\mathcal{P}$, we should first explain how its states are modeled algebraically. The key idea is to model such states as elements of an initial algebra $T_{\Sigma_\mathcal{P}/E_\mathcal{P}}$, where $\Sigma_\mathcal{P}$ is the signature defining the sorts and function symbols for the cryptographic functions and for all the state constructor symbols and $E_\mathcal{P}$ is a set of equations specifying the *algebraic properties* of the cryptographic functions and the state constructors. Therefore, a state is an $E_\mathcal{P}$-equivalence class $[t] \in T_{\Sigma_\mathcal{P}/E_\mathcal{P}}$ with t a ground $\Sigma_\mathcal{P}$-term. However, since the number of states $T_{\Sigma_\mathcal{P}/E_\mathcal{P}}$ is in general infinite, rather than exploring concrete protocol states $[t] \in T_{\Sigma_\mathcal{P}/E_\mathcal{P}}$ we explore *symbolic state patterns* $[t(x_1, \ldots, x_n)] \in T_{\Sigma_\mathcal{P}/E_\mathcal{P}}(X)$ on the free $(\Sigma_\mathcal{P}, E_\mathcal{P})$-algebra over a set of variables X. In this way, a state pattern $[t(x_1, \ldots, x_n)]$ represents not a single concrete state but a possibly infinite set of such states, namely all the instances of the pattern $[t(x_1, \ldots, x_n)]$ where the variables $x_1, \ldots, x_n$ have been instantiated by concrete ground terms.

Let us introduce a motivating example that we will use to illustrate our approach based on exclusive–or. We use an exclusive–or version borrowed from [8] of the Needham-Schroeder-Lowe protocol [29] which we denote NSL⊕. In our analysis we use the protocol based on public key encryption, i.e., operators pk and sk satisfying the equations $pk(P, sk(P, M)) = M$ and $sk(P, pk(P, M)) = M$ and the messages are put together using concatenation and exclusive–or. Note that we use a representation of public-key encryption in which only principal P can compute $sk(P, X)$ and everyone can compute $pk(P, X)$. For exclusive–or we have the associativity and commutativity (AC) axioms for $\oplus$, plus the equations[4] $X \oplus 0 = X,\ X \oplus X = 0,\ X \oplus X \oplus Y = Y$.

1. $A \to B : pk(B, N_A; A)$
   A sends to B, encrypted under B's public key, a communication request containing a nonce N_A that has been generated by A, concatenated with its name.
2. $B \to A : pk(A, N_A; B \oplus N_B)$
   B answers with a message encrypted under A's public key, containing the nonce of A, concatenated with the exclusive–or combination of a new nonce created by B and its name.
3. $A \to B : pk(B, N_B)$
   A responds with B's nonce encrypted under B's public key.

A and B agree that they both know N_A and N_B and no one else does.

In the Maude-NPA [16,17], a *state* in the protocol execution is a term t of sort *state*, $t \in T_{\Sigma_\mathcal{P}/E_\mathcal{P}}(X)_{state}$. A state is a multiset built by an associative and commutative union operator $\_\&\_$. Each element in the multiset can be a strand or the intruder knowledge at that state (intruder knowledge is wrapped by $\{\_\}$). A *strand* [22] represents the sequence of messages sent and received by a principal executing the protocol and is indicated by a sequence of messages $[msg_1^-,\ msg_2^+,\ msg_3^-, \ldots,\ msg_{k-1}^-,\ msg_k^+]$ where each msg_i is a term

[4] The third equation follows from the first two. It is needed for coherence modulo AC.

of sort Msg (i.e., $msg_i \in T_{\Sigma_\mathcal{P}/E_\mathcal{P}}(X)_{\mathsf{Msg}}$), msg^- represents an input message, and msg^+ represents an output message. In Maude-NPA, strands evolve over time and thus we use the symbol $|$ to divide past and future in a strand, i.e., $[msg_1^\pm, \ldots, msg_{j-1}^\pm \mid msg_j^\pm, msg_{j+1}^\pm, \ldots, msg_k^\pm]$ where $msg_1^\pm, \ldots, msg_{j-1}^\pm$ are the past messages, and $msg_j^\pm, msg_{j+1}^\pm, \ldots, msg_k^\pm$ are the future messages ($msg_j^\pm$ is the immediate future message). The *intruder knowledge* is represented as a multiset of facts unioned together with an associative and commutativity union operator `_,_`. There are two kinds of intruder facts: positive knowledge facts (the intruder knows m, i.e., $m {\in} \mathcal{I}$), and negative knowledge facts (the intruder *does not yet know* m but *will know it in a future state*, i.e., $m {\notin} \mathcal{I}$), where m is a message expression. Facts of the form $m {\notin} \mathcal{I}$ make sense in a backwards analysis, since one state can have $m {\in} \mathcal{I}$ and a prior state can have $m {\notin} \mathcal{I}$.

