Lec-40, DC, 24-25, Sec A

linear modulation $\quad \sum a_k p(t - kT_b)$ or

T_b :- bit interval $\quad \sum a_k p(t - kT_s)$

T_s :- symbol ".

Once $s_{b_c, b_s}(t)$ is transmitted & suppose you use M-ary signalling $\equiv$ (b_c, b_s) will have M possibilities. & the waveform is corrupted due to noise. i.e., you rx $y(t) = s_{b_c, b_s}(t) + n(t)$

At the Rx, we are faced with a hypothesis-testing problem.

We wish to design optimal receivers when the rxd. signal is modeled as :-

$$H_i : y(t) = s_i(t) + n(t), \quad i = 1, 2, \ldots M$$

Ingredients of hypothesis testing framework.

1. Hypotheses :- $H_0, H_1, \ldots, H_{M-1}$
2. Observation :- $Y \in \Gamma \rightarrow$ observation space.
3. Conditional densities :- $p(Y|i)$, for $i = 0, 1, \ldots M-1$
4. Prior probabilities :- $\pi_i = P(H_i)$
5. Decision rule $\delta : \Gamma \longrightarrow \{0, 1, 2 \ldots, M-1\}$
6. Decision region $\Gamma_i = \{ y \in \Gamma : \delta(y) = i \}$, $i = 0, 1 \ldots M-1$

ex 6.1.1 Binary hypothesis problem

$$H_0 : Y \sim \mathrm{Exp}(1) \quad ; \quad H_1 : Y \sim \mathrm{Exp}(1/4)$$

$\mathrm{Exp}(\mu) \rightarrow$ exponential distribution with density $\mu e^{-\mu y}$, CDF $1 - e^{-\mu y}$ & CCDF $e^{-\mu y}$

Mean of $\mathrm{Exp}(\mu)$ is $1/\mu$

(a) Find the ML rule.

ML (Maximum Likelihood) says.

$$\delta_{ML}(y) = \arg \max_{0 \le i \le M-1} p(y|i)$$

$$\{ p(y|0), p(y|1), \ldots, p(y|M-1) \}$$

In the problem, we are given

$H_0: Y \sim Exp(1) \Rightarrow p(y|0) = 1 \cdot e^{-y}$

$H_1: Y \sim Exp(\frac{1}{4}) \Rightarrow p(y|1) = \frac{1}{4} e^{-y/4}$

$\Gamma = ?$

$[0, \infty)$

ML rule; If $p(y|0) > p(y|1) \rightarrow H_0$

o.w. $p(y|1) > p(y|0) \rightarrow H_1$

If $y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{4}{3} \log 4 = 1.8484$

$-\ln\frac{1}{4} > \frac{3y}{4} = \ln 4 > \frac{3y}{4}$

$\Rightarrow y < \frac{4}{3} \ln 4$

$e^{-y} > \frac{1}{4} e^{-y/4} \rightarrow H_0 \Rightarrow \ln e^{-y} > \ln\frac{1}{4} + \ln e^{-y/4}$

$-y > \ln\frac{1}{4} - \frac{y}{4}$

Error prob. ML rule is $y > \frac{4}{3}\ln 4 \to H_1$

$y < \frac{4}{3}\ln 4 \to H_0$

— Suppose H_0 is true,

& the ~~rxd~~ observed $y > \frac{4}{3}\ln 4$

CCDF $= e^{-\mu y}$

$H_0 : Y \sim \exp(1)$ $\quad P_e \mid H_0 \text{ is true} = \int_{\frac{4}{3}\ln 4}^{\infty} p(y|0)\,dy$

$= 0.0248 = e^{-\frac{4}{3}\ln 4}$

— other possibility H_1 is true

$H_1 : Y \sim \exp(\frac{1}{4})$ $\quad P_e \mid H_1 \text{ is true} = \int_{0}^{\frac{4}{3}\ln 4} p(y|1)\,dy$

$\to 1 - e^{-\mu y}$

$= 0.6031 = 1 - e^{-\frac{1}{4}\cdot\frac{4}{3}\ln 4}$

→ MAP rule :- Maximum aposteriori probability rule.

$$\delta_{MAP}(y) = \arg\max_{0 \le i \le M-1} P[H_i \mid Y = y]$$

$$= \arg\max_{0 \le i \le M-1} \pi_i \, P(y \mid i)$$