Fundamentals of Quantitative Modeling

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Contents

Code List

Chapter 1

Introduction

1.1 Topics

Some of the topics covered in this module include:

- Exposure to the language of modeling.
- Exploration of different types of models used in business and how to apply them in practice.
- Process of modeling.
- Characteristics of the models.
- Value and limitations of quantitative models. What sort of things they can and cannot do. Understand the limitations of the models.
- Provide a set of foundational for other courses in specialization.

Which model should I use?

Process: map the characteristics of the business into the characteristics of the model.

1.2 Definition, Uses of a Model, Common Functions

In the business context, the models we are talking about are not physical models (different from the idea an arquitect has). We are talking about a **formal description of a business process**.

This description is going to involve a set of mathematical equations and/or random variables.

Observation 1.2.1
A quantitative model it is almost always a simplification of a more complex structure (in particular, of a business process). We do not want to over simplify, but we also do not want to overcomplicate.

Turns out it's going to be really difficult to make an exact and acurate representation of what we really want to model.

Also, there is a set of **assumptions** that underly the model. We have to check weather these assumptions are reasonable or not in our business process.

Observation 1.2.2
A model is usually implemented in Excel or a spreadsheet tool like Google Sheets (or using programming langauges as R).

In a more mathematical way, we can define a model as follows:

Definition 1.2.1 (Model)
A model is a triplet $M = (X, \Theta, F)$, where:

- X is a set of measurable input variables (data).
- Θ is a set of parameters and,
- $F : X \times \Theta \to Y$ is a function, that maps input and parameters to measurable output in Y.

Let's give some examples:

Example 1.2.1
Some examples of models are the following:

- Determine the price of a diamond given its weight.
- The spread of an epidemic over time.
- Relationship between demand for, and price of a product.
- The uptake of a new product in market.

In particular, if I have a product and I want to maximize the gains we acquire when selling it, how can we achieve this goal?

All of these examples are from different areas, but can be addressed by using a quantitative model.

1.2.1 Diamonds and Weight

Let's consider the weight of a diamond and the price it is going to have. We have the following equation that models the price of a diamond given its weight:

$$p(w) = -260 + 3721w$$

This is a linear model. The price is given in dolars and the weight is given in carats. Sometimes we use a visual representation like this:

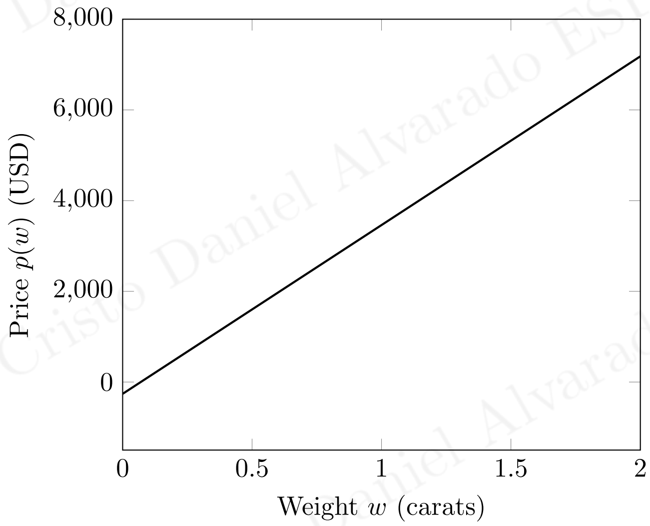

Figure 1.1: Model of the Price of a Diamond

Basically, this model allows us to forcast the price of a certain object given certain properties of it or *data about it*. This is a **linear model**.

Observation 1.2.3
A model doesn't necesarly has to work in all the circustamces. For example, the model given to obtain the price of a diamond doesn't necesarly has to be applicable to a different company.

1.2.2 Spreading of an Epidemic

Depending on the disease and other factors (such as time in which the epidemc ocurred), one model arises.

One of the basic models to start with is the **exponential model**. On the bottom axis we have weeks (since the start of the epidemic), and on the vertical axis we have the number of cases reported.

