1.1

1) Sea $f = x^2 y$ y $g = y\sin z$. Calcular:

a) fg^2 — c) $\frac{\partial f}{\partial x} g + \frac{\partial g}{\partial y} f$

b) $\frac{\partial^2 (fg)}{\partial y \partial z}$ — d) $\frac{\partial}{\partial y}(\sin f)$

Sol.

a) $fg^2 = x^2 y \cdot (y \sin z)^2 = x^2 y^3 \sin^2 z$

b) $\frac{\partial^2}{\partial y \partial z}(fg) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}(fg)\right) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}(x^2 y^2 \sin z)\right) = \frac{\partial}{\partial y}\left(x^2 y^2 \cos z\right) = 2x^2 y \cos z$

c) $\frac{\partial f}{\partial x} g + \frac{\partial g}{\partial y} f = (2xy)(y\sin z) + (\sin z)x^2 y = 2xy^2 \sin z + x^2 y \sin z$

d) $\frac{\partial}{\partial y}(\sin f) = \frac{\partial}{\partial y} \sin(x^2 y) = x^2 \cdot \cos(x^2 y)$

2) Valor de f en cada punto:

a) $(1,1,1)$ — c) $(a,1,1-a)$

b) $(3,-1,1/2)$ — d) (t,t^2,t^3)

$f(x,y,z) = x^2 y - y^2 z$

Sol.

a) $f(1,1,1) = 1 \cdot 1 - 1 \cdot 1 = 1 - 1 = 1$

b) $f(3,-1,1/2) = 3^2 \cdot (-1) - (-1)^2 \cdot \frac{1}{2} = -9 - 1 \cdot \frac{1}{2} = -9 - \frac{1}{2} = -\frac{19}{2}$

c) $f(a,1,1-a) = (a)^2(1) - (1)^2(1-a) = a^2 - 1 + a = a^2 + a - 1$

d) $f(t,t^2,t^3) = (t)^2 \cdot t^2 - (t^2)^2 \cdot t^3 = t^4 - t^4 \cdot t^3 = t^4 - t^7$

3) Calcular $\partial f / \partial x$:

a) $f = x\sin(xy) + y\cos(xz)$

b) $f = \sin g$, $g = e^h$, $h = x^2 + y^2 + z^2$

Sol.

a) $\partial f/\partial x = \frac{\partial}{\partial x}\left(x\sin(xy) + y\cos(xz)\right) = x\frac{\partial}{\partial x}\sin(xy) + \frac{\partial x}{\partial x}\sin(xy) + \frac{\partial}{\partial x} y\cos(xz)$

$= x\cos(xy)\cdot\frac{\partial}{\partial x}(xy) + \sin(xy) - y\sin(xz)\cdot\frac{\partial}{\partial x}(xz)$

$= xy\cos(xy) + \sin(xy) - yz\sin(xz)$

b) $\frac{\partial}{\partial f}\sin(g) = \cos(g)\cdot\frac{\partial}{\partial x}(g) = \cos(g)\cdot\frac{\partial}{\partial x}(e^h) = e^h\cos(g)\cdot\frac{\partial}{\partial x}(h) = e^h\cos(g)\cdot 2x$

$= 2xe^h\cos(e^h)$.

4) Calcular $\partial f/\partial x$ si $h = x^2 - yz$

a) $f = h(x+y, y^2, x+z)$

b) $f = h(x, -x, x)$

c) $f = h(e^z, e^{x+y}, e^x)$

Sol.

a) $f = h(x+y, y^2, x+z) = x^2 + 2xy + y^2 - y^2(x+z) \Rightarrow \frac{\partial f}{\partial x} = 2x + 2y - y^2$

b) $f = h(x, -x, x) = x^2 - (-x)\cdot x = x^2 + x^2 = 2x^2 \Rightarrow \frac{\partial f}{\partial x} = 4x$

c) $f = h(e^z, e^{x+y}, e^x) = e^{2z} - e^{2x+y} \Rightarrow \frac{\partial f}{\partial x} = -2e^{2x+y}$

1.2.

1) Sea $v = (-2, 1, -1)$ y $w = (0, 1, 3)$

a) para $p \in \mathbb{R}^3$, expresar el vector tangente $3v_p - 2w_p$ como combinación lineal de $U_1(p)$, $U_2(p)$ y $U_3(p)$.

Sol.

$3v_p - 2w_p = (3v)_p - (2w)_p = (3v - 2w)_p = ((-6, 3, -3) + (0, -2, -6))_p = (-6, 1, -9)_p$

$= -6U_1(p) + U_2(p) - 9U_3(p)$

2) Sea $V = xU_1 + yU_2$ y $W = 2x^2U_2 - U_3$. Computar $W - xV$ y calcular su valor en $p = (-1, 0, 2)$.

Sol.

$W - xV = 2x^2U_2 - U_3 - x^2U_1 - xyU_2 = -x^2U_1 + (2x^2 - xy)U_2 - U_3$. En $p = (-1, 0, 2)$, su valor será:

$$(W - xV)(p) = -U_1 + (2)U_2 - U_3 = (-1, 0, 0)_p + (0, 2, 0)_p + (0, 0, -1)_p$$

$= (-1,2,-1)_p$

3) Expresar V como $\sum_{i=1}^{3} v_i U_i$.

a) $2z^2 U_1 = 7V + xy U_3$

b) $V(p) = (p_1, p_3 - p_1, 0)_p \; \forall \; p \in \mathbb{R}^3$.

c) $V = 2(xU_1 + yU_2) - x(U_1 - y^2 U_3)$

d) $\forall \; p \in \mathbb{R}^3$, $V(p)$ es el vector que va de (p_1, p_2, p_3) a $(1+p_1, p_2 p_3, p_2)$.

e) A cada punto $p \in \mathbb{R}^3$, $V(p)$ el vector de p al origen.

Sol.

a) $V = \left(\frac{2z^2}{7}\right) U_1 + \left(-\frac{xy}{7}\right) U_3$

b) $V(p) = p_1 U_1 + (p_3 - p_1) U_2 \Rightarrow V = x U_1 + (z - x) U_2$

c) $V = x U_1 + 2y U_2 + x y^2 U_3$

d)