Lec-31, DC, 23_24, Sec A

Study & derivation of Matched Filter (MF)

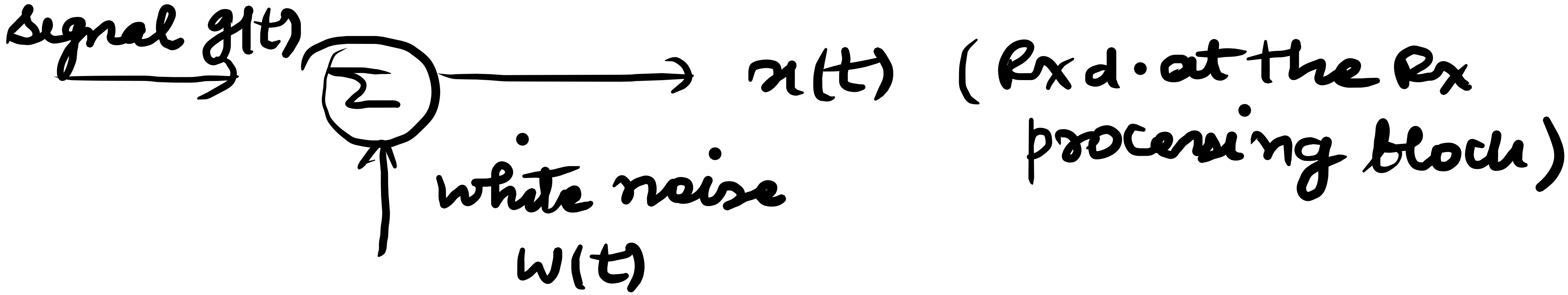
Preliminaries:- Linear modulation → signal space → detour
(Baseband) ✓ $\sum_{k=-\infty}^{\infty} a_k p(t-kT_b)$ for M signal waveforms
Pending - Q below (we will come back to it)

$$g(t) = \sum_k a_k p(t-kT_b) \text{ —}$$

Q. How does receiver retrieves $\{a_k\}$

M.F. — Detecting a pulse transmitted over a channel that is corrupted by channel noise (i.e., additive noise at the front end of Rx)

Q. Do we receive $g(t)$ as it is at the Rx.?



Tx. of digital Data over baseband channel.

1. Effect of I.S.I → due to the finite Tx. B.W. of channel & broadband spectrum of digital data with a low-freq. content.
   → sol. is pulse shaping & equalization
2. Channel noise ($w(t)$):- electrical noise & interference

atmospheric noise

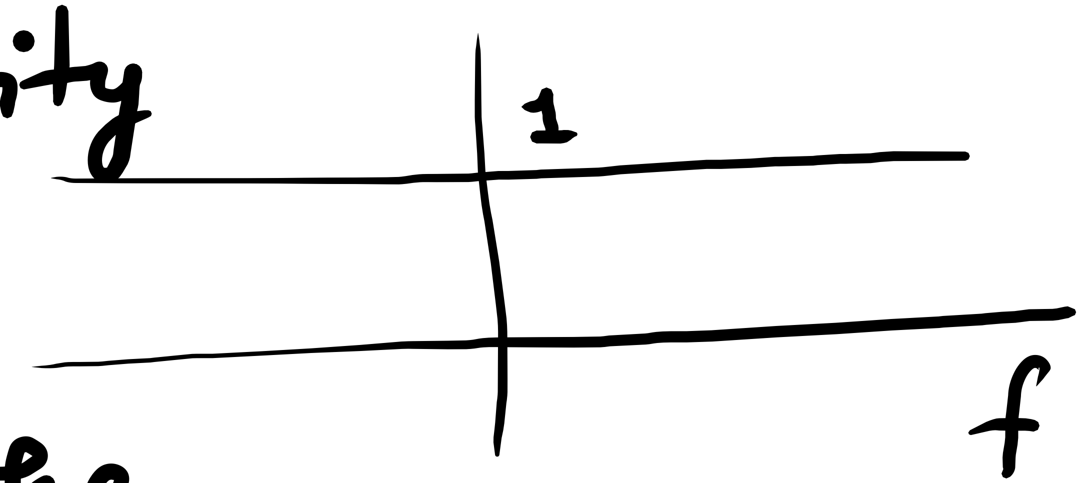
switching transients, interfering signals from other sources.

With proper precautions, much of the noise & interf. entering a Rx. can be reduced in intensity or eliminated.

But one which cannot be eliminated is the noise caused by the thermal motion of electrons in any conducting media. This motion produces thermal noise in amplifiers & circuits & corrupts the signal in an additive fashion. (?)

→ The statistics of thermal noise has been developed & are well known.

White noise :- power spectral density (PSD) is independent of the operating freq. 'white' → as the white light contains equal amounts of all freq. within the visible band of EM radiation.



PSD → $S_W(f) = \frac{N_0}{2}$ Watt / Hz

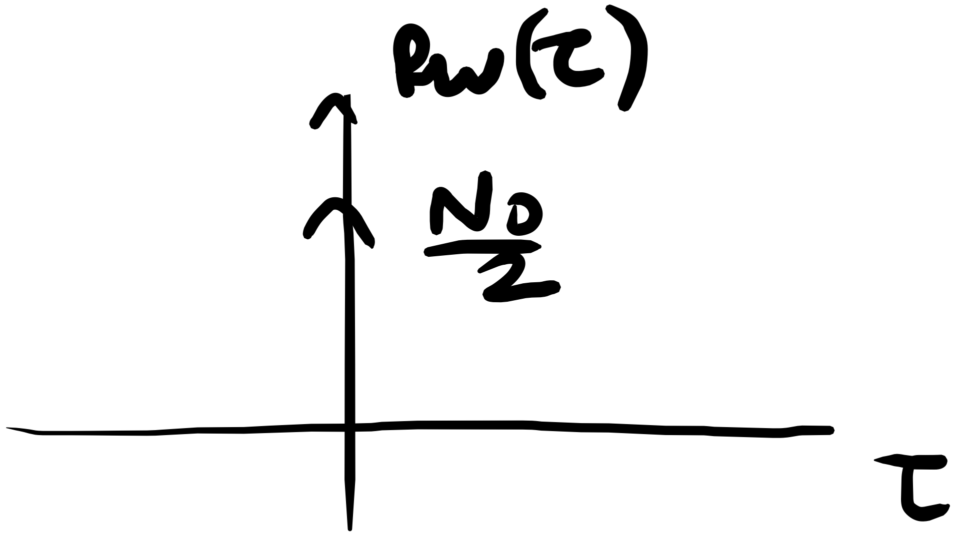
Autocorrelation function → is the Inverse Fourier transform of the PSD. (a few req. exist)

$$R_W(\tau) = \frac{N_0}{2}\,\delta(\tau) \Big\} \begin{array}{l} \frac{N_0}{2}, \ \tau = 0 \\ 0, \ \text{o.w.} \end{array}$$

↳ $E[W(t)W(t+\tau)]$

$FT\left[\frac{N_0}{2}\,\delta(t)\right] \rightarrow \frac{N_0}{2} \ \forall f$

$R_W(\tau) = E[W(t)W(t+\tau)]$. If I fix a t, i.e., $t = t_1$ & view $w(t_1)$ & $w(t_1+\tau)$ for $\tau > 0$, you would have a diff. value.



Two different samples of white noise, no matter how close together in time they are, are uncorrelated.

→ If $W(t)$ is also Gaussian, then the two samples are statistically independent.

Hence, white Gaussian noise represents the **ultimate** in randomness.