THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

ECON 7380 Advanced Macroecnometrics

Assignment II

Replication and extension of Grant and Chan (2017)'s 'A Bayesian model comparison for trend-cycle decomposition of output'

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Abstract

This paper uses Japan data to replace the US data discussed by Grant and Chan (2017) who conducts model comparison for trend-cycle decomposition of output with the Bayesian model. In many ways, the outcomes with alternative specifications have reached the same conclusions as the original results of Grant and Chan (2017). The most crucial one is that the models with the correlated unobserved components fit better than the models with deterministic trend regardless of whether they are with or without a break. The second most important finding is that the permanent shocks plays more decisive role than the transitory shock in explaining the output variation. However, there are also contradictory results. I argue that, with Japan's data, the estimated variance of innovation to GDP trend is far greater than that of innovation to GDP cyclical component. Further, I discover that Japan has two equally significant breaks happened in both 1986 and 2007 and therefore, the UCUR with two breaks is the best model. Moreover, I suggest that the choice of dependent variables has little impact on the estimated correlation between the innovations in Japan's GDP process. I also find that the log marginal likelihoods of all major univariate and bivariate models are pretty close to each other. The discrepancies are most likely attributed to Japan's asset bubble and lost decades from 1986 to 2010. This implies that Japan's long-lasting deflation has caused significant and structural impact especially on the GDP trend component.

JEL classification: C11, C52, E32, F62
Keywords: Bayesian model comparison, deflation, Japan business cycle, unobserved component, structural break

1. Introduction

This paper presents the replication and extension of Grant and Chan (2017)'s 'A Bayesian model comparison for trend-cycle decomposition of output' published in the *Journal of Money, Credit and Banking.* The publication link of Grant and Chan (2017) is https://doi.org/10.1111/jmcb.12388 . I replace the dataset of original paper with Japan's data to examine whether my extension share common results with the US data that covered the quarterly GDP, CPI, and unemployment rate from 1947Q1 to 2014Q4 in Grant and Chan (2017).

2. Background and process for replicating the original paper

2.1. Data and code availability for replicating the original paper

The identical US macroeconomic data used by Grant and Chan (2017) can be easily obtained from the website of Federal Reserve Bank of St. Louis economic database (FRED). Since the corresponding data in csv files and MATLAB codes are already provided in Professor Joshua Chan's website: https://joshuachan.org/research.html, I am able to run all kinds of competing models in the single main file that is connected to other separate files that cover the functions for each model written by Grant and Chan (2017) and succeed in replicating the same main results of the original paper.

2.2. Methodology overview and terminology explanation

Before discussing the results, let's briefly review the main weapons executed by Grant and Chan (2017). The backbone of their methodology is built upon the trend and cycle model: $y_t = \tau_t + c_t$ in which y_t, τ_t, and c_t represent GDP, trend, and stationary cyclical component, respectively. More precisely, τ_t is a stochastic trend or a random walk with drift which shows that $\tau_t = u_1 + \tau_{t-1} + u_t^{\tau}$ and c_t is a AR(p) process with zero mean which indicates that $c_t = \phi_1 c_{t-1} + \cdots + \phi_p c_{t-p} + u_t^c$. The joint distribution of innovations u_t^c and u_t^{τ} can be expressed as:

$$\begin{pmatrix} u_t^c \\ u_t^{\tau} \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_c^2 & \rho\sigma_c\sigma_\tau \\ \rho\sigma_c\sigma_\tau & \sigma_\tau^2 \end{pmatrix}\right).$$

The unobserved components model that permits nonzero correlation between u_t^c and u_t^{τ} (UCUR) is based on Morley *et al.* (2003) in which they set ρ = 2. The restricted unobserved components model (UC0) is invented by Clark (1987) in which he let ρ = 0. Subsequently, the UCUR with one break (UCUR-t_o) is based on Perron and Wada (2009) and can be expressed as: $\tau_t = u_1 1(\mathrm{t} < t_o) + u_2 1(\mathrm{t} \geq t_o) + \tau_{t-1}$.

Additionally, the deterministic model with one break (DT-t_o) is based on c_t which has its innovation $u_t^c \sim N(0, \sigma_c^2)$. The above models will then be fitted with Bayesian estimation based on Chan and Jeliazkov (2009) and Chan (2013). In Grant and Chan (2017)'s paper, the model comparison using Bayes factor is heavily implemented. Krose and Chan (2014) stated that initially, each model denoted by $\mathrm{M_i}$ is defined by a likelihood function $\mathrm{f(x|\theta_i, M_i)}$ and a prior distribution on the model-specific parameter vector θ_i which is $\mathrm{f(\theta_i|M_i)}$.

