ECON 7350 Applied Econometrics for Macroeconomics and Finance

Research Project 1

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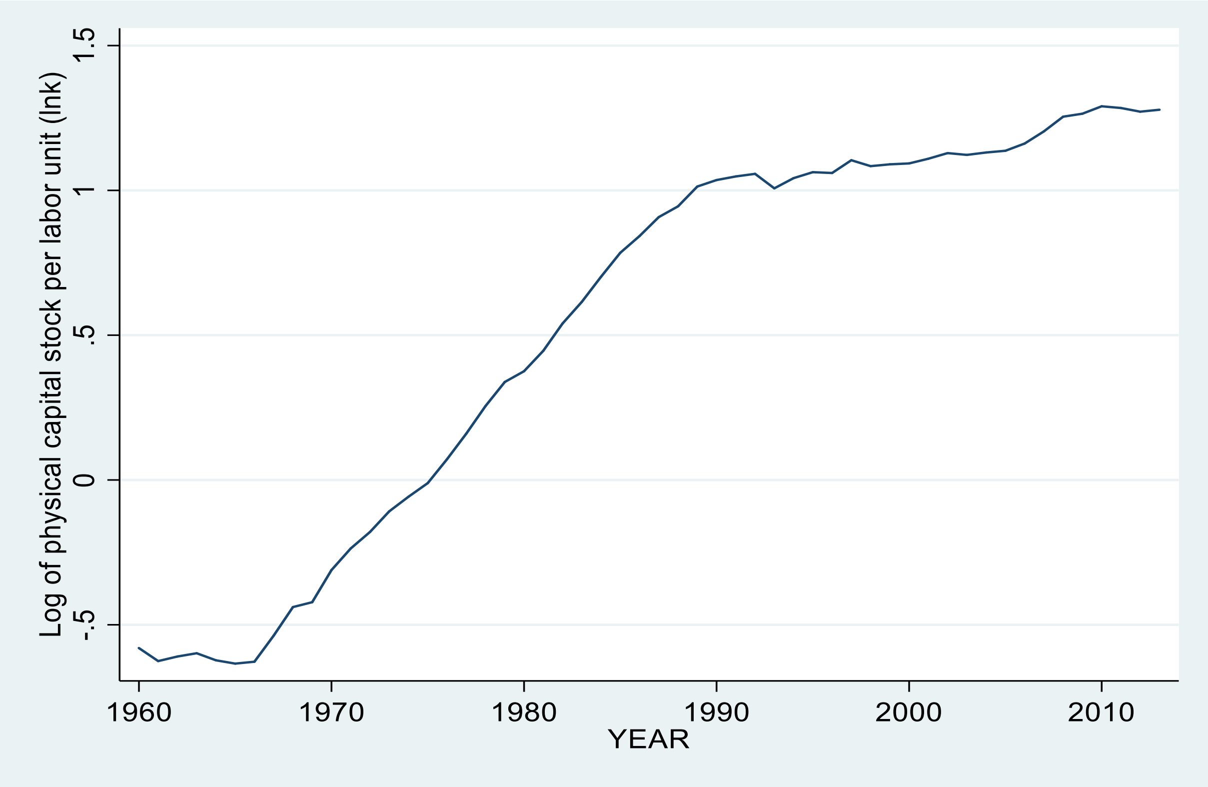
(A)
Figure 1. Time series plot of $\{ln\, k_t\}$



Comment:
Apparently, the time series of log of physical capital stock per labor unit $\{ln\, k_t\}$is explosive. On the annual frequency basis, the data especially did not exhibit any signs of mean reversion from year 1970 to l990. During the 1970~1990 period, it shows a strong deterministic trend. In sum, this time series is not stationary.

