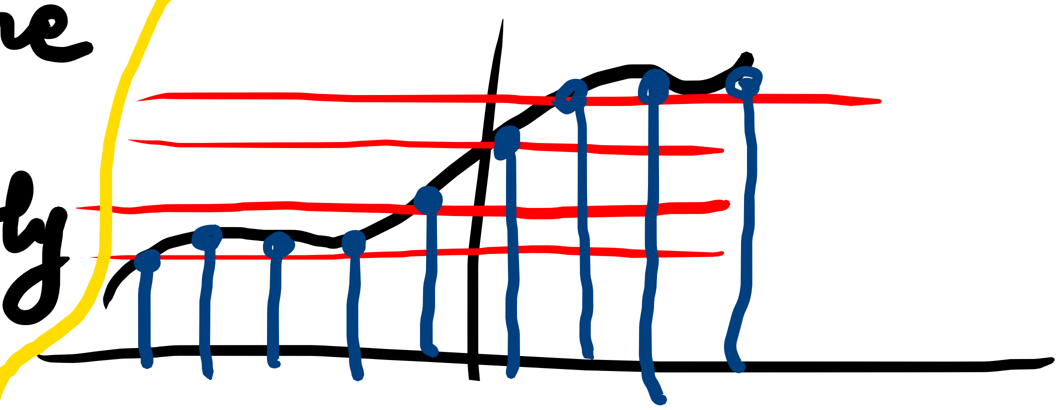
Lec-12, DC, 24-25, Sec A

The quantized samples are coded & transmitted as binary pulses. At the Rx, some pulses maybe detected incorrectly.



There are two sources of error

1. Quantization process.
2. Pulse detection error. (PDE)

In most cases, pulse detection error is quite small compared to quantization error.

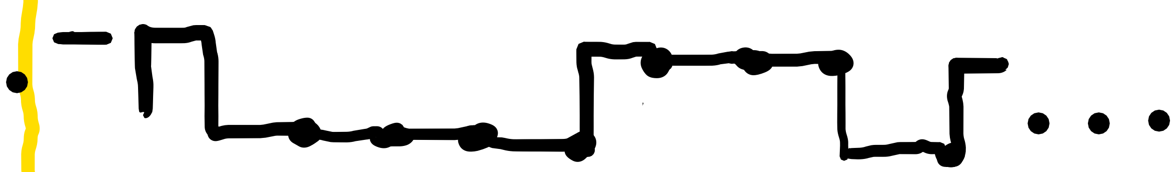
PDE can be handled by power control & through other means.

V_1 : 00
V_2 : 01
V_3 : 10
V_4 : 11

Table is shared b/w Tx & Rx

Take an example, 10 values of the sampled signal above

... $V_3\ V_1\ V_2\ V_4\ V_2\ V_1\ V_1\ V_2$ ...



Let us study the impact of quantization process in the rxd. signal & ignore PDE.

$$m(t) = \sum_k m(kT_s)\,\mathrm{sinc}(2\pi Bt - k\pi)$$

$$\hat{m}(t) = \sum_k \hat{m}(kT_s)\,\mathrm{sinc}(2\pi Bt - k\pi)$$

$m(t) \rightarrow \{m(kT_s)\}$

$[\ldots m(-2T_s), m(-T_s), m(0), m(T_s), m(2T_s)\ldots]$

reconstructed signal at Rx. since it has access to only quantized values of samples.

distortion :- $q(t) = m(t) - \hat{m}(t)$

$$= \sum_k [m(kT_s) - \hat{m}(kT_s)]\,\mathrm{sinc}(2\pi Bt - k\pi)$$

$$\equiv \sum_k q(kT_s)\,\mathrm{sinc}(2\pi Bt - k\pi)$$

$q(t)$:- undesired signal which is called as quantization noise

Let us obtain its power, as the mean square value of $q(t)$.

$$\widetilde{q^2(t)} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} q^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[\sum_k q(kT_s)\, \mathrm{sinc}\,(2\pi Bt - k\pi) \right]^2 dt \quad -(1)$$

Using, $\int_{-\infty}^{\infty} \mathrm{sinc}\,(2\pi Bt - m\pi)\, \mathrm{sinc}\,(2\pi Bt - n\pi)\, dt = \begin{cases} 0, & m \neq n \\ 1/2B, & m = n \end{cases}$

$$\left(\sum_{i=1}^{3} a_i \right)^2 = \sum_{i_1=1}^{3} \sum_{i_2=1}^{3} a_{i_1} a_{i_2} \; ?$$

* There can be a notational diff. in sinc b/w Lathi & Haykin

$$(a_1+a_2)^2 = (a_1+a_2)(a_1+a_2) = \underline{\underline{a_1}}(a_1+a_2) + \underline{\underline{a_2}}(a_1+a_2)$$

$$(a_1+a_2+a_3)^2 = (a_1+a_2+a_3)(a_1+a_2+a_3) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_i a_j$$

$$\left[\sum_k q(kT_s)\,\mathrm{sinc}(2\pi Bt - k\pi)\right]^2 \text{ — } (A)$$

$$= a_1a_2 + a_1a_3 + a_1^2 + a_2a_1 + a_2^2 + a_2a_3 + a_3a_1 + a_3a_2 + a_3^2$$

$$= \sum_{k_1}\sum_{k_2} q(k_1T_s)\,q(k_2T_s)\,\underbrace{\mathrm{sinc}(2\pi Bt - k_1\pi)\,\mathrm{sinc}(2\pi Bt - k_2\pi)}_{B}$$

Integrate (A)

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k_1}\sum_{k_2} q(k_1T_s)\,q(k_2T_s)\,B\,dt$$

$$\lim_{T\to\infty} \frac{1}{T}\sum_{k_1}\sum_{k_2} q(k_1T_s)\,q(k_2T_s)\int_{-T/2}^{T/2} B\,dt$$

now, use eq. (1)

$$\lim_{T\to\infty} \frac{1}{2BT} \sum_k q^2(kT_s) = \text{mean square value or power of } q(t)$$

Because, the sampling rate is 2B, the total no. of samples over the averaging interval T is 2BT

Proof of eq. 1 is H.W. Please refer ex-3.7-2 & 3.7-3 from Chap. 3 in Lathi & Ding