Comparing Methods of Measuring the Earth's Magnetic Field in the Undergraduate Laboratory

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The strength of the Earth's magnetic field was measured via three methods-Helmholtz coil deflection, single-coil deflection, and via a Hall Probe. The results of the single-coil method were found to be within 3σ of the known value. The results of the Helmholtz coil agreed within 1σ, while the value for the Hall Probe was found to be within 1σ for some values, but was more than 3σ off for those at higher elevation. Varied equipment led to numerous difficulties in obtaining reliable data, however in general the single-coil and Helmholtz coil methods agreed, while the Hall Probe provided useful calibration metrics, even if its results for elevation did not match NCEI values.

I. INTRODUCTION

The Earth's magnetic field has been used as a constant for navigators and engineers alike for centuries, and measuring this field is still critical for a variety of equipment that relies on accurate measurements of similarly sized fields.[7] Today, the Earth's magnetic field is well-studied and mapped,[2] however interest in measuring the Earth's magnetic field still remains due to its value as an educational tool. A variety of undergraduate assignments have been created in which students measure the magnetic field of the Earth using a single coil of wire,[6] a Helmholtz coil[5] or a smartphone hall sensor.[7] We take these three experiments and examine the relative ease of performance, effectiveness and potential sources of error for each experiment.

We find no significant difference between the Helmholtz and single coil methods. The smartphone hall sensor appeared somewhat, although not significantly, more accurate than the two coil methods. We argue that the primary source of error in all three of these experimental methods is the variation in the local ambient magnetic field caused by non-Earth magnetic fields.

II. MEASURING AND GENERATING MAGNETIC FIELDS

A. The Earth's Magnetic Field

The Earth's magnetic field is a result of liquid metal in the outer core being spun around the Earth's axis each day. This can be modeled approximately as a rotating sphere of charge which produces the magnetic field seen in Fig. 1. Unfortunately, the Earth is far too misshapen for a calculation with this model to yield a decent approximation. However we do gain a couple useful insights from this theoretical model.

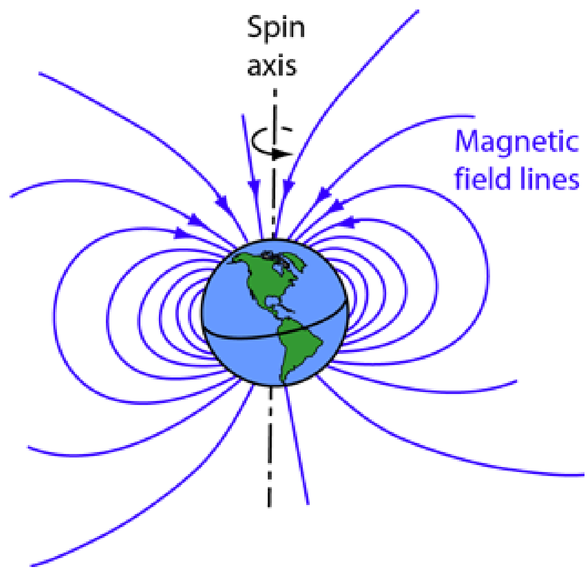

FIG. 1. The magnetic field lines produced by the rotation of the Earth's charged core. The magnetic field lines approximate those of a physical dipole and the magnetic axis is slightly off the spin axis due to deformations in the Earth. This causes declination and inclination in the Earth's field to vary from point to point. Image from HyperPhysics.[1]

The first non-obvious takeaway from Fig. 1 is that the magnetic field doesn't actually just point north! Rather in the northern hemisphere it points down and north while in the southern hemisphere it points up and north. In the northern half of the USA (where the experiments in this paper are taking place), this is pronounced enough that the horizontal component of the field is only about half the strength of the vertical component. This is quantized as an angle measure called the inclination of the field, where a positive inclination is into the Earth. When performing measurements on the strength of the field, the orientation both in terms of the cardinal directions and in terms of the inclination must be taken into account.

