Fundamentals of Quantitative Modeling

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Contents

Code List

Chapter 1

Introduction

1.1 Topics

Some of the topics covered in this module include:

- Exposure to the language of modeling.
- Exploration of different types of models used in business and how to apply them in practice.
- Process of modeling.
- Characteristics of the models.
- Value and limitations of quantitative models. What sort of things they can and cannot do. Understand the limitations of the models.
- Provide a set of foundational for other courses in specialization.

Which model should I use?

Process: map the characteristics of the business into the characteristics of the model.

1.2 Definition, Uses of a Model, Common Functions

In the business context, the models we are talking about are not physical models (different from the idea an arquitect has). We are talking about a **formal description of a business process**.

This description is going to involve a set of mathematical equations and/or random variables.

Observation 1.2.1
A quantitative model it is almost always a simplification of a more complex structure (in particular, of a business process). We do not want to over simplify, but we also do not want to overcomplicate.

Turns out it's going to be really difficult to make an exact and acurate representation of what we really want to model.

Also, there is a set of **assumptions** that underly the model. We have to check weather these assumptions are reasonable or not in our business process.

Observation 1.2.2
A model is usually implemented in Excel or a spreadsheet tool like Google Sheets (or using programming langauges as R).

In a more mathematical way, we can define a model as follows:

Definition 1.2.1 (Model)
A model is a triplet $M = (X, \Theta, F)$, where:

- X is a set of measurable input variables (data).
- Θ is a set of parameters and,
- $F : X \times \Theta \to Y$ is a function, that maps input and parameters to measurable output in Y.

Let's give some examples:

Example 1.2.1
Some examples of models are the following:

- Determine the price of a diamond given its weight.
- The spread of an epidemic over time.
- Relationship between demand for, and price of a product.
- The uptake of a new product in market.

In particular, if I have a product and I want to maximize the gains we acquire when selling it, how can we achieve this goal?

All of these examples are from different areas, but can be addressed by using a quantitative model.

1.2.1 Diamonds and Weight

Let's consider the weight of a diamond and the price it is going to have. We have the following equation that models the price of a diamond given its weight:

$$p(w) = -260 + 3721w$$

This is a linear model. The price is given in dolars and the weight is given in carats. Sometimes we use a visual representation like this:

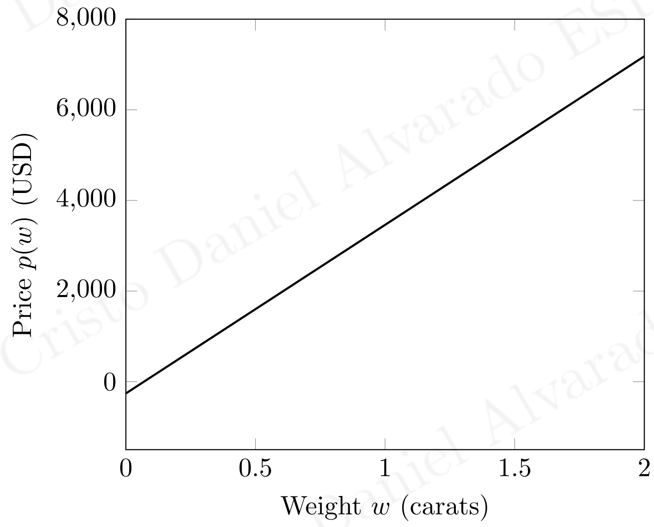

Figure 1.1: Model of the Price of a Diamond

Basically, this model allows us to forcast the price of a certain object given certain properties of it or *data about it*. This is a **linear model**.

Observation 1.2.3
A model doesn't necesarly has to work in all the circustamces. For example, the model given to obtain the price of a diamond doesn't necesarly has to be applicable to a different company.

1.2.2 Spreading of an Epidemic

Depending on the disease and other factors (such as time in which the epidemc ocurred), one model arises.

One of the basic models to start with is the **exponential model**. On the bottom axis we have weeks (since the start of the epidemic), and on the vertical axis we have the number of cases reported.

