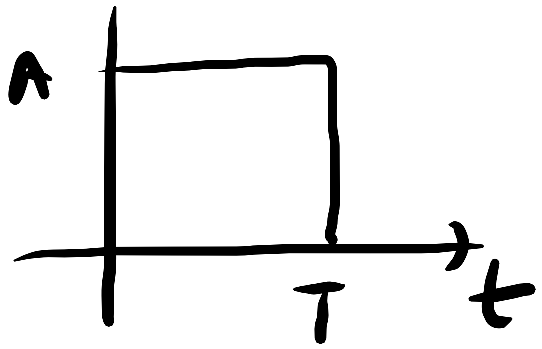
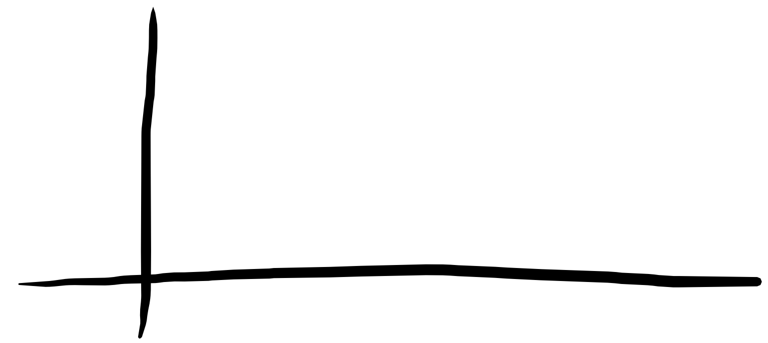
1

