4.3 Sean $f, g : A \longrightarrow \mathbb{R}$ funciones integrables. Probar que la función producto fg es también integrable en A.

5.7 $\int_A (x+y)x\,dx\,dy$, $A=[0,1]\times[0,1]$

$$= \int_0^1\int_0^1 (x+y)x\,dx\,dy = \int_0^1\left(\int_0^1 x^2+xy\,dx\right)dy = \int_0^1\left(\frac{x^3}{3}+\frac{x^2}{2}y\Big|_0^1\right)dy = \int_0^1\left(\frac{1}{3}+\frac{1}{2}y\right)dy$$

$$= \frac{1}{3}y+\frac{1}{4}y^2\Big|_0^1 = \frac{1}{3}+\frac{1}{4} = \frac{7}{12}$$

5.8 Calcular:

$$\int_0^1\int_{e^x}^{e^{2x}} x\log y\,dy\,dx = \int_0^1 x\int_{e^x}^{e^{2x}}\log y\,dy\,dx$$

$u = \ln y$

$\frac{du}{dy} = \frac{1}{y} \Rightarrow du = \frac{dy}{y}$

$\int \ln y\,dy$; $\quad u=\ln(y) \quad dv = dy$

$\quad du = \frac{dy}{y} \quad v = y$

$$\Rightarrow \int \ln(y)\,dy = \ln(y)\cdot y - \int dy = \ln(y)\cdot y - y = y(\ln(y)-1)$$

Luego:

$$\int_0^1 x\int_{e^x}^{e^{2x}}\log y\,dy\,dx = \int_0^1 x\left(y(\ln(y)-1)\Big|_{e^x}^{e^{2x}}\right)dx = \int_0^1 x\left(e^{2x}(2x-1)-e^x(x-1)\right)dx$$

$$= \int_0^1 x\left(2xe^{2x}-e^{2x}-xe^x+e^x\right)dx$$

$$= \int_0^1 2x^2e^{2x}-xe^{2x}-x^2e^x+xe^x\,dx = x^2e^{2x}\Big|_0^1 - \frac{3}{2}\int_0^1 xe^{2x} - \int_0^1 x^2e^x + \int_0^1 xe^x$$

Calculando cada una por separado:

$$= x^2e^{2x} - \frac{3}{4}xe^{2x} + \frac{3}{8}e^{2x}\Big|_0^1 - \int_0^1 x^2e^x + \int_0^1 xe^x$$

$\int xe^{2x}dx \quad u = x \quad dv = e^{2x}dx$

$\quad du = dx \quad v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$$

$$\int_0^1\int_0^{\frac{\arcsin y}{y}} y\cos(xy)\,dx\,dy = \int_0^1 y\int_0^{\frac{\arcsin y}{y}}\cos xy\,dx\,dy$$

$$= \int_0^1 \left(\sin xy\Big|_0^{\frac{\arcsin y}{y}}\right)dy = \int_0^1 y\,dy = \frac{1}{2}$$

$\det: \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m) \to \mathbb{R}$ es continua, con las normas: $\| \|_T: \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m) \to \mathbb{R}$ y $| |: \mathbb{R} \to \mathbb{R}$.

Sea $\varepsilon > 0$, para este, $\exists\, \delta > 0$ tal que si $\| T - L \| < \delta$, entonces: $|\det T - \det L| < \varepsilon$.

Lo probaremos para 2 t. lineales:

$$T: (x_1, \dots, x_j, \dots, x_m) \mapsto (x_1, \dots, Cx_j, \dots, x_m)$$
$$L: (x_1, \dots, x_i, \dots, x_j, \dots, x_m) \mapsto (x_1, \dots, x_i + x_j, \dots, x_m)$$