Lec-45, DC, 24-25, Sec A

→ Energy per symbol, $E_s$ for a constellation is the average of the squared Euclidean distances of the points from the origin.

For an M-ary constellation, each symbol carries $\log_2 M$ bits of information. Hence, we can define the average energy per bit, $E_b$, as $E_b = \frac{E_s}{\log_2 M}$

→ Baseband & passband - going back.

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$
$$= e(t)\cos\theta(t)\cos(2\pi f_c t) - e(t)\sin\theta(t)\sin(2\pi f_c t)$$

$$e(t) = \sqrt{u_c^2(t) + u_s^2(t)}\ , \quad \theta(t) = \tan^{-1}\left(\frac{u_s(t)}{u_c(t)}\right) \quad \left[\ u_p(t) = e(t) \times \cos(2\pi f_c t + \theta(t))\right.$$

If we consider linear modulation, then

$$u_c(t) = b_c\, p(t) \qquad 0 \le t < T_s$$

$$u_s(t) = b_s\, p(t)$$

$$(b_c, b_s)$$

$$e(t) = \sqrt{b_c^2 + b_s^2}\ p(t)$$

$$(0,0)$$

$$= \sqrt{E_s}\ p(t)$$

$$b_c^2 + b_s^2 = E_s$$

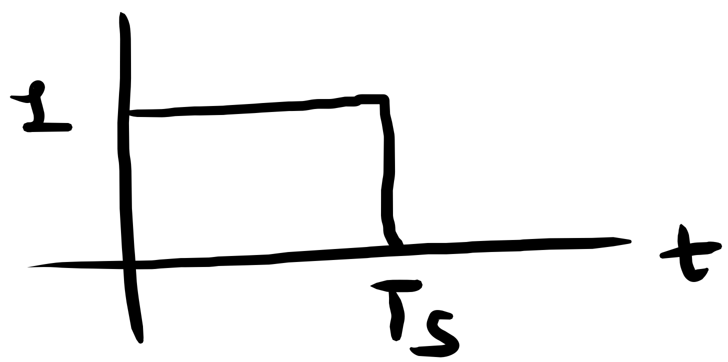
$$\theta(t) = \theta = \tan^{-1}\left(\frac{b_s}{b_c}\right)$$

$$u_p(t) = \sqrt{E_s}\cos\theta\ \underbrace{p(t)\cos(2\pi f_c t)}_{E = \|p\|^2/2} - \sqrt{E_s}\sin\theta\ \underbrace{p(t)\sin(2\pi f_c t)}_{E = \|p\|^2/2}$$

$$u_p(t) = \frac{\sqrt{E_s}\cos\theta}{\|p\|/\sqrt{2}}\, p(t)\cos(2\pi f_c t) - \frac{\sqrt{E_s}\sin\theta}{\|p\|/\sqrt{2}}\, p(t)\sin(2\pi f_c t)$$

$$u_p(t) \equiv \left(\frac{\sqrt{E_s}\cos\theta}{\|p\|/\sqrt{2}},\ \frac{\sqrt{E_s}\sin\theta}{\|p\|/\sqrt{2}}\right)$$

now, if $p(t) =$




$$, \|p\|^2 = \int_0^{T_s} 1^2\, dt = T_s$$

$$\|p\| = \sqrt{T_s}$$

so, for the standard rectangular pulse, we have

$$u_p(t) \equiv \left(\sqrt{\frac{2E_s}{T_s}}\cos\theta,\ \sqrt{\frac{2E_s}{T_s}}\sin\theta\right)$$

constellation for the PB digital Tx.

BPSK :- binary phase shift keying :- phase of the carrier changes & amplitude remains fixed.

$$e(t) = \sqrt{b_c^2 + b_s^2}\, p(t) \ , \ \theta(t) = \tan^{-1}\left(\frac{b_s}{b_c}\right)$$

$\theta(t)$ will have two possible values $\{0, \pi\}$

$$u_p(t) = e(t) \cos\left(2\pi f_c t + \theta(t)\right)$$

case 1 : $\theta(t) = 0$, $\quad u_p(t) = e(t) \cos(2\pi f_c t)$

case 2 : $\theta(t) = \pi$, $\quad u_p(t) = -e(t) \cos(2\pi f_c t)$

equivalently, $\left(\sqrt{\frac{2E_s}{T_s}} \cos 0, \sqrt{\frac{2E_s}{T_s}} \sin 0\right)$ ; $\left(\sqrt{\frac{2E_s}{T_s}} \cos \pi, \sqrt{\frac{2E_s}{T_s}} \sin \pi\right)$

const:- $\left(\sqrt{\frac{2E_s}{T_s}}, 0\right), \left(-\sqrt{\frac{2E_s}{T_s}}, 0\right) \equiv \left(\sqrt{\frac{2E_s}{T_s}}, -\sqrt{\frac{2E_s}{T_s}}\right)$

1-D constellation

In BPSK, $E_s = E_b$

$T_s = T_b$

Const. $= \sqrt{E_s}$

$-\sqrt{E_s}$

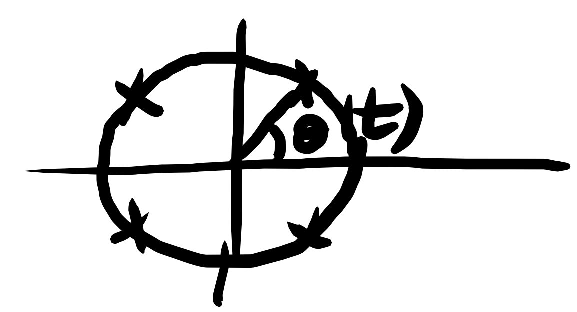
With the basis functions

$\frac{p(t)\cos(2\pi f_c t)}{\|p\|/\sqrt{2}}, \frac{-p(t)\sin(2\pi f_c t)}{\|p\|/\sqrt{2}}$

we have

constellation $= (\sqrt{E_s}\cos\theta, \sqrt{E_s}\sin\theta)$

QPSK:- 4 possibilities for $\theta(t)$. Since $\theta(t)$ varies from 0 to $2\pi$, lets take 4 equidistant points b/w 0 & $2\pi$ :- $(2i-1)\frac{\pi}{4}$, $i = 1, 2, 3, 4$



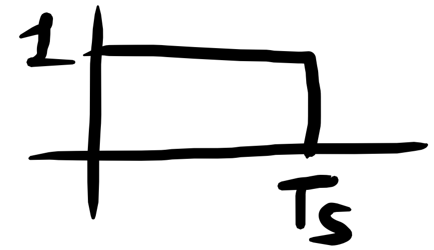
constellation for QPSK $(\sqrt{E_s}\cos\theta, \sqrt{E_s}\sin\theta)$

$\theta = \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$$\left(\frac{p(t)\cos(2\pi f_c t)}{\|p\|/\sqrt{2}}, \frac{p(t)\sin(\,)}{\|p\|/\sqrt{2}}\right)$$

Read lec 41 from the lecture folder of 23-24 batch.

$p(t) \rightarrow$




<u>Coherent detection</u>:- when you assume that perfect & synchronized copy of the carriers $\cos 2\pi f_c t$ & $\sin 2\pi f_c t$ are available at the Rx. And they are used for detection.