Lec-34, DC, 24-25, Sec A

$g(t) =$



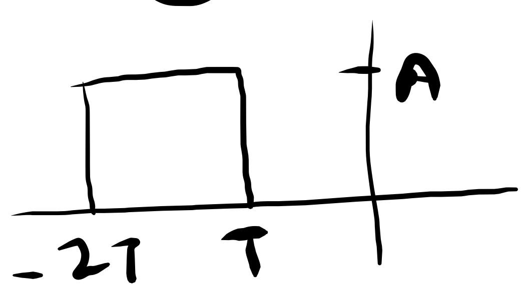
$g(T-t) =$



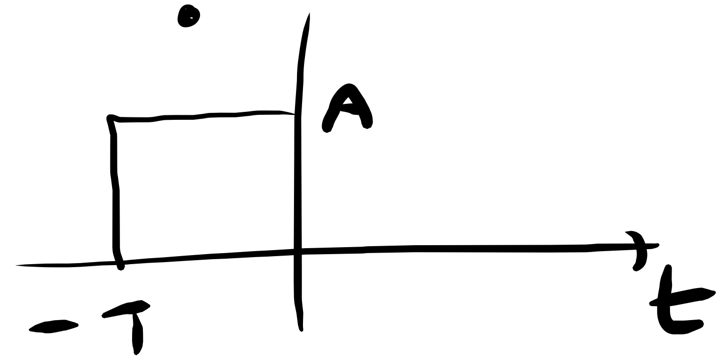
$t = -1.5T$

$g(T+1.5T) = g(2.5T)$

①



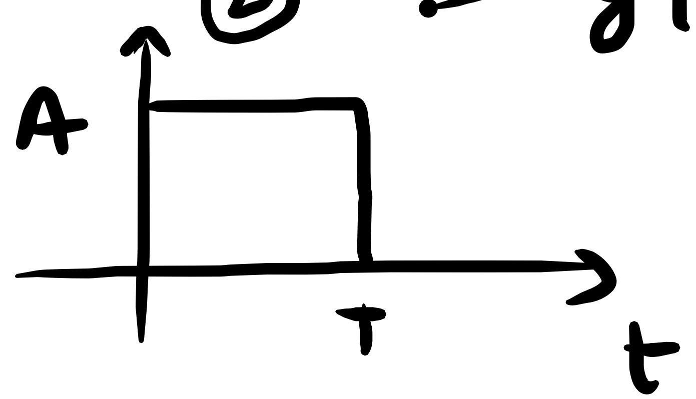
$y(t) = g(-t)$



$g(t) \,\, y_1(t)$

$y_1(t) \triangleq y(t-T) = g(-(t-T))$

② $= g(T-t)$



$y_1(t) = g(T-t)$

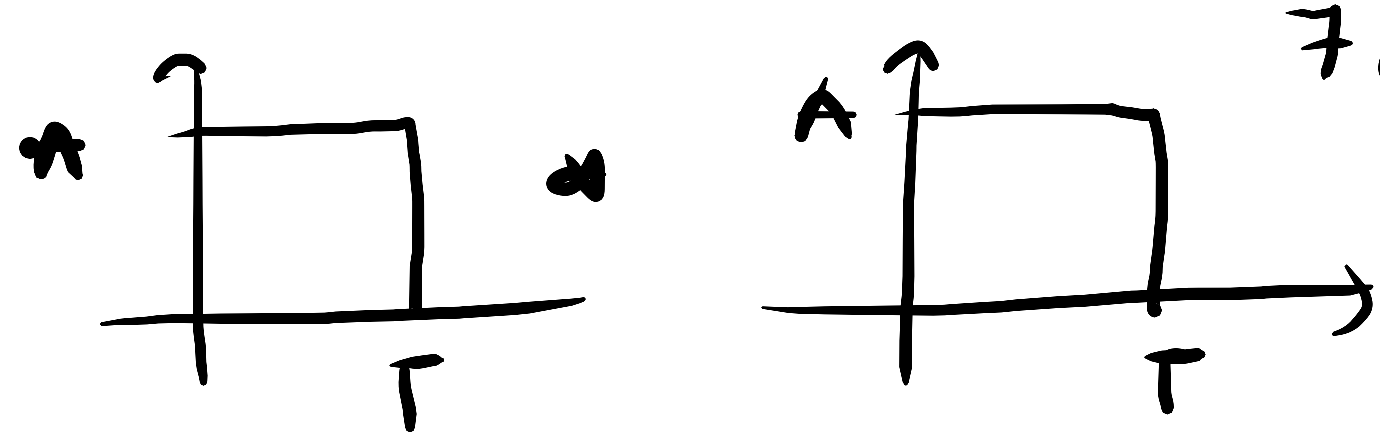
$y_1(-1.5T) = g(T+1.5T)$

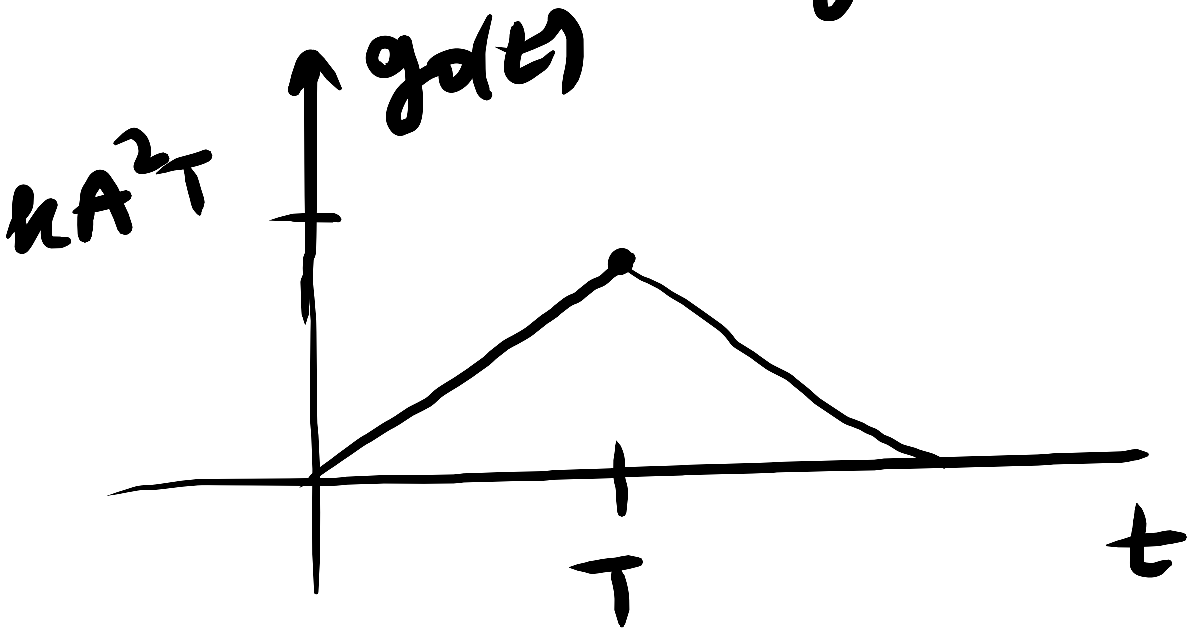
$= g(2.5T)$

$y_1\left(\frac{T}{2}\right) = g\left(T - \frac{T}{2}\right)$

$= g(T/2) = A$

$g(t) * k\,g(T-t)$
$= g_0(t)$





sample $g_0(t)$ at $t = T$

$g_0(T) = kA^2T$

Properties of MF :-

$h_{opt}(t) = k\,g(T-t)$

↳ M.F. corr. to a pulse signal $g(t)$ of duration T

1. $g(t) \longrightarrow \boxed{h_{opt}(t)} \longrightarrow g_0(t)$ —╳— T

$$G_0(f) = k\,G^*(f)\,G(f)\,e^{-j2\pi fT} = k\,|G(f)|^2 e^{-j2\pi fT}$$

$$g_0(T) = \underbrace{\int_{-\infty}^{\infty} G_0(f)\,e^{j2\pi fT}\,df}_{\mathrm{IFT}\{G_0(f)\}_{t=T}} = k\int_{-\infty}^{\infty} |G(f)|^2\,df$$

We know from Rayleigh's energy theorem

