Lec-37, DC, 24-25, Sec A

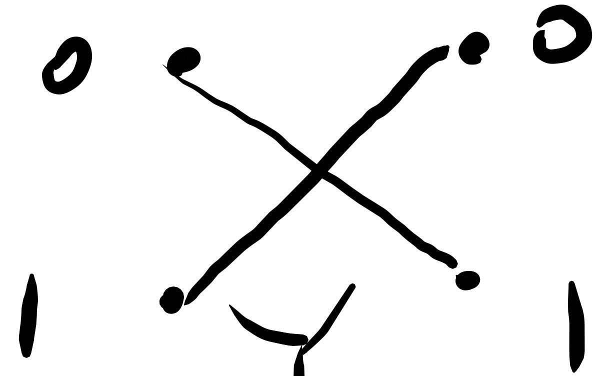
1. Special case:- when 1 & 0 are equiprobable, i.e.,
we have $p_1 = p_0 = \frac{1}{2}$, $\lambda_{opt} = 0$

In this case, will $P_{01} = P_{10}$ (put $\lambda = 0$ in expressions of P_{01} & P_{10} to notice it)

the above result is intuitive as in the case of equiprobable binary symbols, we should choose the threshold at the midpoint b/w the pulse heights $-A$ & A representing the two symbols 0 & 1.

→ channels for which $P_{01} = P_{10}$, are called

Binary Symmetric channels (BSC)

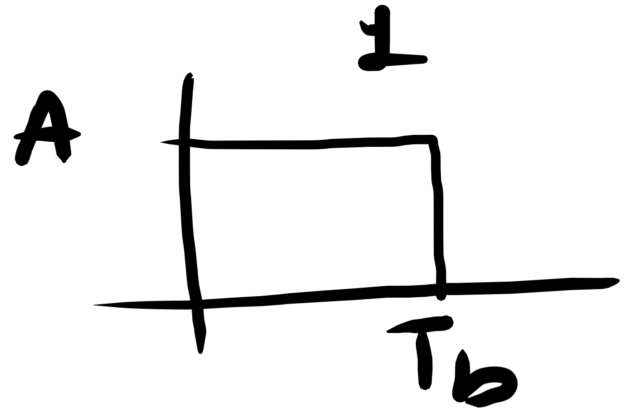


↳ both prob. are same.

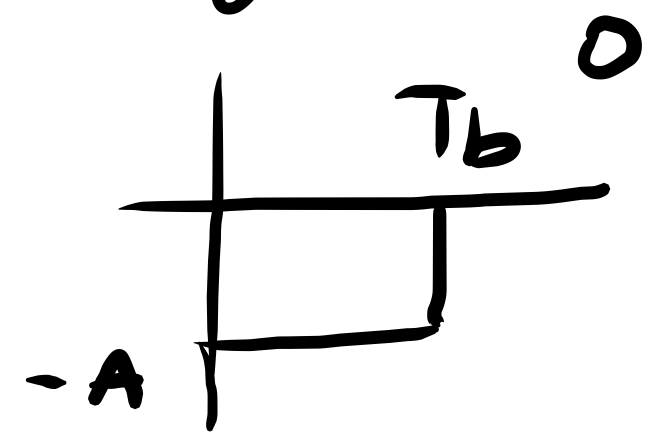
In the special case (1),

$$P_e = \frac{1}{2}\,\text{erfc}\left(A\Big/\sqrt{N_0/T_b}\right)$$

→ What is the Txd. signal energy per bit (E_b)



or



→ $A^2 T_b \triangleq E_b$

(In case of polar NRZ)

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

✓

This shows that for $P_0 = P_1 = \frac{1}{2}$ & $x_{opt} = 0$

Pe depends solely on $\frac{E_b}{N_0}$, ratio of Tx. signal energy per bit to the noise spectral density.