Thereupon, we can compare models with the Bayes factor. For example, if there are two competing models: $\mathrm{M_A}$ and $\mathrm{M_B}$, the Bayes factor in favour of $\mathrm{M_A}$ against $\mathrm{M_B}$ can be described as: $\mathrm{BF_{AB}} \stackrel{\mathrm{def}}{=} \frac{\mathrm{f(x|M_A)}}{\mathrm{f(x|M_B)}}$, where $\mathrm{f(x|M_i)} = \int \mathrm{f(x|\,\theta_i, M_i)\, f(\theta_i|M_i) d\theta_i}$ is the marginal likelihood under $\mathrm{M_i}$. When it comes to comparison, $\mathrm{M_A}$ will be preferred if $\mathrm{BF_{AB}}$ is greater than one which means that $\mathrm{M_A}$ predicts the observed data better than $\mathrm{M_B}$. However, it is not all that simple to obtain the estimation analytically when the marginal likelihood is high dimensional. Grant and Chan (2017) use an improved version of classic cross-entropy method developed by Chan and Eisenstat (2015) who solved this issue by Kullback-Leibler divergence which will search for a density close to the ideal importance sampling density. In addition, the integrated likelihood of unobserved component is estimated with a more efficient approach with band and sparse matrix algorithms that are based on Chan and Jeliazkov (2009) and Chan (2013).

3. Alternative specifications

As mentioned, I will present my extension with Japan's data. I was planning to run the model with the same timeframe from 1947Q1 to 2018Q4 conducted in Grant and Chan (2017). Unfortunately, owing to the data availability from Japan, the earliest data I can get is from 1960; therefore, my results are estimated from the time series samples from 1960Q1 to 2018Q4 which cover the quarterly GDP, CPI, and unemployment rate of Japan.

My motivation for using Japan as alternative specification is that I have observed Japan's economy for quite a long time. Since the launch of "three arrows" in 2013, Japan's financial markets including Nikkei 225 index, JGB, and JPY have turned to become the most volatile in Asia due to its extremely high sensitivity to the monetary policy shock even though its policy rate was already near zero. As we know, even way before the global financial crisis (GFC) in 2008, Japan's interest rate has been experienced the zero lower bound (ZLB) since the mid-1990s due to asset bubble in 1986. Kunieda and Shibata (2016) believed that the asset bubble happens when the equilibrium interest rate is lower than the economic growth rate. During that lost decade, Oda and Ueda (2007) had mentioned that Japan's economic slump was characterized by several deep cyclical downturns followed by some short-term recoveries. Based on these facts, there are no other counties that can be more exciting than Japan in doing trend and cycle research.

Altogether, the purpose of this extension is to check whether Japan has experienced similarities with the US discussed in Grant and Chan (2017). For instance, I will examine whether the stochastic process is still better than deterministic process in explaining the output, identify the structural break, compare the impacts of permanent and transitory shocks, discuss the implication of estimated variance of innovation to GDP trend and cyclical components with the quarterly GDP, CPI, and unemployment rate in Japan.

4. Key results with the extension

4.1. Model comparison results

Table 1. Log marginal likelihoods of the UCUR-t_o and DT-t_o

DT-84	**DT-85**	**DT-86**	**DT-87**	**DT-88**
-1648.5	-1648.5	-1648.5	-1648.5	-1648.5
(0.02)	(0.02)	(0.04)	(0.03)	(0.02)
DT-05	**DT-06**	**DT-07**	**DT-08**	**DT-09**
-1648.5	-1648.5	**-1648.4**	-1648.5	-1648.5
(0.03)	(0.02)	(0.03)	(0.04)	(0.02)
UCUR-84	**UCUR-85**	**UCUR-86**	**UCUR-87**	**UCUR-88**
-1632.3	-1631.9	**-1631.8**	-1631.9	-1631.9
(0.15)	(0.22)	(0.25)	(0.24)	(0.21)
UCUR-05	**UCUR-06**	**UCUR-07**	**UCUR-08**	**UCUR-09**
-1631.9	-1632	**-1631.8**	-1631.9	-1632
(0.26)	(0.15)	(0.22)	(0.22)	(0.17)

