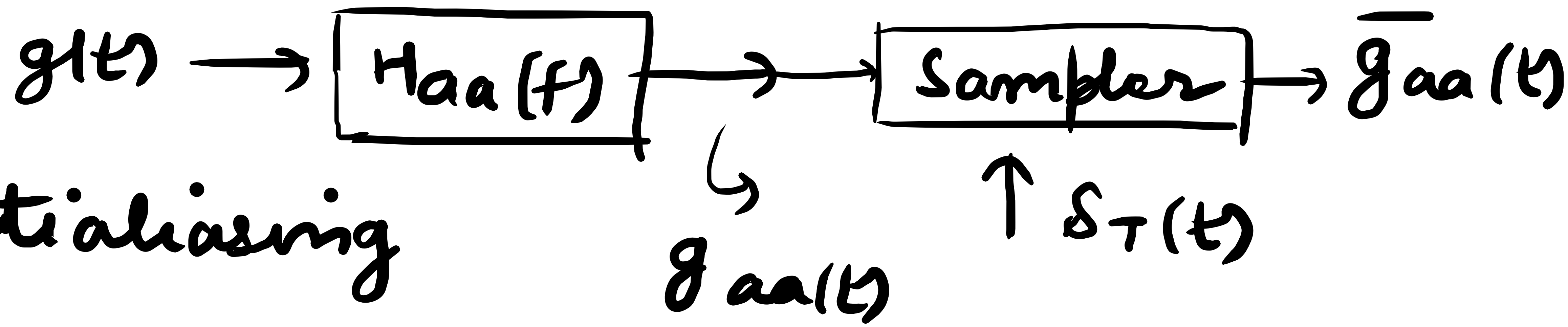
Lec-9, DC, 24_25, SecA

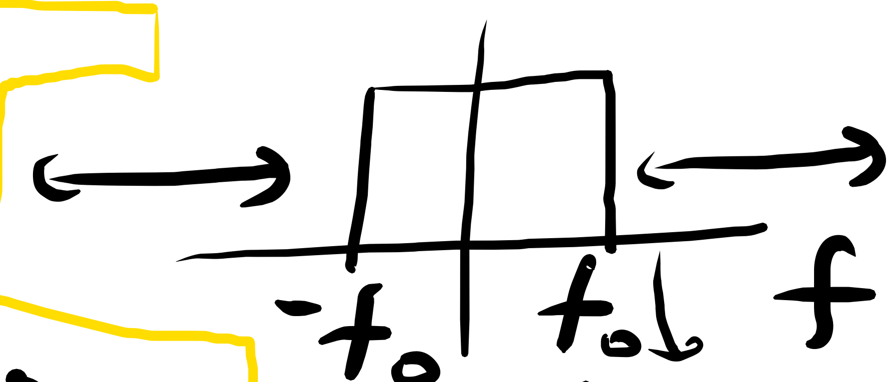
Summarising the solution, we have




aa :- antialiasing

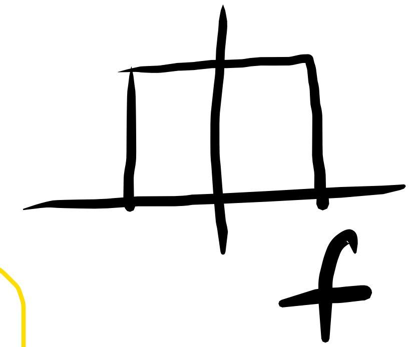
So, our solution relies on ideal filters. Then there is a question :- are ideal filters realizable?

<u>Paley-Wiener</u> :- For a physically realizable system, $H(f)$ may be zero at some discrete frequencies, but it cannot be zero over any finite band.




Strictly zero over a large freq. interval.

One more issue:- Causality.



$\rightarrow \text{sinc}(t)$

so, if $h(t) = \text{sinc}(t)$, it exists for $t<0$, hence non-causal.

Before we move ahead, let detour to another imp. topic.

<u>Distortionless (DL) Tx:-</u>

Tx is said to be DL if the i/p & o/p have identical wave shapes within a multiplicative constant.

$$x(t) \rightarrow \boxed{\phantom{H}} \rightarrow y(t) = a\,x(t)$$

not $a(t)\,x(t)$

Also, a delayed o/p that retains i/p waveform is also considered DL

$$y(t) = k\,x(t-t_d)$$

$$Y(f) = k\,X(f)\,e^{-j2\pi f t_d}$$

$$\Rightarrow \frac{Y(f)}{X(f)} = H(f) = k\,e^{-j2\pi f t_d}$$

So, transfer function (TF) reqd. for DL Tx is

$|H(f)| = k$ $\qquad \therefore\ H(f) = k e^{-j 2\pi f t_d}$

$k > 0$

$\theta_h(f) = -2\pi f t_d$

So, for DL Tx. Amp. res. must be a constant & $\theta_h(f)$ or phase response must be a linear function of f going through origin $(f=0)$

$x(t) = \sin(w_1 t) + \sin(w_2 t) + \sin(w_3 t)$

If $y(t) = k\,x(t-t_d) = k\big[\sin(w_1(t-t_d)) + \sin(w_2(t-t_d)) + \sin(w_3(t-t_d))\big]$