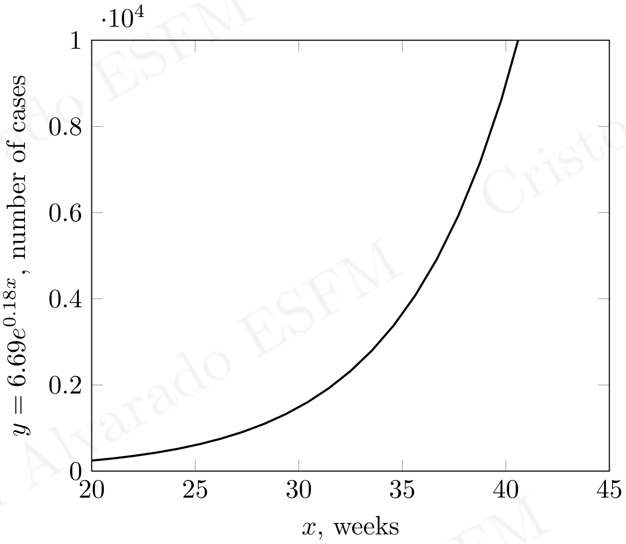

Figure 1.2: Exponential Model for the spread of an epidemic.

The equation to describe this model is:

$$y = 6.69e^{0.18x}$$

Observation 1.2.4
This model is suitable for the start of the pandemic, since there is not an unlimited number of people, we cannot expect that the growth of the graph is unstoppable.

Observation 1.2.5 (Exponential Graphs)
In the context of business they are called **hockey sticks**.

This model is not suitable for a long term, but its suitable for an approximation over the first weeks of the pandemic.

1.2.3 Price and Demand

Let's suppose a situation of price and demand (very common in the industry). Demand models basically tell when we know the price of a certain product, the quantity of it available in the market.

Observation 1.2.6 (Possitive Association)
The graphs in the first two examples are said to have **positive association**.

For products, when there is a huge amount of product, the price decreases and, when the price is too hight, the amout available is too low. The model:

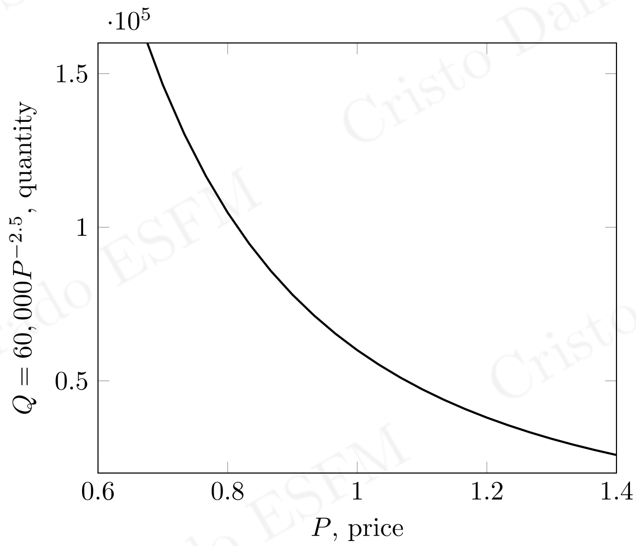

Figure 1.3: Price and Demand Model

This model is given by the equation:

$$Q = 60,000P^{-2.5}$$

In this model we use a power function (which is basically the exponential function in essence, but many people who doesn't have mathematical background calls is *power function*).

Idea 1.2.1
One use of this model is to find what the optimal price for a product should be.

1.2.4 The Uptake of a Product

Definition 1.2.2 (Uptake)
The **uptake of a product**, often referred to as **product adoption** or **market uptake**, is *the process by which individuals or organizations learn about, start using, and ultimately integrate a new product or service into their routine life or business operations.*

The following describes the uptake of a product given the amount of years it has been on the market.

This particular function is called a logistic function, and is given by:

$$P = \frac{e^{2(Y-2.5)}}{1 + e^{2(Y-2.5)}}$$

Here, Y is the number of years and, P is the proportion of target population with product.

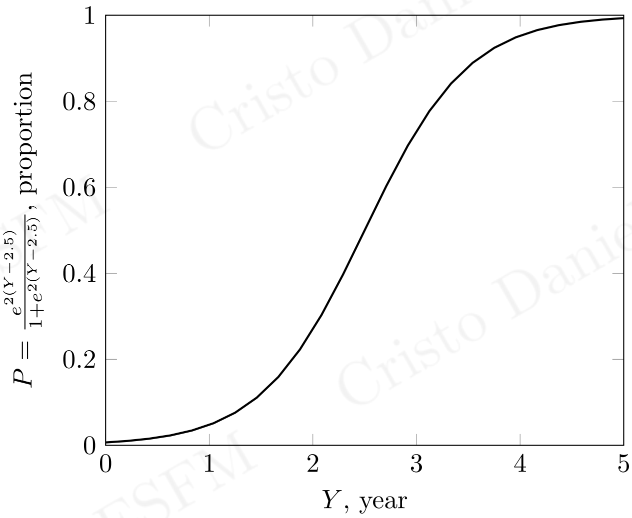

Figure 1.4: Model of the Uptake of a Product

Observation 1.2.7
This model has the potential to model a process where at the first stages we see a slow start (here, it's when target people start adopting the product), and then it has a fast grow that proceeds to slow as time goes by.