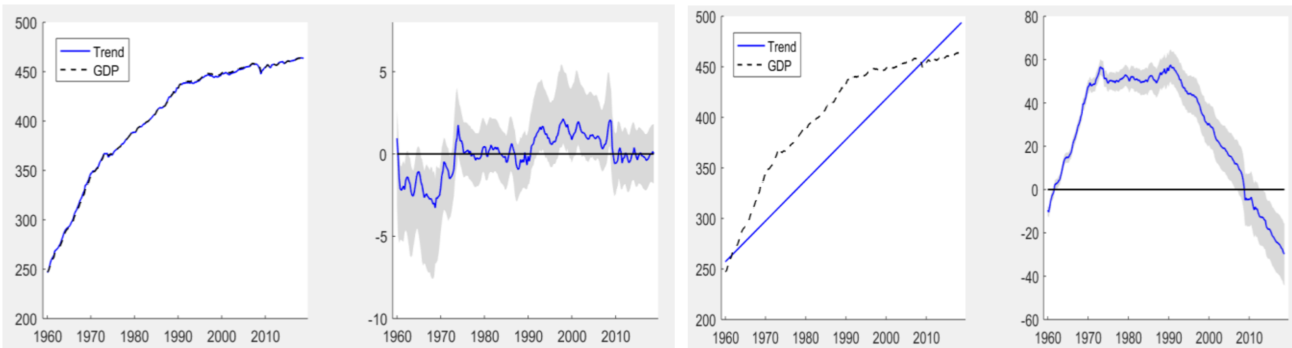
Note: Numerical standard errors are in parentheses. The best model is bolded. The same scheme is applied for all tables in this paper.

Table 2. Log marginal likelihoods of competing models with and without and a break

DT	UC0	UCUR	DT-07	UCUR-07	UCUR (86,07)
-1648.5	-1647.3	-1632.1	-1648.4	-1631.8	**-1621.4**
(0.03)	(0.36)	(0.2)	(0.03	(0.22)	(0.19)

Figure 1. Estimates of trend and cycle of UCUR (LHS) vs DT (RHS)

UCUR: trend **UCUR: cycle** **DT: trend** **DT: cycle**



The entire model comparison are summarized in Table 1. Consistent with Grant and Chan (2017), UCUR-t_0 overwhelmingly dominates DT-t_0. In addition, Figure 1 displays the comparison between UCUR and DT without break. The UCUR model has its trend line tracking the GDP closely and has very small cycle estimates that fluctuated around zero. On the other hand, the DT model has a straight trend line and has large and persistent cycle which climbed to the apex around the mid-1990s. With graphical illustration, we can once again prove that the GDP trend is fitted better with a stochastic process than with a deterministic trend based on the Bayes factor.

Grant and Chan (2017) further argued that once a break is assumed, it is likely to happen in 2007 with the US data. Meanwhile, my result shows that Japan has two equally significant breaks happened in both 1986 and 2007 and this strong evidence is presented in Table 2 that suggests that UCUR with two breaks is the best model.

Furthermore, we should also examine whether allowing for a nonzero correlation between the permanent and transitory shocks greatly enhances model fit by comparing UC0 and UCUR. Table 2 shows that the UCUR clearly outperforms UC0 and is consistent with the findings in Morley *et al.* (2003) and Oh *et al.* (2008), and Grant and Chan (2017). This implies that the model that allows for a nonzero correlation between the permanent and transitory shocks is superior.

4.2. Trend-Cycle estimates and variance decomposition

Table 3. Estimated posterior means

	DT-86	**UCUR**	**UCUR-07**	**UCUR-(86, 07)**
$\boldsymbol{\phi_1}$	1.22	0.74	0.74	0.74
	(0.06)	(0.16)	(0.19)	(0.2)
$\boldsymbol{\phi_2}$	-0.23	-0.06	-0.07	-0.06
	(0.06)	(0.13)	(0.14)	(0.15)
$\boldsymbol{\sigma_c^2}$	1.48	1.03	0.88	0.76
	(0.17)	(0.72)	(0.71)	(0.7)
$\boldsymbol{\sigma_\tau^2}$	-	2.84	2.77	2.54
		(0.14)	(0.19)	(0.32)
ρ	-	-0.83	-0.83	-0.81
		(0.07)	(0.08)	(0.1)

