Lec-25, DC, 24-25, Sec A

2. Sinc pulse decays too slowly. i.e., at a rate of $1/t$. If there is lack of synchronization*, then all contribute to a sampling time

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t} \simeq 1/t$$

& $\sum_n 1/n$ is not summable i.e., can add upto a large value.

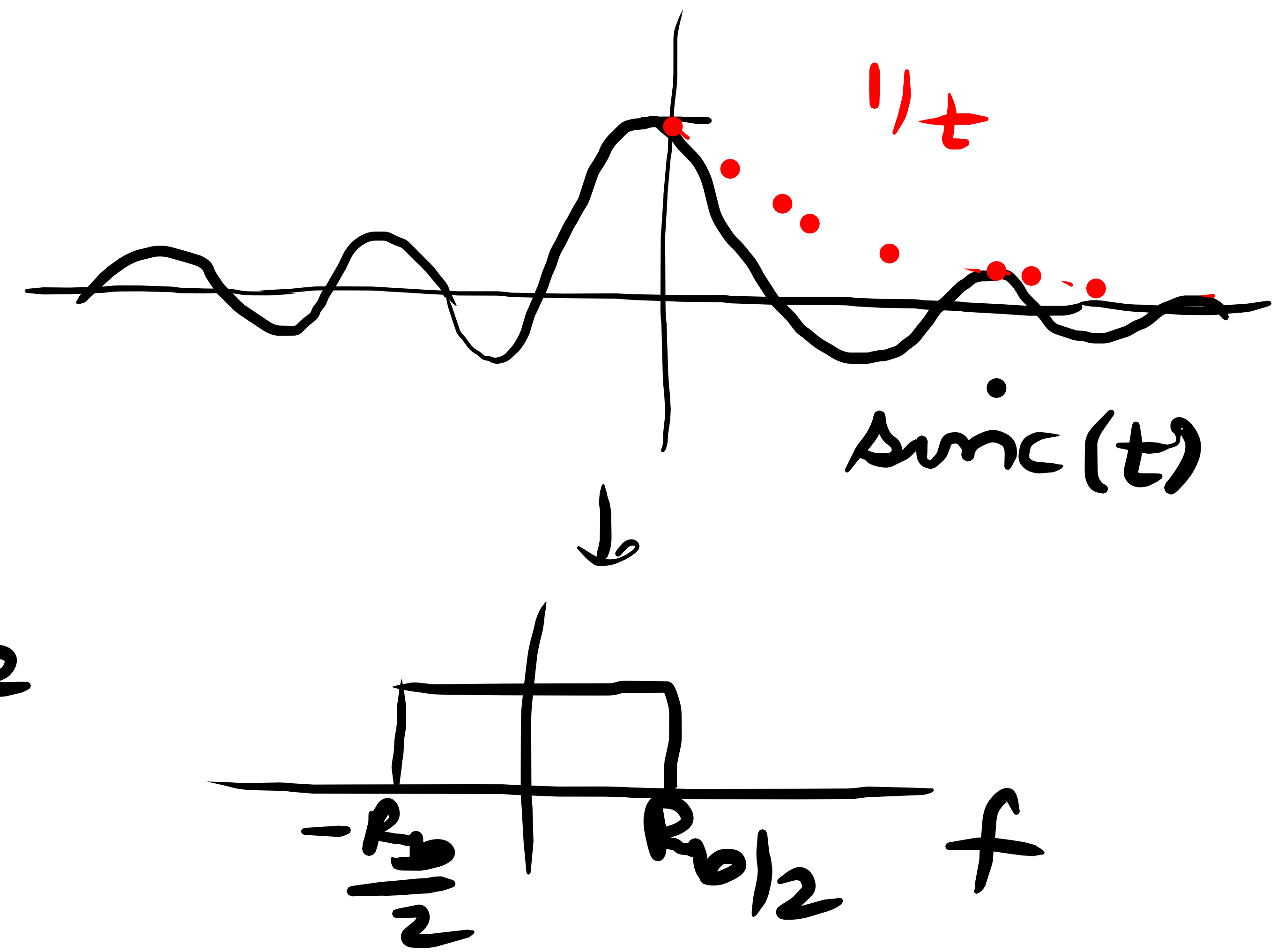
* → i.e., sampling instants change either at Tx or Rx i.e., instead of $\pm nT_b$, you sample at $\pm n(T_b+\delta)$

This may also happen when even the rate (sampling) deviates a little bit.

