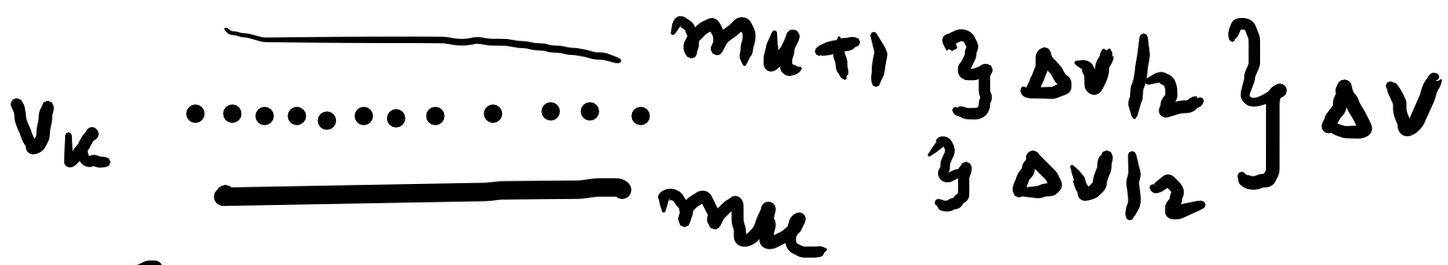
Lec-13, DC, 24-25, Sec A

Hence, $\lim_{T\to\infty} \frac{1}{2BT} \sum_k q^2(kT_s)$ represents the average or the mean of the square of the quantization error.

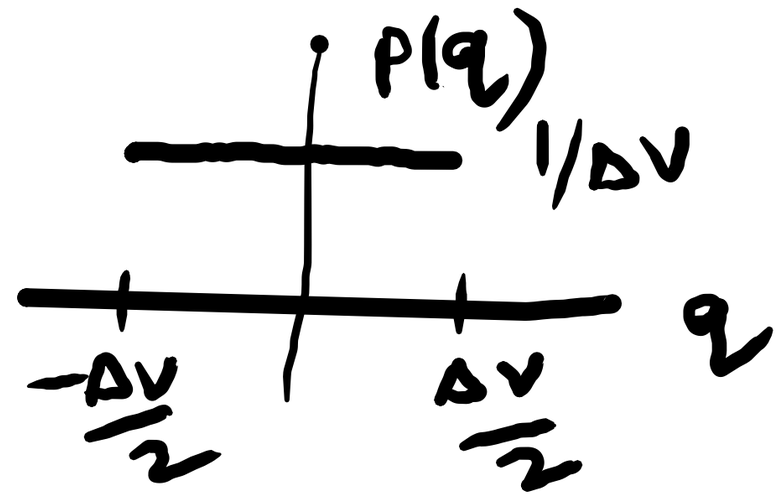
$m(kT_s) - \hat{m}(kT_s)$
$= q(kT_s)$
$k \to -\infty$ to ∞

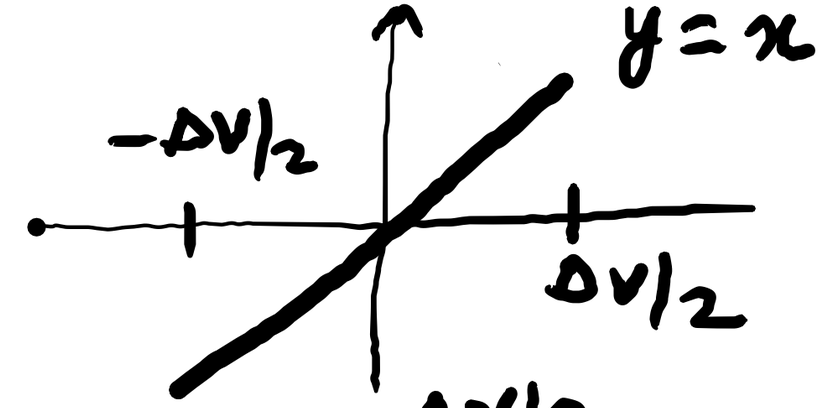


$\underbrace{\left[-\frac{\Delta V}{2}, \frac{\Delta V}{2}\right)}_{A}$ $q(kT_s) \in A$ for all values of k

If ΔV is sufficiently small, we can assume $\{q(kT_s)\}$ takes all possible values in A & an equal number.

$q(kT_s)$ can be assumed to be samples of a r.v. q which is uniformly distributed in $\left[-\frac{\Delta V}{2}, \frac{\Delta V}{2}\right]$





$$E(q^2) = \int_{-\frac{\Delta V}{2}}^{\Delta V/2} q^2 p(q)\, dq = \frac{(\Delta V)^2}{12}$$

$$\frac{\int_{-\Delta V/2}^{\Delta V/2} x^2\, dx}{\Delta V} = \frac{\left.\frac{x^3}{3}\right|_{-\Delta V/2}^{\Delta V/2}}{\Delta V} = \frac{(\Delta V)^2}{12}$$

$$\overline{q^2(t)} = \frac{(\Delta V)^2}{12}$$

$$\Delta V = \frac{2mp}{L}, \quad \overline{q^2(t)} = \frac{mp^2}{3L^2}$$

$$\frac{\sum_{i=1}^{10,000} x_i^2}{10,000} \longrightarrow \frac{(\Delta V)^2}{12}$$

L are the no. of buckets/levels. If we use R bits to represent every level, $L = 2^R$

$$\overline{q^2(t)} = \frac{m_p^2}{3 \cdot 2^{2R}}$$; R → no. of bits per sample used in the construction of the binary code.