Another thing to note is the difference between the spin and magnetic axes. The angle created by these two axes is called the declination, where a positive angle indicates that a compass would point east of true north. For our purposes, this is less important than the inclination as a compass will give you the cardinal direction of the magnetic field at your position and then you can ignore the difference between that and true north for any measurements of field strength. The cause of this non-zero declination angle is mostly inhomogeneity in the shape of the Earth and distributions of materials throughout it. Beyond the declination, these asymmetries also cause significant local fluctuations in the Earth's magnetic field. They can affect the inclination, declination, and field strength locally, so accepted values to compare against must match the location of the experiment. In this paper, we will be using the National Centers for Environmental Information (NCEI) WMM-2020 magnetic field calculator.[2]

B. The Deflection Method of Measurement

The strength of a magnetic field, B_P, can be measured using a secondary magnetic field of strength B_S and then comparing the strengths of the two fields. This comparison can be done using a compass. A compass points in the direction of the projection of the ambient magnetic field onto the plane of the compass. When the B_P field is the only ambient magnetic field, we will call this direction $\hat{p}$. Once the B_S field is added, pointing in the $\hat{s}$ direction, the compass needle deflects to point in the direction of the B_T vector, representing the vector sum of those two magnetic fields $B_P \cos(\phi)\hat{p}$ and $B_S\hat{p}$. The $\cos(phi)$ term represents the projection of the B_P field into the plane of the compass, it is assumed that B_S field is already in the plane of the compass. This can be seen pictorally in Fig. 2. If the B_S magnetic field is also oriented perpendicular to the compass-plane projection of the B_P field ($\dot{\hat{p}}\hat{s} = 0$), then B_T is given by

$$B_T = B_P \cos(\phi)\hat{i} + B_S\hat{j}.^3 \tag{1}$$

Utilizing trigonometry, this can be rewritten in the linearized form

$$\cot(\theta) = \frac{B_P}{B_S}\cos(\phi). \tag{2}$$

C. Magnetic Fields Generated By Coils of Wire

A magnetic field can be created using a single coil of wires. The magnetic field of a current carrying coil with N loops can be modeled along its axis by

$$B(z) = \frac{\mu_0 N R^2 I}{2(z^2+R^2)^{3/2}}\hat{z} \tag{3}$$

where z is distance from the center along the coil's axis, R is the radius of the coil and I is the current through it.

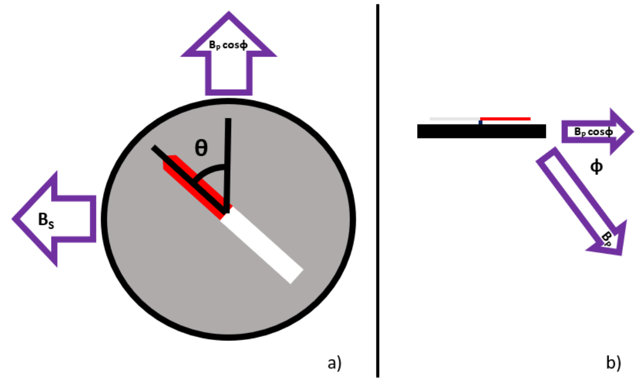

FIG. 2. a) A diagram of a compass exposed to two perpendicular magnetic fields B_P and B_S. θ is defined, as per the figure, as the angle which the compass needle deflects when exposed to the secondary magnetic field, B_S. Notice that B_P is not necessarily in the plane of the compass. b) A diagram demonstrating how the projection of B_P onto the plane of the compass is done. The angle ϕ is defined as the angle between the direction of the primary magnetic field and the plane of the compass, as shown in the figure.

While this set up is more simple than a Helmholtz coil, and thus perhaps easier to understand, its non-uniformity can lead to inaccurate measurements. In particular, a compass needle with non-zero length will experience differing field strengths along its length, which could cause experimental results to diverge from theory.