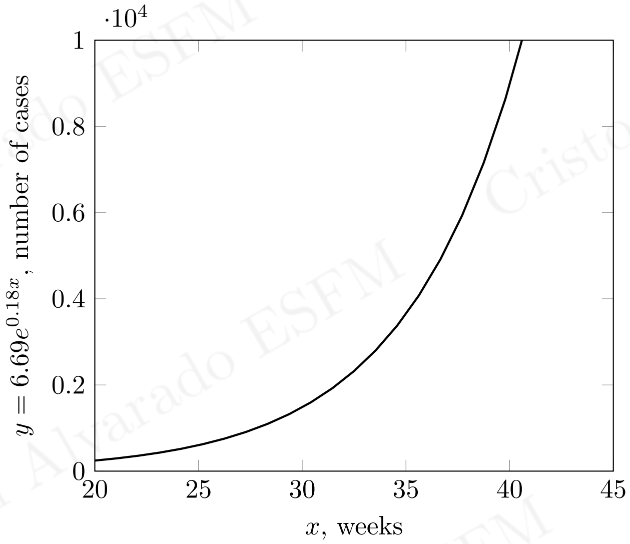

Figure 1.2: Exponential Model for the spread of an epidemic.

The equation to describe this model is:

$$y = 6.69e^{0.18x}$$

Observation 1.2.4
This model is suitable for the start of the pandemic, since there is not an unlimited number of people, we cannot expect that the growth of the graph is unstoppable.

Observation 1.2.5 (Exponential Graphs)
In the context of business they are called **hockey sticks**.

This model is not suitable for a long term, but its suitable for an approximation over the first weeks of the pandemic.

1.2.3 Price and Demand

Let's suppose a situation of price and demand (very common in the industry). Demand models basically tell when we know the price of a certain product, the quantity of it available in the market.

Observation 1.2.6 (Possitive Association)
The graphs in the first two examples are said to have **positive association**.

For products, when there is a huge amount of product, the price decreases and, when the price is too hight, the amout available is too low. The model:

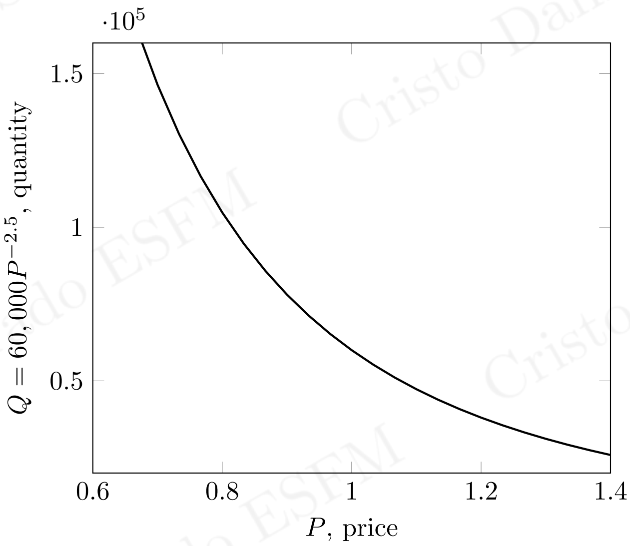

Figure 1.3: Price and Demand Model

This model is given by the equation:

$$Q = 60,000P^{-2.5}$$

In this model we use a power function (which is basically the exponential function in essence, but many people who doesn't have mathematical background calls is *power function*).

Idea 1.2.1
One use of this model is to find what the optimal price for a product should be.

1.2.4 The Uptake of a Product

Definition 1.2.2 (Uptake)
The **uptake of a product**, often referred to as **product adoption** or **market uptake**, is *the process by which individuals or organizations learn about, start using, and ultimately integrate a new product or service into their routine life or business operations.*

The following describes the uptake of a product given the amount of years it has been on the market.

This particular function is called a logistic function, and is given by:

$$P = \frac{e^{2(Y-2.5)}}{1+e^{2(Y-2.5)}}$$

Here, Y is the number of years and, P is the proportion of target population with product.

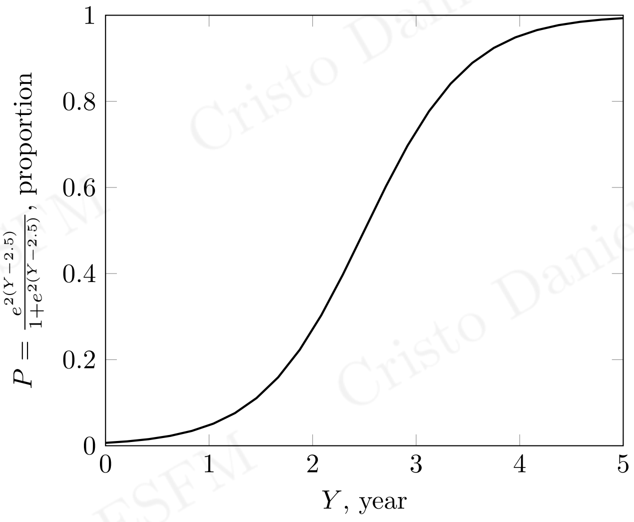

Figure 1.4: Model of the Uptake of a Product

Observation 1.2.7
This model has the potential to model a process where at the first stages we see a slow start (here, it's when target people start adopting the product), and then it has a fast grow that proceeds to slow as time goes by.