1.3 Use of Models in Practice

There are four main usages of a model in the business enviroment:

- **Prediciton**.
- **Forecasting**.
- **Optimization**.
- **Ranking and targeting**.

1.3.1 Prediction

Once we got a quantitative model is **prediction**. Basically is taking a model, puting an input and create a prediction.

Observation 1.3.1
The use of most quantitative models is for predictive analysis.

1.3.2 Forecasting

When talking about forecasting we are talking about time series.

Definition 1.3.1 (Forecasting)
Forecasting is the *process of predicting future developments by analyzing past and present data and trends.*

Example 1.3.1
For example, with the models presented earlier, we can ask the following questions:

- How many people are expected to be infected in 6 weeks?
- Using a scheduling model, who is likely to turn up for their outpatient appointment?

This activity has often a lot to do in businesses and to do with resource planning.

1.3.3 Optimization

How to maximize or minimize something?

1.3.4 Ranking and targeting

When selling a product, we may be interested in which ones we would like to purchase. Since we cannot have a look at all the diamonds in the world, **identify targets of opportunity**. This is a ranking and targeting excersise.

Or, for example, if I am interested in buying, is useful to create a model to help classify places which I can buy.

1.4 How are models used in practice

We use models in real-life scenarios in order to:

- Exploring what if scenarios (this is **called scenario planning**).
- Interpreting the coefficients in a model (what's the meaning behind **coefficients in model equations?**).
- Assesting how sensitive the model is to key-assumptions.

Definition 1.4.1 (Sensitivity Analysis)
A **sensitivity analysis** is the *process where we check how the output of a model is sensitive to some of the assumptions made at the begining.*

Benefits of Modeling
Identify gaps in current understanding
Make assumptions explicit
Have well-defined description of the business process
Create an institution memory
Used as a decision **support** tool
Serendipitous insight generator

Table 1.1: Benefits of Modeling

1.5 Key Steps in the Modeling Process

Every model is different but they share common features in the way the model was created (the workflow).

1. The first steps is always to **stablish what the model is trying to predict**, and what are the **underlying variables** of it (which will help us to predict)?
   What is the scope of the model? Which is, **where is it going to be applied?**.

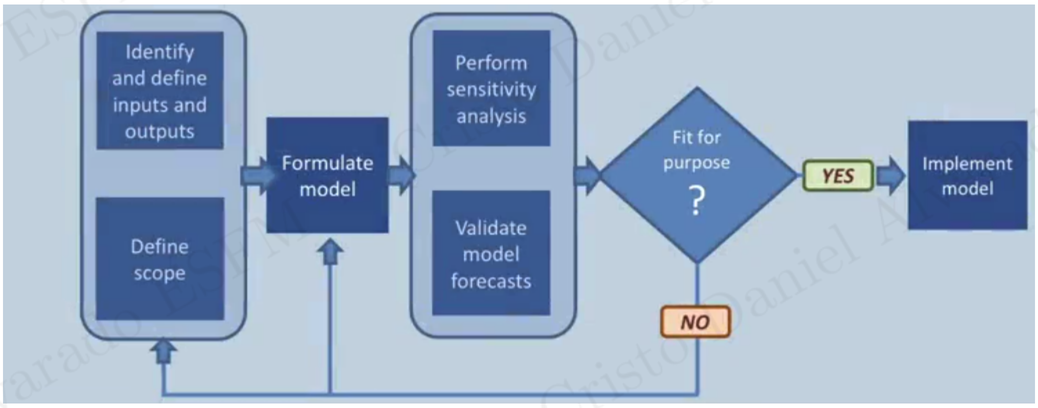

Figure 1.5: Overview of the Modeling Process

2. **Formulate the model**. Its the step where we undestand the business process and al the mathematical ideas come into the process.
3. **Perform a sensitivity analysis** and validate **model forecast**.
4. Is the model fit for purpose? Is it suitable for its purpose?

