ECON 7320 Advanced Microecnometrics

Assignment I

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Q1: Estimating Equations

(A)

Show normal equations for $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$:

Initially, we try to minimize the function below:

$\sum u_i^2 = \sum(Y_i - \widehat{B_1} - \widehat{B_2}X_i)^2$

First order conditions:

$\frac{\partial \sum u^2}{\partial \widehat{B_1}} = 0; \frac{\partial \sum u^2}{\partial \widehat{B_2}} = 0$

Where:

$\frac{\partial \sum u^2}{\partial \widehat{B_1}} = \frac{\partial \sum(Y_i - \widehat{B_1} - \widehat{B_2}X_i)^2}{\partial \widehat{B_1}} = 0$

Partial derivative with respect to $\widehat{B_1}$

$2\sum(Y_i - \widehat{B_1} - \widehat{B_2}X_i)$ (-1) = 0

$\sum(Y_i - \widehat{B_1} - \widehat{B_2}X_i) = 0$

Partial derivative with respect to $\widehat{B_2}$

$\frac{\partial \sum u^2}{\partial \widehat{B_2}} = \frac{\partial \sum(Y_i - \widehat{B_1} - \widehat{B_2}X_i)^2}{\partial \widehat{B_2}} = 0$

$2\sum(Y_i - \widehat{B_1} - \widehat{B_2}X_i)$ ($-X_i$) = 0

$\sum(Y_iX_i - \widehat{B_1}X_i - \widehat{B_2}X_i^2) = 0$

Perform summation:

$\sum(Y_i - \widehat{B_1} - \widehat{B_2}X_{2i}) = 0$

$\sum(Y_iX_i - \widehat{B_1}X_i - \widehat{B_2}X_i^2) = 0$

Obtain the normal equation:

$\sum Y_i = \widehat{B_1}\mathrm{N} + \widehat{B_2}\sum X_{2i}$

$\sum Y_iX_{2i} = \widehat{B_1}\sum X_i + \widehat{B_2}\sum X_i^2$

Finally, we obtain the least square estimates:

$\widehat{B_1} = \frac{\sum X_i{}^2 \sum Y_i - \sum X_i \sum X_i Y_i}{N\sum X_i{}^2 - (\sum X_i)^2}$

$\widehat{B_2} = \frac{N\sum X_i Y_i - \sum X_i \sum Y_i}{N\sum X_i{}^2 - (\sum X_i)^2}$

Alternatively, estimates can be expressed in deviations of the variables from means:

$\widehat{B_1} = \bar{Y} - \widehat{B_2}\bar{X}$

$\widehat{B_2} = \frac{\sum X_i Y_i}{\sum X_i^2}$

(B)

Express equations and solutions in matrix notation:

Given:

$\sum_{i=1}^{N} Y_i$ = N$\widehat{B_1}$ + $\widehat{B_2}\sum_{i=1}^{N} X_{2i}$ (1)

$\sum_{i=1}^{N} Y_i\, X_{2i}$ = $\widehat{B_1}\sum_{i=1}^{N} X_{2i}$ + $\widehat{B_2}\sum_{i=1}^{N} X_{2i}^2$ (2)

Expression in matrix notations are:

$i^T Y_i = \; N(\frac{1}{n}\, i^T Y_i$-$\hat{\beta}_2\,\frac{1}{n}\, i^T X_{2i})$ + $\hat{\beta}_2 i^T X_{2i}$ (1)

$Y_i i^T X_{2i}$ =$\widehat{\beta}_1 i^T\; X_{2i}$ + $\hat{\beta}_2 i^T\; X_{2i}^T\, X_{2i}$ (2)

Where the solutions in matrix notations are:

$\widehat{\beta_2}$ = $(X^T X)^{-1}X^T$y = $\frac{N X_{2i}^T - i^T Y_i X_{2i}^T i}{N X_{2i}^T X_{2i} - i^T X_{2i} X_{2i}^T i}$

$\widehat{\beta_1}$ = $\frac{1}{i^T i}\, i^T Y_i$ - $\frac{1}{i^T i}\, i^T X_{2i}\widehat{\beta_2}$ = $\frac{1}{N}\, i^T Y_i$ - $\frac{1}{N}\, i^T X_{2i}\widehat{\beta_2}$

(C)

Usual moment condition of the linear regression in matrix notation:

Assumption 1: E($u_i|x_i$) = 0 (exogeneity of regressors)

Assumption 2: E($u_i^2|x_i$) = σ^2 (conditional homoskedasticity)

Assumption 3: E($u_i u_j|x_i x_j$) = 0, i ≠ j (conditionally uncorrelated observations)

Y=X β +µ (Linearity)

X is an n x K matrix with rank K (No exact linear relationships among the variables)

β = $\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$

E[u|X] = $\begin{bmatrix}E[u_1|X]\\ E[u_1|X]\\ \vdots\\ E[u_N|X]\end{bmatrix}$ = 0 (The disturbances have conditional expected value zero at every observations)

$E[u_i|X]$ = σ^2 for all i = 1,…,N. (Constant variance or homoscedasticity)

Cov[u_i, u_j|X]=0 for all i ≠ j (No autocorrelation)

Which means:

E[uu^T|X] = σ^2I ; Var[u] = E[Var[u|X] + Var[E[u|X} = σ^2I; Var[u] = E[uu^T] = σ^2

(D)

Show that moment conditions imply the least squares equations (1) – (2):

Since:

E[u_i] = E[E[$u_i|X_i$]] = E[0] = 0

E[$X_i u_i$] = E[E[$u_i X_i|X_i$]] = E[X_iE[$u_i|X_i$]]

= E[$X_i \times 0$] = 0

Given that:

$Y_i = B_1 + B_2 X_{2i} + u_i$

Estimated equation of the bivariate regression:

Y = $\widehat{B}_1$+ $\widehat{B}_2 X_2$ + e_i

Since assumptions or the moment conditions:

$\sum e_i$ = 0 and $\sum$eX_i = 0

We obtain the first normal equation by summing all sample observations and using $\sum e_i$ = 0:

$\sum$Y = N$\hat{B}_1$ + $\hat{B}_2 X_2$

Pre-multiplying the estimated form by X_2:

YX_2 = $\hat{B}_1 X_2$ + $\hat{B}_2 {X_2}^2$ + eX_2

Summing over the N sample observations and use the assumption $\sum$eX_2 = 0, we obtain the second normal equation:

$\sum$ YX_2 = $\widehat{B}_1 \sum X_2$ + $\widehat{B}_2 \sum {X_2}^2$

Q2: Generalized Least Squares

(A)

Verify that Ω = $\boldsymbol{E}$[uu$^{\boldsymbol{T}}$] is a band matrix:

Recapture the assumptions of homoscedasticity:

E[u_i^2] = E[E[u_i^2]X_i] = E[σ^2] = σ^2 ∀ i

Also recapture the assumption of no autocorrelation:

Cov($u_i u_j|X$) = E[$u_i u_j$] = 0 for i ≠ j

If two assumptions are met:

$$\Omega = E[\mathrm{uu}^T] = \begin{bmatrix} E[u_1^2] & E[u_1 u_2] & E[u_1 u_N] \\ E[u_1 u_2] & E[u_2^2] & E[u_2 u_N] \\ E[u_N u_1] & E[u_N u_2] & E[u_N^2] \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \sigma_1^2/\sigma^2 & 0 & 0 \\ 0 & \sigma_2^2/\sigma^2 & 0 \\ 0 & 0 & \sigma_n^2/\sigma^2 \end{bmatrix} = \sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \sigma^2 \mathrm{I}$$

However, E[$u_i u_j$] can also be E[ρσ^2] = ρσ^2 if |i – j| = 1 in which ρ is the residual correlation coefficient and σ^2 is the residual variance. Then Ω can be expressed as:

$$\Omega = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \rho\sigma^2 & \sigma^2 \end{bmatrix}$$

(B)

Show that V[$\widehat{\boldsymbol{\beta}}$|X] = E[$(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T}$ΩX$(\boldsymbol{X^T X})^{-1}$|X]:

V[$\hat{\beta}$|X] = E[($\hat{\beta} - \beta$) (E$(\hat{\beta} - \beta)^T$|X)

= E$(X^T X)^{-1} X^T$ΩX$(X^T X)^{-1}$

= $(X^T X)^{-1} X^T (E\mathrm{uu}^T|X) X (X^T X)^{-1}$

= $\sigma^2 (X^T X)^{-1} X^T (X^T X)\ (X^T X)^{-1}$

= $\sigma^2 (X^T X)^{-1}$

Where:

E[$\hat{\beta}$] = E[E[$\hat{\beta}$|X]] = E[β] = β

$\hat{\beta}$ - E[$\hat{\beta}$|X] = $\hat{\beta}$ - β = $(X^T X)^{-1} X^T$u

Ω = $(E\mathrm{uu}^T|X)$ = E[(u-E[u|X]) (u − E[u|X]$)^T$|X]

= V[u|X] = σ^2I

Therefore:

$\hat{\beta}_{OLS} \sim (\beta, \sigma^2 (X^T X)^{-1})$

(C)

Is the usual OLS estimates $\mathbf{s^2 (X^T X)^{-1}}$ a consistent estimator?

No, although $s^2 (X^T X)^{-1}$ is the default estimator of the VCE, it may not be the consistent estimator due to E[$u_i u_j$] inside the Ω = σ^2I. More precisely, since E[$u_i u_j$] can also be E[ρσ^2] = ρσ^2, so unless ρ is zero, this usual OLS estimates $s^2 (X^T X)^{-1}$ cannot be consistent.

(D)

Is White's heteroskedasticity robust estimator of V[$\hat{\beta}$|X] consistent?

No, the White's heteroskedasticity robust estimator of $V(\hat{\beta}|X)$ will not be consistent if heteroskedasticity exists due to the fact that Ω = diag(σ_1^2, …, σ_n^2). The OLS standard error will be incorrect when X^TE[uu^T]$X \neq \sigma^2(X^T X)$. Since E[$u_i u_j$] can also be E[ρσ^2] = ρσ^2; therefore, ρ must be zero for the White's heteroskedasticity robust estimator of $V(\hat{\beta}|X)$ to be consistent and otherwise.

(E)

How to obtain consistent estimate of V[$\hat{\beta}$|X] that does not depend on unknown parameters?

We should use the feasible generalized least squares estimator (FGLS). As we know, an asymptotically efficient FGLS estimator does not require efficient estimator of θ but a consistent one is required to achieve full efficiency. In FGLS, we will replace the unknown **Ω** with a consistent estimator. We have $\widehat{\Omega} \xrightarrow{p} \Omega$ when $\widehat{\sigma^2} \xrightarrow{p} \sigma^2$ and $\widehat{\rho\sigma^2} \xrightarrow{p} \rho\sigma^2$ and otherwise $\hat{\rho} \xrightarrow{p} \rho$. Further, we let $\widehat{\Omega}$ = **Ω**($\hat{\theta}$) to replace the true **Ω**. The FGLS can be described as $\hat{\beta}_F$ = $[X^T\widehat{\Omega}^{-1}X]^{-1}X^T\,\widehat{\Omega}^{-1}$y. As a result, $\hat{\beta}_F$ is consistent under the general condition.

Since plim $\hat{\theta} = \theta$, conditions for $\hat{\beta}_F$ asymptotically equal to $\hat{\beta}$ are shown below:

Plim [($\frac{1}{N}X^T\widehat{\Omega}^{-1}X$) - ($\frac{1}{N}X^T\widehat{\Omega}^{-1}X$)] = 0

Plim [($\frac{1}{\sqrt{N}}X^T\widehat{\Omega}^{-1}$u) - ($\frac{1}{\sqrt{N}}X^T\widehat{\Omega}^{-1}$u)] = 0

As we know: $\widehat{\sigma^2} = \frac{1}{N}\sum_{i=1}^{N}(\mathrm{Y} - X^T\hat{\beta})^2$. Then to obtain FGLS, we need to estimate for ρ in which the estimator of ρ is: $\hat{\rho} = \frac{\sum_{t=2}^{T}\widehat{u_{t-1}}\widehat{u_t}}{\sum_{t=2}^{T}\widehat{u_{t-1}^2}}$. So then $\widehat{\sigma^2}\hat{\rho} = \frac{1}{N}\sum_{i=1}^{N}(\mathrm{Y} - X^T\hat{\beta})^2\,\frac{\sum_{t=2}^{T}\widehat{u_{t-1}}\widehat{u_t}}{\sum_{t=2}^{T}\widehat{u_{t-1}^2}}$.

Insert the $\hat{\rho}$ into the inverse of autocovariance matrix, Ω_1^{-1}, and then plug this estimate of Ω^{-1} into $[X^T\widehat{\Omega}^{-1}X]^{-1}X^T\,\widehat{\Omega}^{-1}$y should we obtain the FGLS estimator $\hat{\beta}_F$.