The strands associated to the three protocol steps above are given next. There are two strands, one for each principal in the protocol. Note that the first message passing $A \to B : pk(B, N_A; A)$ is represented by a message in Alice's strand sending $(pk(B, n(A, r); A))^+$, together with another message in Bob's strand that receives $(pk(B, N; A))^-$. When a principal cannot observe the contents of a concrete part of a received message (e.g., because a key is necessary to look inside), we use a generic variable for such part of the message in the strand (as with variable N of sort *Nonce* above, and similarly for X, Y below). We encourage the reader to compare the protocol in strand notation to the presentation of the protocol above. We also omit the initial and final *nil* in strands, which are needed in the tool but clutter the presentation.

- (Alice) :: r :: $[(pk(B, n(A, r); A))^+, (pk(A, n(A, r); B {\oplus} Y))^-, (pk(B, Y))^+]$

- (Bob) :: r' :: $[(pk(B, X; A))^-, (pk(A, X; B {\oplus} n(B, r')))^+, (pk(B, n(B, r')))^-]$

Note that r, r' are used for nonce generation (they are special variables handled as *unique constants* in order to obtain an infinite number of available constants, see [17]).

There are also strands for initial knowledge and actions of the intruder, such as concatenation, deconcatenation, encryption, decryption, etc. For example, concatenation by the intruder is described by the strand $[(X)^-, (Y)^-, (X; Y)^+]$. We will show the full list of intruder capabilities in Section 6.

Our protocol analysis methodology is then based on the idea of *backward reachability analysis*, where we begin with one or more state patterns corresponding to *attack states*, and want to prove or disprove that they are *unreachable* from the set of initial protocol states. In order to perform such a reachability analysis we must describe how states change as a consequence of principals performing protocol steps and of intruder actions. This can be done by describing such state changes by means of a set $R_\mathcal{P}$ of *rewrite rules*, so that the rewrite theory $(\Sigma_\mathcal{P}, E_\mathcal{P}, R_\mathcal{P})$ characterizes the behavior of protocol $\mathcal{P}$ modulo the equations $E_\mathcal{P}$. The following rewrite rules describe the general state transitions, where each state transition implies moving rightwards the vertical bar of one strand:

$$SS \ \& \ [L \mid M^-, L'] \ \& \ \{M{\in}\mathcal{I}, IK\} \rightarrow SS \ \& \ [L, M^- \mid L'] \ \& \ \{IK\}$$
$$SS \ \& \ [L \mid M^+, L'] \ \& \ \{IK\} \rightarrow SS \ \& \ [L, M^+ \mid L'] \ \& \ \{IK\}$$
$$SS \ \& \ [L \mid M^+, L'] \ \& \ \{M{\notin}\mathcal{I}, IK\} \rightarrow SS \ \& \ [L, M^+ \mid L'] \ \& \ \{M{\in}\mathcal{I}, IK\}$$

variables L, L' denote lists of input and output messages (m^+,m^-) within a strand, IK denotes a set of intruder facts ($m{\in}\mathcal{I}$,$m{\notin}\mathcal{I}$), and SS denotes a set of strands. An unbounded number of sessions is handled by another rewrite rule introducing an extra strand $[m_1^\pm, \ldots, m_{j-1}^\pm \mid m_j^+, msg_{j+1}^\pm, \ldots, m_k^\pm]$ for an intruder knowledge fact of the form $m_j{\in}\mathcal{I}$. See [16] for further information.

The way to analyze *backwards* reachability is then relatively easy, namely to run the protocol "in reverse." This can be achieved by using the set of rules $R_\mathcal{P}^{-1}$, where $v \longrightarrow u$ is in $R_\mathcal{P}^{-1}$ iff $u \longrightarrow v$ is in $R_\mathcal{P}$. Reachability analysis can be performed *symbolically*, not on concrete states but on symbolic state patterns $[t(x_1, \ldots, x_n)]$ by means of *narrowing modulo* $E_\mathcal{P}$ (see Section 2 and [25,33]).

$E_\mathcal{P}$-unification precisely models all the different ways in which an intruder could exploit the algebraic properties $E_\mathcal{P}$ of $\mathcal{P}$ to break the protocol; therefore, if an initial state can be shown unreachable by backwards reachability analysis modulo $E_\mathcal{P}$ from an attack state pattern, this ensures that, even if the intruder uses the algebraic properties $E_\mathcal{P}$, the attack cannot be mounted. This means that efficient support for $E_\mathcal{P}$-unification is a crucial feature of symbolic reachability analysis of protocols modulo their algebraic properties $E_\mathcal{P}$.