1.3 Use of Models in Practice

There are four main usages of a model in the business enviroment:

- **Prediciton**.
- **Forecasting**.
- **Optimization**.
- **Ranking and targeting**.

1.3.1 Prediction

Once we got a quantitative model is **prediction**. Basically is taking a model, puting an input and create a prediction.

Observation 1.3.1
The use of most quantitative models is for predictive analysis.

1.3.2 Forecasting

When talking about forecasting we are talking about time series.

Definition 1.3.1 (Forecasting)
Forecasting is the *process of predicting future developments by analyzing past and present data and trends.*

Example 1.3.1
For example, with the models presented earlier, we can ask the following questions:

- How many people are expected to be infected in 6 weeks?
- Using a scheduling model, who is likely to turn up for their outpatient appointment?

This activity has often a lot to do in businesses and to do with resource planning.

1.3.3 Optimization

How to maximize or minimize something?

1.3.4 Ranking and targeting

When selling a product, we may be interested in which ones we would like to purchase. Since we cannot have a look at all the diamonds in the world, **identify targets of opportunity**. This is a ranking and targeting excersise.

Or, for example, if I am interested in buying, is useful to create a model to help classify places which I can buy.

1.4 How are models used in practice

We use models in real-life scenarios in order to:

- Exploring what if scenarios (this is **called scenario planning**).
- Interpreting the coefficients in a model (what's the meaning behind **coefficients in model equations?**).
- Assesting how sensitive the model is to key-assumptions.

Definition 1.4.1 (Sensitivity Analysis)
A **sensitivity analysis** is the *process where we check how the output of a model is sensitive to some of the assumptions made at the begining.*

Benefits of Modeling
Identify gaps in current understanding
Make assumptions explicit
Have well-defined description of the business process
Create an institution memory
Used as a decision **support** tool
Serendipitous insight generator

Table 1.1: Benefits of Modeling

1.5 Key Steps in the Modeling Process

Every model is different but they share common features in the way the model was created (the workflow).

1. The first steps is always to **stablish what the model is trying to predict**, and what are the **underlying variables** of it (which will help us to predict)?
   What is the scope of the model? Which is, **where is it going to be applied?**

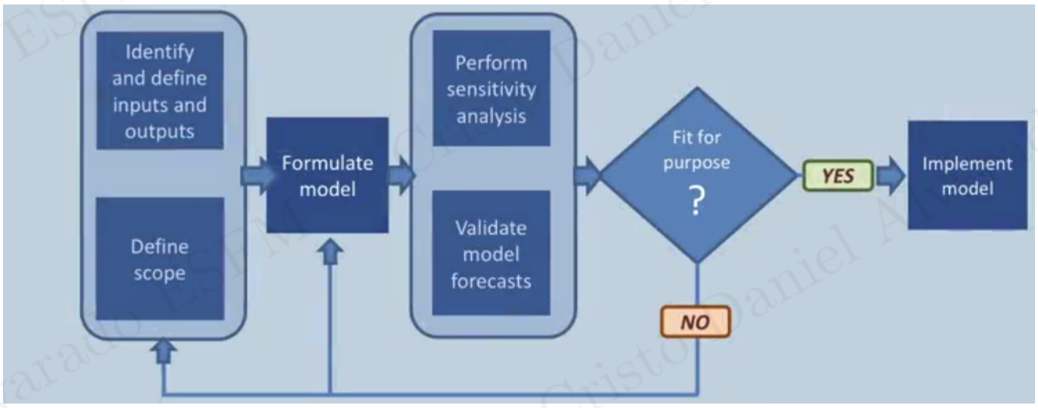

Figure 1.5: Overview of the Modeling Process

2. **Formulate the model**. Its the step where we undestand the business process and al the mathematical ideas come into the process.
3. **Perform a sensitivity analysis** and validate **model forecast**.
4. Is the model fit for purpose? Is it suitable for its purpose?