$$E = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_{-\infty}^{\infty} |G(f)|^2\,df$$

Hence, $g_0(T) = kE$ $\qquad E = \int_{-\infty}^{\infty} g^2(t)\,dt$

$$E[n^2(t)] = \frac{N_0}{2}\,k^2 \int_{-\infty}^{\infty} |G(f)|^2\,df = k^2 N_0 E/2$$

(study linear filtering of WN from Haykin/Couch/Madhow)

$$\eta_{max} = \frac{(kE)^2}{k^2 N_0 E/2} = \frac{2E}{N_0}$$ peak pulse signal power to noise power ratio

MF has completely removed the dependence on the WF of the $g(t)$. The ability of the MF receiver to combat AWN (Additive white noise), we find that all signals that have the same energy are equally effective.

$E \rightarrow$ Joules

$\frac{N_0}{2} \rightarrow$ watts per Hz

$\frac{E}{N_0} \rightarrow$

Error rate due to noise :-

We know that the MF is the optimum detector of a known pulse in AWN, hence we can now study the error rate due to noise in a binary PCM system.

Consider a binary PCM system based on polar NRZ signalling

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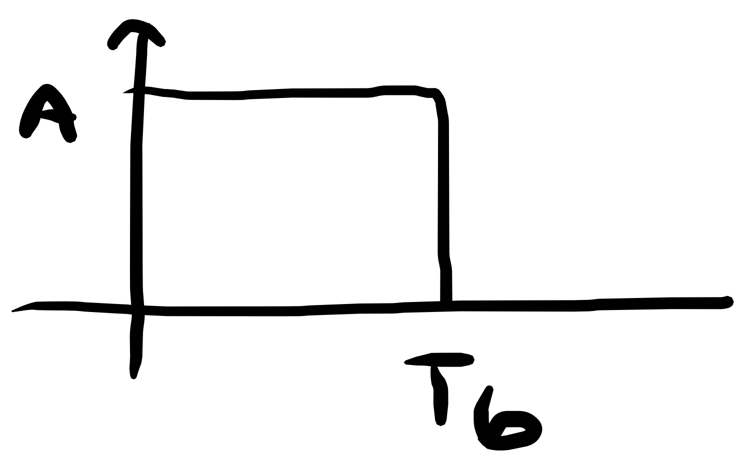
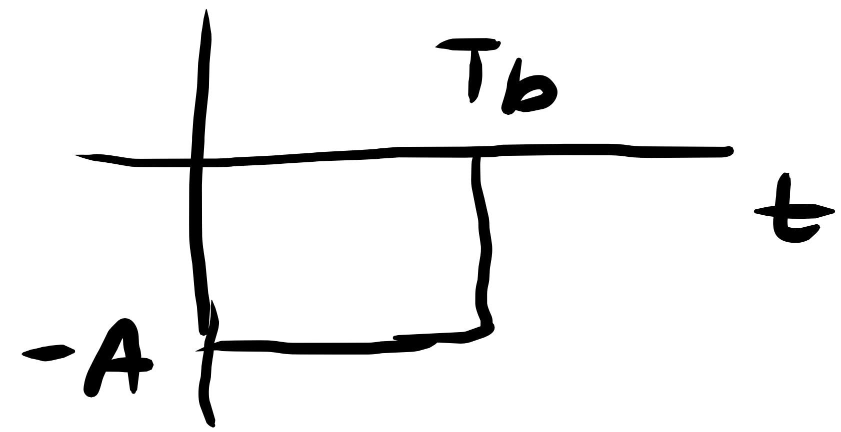