4 A Unification Algorithm for $XOR \cup pk\text{-}sk \cup AC$

In general, combining unification algorithms for a theory $E = E_1 \cup E_2 \cup \ldots \cup E_n$ is computationally quite expensive, and typically assumes that the symbols in E_i and E_j are pairwise disjoint for each $i \neq j$. This is due to the substantial amount of non–determinism involved in the inference systems supporting such combinations (see [3]). In our NSL⊕ example, $E = E_1 \cup E_2 \cup E_3$, where E_1 is the *XOR* theory, E_2 is the theory *pk-sk* given by the two public key encryption equations $pk(K, sk(K, M)) = M$ and $sk(K, pk(K, M)) = M$, and E_3 is the *AC* theory for each of the state constructors **_,_** and _&_ explained in Section 3. To further complicate the matter, we need to combine not just *untyped* unification algorithms, but typed, and more precisely *order-sorted* ones.

Fortunately, the variant–narrowing–based approach that we use in this paper avoids all these difficulties by obtaining the ($XOR \cup pk\text{-}sk \cup AC$)-unification algorithm as an instance of the *variant narrowing* methodology supported by Maude-NPA. The point is that if an equational theory E has the *finite variant property* [12], then a *finitary* E-unification algorithm can be obtained by *variant narrowing* [21,20], as further explained in Section 5. In our case, the equations in the theory *pk-sk* are confluent and terminating and, furthermore, have the finite variant property. Likewise, the equations in the *XOR* theory presented in Section 3 are confluent, terminating and coherent modulo the *AC* axioms of

$\oplus$ and also have the finite variant property. Finally, the theory of AC for the state-building constructors `_,_` and $\_\&\_$ is of course finitary and can be viewed as a trivial case of a theory with the finite variant property (decomposed with no rules and only axioms). Note that all these three equational theories are disjoint, i.e., they do not share any symbols. The good news is that the following disjoint union theory $XOR \cup pk\text{-}sk \cup AC$ with $\Sigma_{NSL\oplus}$ being the entire (order-sorted) signature of our NSL$\oplus$ protocol example is also confluent, terminating and coherent modulo the AC axioms[5], and satisfy the finite variant property:

1. *Rules*:
   - $pk(K, sk(K, M)) = M$, $sk(K, pk(K, M)) = M$,
   - $X \oplus 0 = X$, $X \oplus X = 0$, $X \oplus X \oplus Y = Y$,
2. *Axioms*: AC for $\oplus$, AC for `_,_` and AC for $\_\&\_$

Therefore, Maude-NPA can analyze the NSL$\oplus$ protocol using variant narrowing. In the following we explain variant narrowing in more detail.

5 Variant Narrowing and Variant Unification

Suppose that an equational theory $\mathcal{E}$ is decomposed according to the following definition.

Definition 1 (Decomposition [20]). *Let $(\Sigma, \mathcal{E})$ be an order-sorted equational theory. We call (Σ, Ax, E) a* decomposition *of $(\Sigma, \mathcal{E})$ if $\mathcal{E} = E \uplus Ax$ and (Σ, Ax, E) is an order-sorted rewrite theory satisfying the following properties.*

1. *Ax is* regular*, i.e., for each $t = t'$ in Ax, we have $Var(t) = Var(t')$, and* sort-preserving*, i.e., for each substitution σ, we have $t\sigma \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$ iff $t'\sigma \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$; furthermore all variables in $Var(t)$ have a top sort.*
2. *Ax has a finitary and complete unification algorithm.*
3. *For each $t \to t'$ in E we have $Var(t') \subseteq Var(t)$.*
4. *E is sort-decreasing, i.e., for each $t \to t'$ in E, each $\mathsf{s} \in \mathsf{S}$, and each substitution σ, $t'\sigma \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$ implies $t\sigma \in \mathcal{T}_{\Sigma}(\mathcal{X})_{\mathsf{s}}$.*
5. *The rewrite rules E are confluent and terminating modulo Ax, i.e., the relation $\to_{E,Ax}$ is confluent and terminating.*
6. *The relation $\to_{E,Ax}$ is Ax-coherent.*

[5] All these conditions are easily checkable. Indeed, coherence modulo the combined AC axioms is immediate, and we can use standard methods and tools to check the local confluence and termination of the combined theory; similarly, the method described in [18] can be used to check the finite variant property of the combined theory. Alternatively, one can use *modular* methods to check that a combined theory satisfies all these properties under certain assumptions: see [34] for a good survey of modularity results for confluence and termination. Likewise, the finite variant property can also be checked modularly under appropriate assumptions, but a discussion of this topic is beyond the scope of this paper.