Observation 1.5.1
Some teacher said: some wrong models are useful.

If something went wrong, we start over again. If the model accomplish its purpose, then we implement the model.

Idea 1.5.1
There is always a learning when a model doesn't work properly. We gain insight into what work and what doesn't work in the process.

Modeling is an iterative and evolutionary process.

1.6 Vocabulary for Modeling

1.6.1 Data Driven vs. Theory Driven

There are basically two types of models: theoretical and data-driven. It's more like an spectrum than a certain type of model.

- **Theory**: Given a set of assumptions and relations, then what are the logical consecuences?
- **Data**: Given a set observations, how can we aproximate the underlying process that generated them?

Example 1.6.1
These are two examples of questions that define a model:

- **Theory**: If we assume the markets are efficient, then what should the price of a stock option be?
- **Data**: I've separated out my profitable customers from the unprofitable ones. Now, what features are able to differentiate them?

1.6.2 Deterministic vs Probabilistic

Given a set of inputs, the model always gives the same output or is it different instance from instance? This is the main difference between a deterministic and a probabilistic model, respectively.

1.6.3 Static vs Dynamic

So, in a static model, the model captures a single snapshot of the business process.

Example 1.6.2
Given a website's installed software base, what are the chances that it is compromised today?

And dynamic, in which the evolution of the process itself is of interest. The model describes the movement from state to state.

Example 1.6.3
Given a person's participation in a job training program, how long will it take until he/she finds a job, and, if they find one, for how long will they keep it?

Chapter 2

Linear Models and Optimization

This chapter is dedicated to linear models and optimization.

2.1 Introduction to Linear Models

As stated earlier, in determinstic models we do not have random or uncertain components, so we always have the same output given the same input.

Observation 2.1.1
Due to the fact that we do not have a random component, it´s very hard to asses the uncertainty in the outputs.

As we remember from earlier examples, a linear model (1-imensional) is a model given by the equation:

$$y = mx + b$$

The slope is the constant m, and b is the intercept with the y-axis.

Idea 2.1.1
These kind of models can work in certain circumstances, but may not be ideal every time.

Example 2.1.1 (**Linear Cost Function**)
Let C be the cost of producint q units of a product. If we have a process that must start with 100 dolars cost, then the cost function is given by:

$$C(q) = 100 + 30q$$

where 30 is the cost to produce each unit of the product q.

Observation 2.1.2
The constant b in the latter example is called **fixed cost**. Every time we produce a product, we must have to pay some amount of money, which is independent of the number of units produced.

Example 2.1.2 (**Time to Produce Function**)
If it takes 2 hours to set up a production run, and each incremental unit produced always takes an additional 15 minutes, then the time to produce q units is given by:

$$T(q) = 2 + \frac{15}{60}q$$

Here, $\frac{15}{60}$ can be interpreted as the **work rate** to produce each unit of the product.

Some notes on the latter examples are the following:

1. We are given a description of the process which we have to model, so it´s our work to find the variables and constants that represent the process in the model, describe them and interpret them.
2. In all of the examples, we have the constants involved in the process told to us, but in other cases we may need to find them using data or other information.

Observation 2.1.3 (**Linear Programming**)
Linear Programming is a *method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationships.*

It is used to solve optimization problems, and is one of the work horses of **operations research**.

Linear programming implements something called constraints. When we try to optimize a process, the meaning behind it is to find the best possible solution that satisfies all of the constraints.

Constraints are ideas that we can incorporate into our models to **make them more realistic**.

Example 2.1.3
For example, if we are trying to optimize a process, *we may have constraints* on the amount of resources available, or the maximum number of units that can be produced.

2.2 Growth and Decay (discrete and continuous time)

Growth is a fundamental business concept. Many quantities in business grow over time, such as investments, populations of customers, production capacities, etc.

- The number of customers at a time t.
- The revenue in a quarter q.
- The value of an investment at some time t in the future.

Sometimes, a linear model may be appropiate for a growth process, but an alternative to a linear growth model is a **proportionate** one.

Definition 2.2.1 (**Proportionate Growth**)
Proportionate growth is a *constant percent increase (or decrease) from one period to another.*

2.3 Simple Interest

It is basically interest calculated only on the initial amount (the principal), so the interest does not compound.

