Lec-38, DC, 24-25, Sec A

We will show that $I_1 Q_1$ has no component or value at $f=0$, i.e., DC.

$$I_1 Q_1 = u_c(t)\, u_s(t) \cos(2\pi f_c t) \sin(2\pi f_c t)$$

$$= \frac{u_c(t)\, u_s(t)}{2} \sin(4\pi f_c t) = \frac{q(t)}{2} \sin(2\pi (2f_c) t)$$

Now, if we define $q(t) = u_c(t)\, u_s(t)$, then

$q(t) \rightarrow$ BB? / PB?

Let $u_c(t) \rightarrow$ BB with BW W_1 Hz

$u_s(t) \rightarrow$ BB with BW W_2 Hz

Show $u_c(t) \times u_s(t)$ is BB with B.W. $W_1 + W_2$

$W_1 < f_c$

$W_2 < f_c$

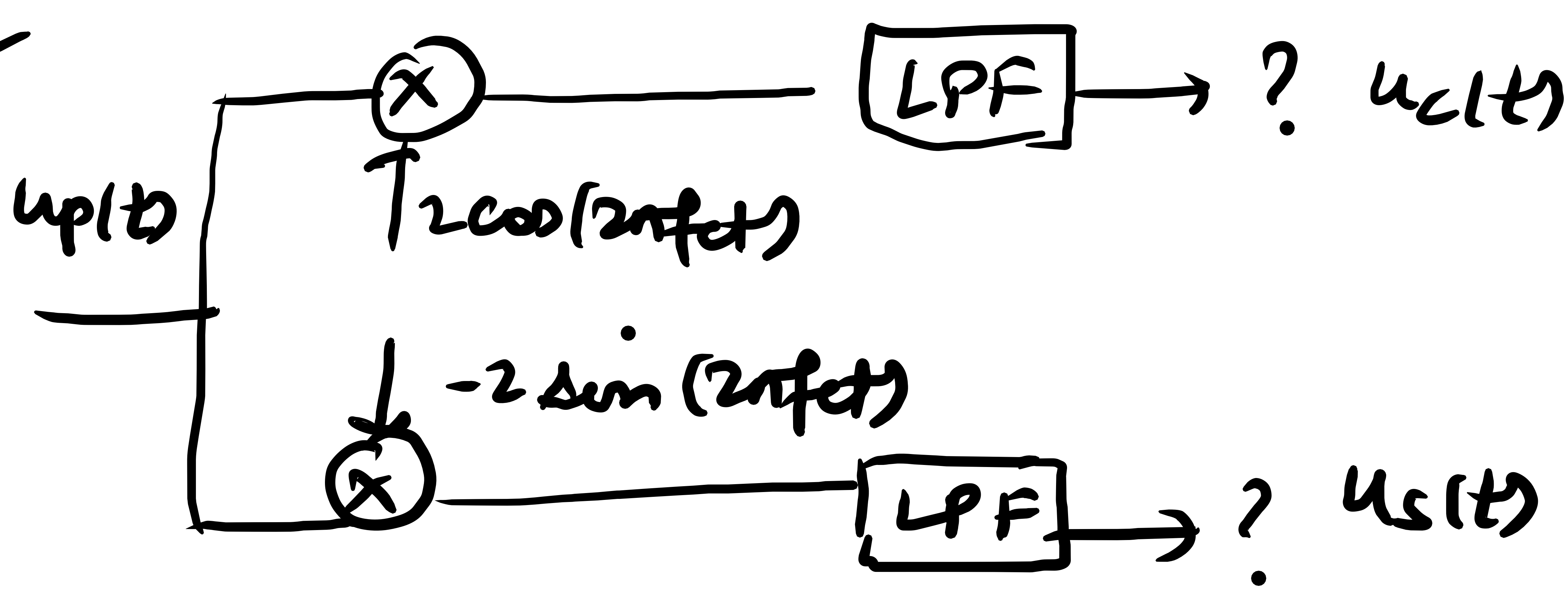
$W_1 + W_2 < 2f_c$

$U_c(f) * U_s(f)$ goes from $-(W_1+W_2)$ to W_1+W_2

$q(t) \rightarrow Q(f)$ lies from $-(W_1+W_2)$ to (W_1+W_2) & we know that $W_1+W_2 < 2f_c$, the signal $I_1 Q_1 = \frac{q(t)}{2}\sin(2\pi(2f_c)t)$ is PB $\Rightarrow \int_{-\infty}^{\infty} I_1 Q_1\,dt = 0$.

Hence, I_1 is orthogonal to Q_1 & viceversa.