Given a term t, an E,Ax-*variant* of t is a pair (t', θ) with t' an E,Ax-canonical form of the term $t\theta$. That is, the variants of a term intuitively give us all the irreducible *patterns* that instances of t can reduce to. Of course, some variants are *more general* than others, i.e., there is a natural preorder $(t', \theta') \sqsubseteq_{E,Ax} (t'', \theta'')$ defining when variant (t'', θ'') is *more general* than variant (t', θ'). This is important, because even though the set of E,Ax-variants of a term t may be infinite, the set of *most general variants* (i.e., maximal elements in the generalization preorder up to Ax-equivalence and variable renaming) may be finite.

The intimate connection of variants with $\mathcal{E}$-unification is then as follows. Suppose that we add to our theory decomposition $E \uplus Ax$ a binary equality predicate *eq*, a new constant $\mathtt{tt}$[6] and for each top sort $[s]$ and x of sort $[s]$ an extra rule $\mathtt{eq}(x, x) \to \mathtt{tt}$. Then, given any two terms t, t', if θ is a $\mathcal{E}$-unifier of t and t', then the E,Ax canonical forms of $t\theta$ and $t'\theta$ must be Ax-equal and therefore the pair $(\mathtt{tt}, \theta)$ must be a variant of the term $\mathtt{eq}(t, t')$. Furthermore, if the term $\mathtt{eq}(t, t')$ has a finite set of most general variants, then we are *guaranteed* that the set of most general $\mathcal{E}$-unifiers of t and t' is *finite*.

For any theory $E \cup Ax$ with E confluent, terminating, and coherent modulo Ax, the *folding variant narrowing* of [21] is a general and effective *complete* strategy. Complete both in the sense of computing a complete set of $E \cup Ax$-unifiers, and of computing a minimal and complete set of variants for any input term t.

In the following, we characterize a notion of variant semantics for equational theories.

Definition 2 (Variant Semantics [21]). *Let (Σ, Ax, E) be a decomposition of an equational theory and t be a term. We define the set of variants of t as* $[\![t]\!]^{\star}_{E,Ax} = \{(t', \theta) \mid \theta \in \mathcal{S}ubst(\Sigma, \mathcal{X}), t\theta \to^{!}_{E,Ax} t''$, and $t'' =_{Ax} t'\}$.

Example 1. Let us consider the equational theory $XOR \cup pk\text{-}sk$, which, together with AC for $\mathtt{\_,\_}$ and $\_\&\_$ is used for our NSL⊕ protocol presented in Section 3. This equational theory is relevant because there are none of our previously defined unification procedures directly applicable to it, e.g. unification algorithms for exclusive–or such as [23] do not directly apply if extra equations are added.

For (Σ, Ax, E) a decomposition of $XOR \ \cup \ pk\text{-}sk$, and for terms $t = M \oplus sk(K, pk(K, M))$ and $s = X \oplus sk(K, pk(K, Y))$, we have that $[\![t]\!]^{\star}_{E,Ax} = \{(0, id), \ldots\}$ and

$$\begin{aligned}[\![s]\!]^{\star}_{E,Ax} = \{&(X \oplus Y, id),\\ &(Z, \{X \mapsto 0, Y \mapsto Z\}), (Z, \{X \mapsto Z, Y \mapsto 0\}),\\ &(Z, \{X \mapsto Z \oplus U, Y \mapsto U\}), (Z, \{X \mapsto U, Y \mapsto Z \oplus U\}),\\ &(0, \{X \mapsto U, Y \mapsto U\}), (Z_1 \oplus Z_2, \{X \mapsto U \oplus Z_1, Y \mapsto U \oplus Z_2\}),\\ &(0, \{X \mapsto V \oplus W, Y \mapsto V \oplus W\}), \ldots\}\end{aligned}$$

[6] We extend Σ to $\widehat{\Sigma}$ by adding a new sort Truth, not related to any sort in Σ, with constant $\mathtt{tt}$, and for each top sort [s] of a connected component, an operator $\mathtt{eq}$: [s] × [s] → Truth.

We write $(t_1, \theta_1) \sqsubseteq_{E,Ax} (t_2, \theta_2)$ to denote that variant (t_2, θ_2) is *more general* than variant (t_1, θ_1).

Definition 3 (Variant Preordering [21]). *Let (Σ, Ax, E) be a decomposition of an equational theory and t be a term. Given two variants $(t_1, \theta_1), (t_2, \theta_2) \in [\![t]\!]^{\star}_{E,Ax}$, we write $(t_1, \theta_1) \sqsubseteq_{E,Ax} (t_2, \theta_2)$, meaning (t_2, θ_2) is* more general *than (t_1, θ_1), iff there is a substitution ρ such that $t_1 =_{Ax} t_2\rho$ and $\theta_1{\downarrow}_{E,Ax} =_{Ax} \theta_2\rho$. We write $(t_1, \theta_1) \sqsubset_{E,Ax} (t_2, \theta_2)$ if for every substitution ρ such that $t_1 =_{Ax} t_2\rho$ and $\theta_1{\downarrow}_{E,Ax} =_{Ax} \theta_2\rho$, then ρ is not a renaming.*