In order to minimize this inhomogeneity, we can create a more uniform magnetic field with a Helmholtz coil. A Helmholtz coil is a pair of coils of conductive wire of radius R, where the separation between the coils is also R. Because of this geometry, the magnetic field with in the Helmholtz coil is particularly uniform, with a field strength of

$$B = \frac{8\mu_0 N}{R\sqrt{125}}I \tag{4}$$

where B represents the strength of the magnetic field between the coils.[5]

D. Smartphone Hall Sensors

In addition to the two deflection methods we've already highlighted, most smart phones posses a magnetic measurement Hall device. iPhones and many other smartphones contain 3-axis Hall sensors which are sensitive to magnetic field strengths as small as $10nT$. In this experiment we use the Gauges app for iPhone to determine the magnitude of local fluctuations in the Earth's magnetic field.

For the iPhone, the only pre-built apps we could find use the CMCalibratedMagneticField interface to retrieve data from the sensor which makes the output adirectional. Additionally, the calibrated interface is filtered with the accelerometer data so that the iPhone won't read change in field strength if it is not moving. There are other interfaces that could be used to get the unfiltered field strengths along three axes[4] but for our purposes that is unnecessary, we only need the overall magnetic field.

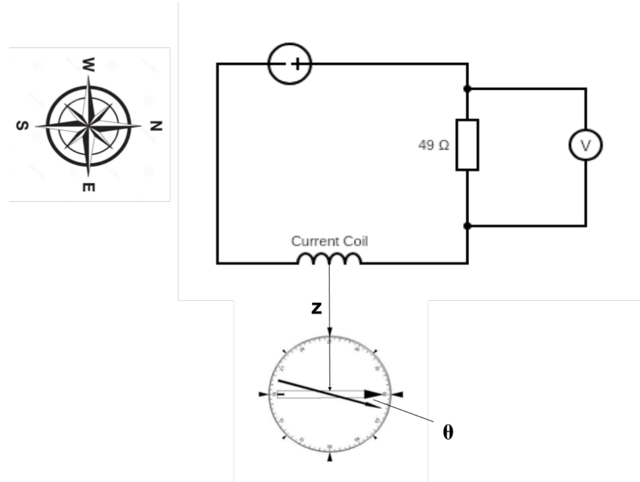

FIG. 3. A schematic of the single coil experimental setup. A $9V$ battery is used as the DC power supply and current is measured through the voltage drop over the 49Ω resistor. The 25 loop coil is shown at the bottom of the circuit and is oriented such that its central axis is aligned with the east-west axis. Below the coil is a compass which is initially aligned along the north-south axis and is then deflected by the coil's field.

III. MEASUREMENT PROCEDURES

A. The Single Coil Deflection Method

The single coil deflection method employs a current carrying coil to create the magnetic field B_1. This coil is held perpendicular to the ground such that the central axis is aligned with east-west (determined by compass to account for declination). This ensures that the field along the central axis of the coil is perpendicular to the horizontal and vertical components of the Earth's magnetic field. When the compass is oriented parallel to the ground, B_0 will be the horizontal component of the Earth's field, whereas when the compass is oriented vertically, B_0 is the vertical component (unfortunately the compass used for this part of the experiment was not well behaved when vertical so only the horizontal field is measured).

Using the single coil setup and operating under the assumption of an ideal compass needle (no length), we can measure B_0 by varying the position of the compass along the z axis and recording the deflection angle. Then B_0 will be the slope of $1/B_1$ plotted against $tan(\theta)$, as is clear from Eq. **??**. This B_1 can be calculated using Eq. 3, where R and n are held constant while z is varied, as seen in Fig. 3. Ideally, I would also be held constant but if that is not possible then both I and z can be recorded for each measurement of the deflection angle. In this experiment, we have used a 9V battery instead of a more typical power supply and while the current is dropping at a steady rate, it has negligible effect on the deflection angle.

