Lec-39, DC, 24-25, SecA

$\cos A \cos B - \sin A \sin B = ?$

2. Complex envelop.

If we have $u(t) = u_c(t) + j\, u_s(t)$, then show $u_p(t) = \mathrm{Re}\{u(t) e^{j2\pi f_c t}\}$

$$u_p(t) = \mathrm{Re}\{\underbrace{(u_c(t) + j\, u_s(t))\left(\cos(2\pi f_c t) + j \sin(2\pi f_c t)\right)}_{\varepsilon_1}\}$$

$\varepsilon_1 = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t) + j\left(u_s(t)\right.$
$\left.\cos(2\pi f_c t) + u_c(t)\sin(2\pi f_c t)\right)$

Given $u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$, we can see that $u_p(t) = \mathrm{Re}\{\varepsilon_1\}$.

$u_p(t) \equiv (\underbrace{u_c(t), u_s(t)}_{\text{BB signals}}) \equiv u_c(t) + j\,u_s(t) = $

$\downarrow$

PB / BP

* $e(t)\, e^{j\theta(t)}$

where

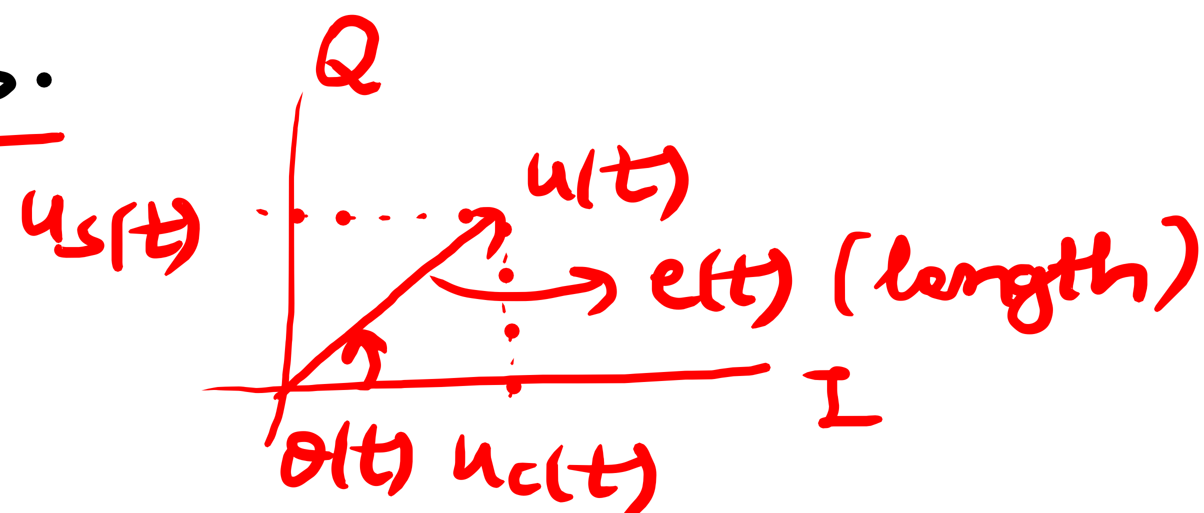
$e(t) = \sqrt{u_c^2(t) + u_s^2(t)}$

$\theta(t) = \tan^{-1}\left(\frac{u_s(t)}{u_c(t)}\right)$

* → polar represe. of a complex no.

$u(t) (= u_c(t) + j\,u_s(t))$ is called the complex baseband corr. to the PB signal $u_p(t)$.

Infor. lies in the complex BB.



↳ angle with I.

$\|u_p\|^2 = \frac{1}{2}\left(\|u_c\|^2 + \|u_s\|^2\right)$

$= \frac{1}{2}\|u\|^2$

$$\|u_p\|^2 = \int_{-\infty}^{\infty} \left[u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)\right]^2 dt$$

$$\int_{-\infty}^{\infty} \Big[\underbrace{u_c^2(t)\cos^2(2\pi f_c t)}_{T_1} + \underbrace{u_s^2(t)\sin^2(2\pi f_c t)}_{T_2} - 2\,\underbrace{u_c(t)u_s(t)\cos(2\pi f_c t)\sin(2\pi f_c t)}_{T_3}\Big]\, dt$$

$$T_3 = \int_{-\infty}^{\infty} \underbrace{u_c(t)u_s(t)\sin(2\pi(2f_c)t)}\, dt = 0$$

↳ PB at 2fc | BB

$$T_1 = \int_{-\infty}^{\infty} u_c^2(t)\left(\frac{1+\cos 2\pi(2f_c)t}{2}\right) dt =$$

$$= \int_{-\infty}^{\infty} \frac{u_c^2(t)}{2} + \int_{-\infty}^{\infty} \frac{1}{2}\, \underbrace{u_c^2(t)\cos(2\pi(2f_c)t)}\, dt \rightarrow 0$$

↳ PB at 2fc

working similarly for T_2, we have

$$T_2 = \int_{-\infty}^{\infty} \frac{u_s^2(t)}{2}\, dt$$

So, $$\|u_p\|^2 = \left(\int_{-\infty}^{\infty} u_c^2(t)\, dt + \int_{-\infty}^{\infty} u_s^2(t)\, dt \right) \frac{1}{2}$$

$$= \frac{1}{2} \left(\|u_c\|^2 + \|u_s\|^2 \right)$$

Energy of PB signal $u_p(t)$

In the case of linear modulation, we have

$u_c(t) = b_c\, p(t)$; $u_s(t) = b_s\, p(t)$

With this, let $u_p(t) \triangleq s_{b_c, b_s}(t)$

$$s_{b_c,b_s}(t) = b_c p(t)\cos(2\pi f_c t) - b_s p(t)\sin(2\pi f_c t)$$

where (b_c, b_s) take M possible values for an M-ary constellation (e.g., $M=4$ for QPSK, $M=16$ for 16QAM)

Let us see how does signal space helps us here?

$$\phi_c(t) = p(t)\cos(2\pi f_c t) \quad ; \quad \mathcal{E}_{\phi_c(t)} = \|p\|^2/2$$

$$\phi_s(t) = -p(t)\sin(2\pi f_c t) \quad ; \quad \mathcal{E}_{\phi_s(t)} = \|p\|^2/2$$

check if $\int_{-\infty}^{\infty} \phi_c(t)\phi_s(t)\,dt = 0$ ✓

$$\psi_c(t) = \frac{\phi_c(t)}{\|\phi_c(t)\|} \quad ; \quad \psi_s(t) = \frac{\phi_s(t)}{\|\phi_s(t)\|}$$

then $$s_{bc,bs}(t) = \frac{1}{\sqrt{2}}\|p\|\, b_c\, \psi_c(t) + \frac{1}{\sqrt{2}}\|p\|\, b_s\, \psi_s(t).$$

$$\psi_c(t) = \frac{p(t)\cos(2\pi f_c t)\sqrt{2}}{\|p\|} \quad ; \quad \psi_s(t) = \frac{-p(t)\sin(2\pi f_c t)\sqrt{2}}{\|p\|}$$

With respect to this basis, the signals $s_{bc,bs}(t)$ can be represented as 2-D Vectors:-

$$s_{bc,bs}(t) \equiv \underline{s}_{bc,bs} = \frac{1}{\sqrt{2}}\|p\| \begin{pmatrix} b_c \\ b_s \end{pmatrix}$$

i.e., up to scaling, the signal-space representation for the transmitted signals are simply the 2-D symbols $\begin{pmatrix} b_c \\ b_s \end{pmatrix}$ or $(b_c \ b_s)^T$

---x---x---