Rx



$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$u_p(t) \times 2\cos(2\pi f_c t)$

$u_c(t) \times 2\cos^2(2\pi f_c t) -$

$u_s(t) \times 2\sin(2\pi f_c t)\cos(2\pi f_c t)$

$= \left[u_c(t)(1+\cos 4\pi f_c t) - u_s(t)\sin(4\pi f_c t)\right] \rightarrow \text{LPF}$

$= \left[\underbrace{u_c(t)}_{\text{BB}} + \underbrace{u_c(t)\cos(2\pi(2f_c)t)}_{\text{PB at } 2f_c} - \underbrace{u_s(t)\sin(2\pi(2f_c)t)}_{\text{PB at } 2f_c}\right]$

here, LPF has a B.W. $> W_1$ & W_2

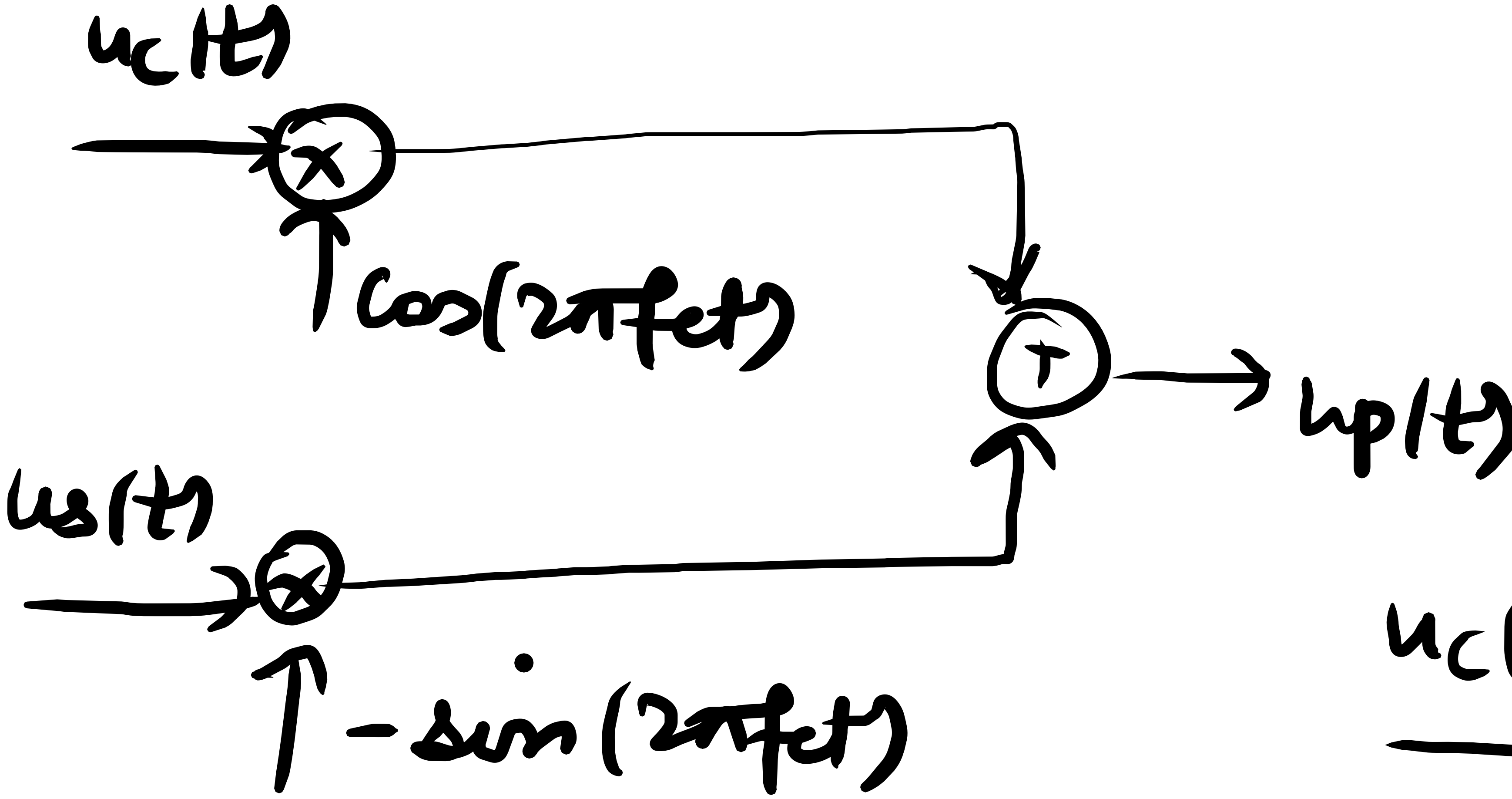
$u_c(t) \leftarrow \text{LPF}$

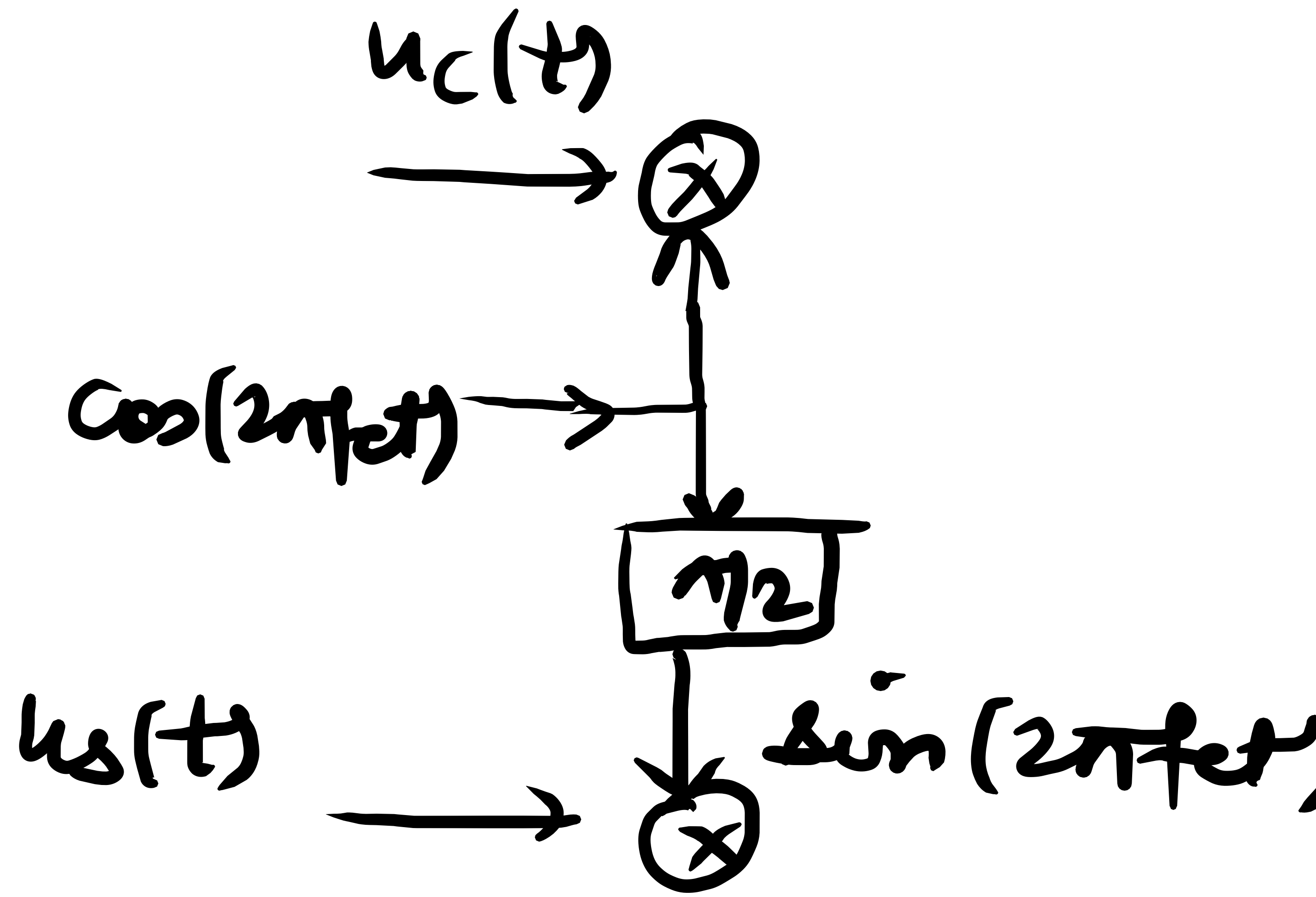
$-u_p(t)\, 2\sin(2\pi f_c t)$

$-u_c(t)\, 2\cos(2\pi f_c t)\sin(2\pi f_c t) + u_s(t)\, 2\sin^2(2\pi f_c t)$

Following similar steps as done for the upper branch, after LPF, we get $u_s(t)$

<u>Tx:</u>





$\boxed{\pi/2}$ phase shift operation is called the Hilbert transform.