This pulse (sinc) fails unless everything is perfect which is impossible with practical implement.

Sol^n:- Find $p(t)$ that satisfies Nyquist's criteria & decays faster than $1/t$

Since we need a faster decay (possible with fast change in time), we need to allow B.W. to be more than the min. $R_b/2$.



→ Nyquist showed that such a pulse (which decays as $1/t^b$ for some $b>1$) requires a B.W. $k R_b/2$

with $1 \leq k \leq 2$

<u>Proof</u>:- Let $p(t)$ be a pulse, with F.T. $P(f)$ where B.W. of $p(t)$ is in the range $\left(\frac{R_b}{2}, R_b\right)$

→ Nyquist criteria for ISI avoidance

$$p(t) = 1, \quad t = 0$$
$$= 0, \quad t = \pm nT_b$$

#1

$$\Rightarrow p(t)\,\delta_{T_b}(t) = \delta(t)$$

$$\delta_{T_b}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_b)$$

$$p(t)\,\delta(t - nT_b) = p(nT_b)\,\delta(t - nT_b)$$

$$p(t)\,\delta_{T_b}(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t - nT_b) = p(0)\,\delta(t)$$
$$= \delta(t)$$

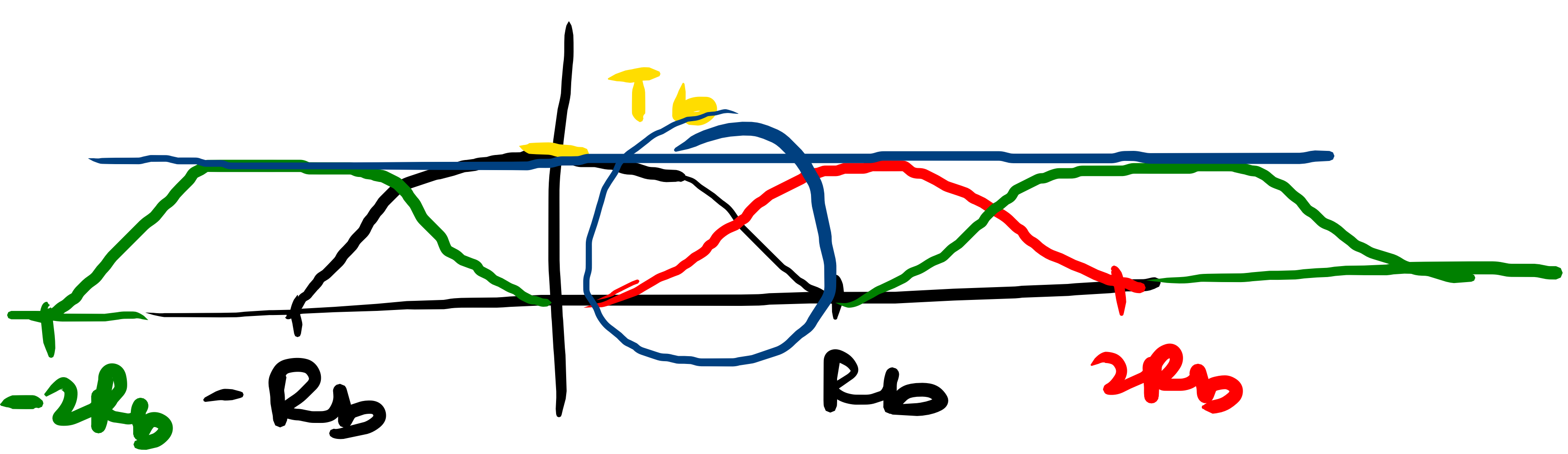
Let's take F.T. of both sides

$$F.T.\{p(t)\delta_{T_b}(t)\} = F.T.\{\delta(t)\}$$

$$\frac{1}{T_b}\sum_{n=-\infty}^{\infty} P(f-nR_b) = 1 \quad , \text{ where } R_b = 1/T_b$$