Q3: Minimizing A Quadratic Form

(A)

Obtain the formula for $\widehat{B}$ which maximizes the objective function $Q_N(\beta)$ = - u^TWu:

Given: y = X β + u

$Q_N(\beta) = -\, u^T$Wu

$u^T w^T\, u = u^T[\frac{1}{2}(w + w^T)]\, u = u^T \widehat{w}\, u$

$$\frac{\partial Q_N(\beta)}{\partial \beta} = -\frac{\partial u}{\partial \beta}\frac{\partial u^T \widehat{w}\, u}{\partial \beta} = -\,(-X^T)\,(2\widehat{w}\, u)$$

$$= 2X^T \widehat{w}\,(\mathrm{y} - \mathrm{X}\,\beta)$$

$\frac{\partial Q_N(\beta)}{\partial B}\,|_{\widehat{B}} = 0$ then $X^T\widehat{w}X\widehat{B} = X^T\widehat{w}y$

$\widehat{B} = (X^T\widehat{w}\,X)^{-1}(X^T\widehat{w}y)$

Where $(X^T\widehat{w}\,X)^{-1}$ exists and has full rank

(B)

For which W does your answer to (a) equal the OLS estimator?

W = I_N

(C)

For which W does your answer to (a) equal the GLS estimator?

W = Ω^{-1} where E[$u^T u$] = Ω

(D)

How you would obtain the Feasible GLS estimator if $\Omega = \mathrm{E}[\mathrm{uu}^{\mathrm{T}}]$?

The W should be $\widehat{\Omega}^{-1}$ in which the variance-covariance matrix of the coefficient vector from the White estimator should be:

$(X^TX)^{-1}X^T(\mathrm{ee}^T)X(X^TX)^{-1}$ rather than:

$(X^TX)^{-1}X^T(\mathrm{uu}^T)X(X^TX)^{-1} = (X^TX)^{-1}X^T(\sigma^2\mathrm{I})X(X^TX)^{-1}$.

In practice, we should test for the existence of heteroskedasticity by regressing the squared residuals e_i^2 on X^TX. If the R^2 is larger than the critical value (n$R^2 \sim \chi_k^2$), then the model will take a toll on heteroskedasticity. Thereafter, we use the White estimator.

Q4: Data Analysis

(A)

Log transformation of all variables:

```
. generate lcost=log(cost)

. generate lkwh=log(kwh)

. generate lpl=log(pl)

. generate lpf=log(pf)

. generate lpk=log(pk)
```

Correlation matrix:

```
. correlate lcost lkwh lpl lpf lpk
(obs=145)

             |    lcost     lkwh      lpl      lpf      lpk
-------------+---------------------------------------------
       lcost |   1.0000
        lkwh |   0.9542   1.0000
         lpl |   0.1174   0.0422   1.0000
         lpf |  -0.0455  -0.1689   0.3319   1.0000
         lpk |  -0.1042  -0.0988  -0.1865   0.1309   1.0000
```

Plot lcosts against lkwh:

```
. plot lcost lkwh

 4.93751 +
         |                                                               *
         |                                                                 *
         |                                                            * *
         |                                                            **
         |                                                      * ** **
         |                                                    ******
         |                                               * *** * * *
    l    |                                              ********
    c    |                                           ********
    o    |                                   *  *** ******
    s    |                                  * **** **
    t    |                              *  *****
         |                            *  * *  **
         |     *       *    *       *  *** *
         |   *          *   *      * *  *
         |              *  *    *  *
         |     *               *
         |       *
         |
-2.50104 + *         *
          +----------------------------------------------------------------+
          .693147                      lkwh                        9.7243
```

Comment:

Firstly, the correlation matrix shows that correlation between lcost and lkwh is 0.95 which is extremely high. The coefficient of lkwh can be biased if there exists omitted variable. Secondly, the plot shows that the relationship between lcost and lkwh is less clear when both are at low regions but becomes more linear and monotonic increasing when both gets higher and higher.