(B)

Figure 2. ACF of $\{ln\, k_t\}$

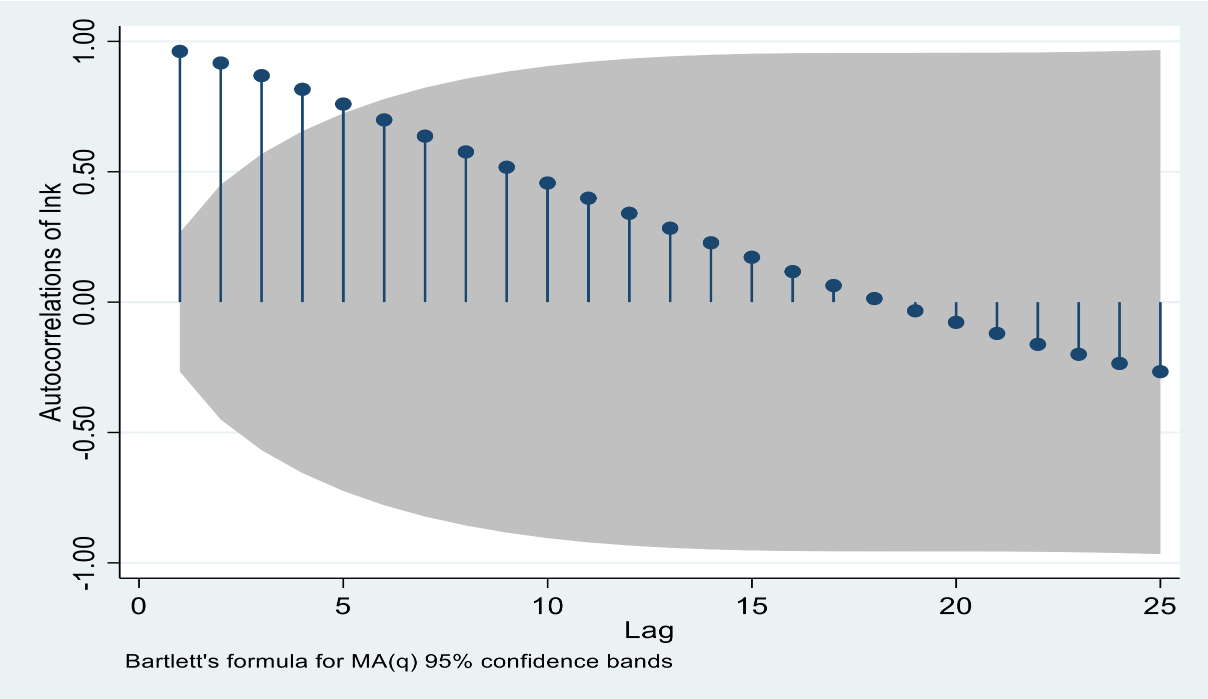
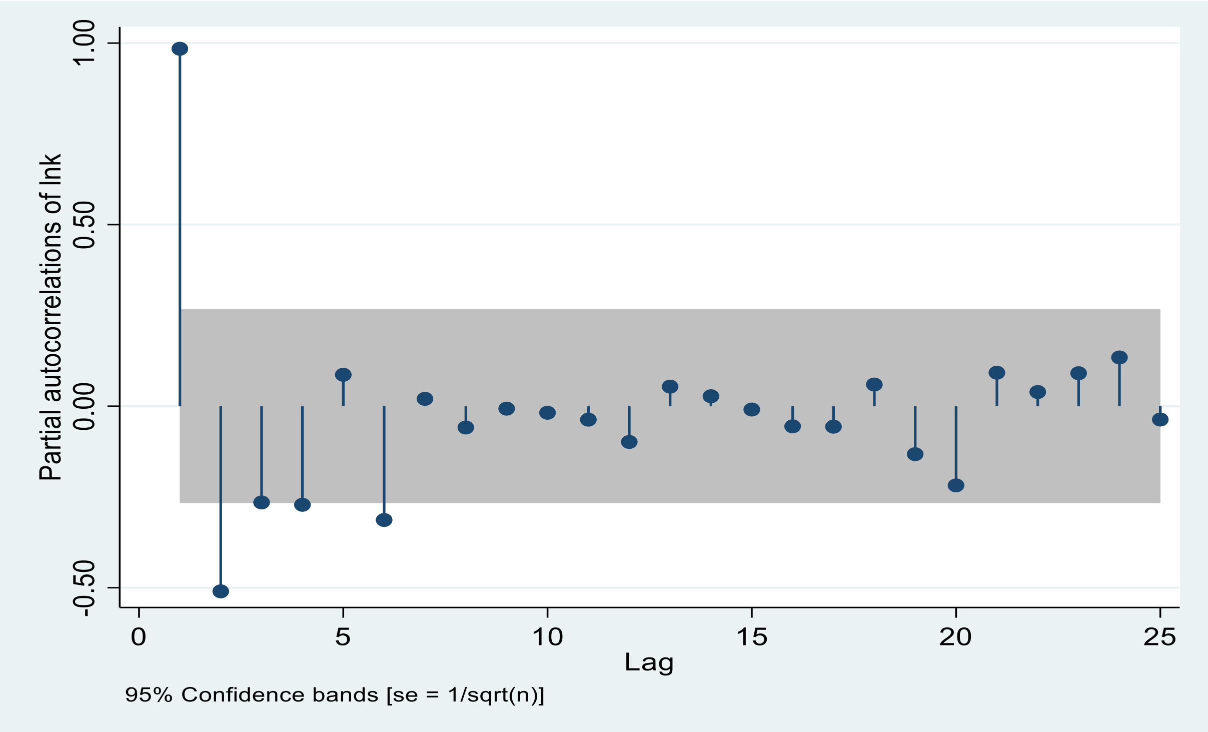