Example 2. Continuing Example 1 we have $v_1 = (0, \{X \mapsto U, Y \mapsto U\})$ as a valid variant of s. Also, $v_2 = (0, \{X \mapsto V \oplus W, Y \mapsto V \oplus W\})$ is a valid variant of s but clearly $v_2 \sqsubseteq_{E,Ax} v_1$, and thus v_2 should not be included in the most general set of variants. On the other hand for $u_1 = (X \oplus Y, id)$ and $u_2 = (Z, \{X \mapsto 0, Y \mapsto Z\})$, we have that neither $u_1 \sqsubseteq_{E,Ax} u_2$ nor $u_2 \sqsubseteq_{E,Ax} u_1$ hold.

We are, indeed, interested in equivalence classes for variant semantics and provide a notion of semantic equality, written $\simeq_{E,Ax}$, based on $\sqsubseteq_{E,Ax}$.

Definition 4 (Variant Equality [21]). *Let (Σ, Ax, E) be a decomposition of an equational theory and t be a term. For $S_1, S_2 \subseteq [\![t]\!]^{\star}_{E,Ax}$, we write $S_1 \sqsubseteq_{E,Ax} S_2$ iff for each $(t_1, \theta_1) \in S_1$, there exists $(t_2, \theta_2) \in S_2$ s.t. $(t_1, \theta_1) \sqsubseteq_{E,Ax} (t_2, \theta_2)$. We write $S_1 \simeq_{E,Ax} S_2$ iff $S_1 \sqsubseteq_{E,Ax} S_2$ and $S_2 \sqsubseteq_{E,Ax} S_1$.*

Despite the previous semantic notion of equivalence, the following, more syntactic notion of equality of variants up to renaming is useful.

Definition 5 (Ax-Equality [21]). *Let (Σ, Ax, E) be a decomposition of an equational theory and t be a term. For $(t_1, \theta_1), (t_2, \theta_2) \in [\![t]\!]^{\star}_{E,Ax}$, we write $(t_1, \theta_1) \approx_{Ax} (t_2, \theta_2)$ if there is a variable renaming ρ such that $t_1\rho =_{Ax} t_2\rho$ and $\theta_1\rho =_{Ax} \theta_2\rho$. For $S_1, S_2 \subseteq [\![t]\!]^{\star}_{E,Ax}$, we write $S_1 \approx_{Ax} S_2$ if for each $(t_1, \theta_1) \in S_1$, there exists $(t_2, \theta_2) \in S_2$ s.t. $(t_1, \theta_1) \approx_{Ax} (t_2, \theta_2)$, and for each $(t_2, \theta_2) \in S_2$, there exists $(t_1, \theta_1) \in S_1$ s.t. $(t_2, \theta_2) \approx_{Ax} (t_1, \theta_1)$.*

The preorder of Definition 3 allows us to provide a most general and complete set of variants that encompasses all the variants for a term t.

Definition 6 (Most General and Complete Variant Semantics [21]). *Let (Σ, Ax, E) be a decomposition of an equational theory and t be a term. A* most general and complete variant semantics *of t, denoted $[\![t]\!]_{E,Ax}$, is a subset $[\![t]\!]_{E,Ax} \subseteq [\![t]\!]^{\star}_{E,Ax}$ such that: (i) $[\![t]\!]^{\star}_{E,Ax} \sqsubseteq_{E,Ax} [\![t]\!]_{E,Ax}$, and (ii) for each $(t_1, \theta_1) \in [\![t]\!]_{E,Ax}$, there is no $(t_2, \theta_2) \in [\![t]\!]_{E,Ax}$ s.t. $(t_1, \theta_1) \not\approx_{Ax} (t_2, \theta_2)$ and $(t_1, \theta_1) \sqsubseteq_{E,Ax} (t_2, \theta_2)$.*

Example 3. Continuing Example 1 it is obvious that the following variants are most general w.r.t. $\sqsubseteq_{E,Ax}$: $[\![t]\!]_{E,Ax} = \{(0, id)\}$ and

$$\begin{aligned}\llbracket s\rrbracket_{E,Ax} = \{&(X\oplus Y, id),\\ &(Z,\{X\mapsto 0, Y\mapsto Z\}),(Z,\{X\mapsto Z, Y\mapsto 0\}),\\ &(Z,\{X\mapsto Z\oplus U, Y\mapsto U\}),(Z,\{X\mapsto U, Y\mapsto Z\oplus U\}),\\ &(0,\{X\mapsto U, Y\mapsto U\}),(Z_1\oplus Z_2,\{X\mapsto U\oplus Z_1, Y\mapsto U\oplus Z_2\})\}.\end{aligned}$$

