Lec-14, DC, 24-25, Sec A + Lec 15

Large amplitudes are relatively rare, only 15% of the time does the voltage exceed the RMS value.

→ Issue:- the SQNR (signal to quantization noise ratio) would be low most of the time

→ Or, SQNR is $\propto$ to the signal power $\frac{3L^2 \overline{m^2(t)}}{m_p^2}$ which varies from speaker to speaker by as much as 40dB (a power ratio of 10^4)

→ even for same speaker, the quality of the rxd. signal will deteriorate markedly when the person speaks softly.

→ Difficulty:- quantizing steps are of uniform value $\Delta v = 2m_p/L$, which makes quantization noise power $\propto (\Delta v)^2/12$ (square of the step size)

→ Uniform Quantizer:- quant. noise same for all magnitudes.

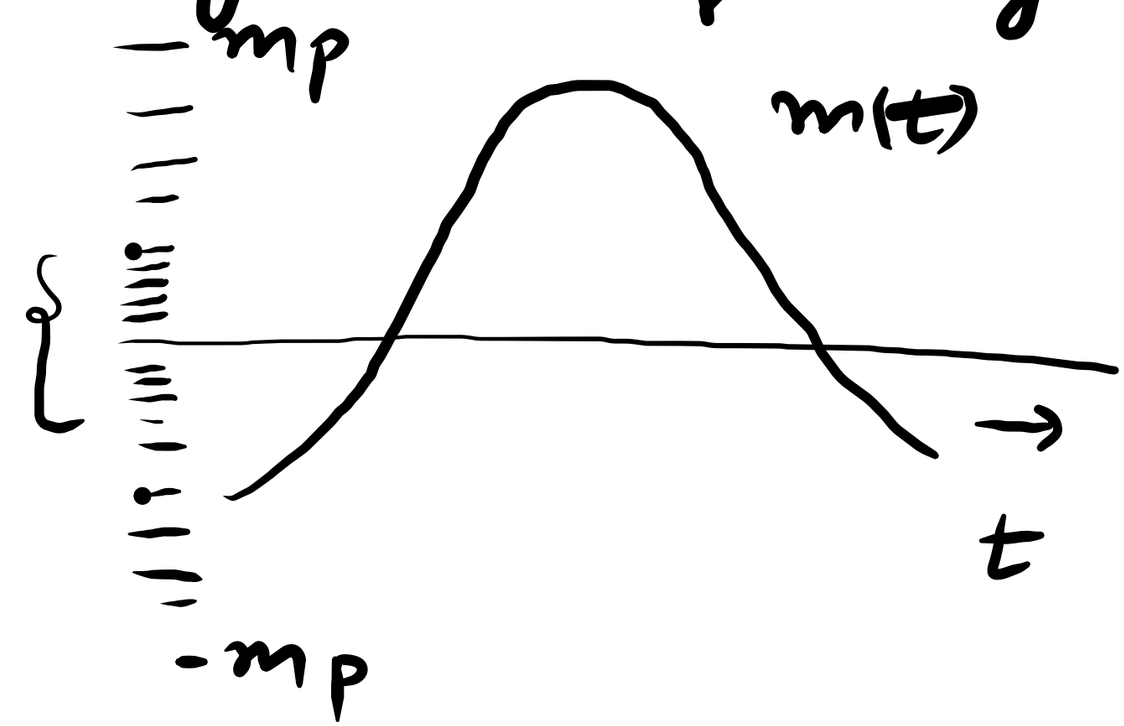
→ SQNR worst for low-level signals than for high-level signals.

→ NUQ:- fine quantization of weak signals & coarse quantiz^n of strong signals.