Year	0	1	2	3	4	5
Amount	\$1000	\$1100	\$1200	\$1300	\$1400	\$1500

2.4 Compound Interest

It is interest calculated on the initial principal, which also includes all of the accumulated interest from previous periods on a deposit or loan.

Year	0	1	2	3	4	5
Amount	\$1000	\$1100	\$1210	\$1331	\$1464.10	\$1610.51

The growth is no longer the same absoulte amount each year, but it is the same proportionate amount (relative) of 10%.

Observation 2.4.1 (**Geometric Progression**)
Lets understand the model. The growth progression is given by:

Year	0	1	2	3	$\cdots$	t
Amount P_t	P_0	$P_0\vartheta$	$P_0\vartheta^2$	$P_0\vartheta^3$	$\cdots$	$P_0\vartheta^t$

Here, P_0 is the initial amount. And:

- If $\vartheta > 1$, then the process is growing.
- If $\vartheta < 1$, then the process is decaying.

This is called a **geometric series** or **geometric progression**.

In this model, we are using a discrete time, because we are only looking at a period of zero, one, two, three, etc...

Example 2.4.1
Lets consider the following example:

- An Indian Ocean nation caught 200,000 tones of fish this year.
- Catch is projected to fall by a constant 5% factor each year for the next 10 years.
- How many fish are predicted to be caught 5 years from now?
- Including this year, what is the total expected catch over the next 5 years?

The model for the amount of fish caught after t years is given by:

$$P_t = 200,000 \times 0.95^t,$$

where t is the time measured in years.

Observation 2.4.2
This particular quantitative model has some nice properties. As we know:

$$S_t = \sum_{n=0}^{t} P_t = P_0 + \cdots + P_t = P_0 \times \left(1 + \vartheta + \cdots + \vartheta^t\right) = P_0 \times \frac{1 - \vartheta^{t+1}}{1 - \vartheta}$$

This sum diverges if $\vartheta \geq 1$ and converges to the finite limit $\frac{P_0}{1-\vartheta}$ if $0 < \vartheta < 1$.

2.5 Present and Future Values

Lets get into some examples.

Example 2.5.1 (Present and Future Value Calculation)
If there is no inflation and the prevaling interest rate is 4% per year, then which of the following options would yoy prefer?

- Receive \$1,000 now.
- Recieve \$1,500 in ten years

In this example we must decide between to investment options. To compare them, we can compute the **future value** of the first option or the **present value** of the second option.

Solution:
To solve this problem, we just have to compute either the future value of \$1,000 or the present value of \$1,500. Lets compute the present value of \$1,000. We know that:

$$P_t = \$1,500 = P_0 \times \vartheta^{10}$$

where $\vartheta = 1.04$. So:

$$P_0 = \frac{\$1,500}{(1.04)^{10}} = \$1,013.53$$

which is bigger than \$1,000. So, it is better to receive \$1,500 in ten years. □

2.5.1 Uses of Present Value

Present value is used:

- In **discounting investments** to the present time.
- Lifetime **customer value** calculations.
- **Present value is also used in lifetiem customer value calculations.**.

One example to use this present value is in annuitys.

Definition 2.5.1 (Annuity)
An **annuity** is a *schedule or fixed payments over a specified and finite time period.*

The present value of an annuity is the sum of the present values of each separate payment.

2.5.2 Continuous Compounding

When compunding, we have a choice of the compounding period.

Observation 2.5.1
Typically, we talk about a yearly basis in compounding.

But, this can be done monthly, daily, or even continuously. In this scenario, we have a neat formula of this model.

Idea 2.5.1 (Exponential Growth)
If a principal amount P_0 of money is continuously componding at a nominal annual interest rate of $\%R$, then at year t the amount will be given by:

$$P_t = P_0 e^{rt},$$

where $r = \frac{R}{100}$.

This is called **exponential growth** or **exponential decay**.

Example 2.5.2 (**Continuous Compunding**)
In this model, t can take on any value in an interval, whereas in the discrete model, t could only take on disctinct values (like integers).

Lets suppose we have $1,000 continuously compounded at a nominal annual interest rate of 4%. How much is this worth after 1 year?

Solution:
The answer is given by pluging t into the equation:

$$P_1 = 1,000 \times e^{\frac{4}{100}} = 1,000 \times e^{0.04} = 1,040.81$$

□

Observation 2.5.2 (**Continuous Compounding vs Discrete Compounding**)
This is a little more than if the 4% was earned at the very end of the time period, in which case you would have exactly $1,040 at the end of the year.

