Lec-16, DC, 24-25, SecA

Output SNR - PCM system

$$\frac{S_o}{N_o} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2}, \text{ as } L = 2^n, \frac{S_o}{N_o} = c\,2^{2n}$$

where

$$C = \begin{cases} 3\dfrac{\widetilde{m^2(t)}}{m_p^2}, & \text{uncompressed quantization} \\ \dfrac{3}{[\ln(1+\mu)]^2}, & \text{compression } (\mu\text{-law}) \end{cases}$$

Now, let us say you are given a max BW of B_T to Tx. your signal. $B_T \geq nB$, $n \leq \dfrac{B_T}{B}$

for time being, assume $n = B_T/B$

$$\frac{S_o}{N_o} = c\,2^{2B_T/B};$$ with Tx B.W. B_T, SNR ↑ exponentially.

$B_T' = B_T + \Delta B_T$, $\frac{S_o'}{N_o} = 2^{2B_T'/B} = 2^{2B_T/B} \, 2^{2\Delta B/B}$

$\left(\frac{S_o}{N_o}\right) \times 2^{2\Delta B/B}$

Small ↑ in Tx B.W. yields a large benefit in terms of SNR.

$$\left(\frac{S_o}{N_o}\right)_{dB} = 10\log_{10}\left(\frac{S_o}{N_o}\right) = 10\log_{10} C + 20n\log_{10} 2$$

$$\simeq (\alpha + 6n)\,dB$$

where $\alpha = 10\log_{10} C$

This shows that ↑ n by 1 (increasing 1 bit in the code word) quadruples the output SNR (a 6dB increase)

$X \to 4X$

$y \to y + 6.020\big|_{dB}$ $\quad \underbrace{10\log_{10} X}_{y} \to 10\log_{10} X + \underbrace{10\log_{10} 4}_{6.020}$

This shows that in PCM, SNR can be controlled by Tx BW.

ex- You are given a signal $m(t)$, which is BL to 4 KHz. It is txd. using a binary companded PCM with $\mu = 100$. Compare $L=64$ with the case of $L=256$ from the pt. of view of Tx BW. (B_T) & output SNR.

Ans. $L=64$, $n_1 = 6$, Tx.B.W. $B_{T_1} = n_1 B = 24$ KHz

$L=256$, $n_2 = 8$, $B_{T_2} = 32$ KHz.

$$\left.\frac{S_0}{N_0}\right|_{L=64} = \alpha + 6n_1 = 10\log_{10} C + 36 \text{ dB}.$$

$$\left.\frac{S_0}{N_0}\right|_{L=256} = \alpha + 6n_2 = 10\log_{10} C + 48 \text{ dB}$$

$\alpha = -8.57\,dB$; $\left.\frac{S_o}{N_o}\right|_{L=64} = 27.49\,dB$

$c = \frac{3}{(\ln(101))^2}$ $\left.\frac{S_o}{N_o}\right|_{L=256} = 39.49\,dB$

The difference b/w the two o/p SNRs is 12dB, which is a ratio of 16. Thus, the SNR for L=256 is 16 times the SNR for L=64. However, the former requires just about 33% more BW compared to later.

$$\frac{32-24}{24} = \frac{8}{24} = \frac{1}{3}$$

in % = 33.33%

As a refresher:- Read "Comments on logarithmic units" on pg 280, Lathi & Ding.

Differential coding:-

Issue:- When serial data is passed through many circuits, the waveform is often un-intentionally inverted. (i.e. data complemented)

To ameliorate the problem diff. coding is employed.

It can be generated as $e_n = d_n \oplus e_{n-1}$

modulo 2 adder or a XOR gate operation

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

decoding:- $\check{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1}$

example,

$e_n = d_n \oplus e_{n-1}$

		d_1	d_2	d_3	d_4	d_5	d_6	d_7
d_n		1	1	0	1	0	0	1
e_n	1	0	1	1	0	0	0	1
	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7

$\hookrightarrow$ ref. bit

$e_1 = d_1 \oplus e_0$

$e_0 \to$ ref. bit

decoding → with correct channel polarity

	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
$\tilde{e}_n$	1	0	1	1	0	0	0	1
$\tilde{d}_n$		1	1	0	1	0	0	1
		$\tilde{d}_1$						

$\tilde{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1}$

$\tilde{d}_1 = \tilde{e}_1 \oplus \tilde{e}_0$

→ with reverse polarity

$\tilde{e}_n$	0	1	0	0	1	1	1	0
$\tilde{d}_n$		1	1	0	1	0	0	1

You can see that inverted polarity does not effect decoded sequence.

→ great adv. when WF is passed through thousands of circuits in a comm. system & the +ve sense of o/p is lost or changes occassionally as the netw. changes, s.as. sometimes occurs during switching b/w. several data paths.