Figure 3. PACF of $\{ln\, k_t\}$



Comment:

For the DGP of $\{ln\, k_t\}$, its ACF is prevailed by AR(1) process which converges toward zero geometrically from lag 1 to lag 19. However, its ACF turns negative from lag 20. Moreover, its PACF has two non-zero peaks and decays in a oscillatory path from lag 2 as MA(1) process dominates. Altogether, the patterns of ACF and PACF imply that $\{ln\, k_t\}$ may contain unit root.

Moreover, it resembles the pattern of ARMA (2,0) because it shows gradual decaying and has two peaks.

(C)

I tested the following AICs and BICs of ARIMA (1,1,0), (1,1,1), (1,1,2), (1,1,3), (1,1,4), (2,1,0), (2,1,1), (2,1,2), (2,1,3), and (2,1,4) models. Among them, ARIMA (2,1,2) is the best model because it has the lowest AIC and BIC.

AIC and BIC tests for ARIMA

Models	AIC	BIC
ARIMA (1,1,0)	-208.9989	-203.088
ARIMA (1,1,1)	-212.5874	-204.7062
ARIMA (1,1,2)	-210.7306	-200.8791
ARIMA (1,1,3)	-210.5381	-198.7164
ARIMA (1,1,4)	-206.8035	-193.0115
ARIMA (2,1,0)	-210.9248	-203.0436
ARIMA (2,1,1)	-210.678	-200.8265
ARIMA (2,1,2)	-216.3281*	-206.4766*
ARIMA (2,1,3)	-214.3194	-202.4976
ARIMA (2,1,4)	-210.4938	-194.7314

Where: * denotes the best model

Below is the estimated model for ARIMA (2,1,2):