Example 2.5.3
Also, continuous compounding can describe the exponential growth or decay, depending on whether r is positive or negative, respectively.

A continuous time model for the initial stages of an epidemic states that the number of cases at week t is $15e^{0.15t}$.

Here, 0.15 can be interpreted as the **growth rate** of the epidemic, which is the poercent change from cases week to week.

2.6 Classical Optimization

The classical tool to optimize is Calculus. Business always try to optimize their performance in some way.

2.6.1 Optimization of the Price of a Product

Lets get into the scenario where we want to optimize the price of a product in order to get maximum profits. For this goal, lets consider the demand model:

$$Q = 60,000P^{-2.5}$$

where Q is the quantity of the product, and P is the price of the product (this is a relation between price and quantity). This model is assuming we will sell Q products and each product will be sold at a price of P.

With this in mind, we may have the following question:

If the price of production is constant at $c = 2$ for each unit, then at what price is profit maximized?

- In this case, profit is the revenue minus the cost of production. So, in this case, profit is the price for which an item is sold minus the cost of production.

$$E = R - C,$$

where R is the revenue, C is the cost, and E is the profit. We want to maximize E.

- In this scenario, the revenue R is given by:

$$R = Q \times P$$

because, revenue can be writen as the price we sell a product times the quantity sold.

- The cost is going to he the cost of production of each product c, times the number of units produced, or the quantity:

$$C = Q \times c$$

- So,

$$E = Q(P - c) = 60,000P^{-2.5}(P - 2)$$

Idea 2.6.1
To solve this problem, we must find the derivative of E with respect to P (to find minimum and maximum values of E), and then check whether these are minimum or maximum.

Solution:
The derivative of E with respect to P can be computed rapidly, and evaluating the values using the second derivative rule we find that the price that maximizes profit is given by:

$$P_{opt} = \frac{5}{3}c \approx 3.33$$

□

Observation 2.6.1 (Elasticity in Demand)
Using calculus one finds that the optimal value of price is:

$$P_{opt} = \frac{cb}{1 + b}$$

Here, b is called the price **elasticity of demand**. In the latter example, $b = -2.5$. This means that as we increase prices by 1%, the quantity demanded decreases by 2.5%.

Chapter 3

Probabilistic Models

3.1 Introduction to Probabilistic Models

Deterministic models assume that outcomes are precisely determined through known relationships among states and events. However, many systems in the real world exhibit inherent randomness and uncertainty. To effectively model and analyze such systems, we turn to **probabilistic models**.

Definition 3.1.1 (**Probabilistic Model**)
A **probabilistic model** is a model that incorporates *random variables* and *probability distrubutions*.

In a a probabilistic model:

- **Random variables** represent the potential outcomes of uncertain events.
- **Probability distrubutions** assign probabilities to the various potential outcomes.

Observation 3.1.1 (**Random Variables and Probability Distrubutions**)
In simple terms:

- A **random variable** *represents the potential outcomes of a certain event.*
- A **probability distrubution** is an *assignation of a probability to a certain outcome*.

Idea 3.1.1
By incorporating the uncertainty explicitly in the model, we can measure the uncertainty asociated with the outputs.

For example, by giving a range to a forecast, which is a more realistic goal than giving just a single outcome.

Definition 3.1.2 (**Forecast**)
Forecast is a *prediction or estimate of future events, especially coming weather or a financial trend.*

Observation 3.1.2 (**Uncertainty in Business Processes**)
In business, by incorporating **uncertainty** is synonymus with understanding and quantifying the **risk** in a business process, and ideally leads to better management decisions.

A key idea is to understand that in some parts of a business process, we have uncertainty, so we must create a model that reflects that uncertainty/risk.

Example 3.1.1
A company has 10 drugs in a development portfolio.

- Given a drug has been aproved, you have to predict its revenue.
- But, whether a drug is approved or not is an uncertain future event (a random variable). We must estimate the probability of approval.
- We wish to invest in the company if the company's total expected revenue for the portfolio is over $10B in 5 years time.

We need to calculate the **probability distrubution of the total revenue** to **understand the investment risk**.

3.2 Examples of Probabilistic Models

Some probabilistic models commonly used in business and finance include:

- **Regression models**.
- **Probability trees**.
- **Monte Carlo simulation**.
- **Markov models**. We look at stages of a certain process.