—————— x ——————————— x

see fig. 4.6 on pg 258 for the plot of Pe vs $\frac{E_b}{N_0}\Big|_{dB}$ on a log scale

Passband signals (PB)

Given a message signal $m(t)$ of B.W. W Hz. how do you send it over a PB channel centered around f_c. ? $f_c \gg W$

1. $u_p(t) = m(t)\cos(2\pi f_c t)$

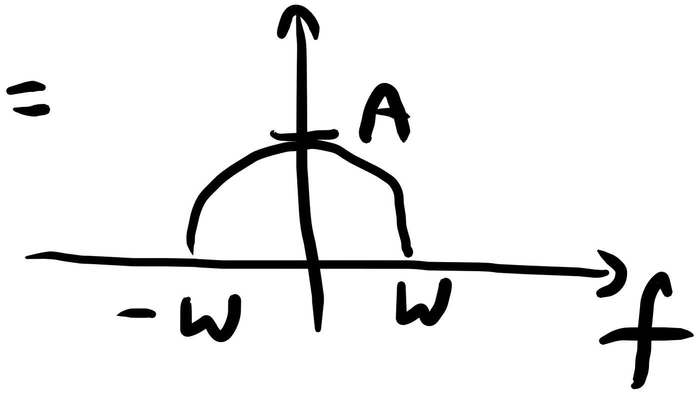
2. $v_p(t) = m(t)\sin(2\pi f_c t)$

Given $M(f)$ which is F.T. of $m(t)$, find $U_p(f)$ & $V_p(f)$

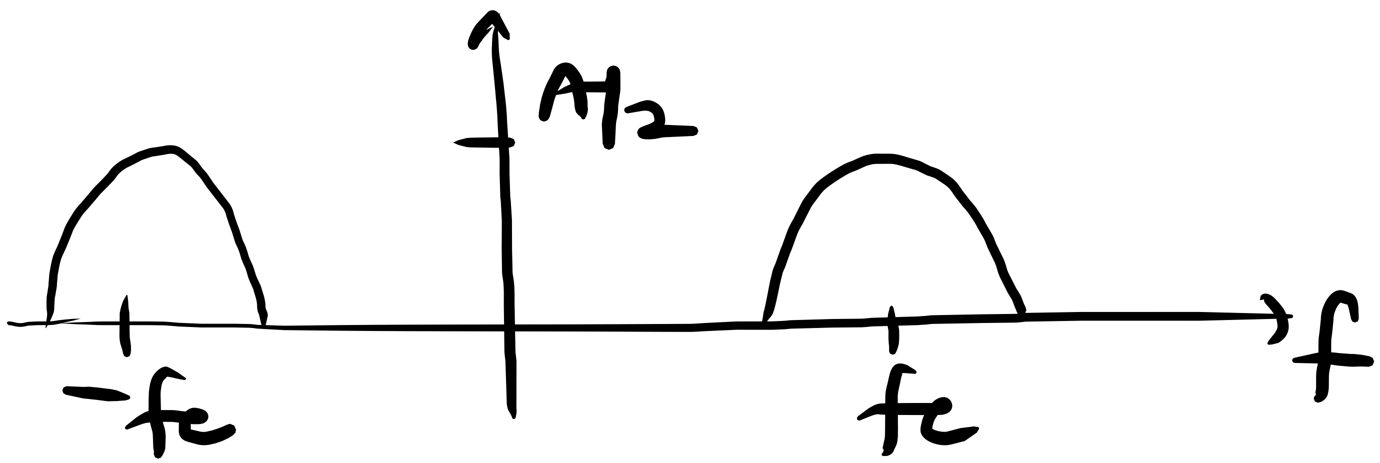
$$U_p(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