0



$w(t)$: channel noise (AWGN) of zero mean & PSD $N_0/2$.

In the signalling interval $0 \le t \le T_b$, Rxd. sig. is thus written as

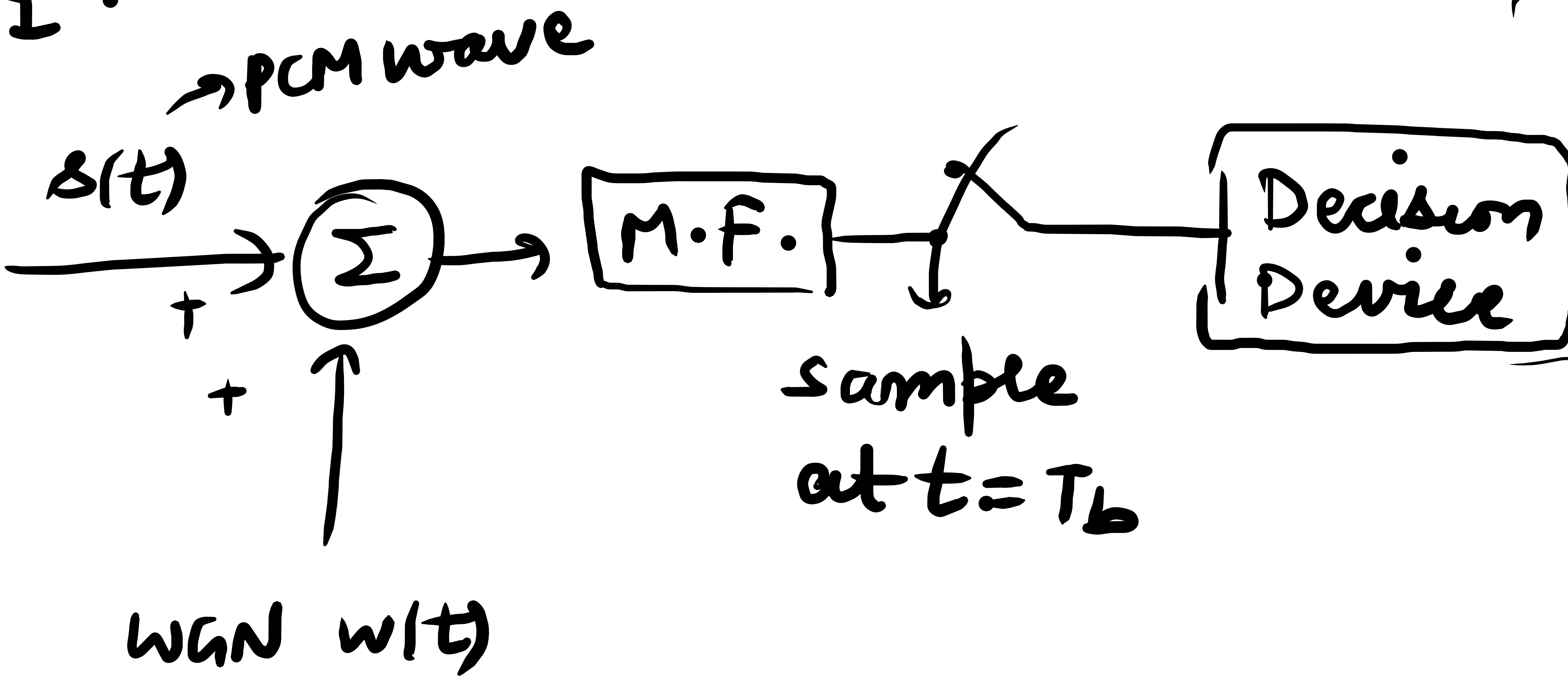
$$x(t) = \begin{cases} +A + w(t), & \text{if symbol 1 is sent} \\ -A + w(t), & \text{if " 0 is sent} \end{cases}$$