B. The Helmholtz Deflection Method

A Helmholtz coil was placed on a level table. A compass was then suspended using cardboard, wood and duck tape such that the compass was centered between the coils and along the central axis of the two coils. The plane of the compass was parallel to the table. The Helmholtz coil and compass were then rotated, such that they faced magnetic north, with any magnetic field created by the Helmholtz coil facing in the east-west direction. A current was then induced through the Helmholtz coil, ranging from 20mA to 150mA. The resultant angle of deflection of the compass needle was then recorded. A linearized plot of $\frac{\cos(\phi)}{B_S}$ vs $\cot(\theta)$, relying on Eq. 2, was then created, and the strength of the Earth's magnetic field was obtained from the slope of that linearized plot. A value for ϕ was determined using a dip needle.

Systematic error from ambient magnetic fields was minimized by conducting the experiment outdoors, away from most man-made sources of magnetic fields. The power source was also kept approximately one meter away from the compass (the maximum practical distance), in order to minimize interference from that source. The magnetic fields created by the wires carrying current to and from the Helmholtz coil were neglected, as the hundred-plus wire coils of the Helmholtz coil likely dominated that effect. Interference in the dip needle measurement from the wind was minimized by performing that measurement inside a cardboard box.

C. Hall Probe Method

Measurements of overall magnetic field were taken at different elevations above sea level, at least $1m$ away from any clearly visible sources of electromagnetic fields. When taking data for temperature data, data was taken at the same location for multiple days.

To minimize error from magnetic sources that may be in the ground or near the point of measurement, the phone was rotated, and the data was discarded unless there was a difference of less than $1\mu T$ during rotation. The phone was also kept $1m$ off the ground, since it was earlier confirmed that most elemental sources of ambient fields fall off at that distance for the purposes of this experiment, and

IV. MAGNETIC FIELD MEASUREMENTS

	Single	**Helmholtz**	**Smart Phone**
Measured (μT)	21.8 ±0.5	53 ±2	56 ±0.5
Theoretical (μT)	20.33	55.78	54.88
Percent Error:	7%	5 %	2 %

TBL. 1. A table of the experimental results of this paper. The theoretical values and percent deviation from theoretical values are also included.

A. Single Coil Results

The horizontal component of the Earth's magnetic field was measured to be $(21.8 \pm 0.5)\mu T$, which is $1.5\mu T$ greater than the value provided by the NCEI calculator of $20.33\mu T$. The linearization and accepted values can be seen in Fig. 6.

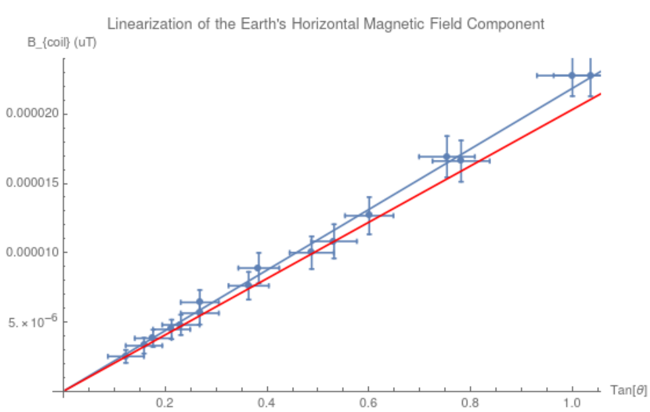

FIG. 4. This plot shows a linearization of the tangent of the deflection angle vs the added field strength which yields a slope of the horizontal component of the Earth's magnetic field (in blue). The expected value is shown as the slope of the red line. The expected value is 20.3 μT and the measured value is $(21.8 \pm 0.5)\mu T$.