$$\{\widehat{\Delta \ln k_t}\} = \underset{(0.0039)}{0.0366} + \underset{(0.0091)}{1.9768}\Delta \ln k_{t-1} - \underset{(0\ .0115)}{0.9948}\Delta \ln k_{t-2} - \underset{(0.0206)}{1.9976}\Delta\varepsilon_{t-1} + 1.0000\Delta\varepsilon_{t-2}$$

```
ARIMA regression

Sample:  1961 - 2013                            Number of obs     =         53
                                                Wald chi2(3)      =  438701.83
Log likelihood =   113.164                      Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
       D.lnk |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnk          |
       _cons |   .0365961   .0038975     9.39   0.000     .0289572    .0442349
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   1.976776   .0090572   218.26   0.000     1.959024    1.994528
         L2. |  -.9948172   .0114776   -86.67   0.000    -1.017313   -.9723214
             |
          ma |
         L1. |  -1.997584   .0206332   -96.81   0.000    -2.038025   -1.957144
         L2. |    1.00001          .        .       .            .           .
-------------+----------------------------------------------------------------
      /sigma |    .026253   .0031816     8.25   0.000     .0200173    .0324888
------------------------------------------------------------------------------
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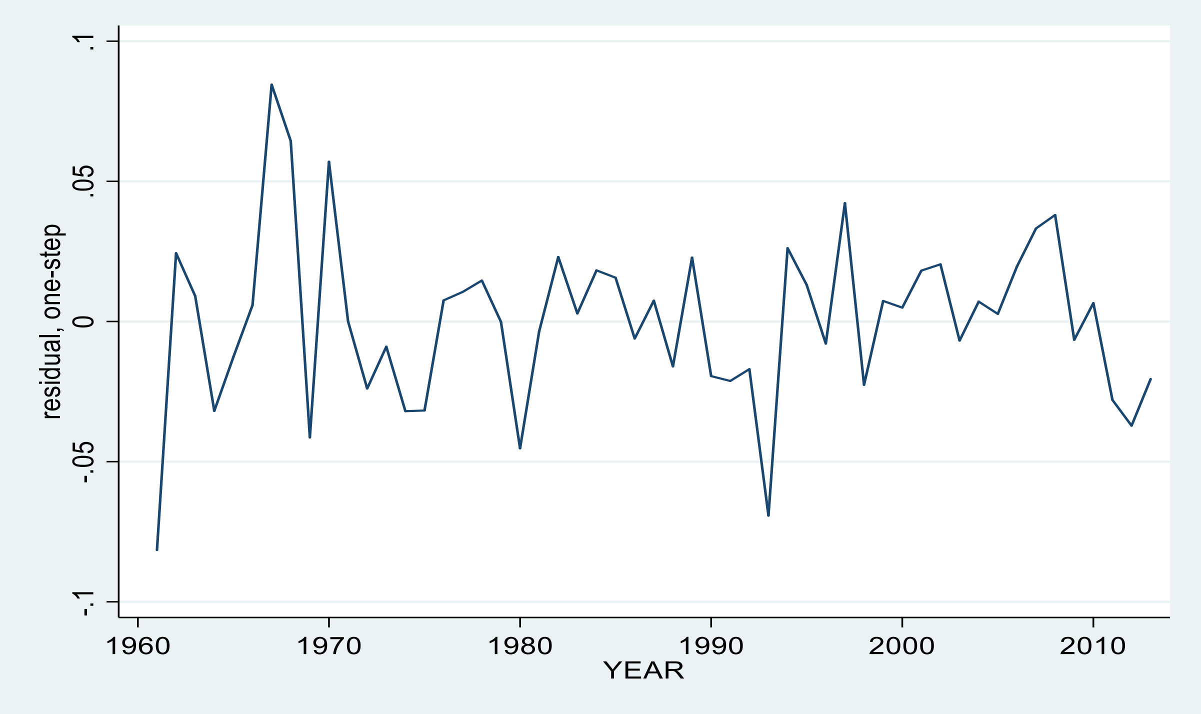
(D)

Figure 4. Residuals of ARIMA (2,1,2) for $\{ln\, k_t\}$



The process $\{\widehat{\varepsilon_t}\}$ of residuals had been mean reverted around zero. Then I run Ljung-Box test from lag 5 to 9 to check for autocorrelation.

The test of Ljung-Box:

Ho: The variable is independently distributed; so exhibits white noise process.
Ha: The variable is not independently distributed; so exhibits autocorrelation.

Lung-Box test results:

Lag	Q-statistics	Critical Value	Decision @ 95 % significance
5	3.8878	3.8415	Reject the null
6	6.3003	5.9915	Reject the null
7	8.8939	7.8147	Reject the null
8	9.1988	9.4877	Do not reject the null
9	12.1670	11.0705	Reject the null

Explanation:

I started the test from lag 5 because Q-statistic requires (k-2-2) degrees of freedom since ARIMA (2,1,2) has p=2 and q=2. I tested from lag 5 to 9; the result shows that except for lag 8, we can reject the Ho that $\{\widehat{\varepsilon_t}\}$ is a white noise process at 95 percent significance level and conclude that $\{\widehat{\varepsilon_t}\}$ exhibits autocorrelation. Overall, this ARIMA is inadequate due to presence of autocorrelation which is likely to cause inaccurate computed variance and inaccurate standard error of forecast.

(E)

The ADF test is conducted for $\{ln\, k_t\}$, $\{ln\, y_t\}$, $\{ln\, w_t\}$, and $\{ln\, O_t\}$ and each with three different equations.

The three equations are:
Equation (1): the pure random walk model.
Equation (2): adds a drift term to the previous model, Equation (1).
Equation (3): adds a drift and a linear time trend to the random walk model.

The hypothesis test in ADF is described below:
Ho: The variable contains a unit root
Ha: The variable is stationary

(E-1)

For $\{ln\, k_t\}$, the test is conducted with lag 2. We failed to reject the null for equations (1) and (3) but can reject the null for equation (2) at 95 percent significance level. Although the results are ambiguous, the ACF and PACF of $\{ln\, k_t\}$ show that there exists at least one unit root because $\rho_1 \geq 0.95$. To determine the most suitable specification, we move on to conduct Ø3 and Ø2 tests subsequently. As a result, we cannot reject the null of Ø3 and Ø2 tests at 95 percent significance level. Altogether, we can speculate that $\{ln\, k_t\}$ contains unit roots and a drift. To be cautious, we should try further with the first differences$\{\Delta ln\, k_t\}$. However, we still get the same decision as the previous result. So I try again with second differences$\{\Delta^2\, ln\, k_t\}$; consequently, we can see that the unit roots of $\{ln\, k_t\}$ for equations (1) and (3) will not be eliminated until second differences.

	F-test	Critical Value	Decision@95%
phi 3	2.24	6.73	Do not reject
phi 2	2.63	5.13	Do not reject

lnk	Test Statistics	5% Critical Value	P-Value
Equation (1)	-0.451	-1.950	N.A.
Equation (2)	-2.132*	-1.678	0.0191
Equation (3)	-0.806	-3.499	0.9652

Δlnk	Test Statistics	5% Critical Value	P-Value
Equation (1)	-1.130	-1.950	N.A.
Equation (2)	-1.715*	-1.678	0.0466
Equation (3)	-2.503	-3.499	0.3262

Δ^2lnk	Test Statistics	5% Critical Value	P-Value
Equation (1)	-4.640*	-1.950	N.A.
Equation (2)	-4.589*	-1.678	0.0000
Equation (3)	-4.749*	-3.499	0.0006

Note: * denotes statistically significant at 5%

(E-2)
For $\{ln\, y_t\}$, the test is conducted with lag 2. We failed to reject the null for all three equations at 95 percent significance level. After trying the steps of roadmap, we may conclude that the series is trend stationary and has a quadratic trend. As a result, the next step may require second differences$\{\Delta^2\ ln\, y_t\}$. Initially, I estimated it with $\{\Delta\, ln\, y_t\}$ and found that we still failed to reject the null for equation (1) at 95 percent significance level. But after second differences, we can certainly eliminate the unit roots for all three equations at 95 percent significance level.

lny	Test Statistics	5% Critical Value	P-Value
Equation (1)	4.315	-1.950	N.A.
Equation (2)	-1.110	-1.678	0.1363
Equation (3)	-1.704	-3.499	0.7489

Δlny	Test Statistics	5% Critical Value	P-Value
Equation (1)	-1.880	-1.950	N.A.
Equation (2)	-4.386*	-1.678	0.0000
Equation (3)	-4.490*	-3.499	0.0016

Δ^2lny	Test Statistics	5% Critical Value	P-Value
Equation (1)	-7.804*	-1.950	N.A.
Equation (2)	-7.765*	-1.678	0.0000
Equation (3)	-7.677*	-3.499	0.0000

Note: * denotes statistically significant at 5%

(E-3)
For $\{ln\, w_t\}$, the test is conducted with lag 2. We failed to reject the null for all three equations at 95 percent significance level. Similar to $\{ln\, y_t\}$, we may conclude that the series is trend stationary and has a quadratic trend. Thus, we continue to try the first differences but still failed to reject the null for equation (3). Then we should try second differences$\{\Delta^2\ ln\, w_t\}$; eventually, we can reject the null for all three equations at 95 percent significance level.