$$V_p(f) = \frac{1}{2j}\left[M(f-f_c) - M(f+f_c)\right]$$

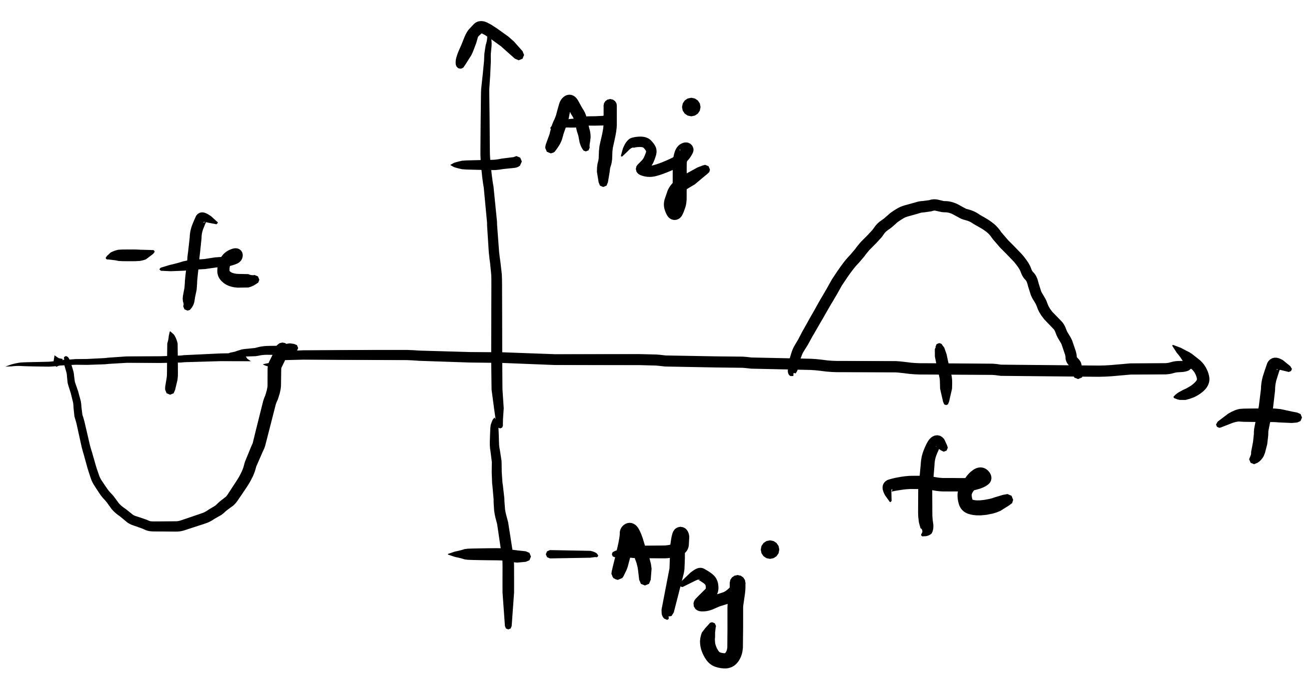
$M(f) =$



$U_p(f) =$



$V_p(f)$



Baseband (BB) Signal

$u(t)$ - if the energy is concentrated in a band around DC &

$$U(f) \approx 0, \quad |f| > W$$

Passband (PB) Signal

$u(t)$ - if the energy

is concentrated in a band away from DC, with

$$U(f) \approx 0, \quad |f \pm f_c| > W \quad \text{— (2)}$$

where $f_c > W > 0$

$f_c + W$

$f_c - W$

$-f_c - W$

$-f_c + W$

* A channel modeled as a linear time-invariant system is said to be PB if its T.F. satisfies (2).

→ $|U_p(f)|$ & $|V_p(f)|$ have freq. content in a band around f_c & are PB signals

→ If we use both sine & cosine carriers, we can construct a PB signal of the form –

$$u_p(t) = \underbrace{u_c(t)\cos(2\pi f_c t)}_{I_1} - \underbrace{u_s(t)\sin(2\pi f_c t)}_{Q_1} \quad \text{--- (4)}$$

where $u_c(t)$ & $u_s(t)$ are real BB signals of B.W. at most W, with $f_c > W$.

$u_c(t) \rightarrow$ In-phase component (I)

$u_s(t) \rightarrow$ quadrature component (Q)

orthogonality of I & Q channels. (this is what allows you to construct $u_p(t)$ in (4))

$$\int_{-\infty}^{\infty} \underset{\sim\sim}{I_1 Q_1}\, dt = 0$$

zero DC component

$$x(t) \rightarrow X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$\underset{\sim\sim\sim}{\int_{-\infty}^{\infty} x(t)\, dt = 0} \Rightarrow X(0) = 0$$