Representations of $u_p(t)$

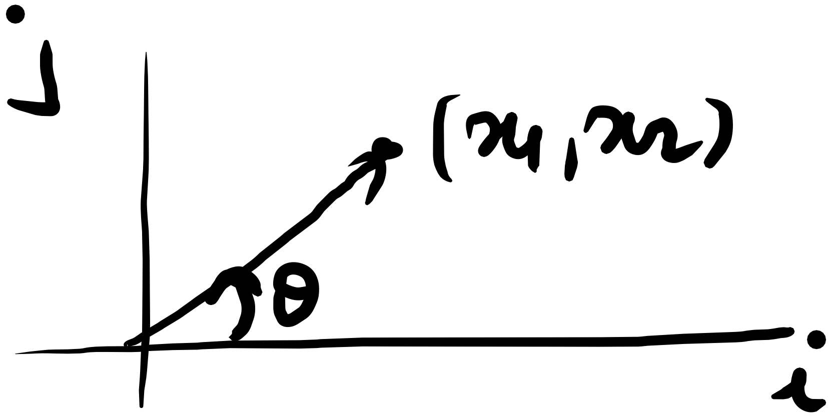
1. Envelop & phase. $u_p(t) \equiv (u_c(t), u_s(t))$
given f_c

Hence, PB modulation is often called two-dimensional modulation. $\cos(2\pi f_c t)$ & $-\sin(2\pi f_c t)$, since they are orthogonal to each other, we have $u_p(t)$ as a vector in a 2D plane with $u_c(t)$ as a component along $\cos(2\pi f_c t)$ axis, while $u_s(t)$ " " " " $-\sin(2\pi f_c t)$

$$e(t) = \sqrt{u_c^2(t) + u_s^2(t)}, \quad \theta(t) = \tan^{-1}\left(\frac{u_s(t)}{u_c(t)}\right)$$

where $e(t) \geq 0 \rightarrow$ envelop & $\theta(t)$ is called the phase

$$x \in \mathbb{R}^2$$
$$\equiv$$
$$(x_1, x_2)$$

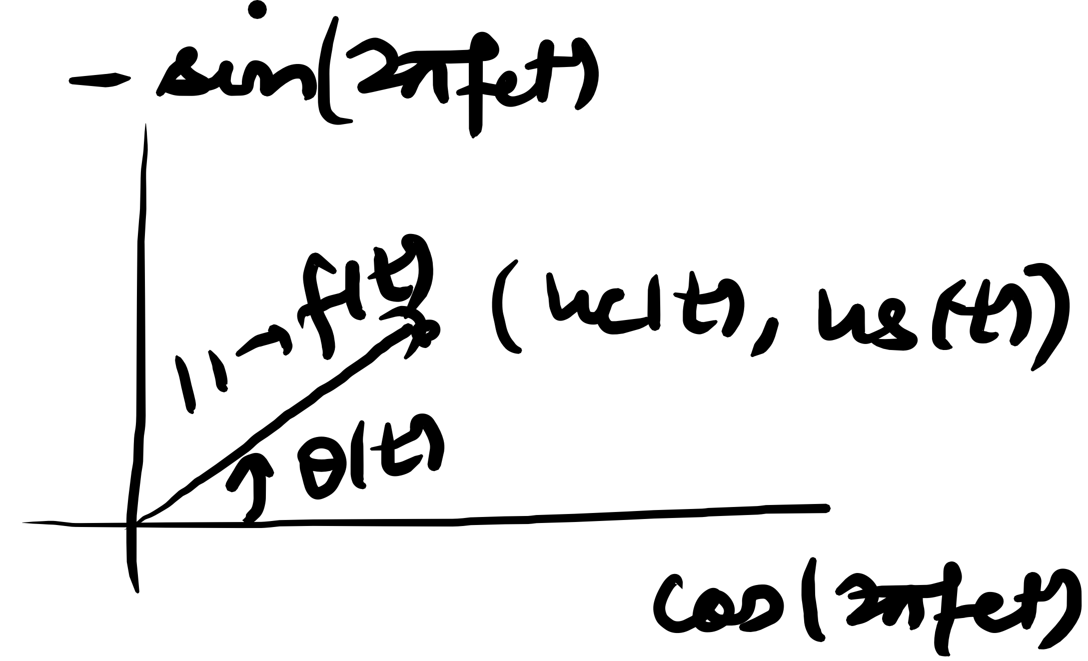


$$|x| = \sqrt{x_1^2 + x_2^2}$$

$$\tan\theta = x_2 / x_1$$

$$x_1 = |x| \cos\theta$$
$$x_2 = |x| \sin\theta$$

analogous $\Rightarrow$



$$u_c(t) = e(t) \cos\theta(t)$$
$$u_s(t) = e(t) \sin\theta(t)$$

$$u_p(t) = e(t)\cos\theta(t)\cos(2\pi f_c t) - e(t)\sin\theta(t)\sin(2\pi f_c t)$$

$$= e(t)\cos(2\pi f_c t + \theta(t))$$