or $\Rightarrow$ $$\boxed{\sum_{n=-\infty}^{\infty} P(f-nR_b) = T_b}$$ #2

We assumed that $P(f)$ exists from $-\frac{R_b}{2}-y$ to $\frac{R_b}{2}+y$



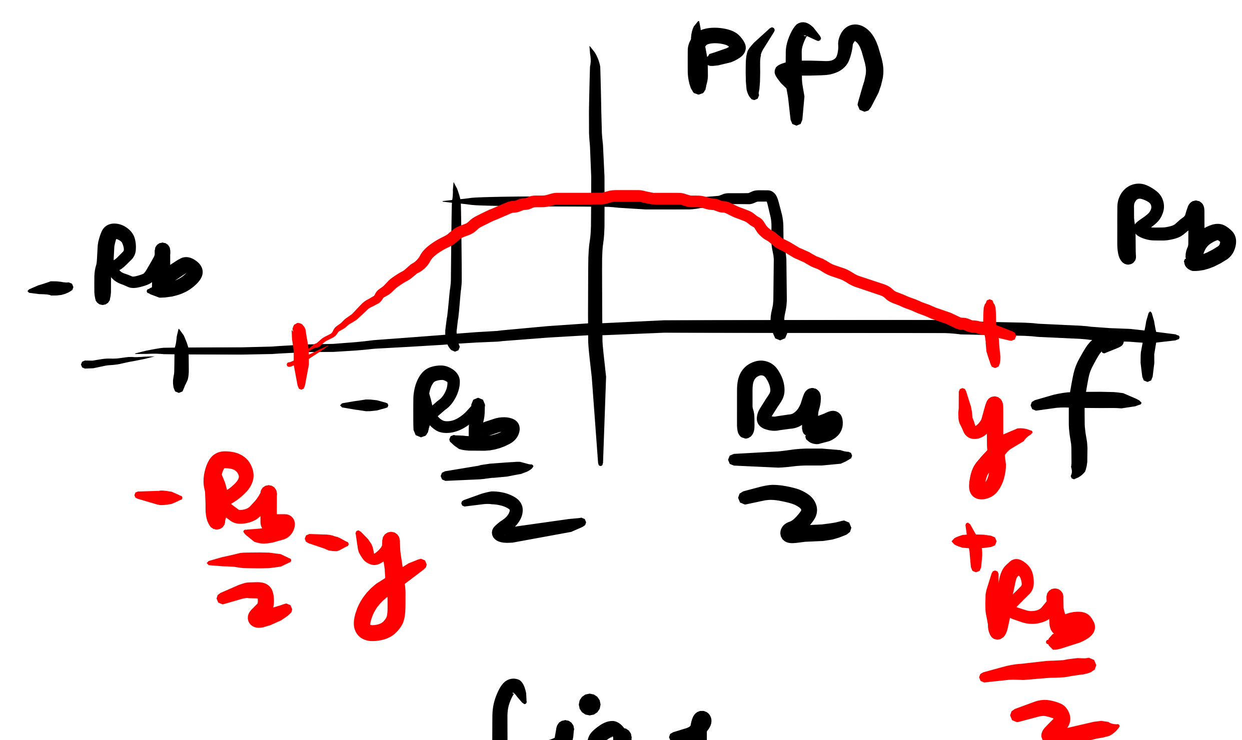


fig 1.