lnw	Test Statistics	5% Critical Value	P-Value
Equation (1)	-1.033	-1.950	N.A.
Equation (2)	-1.213	-1.678	0.1156
Equation (3)	-1.912	-3.499	0.6483

Δlnw	Test Statistics	5% Critical Value	P-Value
Equation (1)	-2.190*	-1.950	N.A.
Equation (2)	-2.383*	-1.678	0.0107
Equation (3)	-2.294	-3.499	0.4374

Δ^2lnw	Test Statistics	5% Critical Value	P-Value
Equation (1)	-5.079*	-1.950	N.A.
Equation (2)	-5.023*	-1.678	0.0000
Equation (3)	-5.061*	-3.499	0.0002

Note: * denotes statistically significant at 5%

(E-4)

For $\{ln\,O_t\}$, the test is conducted with lag 2. We failed to reject the null for equations (1) and (3) but can reject the null for equation (2) at 95 percent significance level. Although the results are ambiguous, the ACF and PACF of $\{ln\,O_t\}$ show that there exists at least one unit root because $\rho_1 \geq 0.95$. Moreover, to determine the most suitable specification, we move on to conduct Ø3 and Ø2 subsequently. As a result, we cannot reject the null of Ø3 and Ø2 tests at 95 percent significance level. To be cautious, we should try further with first differences $\{\Delta\,ln\,O_t\}$; eventually, we can reject the null for all three equations at 95 percent significance level.

	F-test	Critical Value	Decision@95%
phi 3	2.53	6.73	Do not reject
phi 2	1.70	5.13	Do not reject

lno	Test Statistics	5% Critical Value	P-Value
Equation (1)	-0.830	-1.950	N.A.
Equation (2)	-1.926*	-1.678	0.0301
Equation (3)	-2.231	-3.499	0.4723

Δlno	Test Statistics	5% Critical Value	P-Value
Equation (1)	-4.179*	-1.950	N.A.
Equation (2)	-4.136*	-1.678	0.0001
Equation (3)	-4.087*	-3.499	0.0066

Note: * denotes statistically significant at 5%

(F)

Based on BIC, we choose the **ARDL (3,0,1,0)** regression model. **The estimated model** is shown below:

$$\{\widehat{ln\,y_t}\} = \underset{(1.0844)}{2.7321} + \underset{(0.0046)}{0.0048t} - \underset{(0\ .0223)}{0.0313D_{01}} - \underset{(0.0166)}{0.0289D_0} + \underset{(0.0267)}{0.0023D_{77}} + \underset{(0.1475)}{1.2466ln\,y_{t-1}}$$

$$- \underset{(0.2126)}{0.7763ln\,y_{t-2}} + \underset{(0.1421)}{0.3286ln\,y_{t-3}} + \underset{(0.0387)}{0.0493\,ln\,k_t} - \underset{(0\ .0730)}{0.1471ln\,w_t}$$

$$+ \underset{(0.0751)}{0.1880\,ln\,w_{t-1}} + \underset{(0.0220)}{0.0101ln\,O_t}$$

R^2= 0.9993

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Sample: 1964 - 2013                               Number of obs     =         50
                                                  F(  11,      38)  =    4742.94
                                                  Prob > F          =     0.0000
                                                  R-squared         =     0.9993
                                                  Adj R-squared     =     0.9991
Log likelihood =  129.45628                       Root MSE          =     0.0208
```