Assuming pulse detection error at the Rx is negligible, $\hat{m}(t)$ at the Rx O/P can be written as $\hat{m}(t) = m(t) + q(t)$

Let the power (average) of the message signal be denoted as $\overline{m^2(t)}$, then

$$S_o = \overline{m^2(t)}, \quad N_o = N_q = \frac{m_p^2}{3L^2}$$

[Here, it is assumed that $q(t) = \hat{m}(t) - m(t)$ but we take $q(t) = m(t) - \hat{m}(t)$ both are equivalent]

$$SNR = \frac{S_o}{N_o} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2}$$

Q. Consider a full-load sinusoidal modulating signal of amplitude A_m, which uses all the represent. levels. Find SNR in terms of R. & express it in dB.

A. $m(t) = A_m \sin(2\pi f t)$, average power. find it over a period of $m(t)$

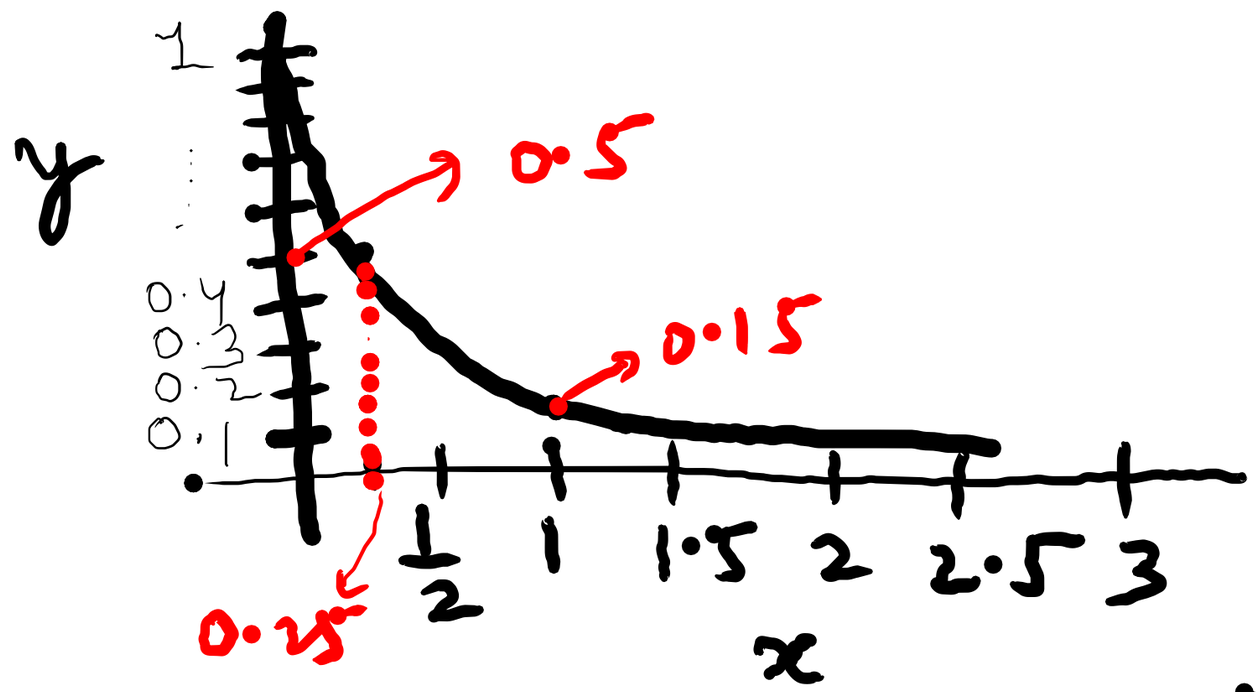
$$\overline{m^2(t)} = \frac{A_m^2}{2}$$

$$\frac{\int_{-T_p/2}^{T_p/2} m^2(t)\,dt}{T_p}$$

$$SNR = \frac{3 \cdot 2^{2R} A_m^2}{2 \cdot A_m^2} = \frac{3 \cdot 2^{2R}}{2}$$

$$10 \log_{10} SNR = 1.8 + 6R$$

* analog input takes on all values with equal probability & we assume the rep. level (no. of) to be sufficiently high for quantization noise's uniform assumption.



$y \rightarrow$ prob. that x value is exceeded

$x \rightarrow$ speech sig. mag. rel. to their RMS value

Non-uniform Quantizer (NUQ)

$V_0 \quad V_1 \quad V_2 \quad V_3$

$x_{01} \quad x_{12} \quad x_{13}$

$x_{01} = x_{12} = x_{13} \rightarrow$ UQ

$x_{01} \neq x_{12} \neq x_{13} \rightarrow$ NUQ

Basically, very low speech volumes dominate 50% of the time, the voltage charac. detected speech energy is less than 1/4 of the RMS value.