Observation 1.5.1
Some teacher said: some wrong models are useful.

If something went wrong, we start over again. If the model accomplish its purpose, then we implement the model.

Idea 1.5.1
There is always a learning when a model doesn't work properly. We gain insight into what work and what doesn't work in the process.

Modeling is an iterative and evolutionary process.

1.6 Vocabulary for Modeling

1.6.1 Data Driven vs. Theory Driven

There are basically two types of models: theoretical and data-driven. It's more like an spectrum than a certain type of model.

- **Theory**: Given a set of assumptions and relations, then what are the logical consecuences?
- **Data**: Given a set observations, how can we aproximate the underlying process that generated them?

Example 1.6.1
These are two examples of questions that define a model:

- **Theory**: If we assume the markets are efficient, then what should the price of a stock option be?
- **Data**: I've separated out my profitable customers from the unprofitable ones. Now, what features are able to differentiate them?

1.6.2 Deterministic vs Probabilistic

Given a set of inputs, the model always gives the same output or is it different instance from instance? This is the main difference between a deterministic and a probabilistic model, respectively.

1.6.3 Static vs Dynamic

So, in a static model, the model captures a single snapshot of the business process.

Example 1.6.2
Given a website's installed software base, what are the chances that it is compromised today?

And dynamic, in which the evolution of the process itself is of interest. The model describes the movement from state to state.

Example 1.6.3
Given a person's participation in a job training program, how long will it take until he/she finds a job, and, if they find one, for how long will they keep it?

Chapter 2

Linear Models and Optimization

This chapter is dedicated to linear models and optimization.

2.1 Introduction to Linear Models

As stated earlier, in determinstic models we do not have random or uncertain components, so we always have the same output given the same input.

Observation 2.1.1
Due to the fact that we do not have a random component, it´s very hard to asses the uncertainty in the outputs.

As we remember from earlier examples, a linear model (1-imensional) is a model given by the equation:

$$y = mx + b$$

The slope is the constant m, and b is the intercept with the y-axis.

Idea 2.1.1
These kind of models can work in certain circumstances, but may not be ideal every time.

Example 2.1.1 (**Linear Cost Function**)
Let C be the cost of producint q units of a product. If we have a process that must start with 100 dolars cost, then the cost function is given by:

$$C(q) = 100 + 30q$$

where 30 is the cost to produce each unit of the product q.

Observation 2.1.2
The constant b in the latter example is called **fixed cost**. Every time we produce a product, we must have to pay some amount of money, which is independent of the number of units produced.

Example 2.1.2 (**Time to Produce Function**)
If it takes 2 hours to set up a production run, and each incremental unit produced always takes an additional 15 minutes, then the time to produce q units is given by:

$$T(q) = 2 + \frac{15}{60}q$$

Here, $\frac{15}{60}$ can be interpreted as the **work rate** to produce each unit of the product.

Some notes on the latter examples are the following:

1. We are given a description of the process which we have to model, so it´s our work to find the variables and constants that represent the process in the model, describe them and interpret them.
2. In all of the examples, we have the constants involved in the process told to us, but in other cases we may need to find them using data or other information.

Observation 2.1.3 (**Linear Programming**)
Linear Programming is a *method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationships*.

It is used to solve optimization problems, and is one of the work horses of **operations research**.

Linear programming implements something called constraints. When we try to optimize a process, the meaning behind it is to find the best possible solution that satisfies all of the constraints.

Constraints are ideas that we can incorporate into our models to **make them more realistic**.

Example 2.1.3
For example, if we are trying to optimize a process, *we may have constraints* on the amount of resources available, or the maximum number of units that can be produced.

2.2 Growth and Decay (discrete and continuous time)

2.3 Classical Optimization

Appendix A

Operations Research

A.1 Introduction

The discipline of operations research develops and uses mathematical and computational methods for decision-making. The field revolves around a mathematical core consisting of several fundamental topics including optimization, stochastic systems, simulation, economics and game theory, and network analysis.

The broad applicability of its core topics places operations research at the heart of many important contemporary problems such as communication network management, statistical learning, supply-chain management, pricing and revenue management, financial engineering, market design, bio-informatics, production scheduling, energy and environmental policy, and transportation logistics, to name a few.

Operations research offers a wide variety of career opportunities in industry, public service, and academia—applying operations research methods to improve how organizations or engineering systems perform, developing products that leverage operations research tools, consulting, conducting research, or teaching.

A.2 Linear Programming