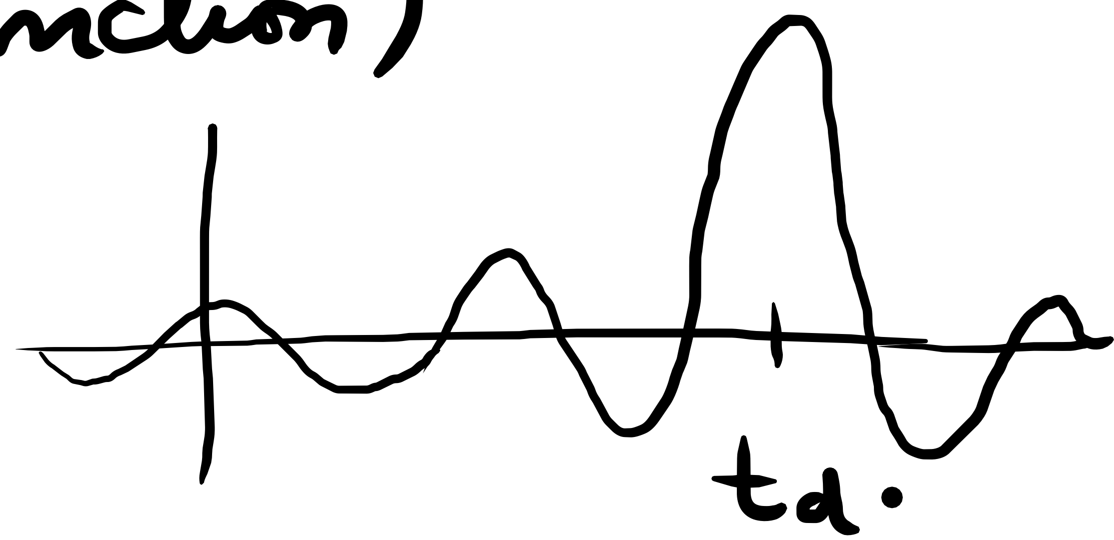
$k\big[\sin(w_1 t - w_1 t_d) + \sin(w_2 t - w_2 t_d) + \sin(w_3 t - w_3 t_d)\big]$

phase shift is a multiple of freq. of sinusoid.

See fig. 3.26(a)&(b) from Lathi & Ding's book for the $H(f)$ & $h(t)$ of an ideal LPF which is DL.

With the help of large $t_d$, we can overcome the non-causality of $h(t)$ by multiplying it with $u(t)$. (unit-step function)

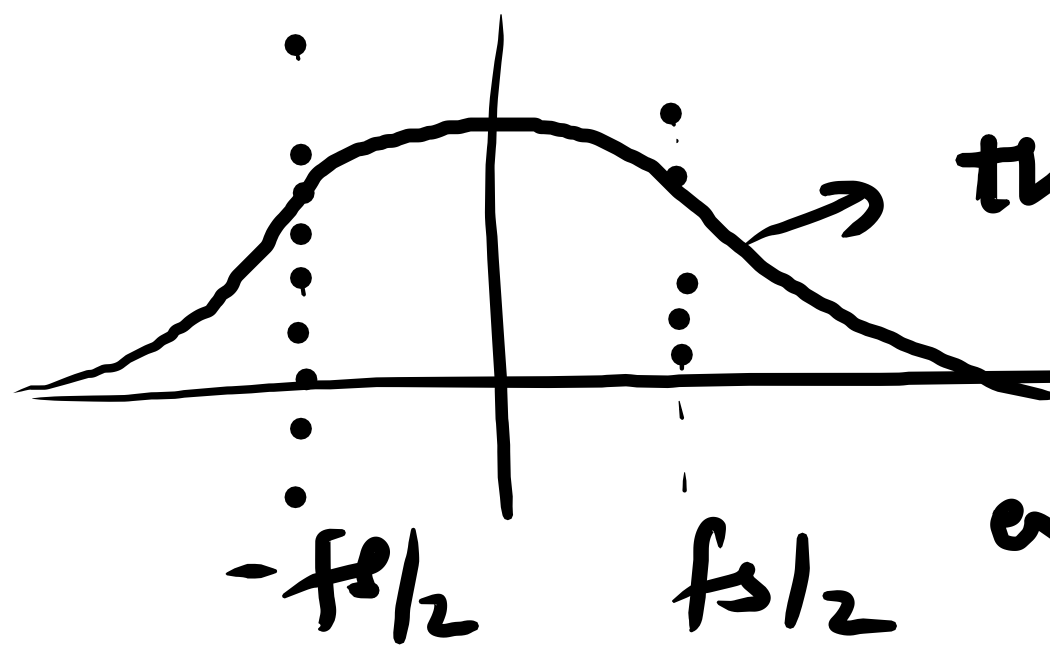
→ keep $t_d$ large & then × with $u(t)$. This will make $h(t)$ causal with limited or no distortion.




→ We can conclude that ideal filters allow DLTx of a certain band of freq. & suppress all the remain.

freq. Then what about Paley-Wiener theorem.

So, we can have filters with no sharp cutoff but gradual & perpetual fall to zero outside band of interest




this fall can be customized.

ex - Chebyshev filters.

**Sampling of band pass signal :-**

A BP signal whose spectrum exists over a freq. band $f_c - B/2 < |f| < f_c + B/2$ has a BW B Hz. Such a signal is also uniquely determined by samples taken at above the Nyquist freq · 2B .