Note that, for any term t, $\llbracket t\rrbracket^{\star}_{E,Ax} \simeq_{E,Ax} \llbracket t\rrbracket_{E,Ax}$ but, in general, $\llbracket t\rrbracket^{\star}_{E,Ax} \not\approx_{Ax} \llbracket t\rrbracket_{E,Ax}$. Also, by definition, all the substitutions in $\llbracket t\rrbracket_{E,Ax}$ are E,Ax-normalized. Moreover, $\llbracket t\rrbracket_{E,Ax}$ is unique up to $\approx_{Ax}$ and provides a very succinct description of $\llbracket t\rrbracket^{\star}_{E,Ax}$. Indeed, up to Ax-equality, $\llbracket t\rrbracket_{E,Ax}$ characterizes the set of *maximal elements* (therefore, most general variants) of the preorder $(\llbracket t\rrbracket_{E,Ax}, \sqsubseteq_{E,Ax})$.

Again, let us make explicit the relation between variants and $\mathcal{E}$-unification.

Proposition 1 (Minimal and Complete $\mathcal{E}$-unification [21]). *Let (Σ, Ax, E) be a decomposition of an equational theory $(\Sigma, \mathcal{E})$. Let t, t' be two terms. Then, $S = \{\theta \mid (\mathtt{tt}, \theta) \in \llbracket \mathtt{eq}(t,t')\rrbracket_{\widehat{E},Ax}\}$ is a minimal and complete set of $\mathcal{E}$-unifiers for $t = t'$, where* eq *and* tt *are new symbols defined in Footnote 6 and $\widehat{E} = E \cup \{\mathtt{eq}(X,X) \to \mathtt{tt}\}$.*

The *finite variant property* defined by Comon-Lundh and Delaune [12], provides a useful sufficient condition for finitary $\mathcal{E}$-unification. Essentially, it determines whether every term has a finite number of most general variants.

Definition 7 (Finite variant property [12]). *Let (Σ, Ax, E) be a decomposition of an equational theory $(\Sigma, \mathcal{E})$. Then $(\Sigma, \mathcal{E})$, and thus (Σ, Ax, E), has the* finite variant property *iff for each term t, the set $\llbracket t\rrbracket_{E,Ax}$ is finite. We will call (Σ, Ax, E) a* finite variant decomposition *of $(\Sigma, \mathcal{E})$ iff (Σ, Ax, E) has the finite variant property.*

In [19] a technique is proposed to check whether an equational theory has the finite variant property. Using this technique it is easy to check that Example 1 has the finite variant property, as every right–hand side is a constant symbol or a variable. See [19, Example 2] for more details.

Finally, it is clear that when we have a finite variant decomposition, we also have a finitary unification algorithm.

Corollary 1 (Finitary $\mathcal{E}$-unification [21]). *Let (Σ, Ax, E) be a finite variant decomposition of an equational theory $(\Sigma, \mathcal{E})$. Then, for any two given terms t, t', $S = \{\theta \mid (\mathtt{tt}, \theta) \in \llbracket \mathtt{eq}(t,t')\rrbracket_{\widehat{E},Ax}\}$ is a* finite, minimal, and complete *set of $\mathcal{E}$-unifiers for $t = t'$, where $\widehat{E}$,* eq*, and* tt *are defined as in Proposition 1.*

Note that the opposite does not hold: given two terms t, t' that have a finite, minimal, and complete set of $\mathcal{E}$-unifiers, the equational theory $(\Sigma, \mathcal{E})$ may not have a finite variant decomposition (Σ, Ax, E). An example is the unification under homomorphism (or one-side distributivity), where there is a finite number of unifiers of two terms but the theory does not satisfy the finite variant property (see [12,19]); the key idea is that the term $\mathtt{eq}(t, t')$ may have an infinite number of variants even though there is only a finite set of most general variants of the form $(\mathtt{tt}, \theta)$. We refer the reader to [21] for further information.

6 Finding attacks modulo $\boldsymbol{XOR} \cup \boldsymbol{pk\text{-}sk} \cup \boldsymbol{AC}$ using Maude-NPA

We have analyzed the NSL⊕ protocol presented in Section 3 modulo its equational theory $XOR \cup pk\text{-}sk \cup AC$ in Maude-NPA using variant narrowing.

We now explain in more detail all the operations available to the intruder. Its capabilities are all given in strand notation. Note that we are omitting the position marker | which is assumed to be at the beginning.

(s1) $[(X)^-, (Y)^-, (X;Y)^+]$ Concatenation
(s2) $[(X;Y)^-, (X)^+]$ Left-deconcatenation
(s3) $[(X;Y)^-, (Y)^+]$ Right-deconcatenation
(s4) $[(X)^-, (Y)^-, (X \oplus Y)^+]$ Exclusive–or
(s6) $[(X)^-, (sk(i, X))^+]$ Encryption with i's private key
(s7) $[(X)^-, (pk(A, X))^+]$ Encryption with any public key
(s8) $[(0)^+]$ Generate the exclusive–or neutral element
(s9) $[(A)^+]$ Generate any principal's name.