(B)

Regress lcosts on lkwh and an intercept:

```
. reg lcost lkwh, robust

Linear regression                               Number of obs     =        145
                                                F(1, 143)         =     423.43
                                                Prob > F          =     0.0000
                                                R-squared         =     0.9104
                                                Root MSE          =     .42699

------------------------------------------------------------------------------
             |               Robust
       lcost |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lkwh |   .7092017   .0344649    20.58   0.000     .6410752    .7773282
       _cons |  -2.925325    .246447   -11.87   0.000    -3.412474   -2.438175
------------------------------------------------------------------------------
```

Regress lcosts on lkwh, lpl, lpk, lpf and an intercept:

```
. reg lcost lkwh lpl lpf lpk, robust

Linear regression                               Number of obs     =        145
                                                F(4, 140)         =     175.79
                                                Prob > F          =     0.0000
                                                R-squared         =     0.9260
                                                Root MSE          =     .39236

------------------------------------------------------------------------------
             |               Robust
       lcost |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lkwh |   .7203941   .0325975    22.10   0.000      .655947    .7848411
         lpl |   .4363412   .2456358     1.78   0.078     -.049294    .9219763
         lpf |   .4265169   .0754827     5.65   0.000     .2772835    .5757503
         lpk |  -.2198882   .3238121    -0.68   0.498    -.8600822    .4203057
       _cons |  -3.526503     1.7186    -2.05   0.042    -6.924269    -.128738
------------------------------------------------------------------------------
```

Explain why the two estimates are different:

The coefficients of lkwh in the second regression is larger than that in the first regression. This difference is caused by the omitted variable bias (OVB) in the first regression in which the OVB may results in violations of unbiasedness and consistency in the OLS regression.

(C)

Using the R^2 measure of goodness of fit would you say that the first regression in (B) provides a satisfactory fit to the data? Explain:

The R^2 of the first regression is 0.91 which implies that lkwh and lcost are highly correlated. However, R^2 has limitation in which it fails to tell whether the coefficient and prediction are biased and cannot indicate whether the regression has an adequate fit.

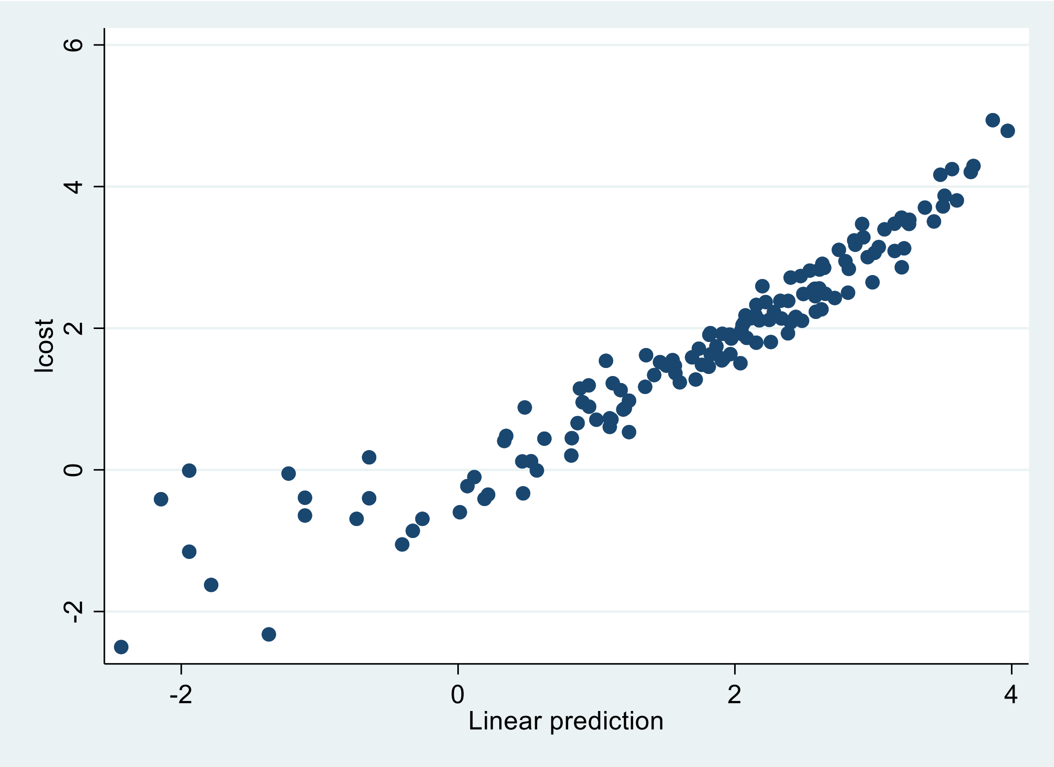
(D)

