Furhter Thesis Research

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Capítulo 1

Goal

1.1. Fundations

So, in my Thesis I wrote a proof of the following result, in that work the result is divided in two propositions:

Teorema 1.1.1 (**Nyikos 1981**)
$\omega_1 < \mathfrak{p}$ or $\mathfrak{p} = \mathfrak{b}$ imply the existence of a topology over the boolean group $([\omega]^{<\omega}, \Delta)$ such that this group becomes a T_0 topological group that is Fréchet and non-metrizable.

The following question arises: Is it possible to construct (at some extent) given this result, countable T_0 Fréchet and non-metrizable topological groups? The question by itself seems to be complicated to answer, so maybe we need a little bit of restrictions on our question. What if we want topological groups G such that:

- Countable.
- Are T_0.
- Are Fréchet.
- Are non-metrizable.
- Are abelian.

Given the construction made by Nyikos, is there any way to generalize this in order to produce different topological groups T_0 Fréchet-Urysohn and non-metrizable?

Observación 1.1.1
Given the fact that:

$$[\omega]^{<\omega} = \bigoplus_{n<\omega} \mathbb{Z}/2\mathbb{Z}$$

is it possible to build different Fréchet-Uryson and non-metrizable topological groups? What if we find a set X such that $[\omega]^{<\omega}$ is a dense topological subgroup of $[X]^{<\omega}$, being $[X]^{<\omega}$ Fréchet-Urysohn and non-metrizable.