While this run of the experiment was performed outdoors, earlier variants were performed indoors and showed results of $35 - 33\mu T$, almost certainly due to effects of structural metals and wiring. Interestingly though, the 95% confidence intervals on these linearizations were similarly small, reaching as high as $2.1\mu T$, indicating that it is still a steady field being measured.

B. Helmholtz Coil Results

A Helmholtz Coil with coils $N = 135$ was used. The inclination, ϕ, of the Earth's magnetic field was measured to be $70° \pm 1°$. This compares to an accepted value of $72°$. The strength of the Earth's magnetic field was then found to be, once an outlier data point was removed and the linearization y-intercept was set to zero, to be $53 \pm 2\mu T$. This compares to an accepted value of $55.8\mu T$. Uncertainty was calculated using standard methods. The linearized plot can be seen in Fig. 5.

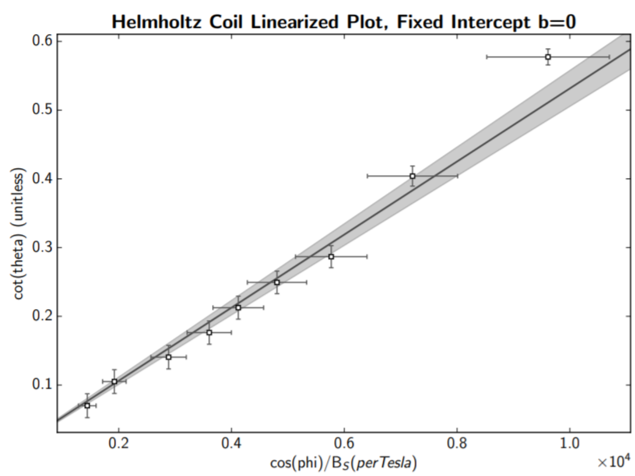

FIG. 5. The linearized plot of $\frac{\cos(\phi)}{B_{coil}}$ vs. $\cot(\theta)$ where ϕ is the inclination of the Earth's magnetic field, B_{coil} is the strength of the magnetic field created by the Helmholtz coil and θ is the angle which the compass needle was deflected when B_{coil} was applied. The slope of the line, which corresponds to the magnitude strength of the Earth's magnetic field, was found to be $53 \pm 2\mu T$. The y-intercept of this plot was fixed at $b = 0$.

C. Smartphone Results

The presence of metallic objects and electronic devices greatly influenced, usually positively, the magnetic field reading of the smart phone, with the reading returning to the standard $55 - 60\mu T$ almost universally when moved more than $1m$ away. Temperature was found to have no effect on the measured magnetic field. The temperatures used ranged from $288K - 303K$. For each temperature, the readings was identical to the last. Likewise, there was no effect on measurement from the height of the smartphone above the surface of the earth, ranging from $0-10m$. The measured value remained constant at all heights.

This was not the case for total elevation. When the magnetic field was measured at varying locations with varying elevations above see level, the magnetic field measured increased at a rate of $53 \pm 7nT/m$. This trend is shown in Fig. 6.

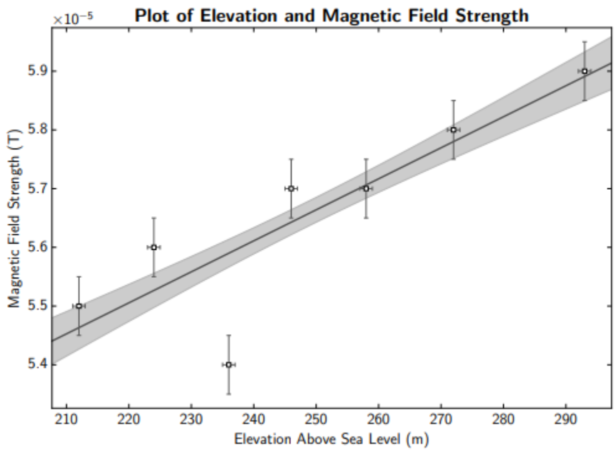

FIG. 6. This plot shows the magnetic field strength plotted against the elevation above sea level. This yields a slope of $5.3(10^{-8}) \pm 0.7(10^{-8})T/m$