lny	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lny						
L1.	1.246609	.1474874	8.45	0.000	.9480363	1.545182
L2.	-.7762955	.2126313	-3.65	0.001	-1.206745	-.3458458
L3.	.3285665	.1420503	2.31	0.026	.0410006	.6161323
lnk	.0492925	.0386811	1.27	0.210	-.0290134	.1275983
lnw						
--.	-.1470679	.0729489	-2.02	0.051	-.2947453	.0006095
L1.	.1879981	.075057	2.50	0.017	.0360532	.339943
lno	.0101193	.0220055	0.46	0.648	-.0344284	.0546671
d01	-.0312674	.0222532	-1.41	0.168	-.0763166	.0137818
d0	-.0289431	.0166422	-1.74	0.090	-.0626335	.0047473
d77	.0022812	.0266803	0.09	0.932	-.0517303	.0562927
t	.0047761	.004608	1.04	0.307	-.0045523	.0141046
_cons	2.732141	1.084377	2.52	0.016	.5369344	4.927347

(G)

The estimated ECM can be represented as:

$$\{\widehat{\Delta \ln y_t}\} = \underset{(1.0580)}{2.4168} + \underset{(0.0046)}{0.0037}t - \underset{(0.0884)}{0.1768}\,[\ln y_{t-1} - \underset{(0.3103)}{0.3138}\ln k_{t-1} - \underset{(0.0975)}{0.2074}\ln w_{t-1}$$

$$- \underset{(0.1269)}{0.0419}\ln O_{t-1}\,] + \underset{(0\ .1236)}{0.4107}\Delta \ln y_{t-1} - \underset{(0.1402)}{0.2959}\Delta \ln y_{t-2} + \underset{(0.0386)}{0.0555}\,\Delta \ln k_t$$

$$- \underset{(0.0733)}{0.1492}\Delta \ln w_t + \underset{(0.0220)}{0.0074}\,\Delta \ln O_t - \underset{(0.0224)}{0.0306}\Delta D_{01t-1}$$

$$- \underset{(0.0167)}{0.0286}\Delta D_{0t-1} + \underset{(0.0268)}{0.0001}\Delta D_{77t-1}$$

R^2= 0.5889

Comment:

After estimating the ECM, I find that the speed of adjustment is slow based on the magnitude of coefficient and if we incorporate the result from part (E), we learn that $\ln y_t$ is possibly a trend stationary time series. Thus, $\ln y_t$ may take quite some time to return to the long run equilibrium or steady state.

