$$
\begin{array}{ccc}
\tilde{H}_0(X) & \xrightarrow{\ \zeta\ } & H_0(X) \\
\downarrow{\scriptstyle f_*} & & \downarrow{\scriptstyle f_*} \\
\tilde{H}_0(Y) & \xrightarrow{\ \zeta\ } & H_0(Y)
\end{array}
\qquad
\begin{array}{ccc}
H_0(X) & & \\
& \searrow{\scriptstyle \varepsilon_*} & \\
\downarrow{\scriptstyle f_*} & & \mathbb{Z} \\
& \nearrow{\scriptstyle \varepsilon_*} & \\
H_0(Y) & &
\end{array}
$$