3.3 Regression Model

Definition 3.3.1 (Regression Model)
A **regression model** is based on a set of data. We use the data to *make a reverse engineering of the data to find a relationship between the variables in order to capture a realistic description of the process*.

The following is an example of a regression model.

Example 3.3.1
Lets suppose we have data from several diamond shops in a certain street in NY. In the picture, we draw the dots corresponding to the price vs number of carats of a diamond sold in that street.

We can see that there is a pattern which is linear, this is, there is a linear relation between the price of a diamond and its weight in carats. We use linear regression in order to find the best fitting line (we may use the best fitting curve, etc...) that predicts the price of a diamond given its weight in carats.

$$P = -259.2 + 3721 \times C$$

where P is the price in dollars and C is the weight in carats. In the graph, we have a band that is a **prediction inverval** which *captures the range of uncertainty of the model.*

If we check the points in the graph, we see that not all of them lie in a line, but some of them are outside of it, these points are called noise in the system. We must incorporate them in our prediction interval and forecast.

Figure 3.1: Linear Regression Model for Diamond Prices.

3.4 Probability Trees

Definition 3.4.1 (Probability Tree)
A **probability tree** (or **tree diagram**) is a *visual tool in math that maps out all possible outcomes of a series of events* (like coin flips or drawing cards) *and their likelihood*, using *branches for each outcome and labeling them with probabilities* (fractions/decimals).

Probability trees allow to propagate probabilities through a sequence of events.

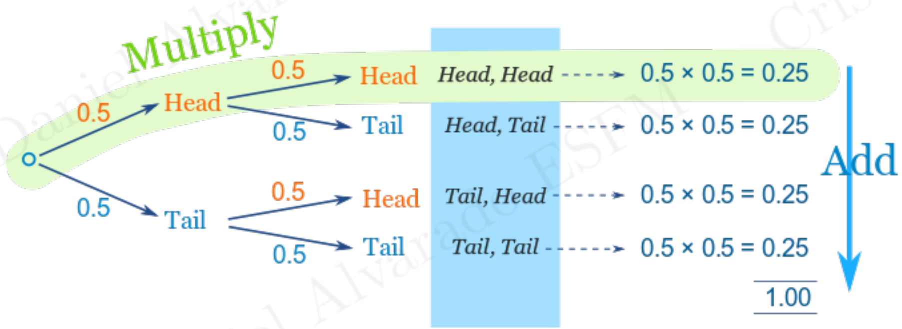

Figure 3.2: Probability Three for the Event of Flipping a Coin Twice in a Row.

Example 3.4.1
Lets consider the following scenario. Imagine that we want to predict the probability of certain people of stop sharing copyright protected files online.

A team made an estimation that:

- When pirates get a notice from the goverment to stop infringing the law, 10% stop doing it.
- If they continue infringing it and they get a second notice, 15% of them stop infringing the law.
- If they continue infringing it and they get a third notice, 20% of them stop infringing the law.

This can be visualized in the tree diagram in Figure 3.3.

Lets suppose that we want to measure the probability of a pirate to stop infringing. We can calculate this by doing the multiplications in the diagram:

$$P_{stop} = 0.1 + 0.9 \times 0.15 + 0.9 \times (0.85 \times 0.2) = 0.388$$

So, there is a 39% change for a pirate to stop infringing the law.

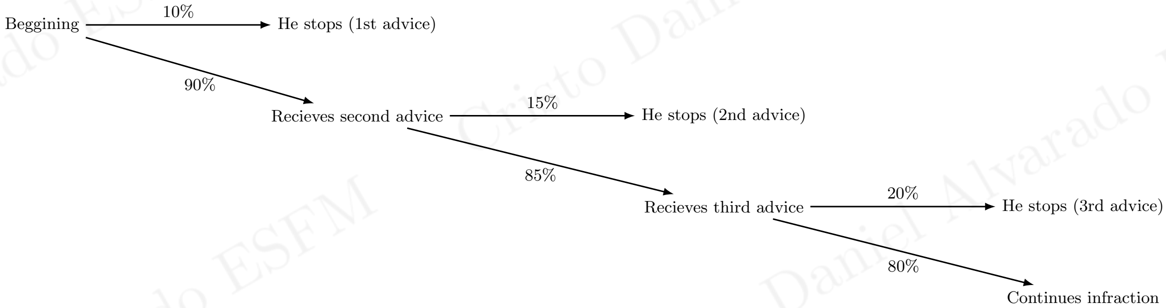

Figure 3.3: Probability Tree of the Example

3.5 Monte Carlo Simulations

Monte Carlo simulations are very useful for modelating complicated scenarios. To give an example, lets go back to the demand model.

$$Q = 60,000P^{-2.5}$$

Here, Q is the quantity of the product and P is the price at which the product is sold.