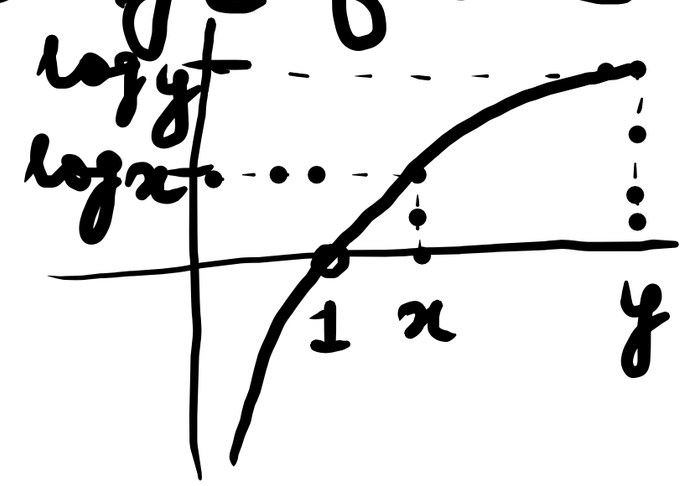
so, quanti^n noise $\propto$ signal size

∵ effect is to improve overall SQNR by ↓ the noise for the predominant weak signals, at the expense of an ↑ in noise for the rarely occurring strong signals.

How will it be done?



→ first pass samples through a non-linear element that reduces dynamic range of the signal. (possibly a compressor)



→ perform uniform quantization on the output.

1. μ - law — compression law (used in North America introduced by Bell labs)

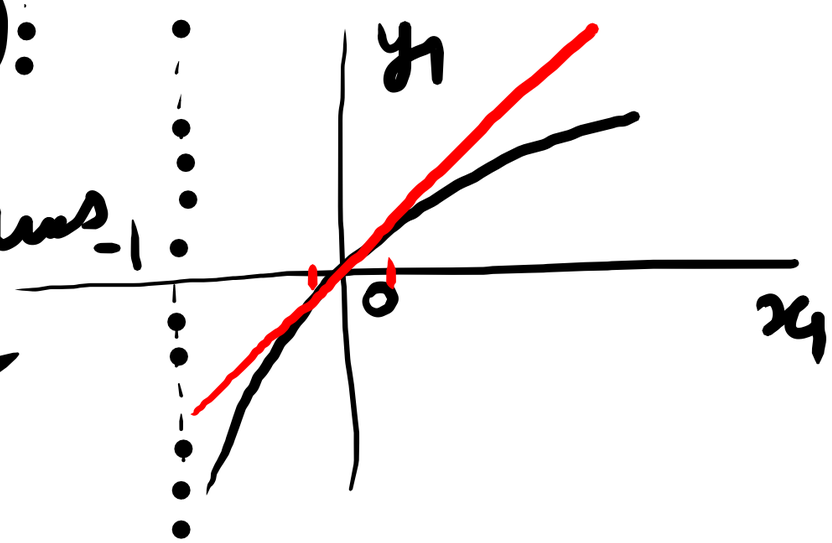
$$y = y_{max} \frac{\log_e\left[1 + \mu(|x|/x_{max})\right]}{\log_e(1+\mu)} \operatorname{sgn}(x)$$

$-y_{max} \le y \le y_{max}$

$y_1 = \log_e(1+x_1)$:

for low values of x_1, can we write

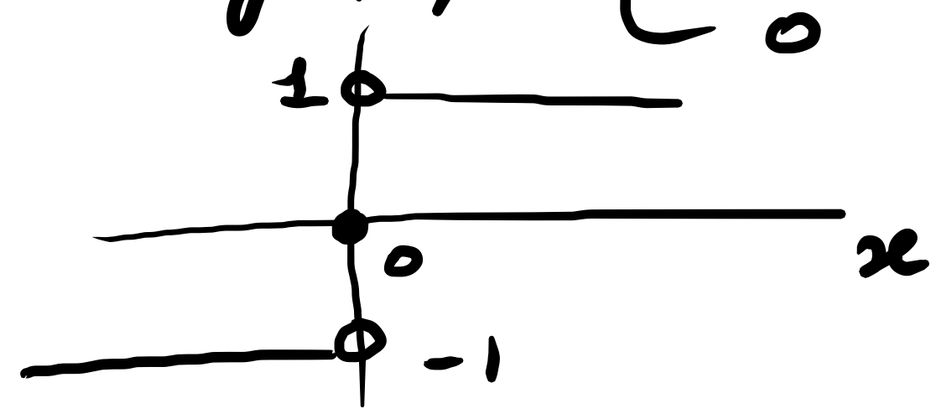
$y_1 \approx x_1$?