Table 3 provides the summary of the parameter estimates of three major models: DT-86, UCUR, and UCUR-07. Besides them, I add another UCUR model with two breaks to make easy comparison. As we can see, the estimates of σ_τ^2 and σ_c^2 for UCUR-07 are 2.77 and 0.88, respectively. Since the former is much larger, this phenomenon indicates that the permanent shocks are way more important than the transitory shocks. Likewise, the same phenomenon appears in both UCUR and UCUR-(86, 07). This result is consistent with Morley *et al.* (2003) and Grant and Chan (2017). Moreover, similar to Grant and Chan (2017), the estimates of ρ, which indicates the correlation of unobserved components, are all large in magnitude and negative. This phenomenon justifies the relevance of allowing for nonzero correlation between the permanent and transitory shocks.

4.3. Breaks in trend output growth

Table 4. Estimated trend output growth rates (μ)

	DT-86	**UCUR**	**UCUR-07**	**UCUR(86,07)**
μ: 1960-2018	-	0.88	-	-
		(0.11)		
μ: 1960-2006	-	-	1.06	-
			(0.12)	
μ: 1960-1985	1.52	-	-	1.15
	(0.1)			(0.22)
μ: 1986-2018	0.4	-	-	-
	(0.06)			
μ: 1986-2006	-	-	-	0.46
				(0.15)
μ: 2007-2018	-	-	-0.24	-0.26
			(0.32)	(0.31)

Table 4 clearly suggests that Japan had went through a period of strong growth since the 1960s and then slowed down sharply since 1986. More recently, Japan's trend output growth rate or μ turned out to be negative during the period from 2007 to 2018. Meanwhile, Grant and Chan (2017) shows that the US only experiences substantial slowdown of positive growth right after the shock of GFC.

Table 5. Estimated trend output growth rates under four bivariate models

	2007 Break		**1986 + 2007 Breaks**	
	Inflation	**Unemployment**	**Inflation**	**Unemployment**
μ: 1960-2006	1.06	1.06	-	-
	(0.12)	(0.12)		
μ: 1960-1985	-	-	1.12	1.15
			(0.21)	(0.22)
μ: 1986-2006	-	-	0.47	0.47
			(0.15)	(0.15)
μ: 2007-2018	-0.21	-0.25	-0.21	-0.24
	(0.31)	(0.31)	(0.32)	(0.31)

Figure 2. Estimates of the output gap for the two bivariate unobserved components models with a break in 2007: GDP and inflation (LHS) vs GDP and unemployment (RHS)