Generate the fitted values of the dependent variable in the first regression of part (B):

```
. quietly reg lcost lkwh, robust

. predict lyhat, xb
```

Scatter plot of the fitted and observed values of lcosts:

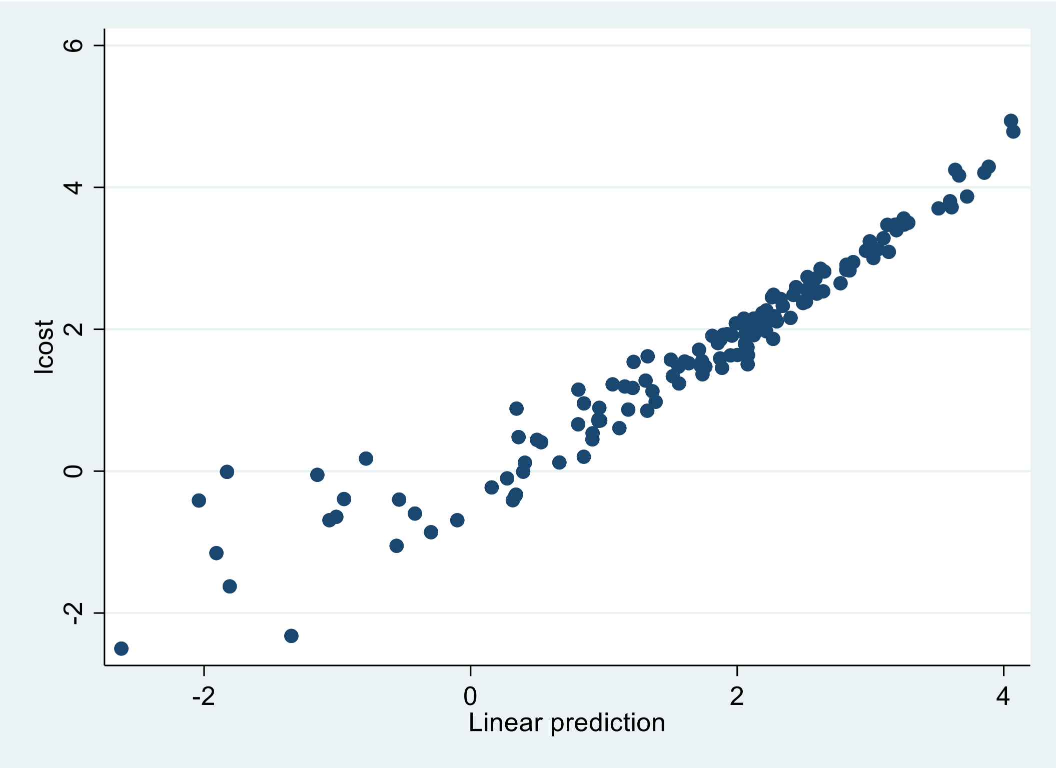


Comment on the goodness of fit of the model:

Generally speaking, this is a good fit since the plot is nearly 45 degrees after lcost becomes positive. However, when lcost is in negative region, the discrepancy between the observed and the fitted values is quite large. The fit gets better as lcost moves past zero.

(E)

Scatter plot of observed values of lcosts and the fitted values of the second regression in part (B):



Interpret the results:

The points generated from the second regression are more concentrated around the 45-degree line than the first regression in part (B). Although it is difficult to distinguish them visually, the R^2 of the second regression is higher than that of the first regression. As a result, the second regression has better fit.

(F)

What advantages does this assumption (error term u has log-normal distribution) have?

A random variable which is log-normally distributed takes only positive real values. As a result, ln(u) will be normally distributed. The main advantage is that it can be used to apply on the Cobb-Douglas function which requires log linearization. Moreover, we should use log-normal distribution to deal with any data that exhibits low average and large variance.

(G)

Suppose we change the functional form of the cost function in which the error term enters additively and is assumed to be normally distributed. Is ordinary least squares an appropriate estimator? Justify your answer:

No, in this case, the Nonlinear Least Squares (NLS) estimator should be used for the Cobb-Doubles production function which is nonlinear. The estimators are assumed to be jointly normal. The NLS is achieved by means of minimizing the sum of squares However, this minimization process cannot be solved analytically and instead, it must be solved with numerical methods that requires iterative algorithm. Another way to tackle this type of nonlinear function is by using the Maximum Likelihood Estimation (MLE).