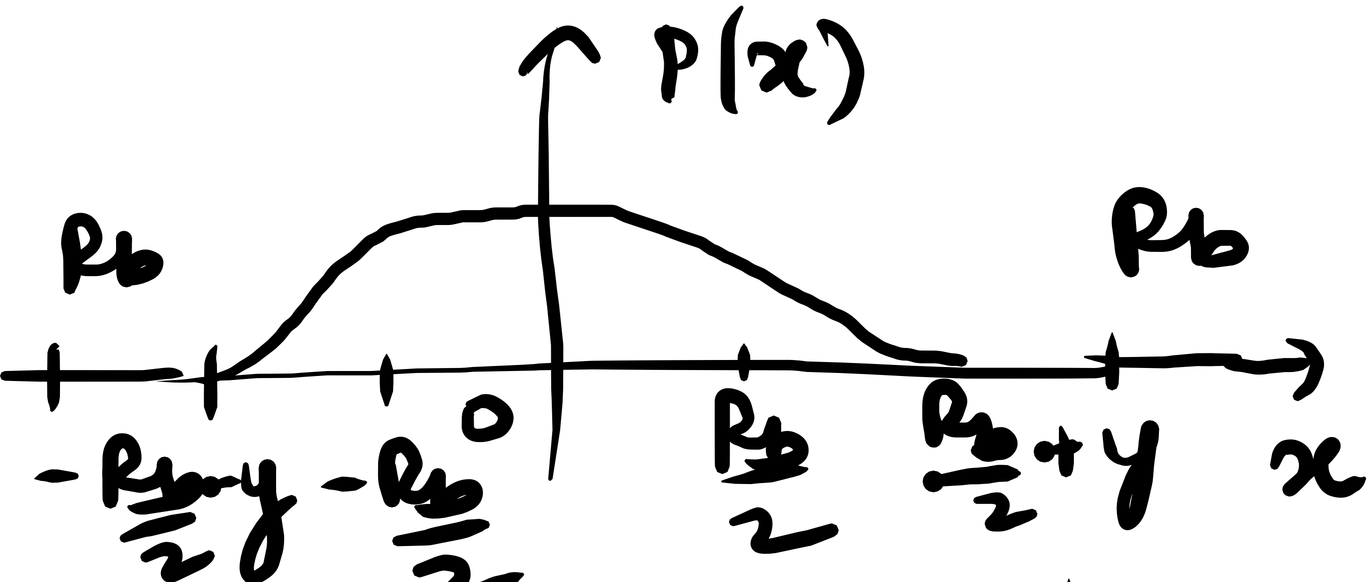
Let's take the range $0 < f < R_b$. Over this range only two terms $P(f)$ & $P(f-R_b)$ in the summation are involved.

(In context of fig. 1)

$$P(f) + P(f - R_b) = T_b$$

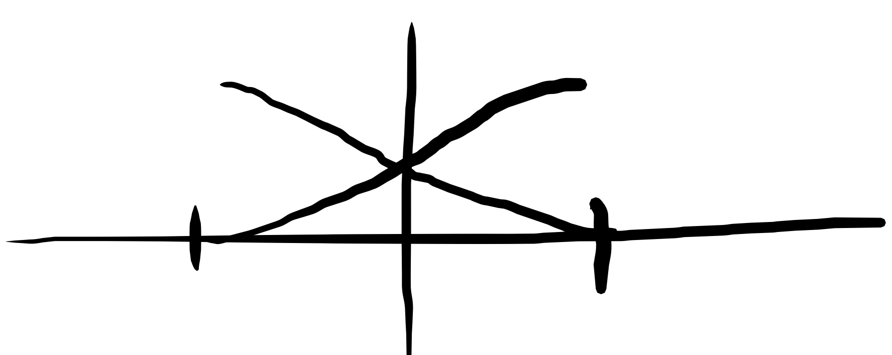
let $f = x + R_b/2$ or $x = f - R_b/2$

we have, $P\left(x + \frac{R_b}{2}\right) + P\left(x - \frac{R_b}{2}\right) = T_b$ for $|x| < R_b/2$



$P\left(x + \frac{R_b}{2}\right)$ & $P\left(x - \frac{R_b}{2}\right)$

$-\frac{R_b}{2} < x < R_b/2$



Draw the above signals

$$\Rightarrow \quad P\left(x + \frac{R_b}{2}\right) + P^*\left(-x + \frac{R_b}{2}\right) = T_b \; ; \; |x| < R_b/2$$

If we choose $P(f)$ to be real-valued & +ve then,

$$\left|P\left(x + \frac{R_b}{2}\right)\right| + \left|P\left(\frac{R_b}{2} - x\right)\right| = T_b$$

for $|x| < R_b/2$

$P(t) \rightarrow$ real then

$P(f) = P^*(-f)$

F.T. is conjugate symmetric

$\Rightarrow$ $P(f)$ has an odd symmetry about the set of axis intersecting at pt. $f = R_b/2$