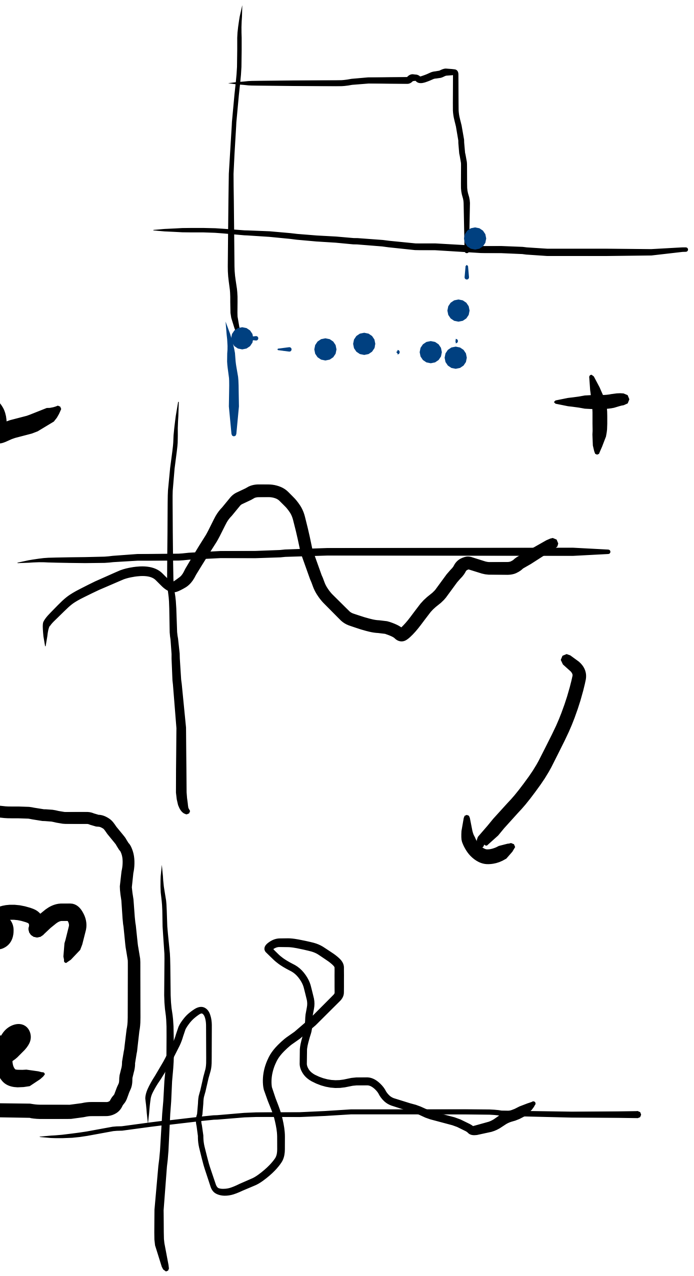
$T_b \rightarrow$ bit duration & $A \rightarrow$ Txd. pulse amplitude

$\rightarrow$ Assumption:- ① Tx. takes place over infinite BW channel, i.e., channel right now is not B.L. $\rightarrow$ no I.S.I.

② Rx. has acquired knowledge of the starting & ending times of each Txd. pulse. Basically, pulse shape is known, but polarity is unknown.

Prob: Given the noisy signal $x(t)$, the Rx· is reqd· to make a decision in each signalling interval as to whether the txd· symbol is 0 or 1·





Let y denote the sample value obtained at the end of a signalling interval.