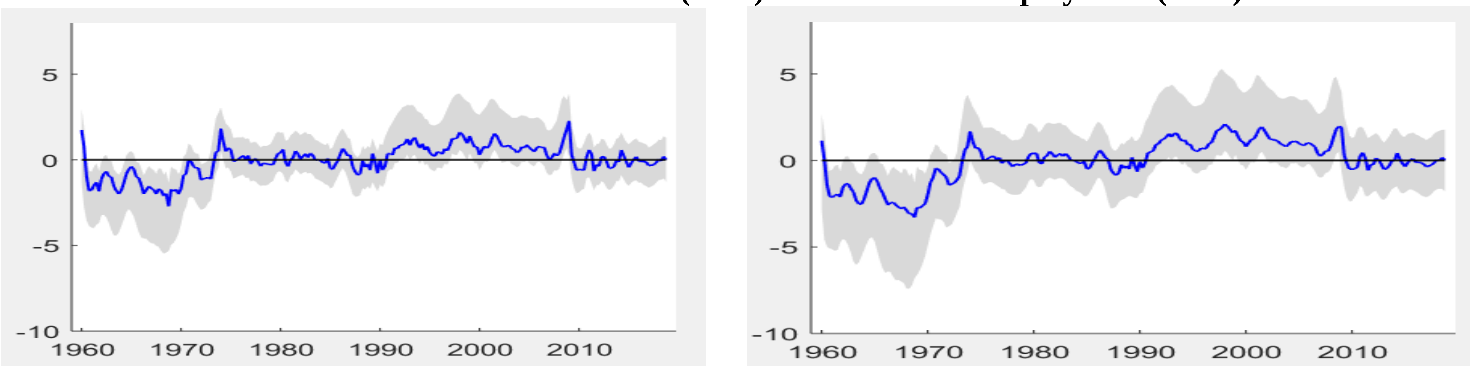


Table 6. Selected parameters for the GDP process under four bivariate models

	2007 Break		1986 + 2007 Breaks	
	Inflation	**Unemployment**	**Inflation**	**Unemployment**
σ^2 of innovation to GDP cycle	1.04	1.02	0.72	0.93
	(0.75)	(0.67)	(0.58)	(0.68)
σ^2 of innovation to GDP trend	2.84	2.84	2.84	2.84
	(0.14)	(0.14)	(0.14)	(0.14)
Correlation between innovations	-0.83	-0.82	-0.86	-0.83
	(0.07)	(0.07)	(0.07)	(0.07)

Table 5 exhibits the estimates from the bivariate model of GDP and inflation with a break in 2007 and shows that the growth rate from the model with employment is larger than the one from the model with inflation. All four specifications have similar trend output growth estimates after 2007. The growth rates from models with two breaks are only slightly lower from 2007 to 2018.

Figure 2 displays the estimates of the output gap for the two bivariate unobserved components models with a break in 2007; however, both look identical which is very different from the results in Grant and Chan (2017) whose output gap of GAP and inflation is much larger. Table 6 aids in explaining the output gap differences by providing the contributions of estimated variance to both GDP cycle and trend components. Grant and Chan (2017) believe that their result can be explained by the trade-off in which a smaller contribution of estimated variance of the innovation to the trend component of GDP will be offset by a larger contribution of the cyclical component of GDP.

In contrast, my result with Japan's data shows that estimated variance of innovation to GDP trend component is much larger than that of innovation to GDP cyclical component. This result is likely caused by the deflation due to the asset bubble which has significant and structural impact on the GDP trend. Moreover, the correlations between innovations are all very large in magnitude and negative throughout all four specifications and similar to those of univariate unobserved components models. Nevertheless, the same phenomenon in Grant and Chan (2017) only happens when GDP pairs with unemployment.

Grant and Chan (2017) also argue that the estimated correlation between the innovations in the GDP process can be sensitive to the choice of dependent variables. Conversely, the choice of dependent variables has little impact on the estimated correlation between the innovations in the GDP process in Japan.

Table 7. Log marginal likelihoods of various univariate and bivariate models

	2007 Break			1986 + 2007 Breaks	
UCUR	**Inflation**	**Unemployment**	**UCUR**	**Inflation**	**Unemployment**
-1632.3	-1632	-1631.6	-1631.8	-1632.2	-1631.8
(0.13)	(0.21)	(0.26)	(0.24)	(0.18)	(0.36)

Table 7 summarizes the log marginal likelihoods of UCUR and the two bivariate models under the assumptions of one and two breaks. Among them, the bivariate model of GDP and unemployment under UCUR-07 is the winner but only outperforms others by a tiny margin. This result yields the same conclusion as Grant and Chan (2017) but the results of differentials among log marginal likelihoods are not consistent. With the US data, Grant and Chan (2017) show that each log marginal likelihood is substantially different from one another. To contrast, as Table 7 suggests, the log marginal likelihoods from all six specifications are very close to each other with Japan data.

5. Conclusion

I use the data of Japan to replace the data of the US discussed by Grant and Chan (2017) which conducts model comparison for trend-cycle decomposition of output with the Bayesian model. In several ways, my results with alternative specifications have reached the same conclusions as the original results of Grant and Chan (2017). The most crucial one is that the models with the correlated unobserved components fit better than the models with deterministic trend regardless of whether they are with or without a break. The second most important and consistent finding is that the permanent shocks plays more decisive role than the transitory shock in explaining the output variation.

However, there are also contradictory results. I argue that, with Japan's data, the estimated variance of innovation to GDP trend is far greater than that of innovation to GDP cyclical component. Further, I discover that Japan has two equally significant breaks happened in both 1986 and 2007 and therefore, the UCUR with two breaks is the best model. Moreover, I suggest that the choice of dependent variables has little impact on the estimated correlation between the innovations in Japan's GDP process. I also find that the log marginal likelihoods of all major univariate and bivariate models are pretty close to each other.

The above discrepancies are most likely attributed to Japan's asset bubble and lost decades from 1986 to 2010. This implies that Japan's long-lasting deflation has caused significant and structural impact especially on the GDP trend component.

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