This is a demand model, which describes, given the amount of a certain product, the price it must have.

Observation 3.5.1
In the latter chapter, we found out the optimal price given the cost of production of $2 per product is $3.33.

Here, we know the elasticity of demand, which is $b = -2.5$, but what if we dont know about it? In order to find this particular value, we can use Monte Carlo to find a suitable value of b.

Appendix A

Operations Research

A.1 Introduction

The discipline of operations research develops and uses **mathematical and computational methods** for decision-making. The *field revolves around a mathematical core consisting of several fundamental topics including optimization, stochastic systems, simulation, economics and game theory, and network analysis*.

Observation A.1.1
The *broad applicability of its core topics places operations research at the heart of many important contemporary problems such as* **communication network management, statistical learning, supply-chain management, pricing and revenue management, financial engineering, market design, bio-informatics, production scheduling, energy and environmental policy, and transportation logistics, to name a few**.

Operations research offers a wide variety of career opportunities in industry, public service, and academia applying operations research methods to improve how organizations or engineering systems perform, developing products that leverage operations research tools, consulting, conducting research, or teaching.

Idea A.1.1
The major sub-disciplines (*but not limited to*) in modern operational research, as identified by the journal Operations Research and The Journal of the Operational Research Society are:

- Computing and information technologies
- Financial engineering
- Manufacturing, service sciences, and supply chain management
- Policy modeling and public sector work
- Revenue management
- Simulation
- Stochastic models
- Transportation theory
- Game theory for strategies
- Linear programming
- Nonlinear programming
- Integer programming in NP-complete problem specially for 0-1 integer linear programming for binary
- Dynamic programming in Aerospace engineering and Economics

- Information theory used in Cryptography, Quantum computing
- Quadratic programming for solutions of Quadratic equation and Quadratic function

A.2 Linear Programming

Many problems in operations research can be modeled as **linear programming** problems.

Definition A.2.1 (**Linear Programming**)
The maximum flow and minimum cut problems are examples of a general class of problems called **linear programming**.

Observation A.2.1
Many other optimization problems, such as:

- Minimum spanning trees.
- Shortest paths.
- Problems in scheduling, logistics and economics.

It was first formalized and applied to problems in economics in the 1930s by Leonid Kantorovich. Linear programming was rediscovered and applied to shipping problems in the late 1930s by Tjalling Koopmans. The first complete algorithm to solve linear programming problems, called the simplex method, was published by George Dantzig in 1947.

Idea A.2.1 (**Focus of Linear Programming**)
A linear programming problem, or more simply a **linear program**, asks for a vector $\vec{x} \in \mathbb{R}^n$ that satisfies a linear set of inequalities.

The general form of a linear programing problem is:

- Maximize $c_1x_1 + c_2x_2 + \cdots + c_nx_n$,
- subject to $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$ for $i = 1, \ldots, k$,
- and $\sum_{j=1}^{n} a_{ij}x_j = b_i$ for $i = k+1, \ldots, k+l$,
- and $\sum_{j=1}^{n} a_{ij}x_j \geq b_i$ for $i = k+l+1, \ldots, m$,

Here, the input consist of a matrix called **constraint matrix** $A = (a_{ij}) \in \mathbb{R}^{m \times n}$, an **offset vector** $\vec{b} \in \mathbb{R}^m$, and a **objective vector** $\vec{c} \in \mathbb{R}^n$.

Each coordinate of $\vec{x}$ is called a **variable**. Each of the linear inequalities is called a **constraint**. The function:

$$\vec{x} \mapsto \vec{c} \cdot \vec{x} = c_1x_1 + c_2x_2 + \cdots + c_nx_n = \sum_{i=1}^{n} c_ix_i$$

is called the **objective function**.

Observation A.2.2 (**Cannonical Form of a Linear Progam**)
A linear program is said to be in **cannonical form** if it has the following form:

- Maximize $c_1x_1 + c_2x_2 + \cdots + c_nx_n$,

- subject to $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$ for $i = 1, \ldots, m$,
- and $x_i \geq 0$ for $i = 1, \ldots, n$ (in other words, $\vec{x} \geq 0$).