V. ANALYSIS OF FIELD DATA

Both the single coil and Helmholtz coil methods for measuring the Earth's magnetic field were found to deviate from expected values on the order of 5-10% without meaningful difference between the two. Given the small confidence interval in the single coil method, the discrepancy between experiment and the accepted value is likely explained by some systematic error which may be unavoidable with the single coil method. However, our results show that this lack of uniformity is not large enough to meaningfully affect the experiment.

Our certainty in this statement does include some reservations. Firstly, due to the danger posed by the SARS-CoV-2 virus, these experiments were conducted in separate laboratories using different equipment. It is possible that the ambient magnetic fields varied in strength from the reported NCEI magnetic field value, our theoretical benchmark, by differing amounts in the two labs. This could either be due to some non-Earth based ambient magnetic field or due to variance of the Earth's magnetic field from the value reported by the NCEI. Secondly, the two experiments also used differing equipment, which may make comparisons between the two more difficult. In the Helmholtz coil experiment, the data points corresponding to low current deflected the compass needle more than would be expected by theory, so much so that the most extreme data point was thrown out. This suggests that the power source used may have not been properly calibrated at the extreme low end of its range of possible currents. Finally, the choice to either fix or leave free the intercept of the linearized plot of the Helmholtz coil also had some effect on the measured value, unfixing the intercept caused the experimental value to overshoot the theoretical by a similar margin.

Despite these reservations, we remain confident that the single coil and Helmholtz coil methods are comparable experiments, both in difficulty and results given. The differences in error between the two methods is minor when compared to the considerable variation in the Earth's magnetic field with elevation and environmental interference. The single coil method is as effective in the undergraduate laboratory from a purely experimental perspective. From a teaching perspective, the single coil method may be preferred, as the requisite theory involved is both simpler to understand and more in-line with a typical undergraduate electricity and magnetism curriculum. Of course, which experiment is ultimately chosen will rely on the specific educational goals and needs of the respective department.

For the elevation measurement, there was a slight positive slope. It seems unlikely that this measurement was just an anomaly, because five measurements were taken for each elevation, and interference was generally avoided. The elevations were in a relatively narrow range, however, so it seems exceptionally strange that there would be such a dramatic shift over this elevation change. Moreover, there is no physical reason why the fields would increase the higher the elevation became. When comparing to data from the NCEI, we found that the data was actually constant at a value of 54.88 ± 0.01 across all of the areas where the data for the elevation was taken, which means that our data was a strict overestimate for field strength. Thus it seems that there was some form of systematic error in the increasing field strength. Given that most of the data was taken over the course of a single day, it's unlikely that this was due to weather patterns, but it's possible that the phone used to take the data heated up internally, which caused it measure a higher magnetic field the more measurements were taken. If the elevation data were retaken, it would be better to spread out across more diverse elevations, and try harder to keep the phone at the same temperature.

The outside temperature measurements not varying was not a surprise, since again they were taken in a relatively narrow band, and not enough to affect the sensor itself. This did help us confirm that regardless of where the deflection measurements were being taken they almost certainly were not affected by temperature. Likewise, the field strength falloff was also not surprising, since the Earth's ambient field was so small compared to the other sources of fields (such as electronic devices).

[1] HyperPhysics, "Magnetic Field of the Earth", hyperphysics.phy-astr.gsu.edu/hbase/magnetic/MagEarth

[2] National Centers for Environmental Information, "Magnetic Field Calculators," ngdc.noaa.gov/geomag/calculators/magcalc

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[7] Helesund, Simon. "Measuring Earth's Magnetic Field Using a Smartphone Magnetometer" arXiv. 13 Dec. 2018.