Lec-43, DC, 24-25, SecA

If $\underline{n} = \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix}$ is a collection of i.i.d. R.Vs. what is the joint pdf? i.e.,

$$p(\underline{n}) = p(n_0, n_1, n_2 \ldots n_{N-1}) = \prod_{i=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n_i^2/2\sigma^2}$$

$$= \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\sum_{i=0}^{N-1} n_i^2/2\sigma^2}$$

$-\infty < n_0 < \infty$
$-\infty < n_1 < \infty$
$\vdots$
$-\infty < n_{N-1} < \infty$

$n_i \sim \mathcal{N}(0, \sigma^2)$

$E[n_i n_j] = 0, \; i \neq j$

$$p(n_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n_i^2/2\sigma^2}$$

$-\infty < n_i < \infty$

X & Y are independent hence, $p(X, Y) = p(X)\,p(Y)$

The hypotheses testing framework at Rx is reduced

to $\quad y(t) = s_i(t) + n(t) \; ; \; i = 0, 1, 2, \ldots, M-1 \qquad 0 \leq t \leq T_s$

$$\equiv$$

$$\underline{y} = \underline{s_i} + \underline{n} \quad , \; i = 0, 1, 2, \ldots, M-1$$

$$\underline{y} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix}, \quad \underline{s_i} = \begin{bmatrix} s_{i0} \\ s_{i1} \\ \vdots \\ s_{iN-1} \end{bmatrix}, \quad \underline{n} = \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

ML receiver/rule, $\quad p(\underline{y} \mid \underline{s_i}) \quad , \; i = 0, 1, 2, \ldots, M-1$

$$s(\underline{y}) = \arg\max_{i} \; p(\underline{y} \mid \underline{s_i})$$

$\underline{y} = \underline{s}_i + \underline{n}$ $\underline{y}|s_i \rightarrow$ implies that $\underline{s}_i$ is a deterministic vector quantity

$p(\underline{n}) = \mathcal{N}(\underline{0}, \sigma^2 I)$ $p(\underline{y}|\underline{s}_i) = \mathcal{N}(\underline{s}_i, \sigma^2 I)$

$\underline{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{N\times 1}$ $I = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}_{N\times N}$

$\hookrightarrow \dfrac{1}{(2\pi\sigma^2)^{N/2}} e^{-\sum_{j=0}^{N-1} (y_j - s_{ij})^2 / 2\sigma^2}$ — ③

$\dfrac{1}{(2\pi\sigma^2)^{N/2}} e^{-\sum_{i=0}^{N-1} n_i^2 / 2\sigma^2}$

$\sum_{j=0}^{N-1} (y_j - s_{ij})^2 \overset{?}{=} \|\underline{y} - \underline{s}_i\|_2^2$

$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \underline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ $\underline{a} - \underline{b} = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \end{bmatrix}, \|\underline{a} - \underline{b}\|_2 = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

$$\|\underline{a}-\underline{b}\|_2^2 = (a_1-b_1)^2+(a_2-b_2)^2 = \sum_{i=1}^{2}(a_i-b_i)^2 = (\underline{a}-\underline{b})^T(\underline{a}-\underline{b})$$

expression in 3 $\equiv$

$$p(\underline{y}|\underline{s}_i) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\|\underline{y}-s_i\|^2/2\sigma^2}$$

$$\begin{array}{cccc} & 1 & 2 & 3 \\ A = \{ & -2, & -3, & -5 \} \end{array}$$

$$\arg\min_i A$$

$$\arg\max_i -A$$

$$s(\underline{y}) = \arg\max_i -\|\underline{y}-s_i\|^2 =$$

$$\arg\min_i \|\underline{y}-\underline{s}_i\|^2 = \arg\min_i \underline{s}_i^T\underline{s}_i - 2\underline{y}^T\underline{s}_i$$

$$= -2\left[\underline{y}^T\underline{s}_i - \|s_i\|^2/2\right]$$

$$\|\underline{y}-\underline{s}_i\|^2 = (\underline{y}-\underline{s}_i)^T(\underline{y}-\underline{s}_i)$$

$$= (\underline{y}^T-\underline{s}_i^T)(\underline{y}-\underline{s}_i) = \underline{y}^T\underline{y} - \underline{y}^T\underline{s}_i - \underline{s}_i^T\underline{y} + s_i^T s_i$$

$$s_{ML}(\underline{y}) = \arg\max_i \underbrace{\langle \underline{y}, \underline{s}_i \rangle}_{\hookrightarrow\, \underline{y}^T \underline{s}_i} - \|\underline{s}_i\|^2/2$$

MAP rule:– $\arg\max_i \; p(\underline{s}_i \mid \underline{y})$

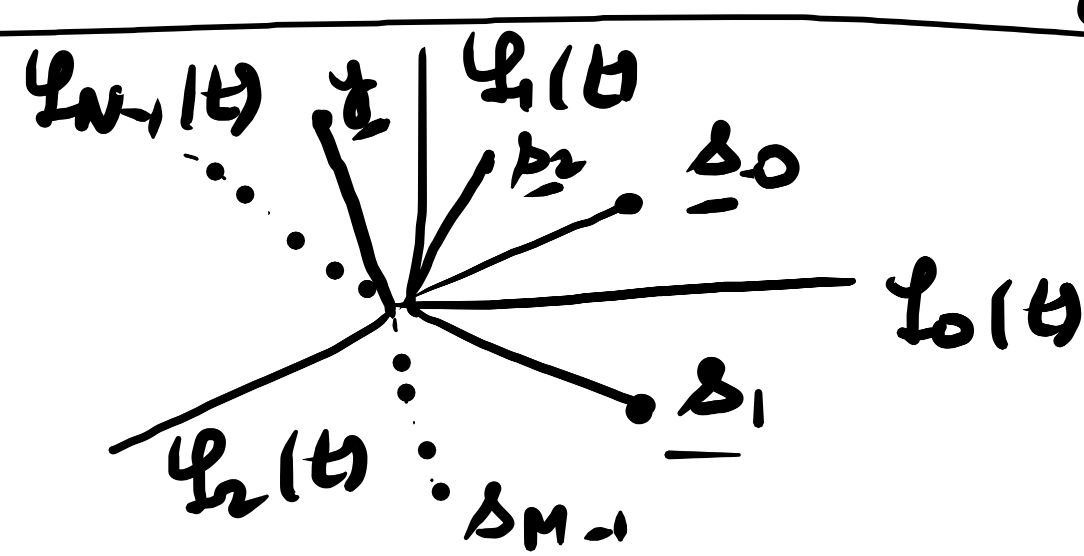
$$s_{MAP}(\underline{y}) = \arg\min_i \left(\|\underline{y} - \underline{s}_i\|^2 - 2\sigma^2 \log \pi_i \right)$$

These two steps are H.W.

$$= \arg\max_i \langle \underline{y}, \underline{s}_i \rangle - \|\underline{s}_i\|^2/2 + \sigma^2 \log \pi_i$$

In the signal space;

$$s_i(t) \equiv \underline{s}_i$$



In $\mathbb{R}^2$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, what is the dist b/w two.

$$\sqrt{(x-y)^T(x-y)} = \|x-y\|$$

$$= \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$$

ML rule, $\delta_{ML}(\underline{y})$ is a min. sq. of distance rule

but MAP rule, takes min of dist sq. after subtracting $2\sigma^2 \log \pi_i$

See figs. 5.9 (a) & (b)

#1 from Haykin's T.B for the diagram of ML receiver.

#2 Also, draw the diag. of MAP receiver

#1, #2 :- Imp. for the end-sem exam.