1. Each basis function is normalized to have unit energy.

2. Basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ are orthogonal w.r.t. each other over the interval $0 \leq t \leq T$

Inner product b/w complex valued signals $x(t)$ & $y(t)$ is defined as $\langle x(t), y(t)\rangle = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt$ leading to $\langle x(t), y(t)\rangle \neq \langle y(t), x(t)\rangle$

3. Set of coeff. $\{s_{ij}\}_{j=1}^{N}$ may be viewed as an N-dimensional vector denoted as $\bar{s}_i$. $\bar{s}_i$ has a one-to-one relationship with the signal $s_i(t)$.

see
Synthesizer & Analyzer in fig 5.3 in Haykin's T.B.

In each arm of analyzer, we have the input
ex- $\phi_2(t)$.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\, \phi_j(t)$$

The processing $\rightarrow \int_0^T s_i(t)\,\phi_2(t)\,dt$

$$= \sum_{j=1}^{N} s_{ij} \underbrace{\int_0^T \phi_j(t)\,\phi_2(t)\,dt}_{} = s_{i2}$$

$\rightarrow$ 0 if $j \neq 2$
1 if $j = 2$

Synthesizer :- Multipliers & summer
Analyzer :- 'product - integrators' or 'correlators'

$\bar{s}_i$ is called signal vector. If we conceptually extend our conventional notation of 2-D or 3-D Euclidean space to an N-dimensional Euclidean space, $\{\bar{s}_i \mid i=1,2,\ldots,M\}$ is a set of M points in an N-dim. Euclidean space, with N mutually orthogonal axes labeled $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$.

This N-dim. Euclidean space is called the **signal space**.

length of any vector in Euclidean space

$$\|\bar{s}_i\| = \sqrt{\bar{s}_i^T \bar{s}_i} = \sqrt{\sum_{j=1}^{N} s_{ij}^2}, \quad i=1,2,\ldots M$$

Consider a vector $t = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ or $(2,3)$ as a point in the 2-D Euclidean space. Then $\sqrt{t^T t}$ is the length of the vector.

Squared length of any vector in Euclidean space = inner product or dot product of $\bar{s}_i$ with itself. In the N-dim Euclidean space of signal vector, this is the energy of the " corr. to that vector.

→ Note that energy of a signal $s_i(t)$ of duration T seconds is —

$$E_i = \int_0^T s_i^2(t)\,dt \overset{(a)}{=} \|\bar{s}_i\|^2$$

Prove (a).

$$E_i = \int_0^T \left(\sum_{j=1}^{N} s_{ij} \phi_j(t) \times \sum_{k=1}^{N} s_{ik} \phi_k(t) \right) dt$$

$$= \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij} s_{ik} \int_0^T \phi_j(t) \phi_k(t) dt = \sum_{j=1}^{N} s_{ij}^2 = \|\bar{s}_i\|^2$$

$$\because \int_0^T \phi_j(t) \phi_k(t) dt = \delta_{jk} \; (= \delta[j-k])$$

$$\delta_{jk} = \begin{cases} 1, & j = k \\ 0, & \text{otherwise} \end{cases}$$

$\delta(t)$

$\delta[n]$

$\delta[j-k]$

$\rightarrow$ We can also show that

$$\int_0^T s_i(t) s_k(t) dt = \bar{s}_i^T \bar{s}_k \quad (\text{proof H.W.})$$

Inner product of signals $s_i(t)$ & $s_k(t)$ over the interval $[0, T]$, using time domain rep $=$ inner product of resp. vector representation $\bar{s}_i$ & $\bar{s}_k$

→ Note that inner product of $s_i(t)$ & $s_k(t)$ is invariant to the choice of basis functions $\{\phi_j(t)\}_{j=1}^{N}$. It depends only on the components of the signals projected onto each of the basis funcn.