$\operatorname{sgn}(x) = x/|x| \Rightarrow$

$$\begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0 & x = 0 \end{cases}$$

$\operatorname{sgn}(x)$



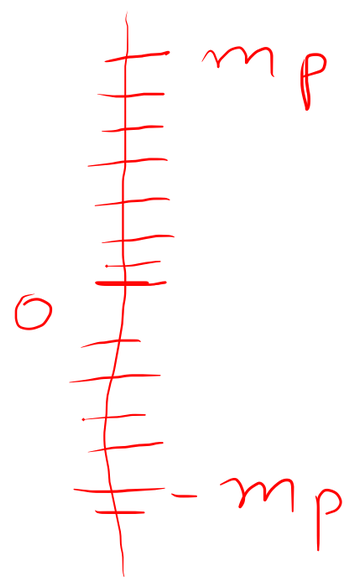
$x_{max} = 10$, $x = -4$

$$y = \begin{cases} \dfrac{y_{max}\, \mu(|x|/x_{max})}{\log_e(1+\mu)} \operatorname{sgn}(x); & \dfrac{\mu|x|}{x_{max}} << 1 \\[2ex] \dfrac{y_{max} \log_e(\mu|x|/x_{max})}{\log_e(1+\mu)} \operatorname{sgn}(x); & \dfrac{\mu|x|}{x_{max}} >> 1 \end{cases}$$

$\log(100)$

$\log(10)$

This shows that μ-law leads to approximately linear charac. at low input levels & approx. logarithmic at high input ".



sgn(x) is to return the polarity of the input

$\mu = 0$ corresponds to uniform quantization - show. C.W./H.W.

A-law- compression (used in Europe)

$$y = \begin{cases} \dfrac{A\,|x|/x_{max}}{1+\log_e A}\,\mathrm{sgn}(x), & 0 < \dfrac{|x|}{x_{max}} < \dfrac{1}{A} \\[2ex] \dfrac{1+\log_e\left[A\,|x|/x_{max}\right]}{1+\log_e A}\,\mathrm{sgn}(x), & \dfrac{1}{A} < \dfrac{|x|}{x_{max}} < 1 \end{cases}$$

Parameters:- $\mu = 255$ for use with 8-bit converter

$A = 87.56$ " "

* An approx. log. compression charac. yields a quant^n noise nearly $\propto$ to the sig. pow. $\widetilde{m^2(t)}$

thus making the SQNR practically indep. of the i/p signal power over a large dynamic range.

At the rx, compressed samples must be restored to their original values at the Rx. by using an expander with a charac. complimentary to that of compressor.

compressor $\longrightarrow$ expander

together called 'compander'