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ARDL(3,0,1,0) regression

Sample: 1963 - 2013                             Number of obs     =         51
                                                R-squared         =     0.5889
                                                Adj R-squared     =     0.4730
Log likelihood =  131.60073                     Root MSE          =     0.0210
```

D.lny	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ADJ						
lny L1.	-.1767611	.0884139	-2.00	0.053	-.3555952	.0020729
LR						
lnk L1.	.3138241	.3103313	1.01	0.318	-.3138803	.9415285
lnw L1.	.2073692	.0974537	2.13	0.040	.0102504	.4044879
lno L1.	.0418554	.1268715	0.33	0.743	-.2147663	.2984772
SR						
lny LD.	.4106602	.1235888	3.32	0.002	.1606783	.6606422
L2D.	-.2959411	.1402152	-2.11	0.041	-.579553	-.0123292
lnk D1.	.0554719	.0385538	1.44	0.158	-.0225106	.1334544
lnw D1.	-.1491997	.0733392	-2.03	0.049	-.2975423	-.0008571
lno D1.	.0073984	.0220121	0.34	0.739	-.0371253	.0519221
d01	-.0305867	.0223716	-1.37	0.179	-.0758376	.0146641
d0	-.0286444	.0167344	-1.71	0.095	-.0624929	.005204
d77	.0001318	.0267706	0.00	0.996	-.0540168	.0542804
t	.0037351	.0045513	0.82	0.417	-.0054707	.012941
_cons	2.416759	1.05803	2.28	0.028	.2766915	4.556827

(H)

I conducted the Engle-Granger test with lag 2 to see whether ($\{ln\, k_t\}$,$\{ln\, y_t\}$,$\{ln\, w_t\}$, and $\{ln\, O_t\}$) are cointegrated. Hypothesis test and test results are represented below.

Engle-Granger test

Ho: No cointegration among the variables
Ha: Cointegration exist among the variables

Explanation:

The pre-test shows that none of $\{ln\, k_t\}$, $\{ln\, y_t\}$, and $\{ln\, w_t\}$ can be certainly stationary even after taking the first difference. We then conduct cointegration test with Engle-Granger to see whether the system of multivariate equation can be stationary. The Engle-Granger shows that we failed to reject the null hypothesis and; thus there is no cointegration among them at 95 percent significance level. For the purpose of robustness check, I also estimated the Engle-Granger test with eight different orders among ($\{ln\, k_t\}$,$\{ln\, y_t\}$,$\{ln\, w_t\}$, and $\{ln\, O_t\}$). As a result, they return the same decision which concludes that there is no cointegration among them at 95 percent significance level; in other words, there is no existence of long run stable relationship among them. Since there is no evidence of cointegration, we can then safely move on to do multivariate time series model like the vector autoregression model.

Pre-Test

Δlnk	**Test Statistics**	**5% Critical Value**	**P-Value**
Equation (1)	-1.130	-1.950	N.A.
Equation (2)	-1.715*	-1.678	0.0466
Equation (3)	-2.503	-3.499	0.3262
Δlny	**Test Statistics**	**5% Critical Value**	**P-Value**
Equation (1)	-1.880	-1.950	N.A.
Equation (2)	-4.386*	-1.678	0.0000
Equation (3)	-4.490*	-3.499	0.0016
Δlnw	**Test Statistics**	**5% Critical Value**	**P-Value**
Equation (1)	-2.190*	-1.950	N.A.
Equation (2)	-2.383*	-1.678	0.0107
Equation (3)	-2.294	-3.499	0.4374
Δlno	**Test Statistics**	**5% Critical Value**	**P-Value**
Equation (1)	-4.179*	-1.950	N.A.
Equation (2)	-4.136*	-1.678	0.0001
Equation (3)	-4.087*	-3.499	0.0066

Where: * denotes statistically significant at 5%
Equation (1): the pure random walk model.
Equation (2): adds a drift term to the previous model, Equation (1).
Equation (3): adds a drift and a linear time trend to the random walk model.

Engle-Granger Cointegration test table

Orders	Test Statistics	5% Critical Value	Decision
lnk lny lnw lno	-2.3260	-4.316	Do not reject the null
lny lnk lnw lno	-1.7240	-4.316	Do not reject the null
lnw lnk lny lno	-1.8030	-4.316	Do not reject the null
lno lnk lny lnw	-3.3500	-4.316	Do not reject the null
lny lno lnw lnk	-1.7240	-4.316	Do not reject the null
lnw lno lny lnk	-1.8030	-4.316	Do not reject the null
lno lnw lny lnk	-3.3500	-4.316	Do not reject the null
lno lnw lnk lny	-3.3500	-4.316	Do not reject the null

(I)

The first differences model of the form:

$$\{\widehat{\Delta\, ln\, y_t}\} = \underset{(0.0102)}{0.0660} - \underset{(0.0300)}{0.0220}D_{01} - \underset{(0.0163)}{0.0323}D_0 - \underset{(0.0108)}{0.0148}D_{77}$$

$$- \underset{(0.1169)}{0.1384}\, \Delta ln\, k_t - \underset{(0.0622)}{0.0251}\, \Delta ln\, w_t + \underset{(0.0339)}{0.0243}\, \Delta ln\, O_t$$

R^2= 0.1394

```
      Source |       SS           df       MS      Number of obs   =        52
-------------+----------------------------------   F(6, 45)        =      1.22
       Model |  .006251543         6  .001041924   Prob > F        =    0.3163
    Residual |  .038585289        45  .000857451   R-squared       =    0.1394
-------------+----------------------------------   Adj R-squared   =    0.0247
       Total |  .044836831        51  .000879154   Root MSE        =    .02928

------------------------------------------------------------------------------
        dlny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         d01 |  -.0220242   .0300027    -0.73   0.467    -.0824526    .0384042
          d0 |  -.0322923   .0162541    -1.99   0.053    -.0650297     .000445
         d77 |  -.0148066   .0108343    -1.37   0.179    -.0366279    .0070147
        dlnk |  -.1383957    .116944    -1.18   0.243    -.3739329    .0971415
        dlnw |  -.0250926   .0622368    -0.40   0.689    -.1504438    .1002587
        dlno |   .0242653   .0339092     0.72   0.478    -.0440314    .0925621
       _cons |    .065984   .0102216     6.46   0.000     .0453967    .0865714
------------------------------------------------------------------------------
```