The attack state pattern from which we start the backwards narrowing search in this example is given by one strand, representing Bob (b) wanting to communicate with Alice (a)

$$:: r :: [(pk(b, X; a))^-, (pk(a, X; b \oplus n(b, r)))^+, (pk(b, n(b, r)))^- | nil]$$

together with requiring the intruder (i) to have learned Bob's nonce, i.e., $n(b, r) \in \mathcal{I}$. What this represents is an attack in which Bob has properly executed the protocol and believes to be talking to Alice, while the intruder has obtained the nonce that Bob created and that he considers a secret shared between Alice and him.

See Figure 1 for a pictorial representation of the strand space and messages sent and received, depicting the attack found by Maude-NPA. This attack agrees with the one described in [8]. The figure has been created with the help of the Maude-NPA GUI [35], with the exclusive–or symbol ⊕ textually represented as $*$ in the figure. Let us explain here how the attack proceeds, based on Figure 1.

7 Related Work

There is a substantial amount of research on formal verification of cryptographic protocols. Much of it abstracts away from any equational theories obeyed by the cryptographic operators, but there is a growing amount of work addressing this problem. The earliest was the NRL Protocol Analyzer [30], which, like Maude-NPA, was based on unification and backwards search, implemented via narrowing over confluent equational theories. This was sufficient to handle, for example, the cancellation of encryption and decryption, although there were many theories of interest it did not address, such as exclusive-or and other Abelian group operators.

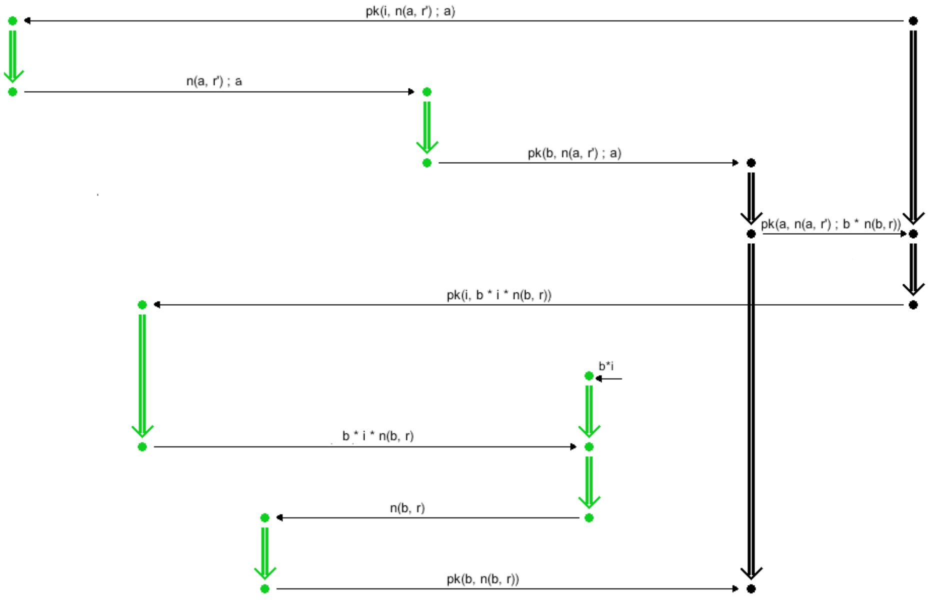

Fig. 1. Pictorial representation of the initial state, leading to an attack

More recently, tools have begun to offer support for specification and, to some degree, analysis of protocols involving equational theories. These tools include, for example, ProVerif [6], OFMC [4], and CL-Atse [38]. Both OFMC and CL-Atse work in the bounded session model, while ProVerif uses abstraction and unbounded sessions. Both OFMC and CL-Atse support exclusive-or and Diffie-Hellman exponentiation. ProVerif can also be used to analyze these, but the equational theories it is known to work well with are more limited, e.g. not supporting associativity-commutativity or Diffie-Hellman exponentiation. However, Küsters and Truderung [26,27] have developed algorithms that, can translate protocols using exclusive-or or Diffie-Hellman exponentiation to protocols that can be analyzed by ProVerif in a free algebra model; for exclusive-or they can handle protocols satisfying the ⊕-linearity property. According to a study by Lafourcade et al. [28], this produces analysis times that are only slightly slower than analyses by OFMC and CL-Atse, mainly because of the translation time.