(H)

Given the alternative specification, what advantages if any does the original (with multiplicative error) functional form have relative to this one?

The original model (with multiplicative error) functional form will indicate whether the inputs have increasing $((\alpha_1 + \alpha_2 + \alpha_3) > 1)$, decreasing $((\alpha_1 + \alpha_2 + \alpha_3) < 1)$, or constant returns to scale $((\alpha_1 + \alpha_2 + \alpha_3) = 1)$. On the contrary, the alternative model strictly imposes constant return to scale which is unfavored (too little R^2 and insignificant F-test) by the regression which will be shown in part (I). Furthermore, if we conduct test on whether the inputs result in constant returns to scale on the output, the test shows that we can reject the null hypothesis that the output, lkwh, is generated with constant returns to scale at the five percent significance level.

```
. test lkwh==1

 ( 1)  lkwh = 1

       F(  1,   140) =   73.57
            Prob > F =    0.0000
```

(I)

Estimate this alternative model and interpret the regression results:

Alternative model:

```
. generate cpkwh =lcost/lkwh

. reg cpkwh lpl lpf lpk, robust

Linear regression                               Number of obs     =        145
                                                F(3, 141)         =       1.13
                                                Prob > F          =     0.3409
                                                R-squared         =     0.0203
                                                Root MSE          =     .39831

------------------------------------------------------------------------------
             |               Robust
       cpkwh |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lpl |  -.1455475   .3134051    -0.46   0.643    -.7651279     .474033
         lpf |   .0496846    .116223     0.43   0.670    -.1800803    .2794495
         lpk |  -.5854167   .4010958    -1.46   0.147    -1.378356    .2075222
       _cons |   3.137474   1.972547     1.59   0.114    -.7621171    7.037064
------------------------------------------------------------------------------
```

Original model (multiplicative error):

```
. reg lcost lkwh lpl lpf lpk,robust

Linear regression                               Number of obs     =        145
                                                F(4, 140)         =     175.79
                                                Prob > F          =     0.0000
                                                R-squared         =     0.9260
                                                Root MSE          =     .39236

------------------------------------------------------------------------------
             |               Robust
       lcost |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lkwh |   .7203941   .0325975    22.10   0.000      .655947    .7848411
         lpl |   .4363412   .2456358     1.78   0.078     -.049294    .9219763
         lpf |   .4265169   .0754827     5.65   0.000     .2772835    .5757503
         lpk |  -.2198882   .3238121    -0.68   0.498    -.8600822    .4203057
       _cons |  -3.526503     1.7186    -2.05   0.042    -6.924269    -.128738
------------------------------------------------------------------------------
```

Interpretation of the regression results:

The R^2 of the alternative model is nearly zero while the R^2 of the original model with multiplicative error is above 0.90. The F-test of the alternative model also indicates that we fail to reject the null hypothesis that nothing is going on at the five percent significance level. Moreover, none of p-values for each independent variable in the alternative model can be statistically significant at the five percent level.

Appendix: STATA code for Q4:

```
clear
capture log close
set logtype text
log using A1Q4CSLO.txt, replace
cd "D:\UQ\2020 S1\ECON7320 Advanced MicroMetrics\Assignment"
use nerlove63.dta
//A//
generate lcost=log(cost)
generate lkwh=log(kwh)
generate lpl=log(pl)
generate lpf=log(pf)
generate lpk=log(pk)
correlate lcost lkwh lpl lpf lpk
//plot lcost lkwh//
//scatter lcost lkwh//
//graph export a.png, replace//
//B//
reg lcost lkwh, robust
reg lcost lkwh lpl lpf lpk, robust
//C//
//Explain r-squared from part B//
//D//
quietly reg lcost lkwh, robust
predict lyhat, xb
//scatter lcost lyhat//
//E//
quietly reg lcost lkwh lpl lpf lpk, robust
predict lyhat2, xb
//scatter lcost lyhat2 //
```

```
//graph export E.png, replace//
//F//
//See word or pdf//
//G//
//See word or pdf//
//H//
reg lcost lkwh lpl lpf lpk,robust
test lkwh==1
//I//
generate cpkwh =lcost/lkwh
reg cpkwh lpl lpf lpk, robust
log cl
```