<u>Inverse laws:-</u>

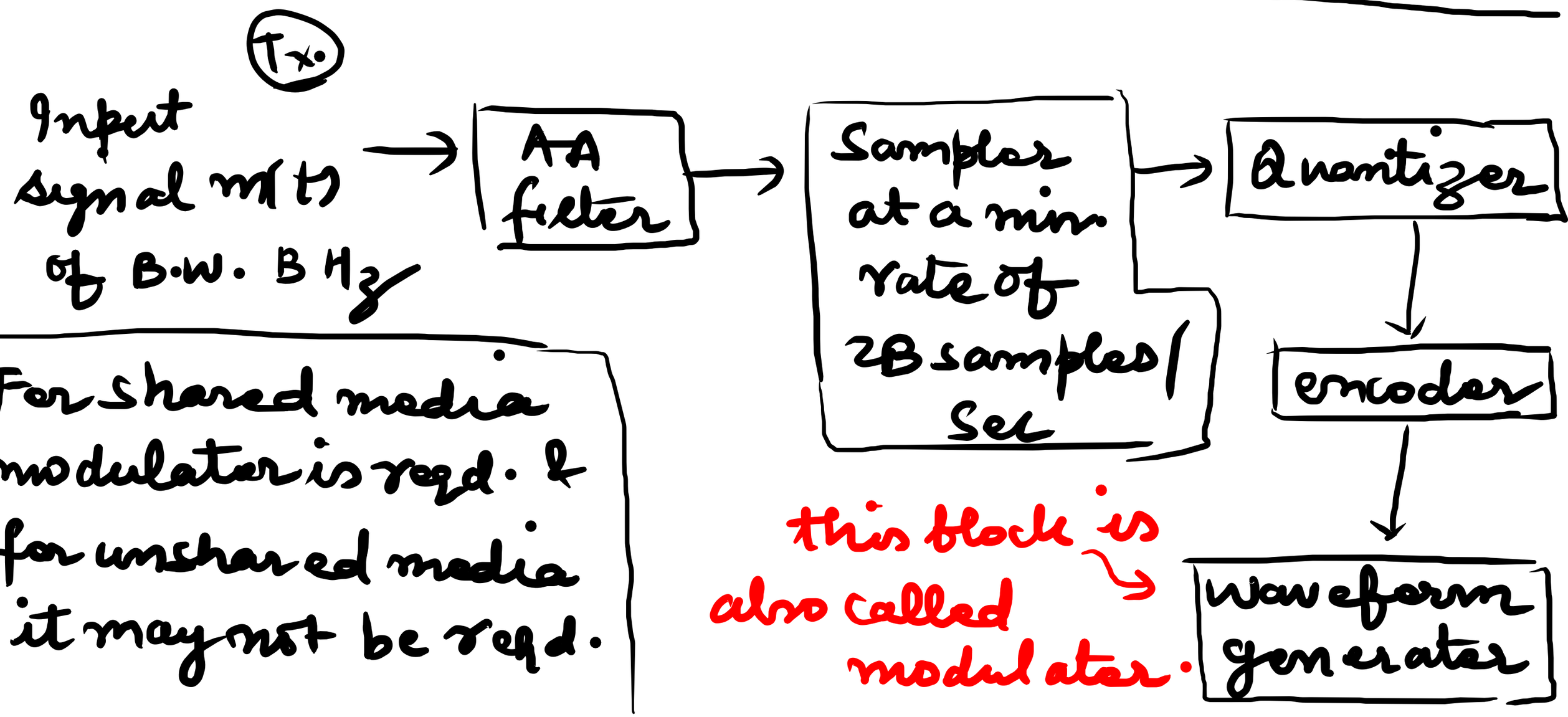
$$F^{-1}(y') = \operatorname{sgn}(y')\left(\frac{1}{\mu}\right)\left[(1+\mu)^{|y'|}-1\right]$$

$y' = y/y_{max}$ is the i/p $\qquad -1 \le y' \le 1$

A-law,

$$F^{-1}(y) = \mathrm{sgn}(y) \begin{cases} \dfrac{|y|\,(1+\ln(A))}{A}, & |y| < \dfrac{1}{1+\ln A} \\[2ex] \dfrac{e^{|y|(1+\ln A)-1}}{A}, & \dfrac{1}{1+\ln A} \le |y| < 1 \end{cases}$$

PCM:- Pulse code modulation



For shared media modulator is reqd. & for unshared media it may not be reqd.

Code :- representing each of the discrete set of values as a particular arrangement of discrete event is called a code. It contains codewords.

101010...

000 1101100

for ex- binary code, ternary code.

→ 16 possible rep. levels & two events {0, 1}
three " {0, 1, 2}

→ Transmission BW. of PCM

Since $L = 2^n$ or $n = \log_2 L$

L ↓ no. of quantizn levels.

n :- no. bits to encode each level

You can read sec. on PCM from Lathi as well as → See the Tx. & Rx. for a PCM system from Haykin

$m(t) \rightarrow$ being BL to B Hz requires a min. of 2B samples/sec.

So, 2nB bits/sec or 2nB pieces of information/sec.

Result:- 1 Hz of BW can at most carry 2 bits of information/sec. (Lathi 6.4.3)

2 bits/sec — 1 Hz

2nB " $\rightarrow$ nB Hz.