There is also a growing amount of theoretical work on cryptographic protocol analysis using equational theories, e.g. [1,9,7,11,5]. This concentrates on the decidability of problems of interest to cryptographic protocol analysis, such as deducibility, which means that it is possible (e.g. for an intruder) to deduce a term from a set of terms, and static equivalence, which means that an intruder cannot tell the difference between two sets of terms. However, there is much less work on the combination of different theories, although Arnaud, Cortier, and Delaune [13] have considered the problem in terms of decidability of the problem for combination of disjoint theories, showing that if any two disjoint theories have decidable static equivalence problems, then so does their combination. More recently Chevalier and Rusinowitch analyze the security of cryptographic protocols via constraint systems and have also studied composition of theories. In [10], they give a general method for combining disjoint theories that is based on the Baader-Schulz combination algorithm for unification algorithms for different theories [3]. This can be thought of as a constraint-based analogue of the Maude-NPA combination framework we describe in [15], which is also based on the Baader-Schulz combination algorithm [3].

8 Conclusions and Future Work

To gain high assurance about cryptographic protocols using formal methods requires reasoning modulo the algebraic properties of the underlying cryptographic functions. In symbolic analyses this typically necessitates performing unification *modulo* such algebraic properties. However, since a protocol may use a variety of different functions —so that different protocols typically require reasoning modulo different theories— it is unrealistic to expect that a fixed set of unification algorithms will suffice for such analyses. That is, *combination methods* that obtain unification algorithm for a composition of theories out of a family of such algorithm for each of them, are unavoidable. Standard methods for obtaining a unification algorithm for a combined theory $E_1 \cup \ldots \cup E_n$ [3] are computationally costly due to the high degree of non-determinism in the combination method; furthermore, they require the existence of a unification algorithm for each individual theory E_i, which in practice may not be available in a tool's infrastructure. In this work we have proposed an alternative method based on *variant narrowing* to obtain a $(E_1 \cup \ldots \cup E_n)$-unification algorithm under simpler requirements. Specifically, dedicated implementations of unification algorithms for each of the theories E_i are not needed: in our example, only a dedicated *AC*-unification algorithm was used: no dedicated algorithms for *XOR* of *pk-sk* were needed. Furthermore, even though narrowing is less efficient than a dedicated algorithm for each individual theory E_i, the costly computational overhead of a standard combination method is avoided. The case study presented has shown that variant narrowing, as supported by the Maude-NPA, is indeed an effective method to deal with nontrivial combinations of equational theories; and for analyzing many protocols with even a modest infrastructure of built-in unification algorithms. The case study was chosen as a well-known protocol for illustration purposes, but many other examples could have been given.

We should emphasize that standard combination methods such as those described in [3], and the alternative variant narrowing method presented here are not "rival" methods. Instead they are highly *complementary* methods which, when used *in tandem*, allow a tool to analyze a much wider range of protocols than those analyzable by each method in isolation. Let us use our example theory $XOR \cup pk\text{-}sk \cup AC$ to illustrate this important point. Variant narrowing decomposed this combined theory into: (i) three rewrite rules for *XOR* and two rewrite rules for *pk-sk* plus, (ii) three instances of *AC*: one for $\oplus$, another for

$\_,\_$ and another for $\_\&\_$. That is, variant narrowing with the rules in (i) was performed *modulo* the axioms in (ii). But the axioms in (ii) are themselves a *combined theory* (in fact, also combined with all the other function symbols in the protocol specification as free function symbols). The Maude infrastructure used by Maude-NPA has in fact used an order-sorted version of a standard combination method in the style of [3] to support unification with the combined axioms of (ii). Therefore, the advantage of using standard combination methods and variant narrowing in tandem is the following:

1. A given tool infrastructure can only have a finite number of predefined (finitary) unification algorithms for, say, theories $T_1, \ldots, T_k$; however, it should also be able to support any combination of such built-in theories by a standard combination method.
2. A given protocol may require performing unification modulo a combination of theories $E_1 \cup \ldots \cup E_n$, but some of the E_i may not belong to the library $T_1, \ldots, T_k$, so that the standard combination method cannot be used.
3. However, if $E_1 \cup \ldots \cup E_n$ can be *refactored* as a theory decomposition (Σ, B, R) that: (i) it has the finite variant property; and (ii) B is a combination of the theories $T_1, \ldots, T_k$ supported by the current library, then a *finitary* $(E_1 \cup \ldots \cup E_n)$-unification algorithm can be obtained by variant narrowing.

A very important direction for future work in formal tools supporting symbolic protocol analysis modulo equational properties consists in: (i) developing methods for expanding a tool's built-in unification infrastructure as described in (1) above to make it as efficient and extensible as possible; and (ii) improving and optimizing the methods for efficient variant narrowing modulo such infrastructure. Good candidates for new theories T_j to be added to the built-in infrastructure include commonly used theories, with high priority given to theories that lack the finite variant properties. For example, in [15] the theory of homomorphic encryption, which lacks the finite variant property, has been recently added to Maude-NPA for exactly this purpose.

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