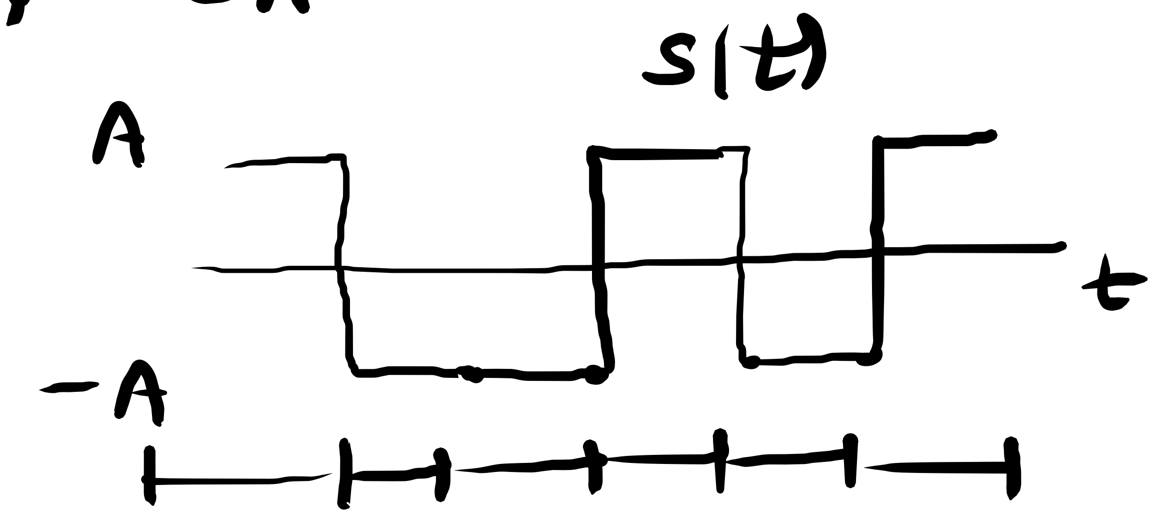
Lec-35, DC, 24-25, Sec A

$s(t) \rightarrow$ PCM wave. ex- 100101



bit interval. (T_b)

Suppose symbol '0' is sent,

then $\quad x(t) = -A + w(t)$

$$y = A\int_0^{T_b} x(t)\,dt = A\int_0^{T_b} [-A + w(t)]\,dt = -A^2 T_b + A\int_0^{T_b} w(t)\,dt$$

$y(t) = x(t) * M\cdot F$

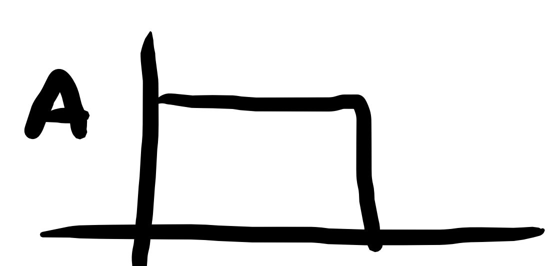
$y(T_b) = \quad x(t) * M\cdot F \Big|_{t=T_b}$

$\quad = x(t) * g_{MF}(t) \Big|_{t=T_b}$

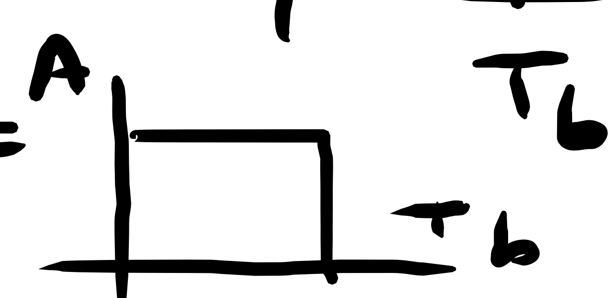
$g_{MF}(t) = k\, g(T_b - t)$.

M.F. corr. to

$g(t) = $



$g(T_b - t) = $



$$y(t) = x(t) * g_{MF}(t) = \int_{-\infty}^{\infty} x(\tau)\, g_{MF}(t-\tau)\, d\tau$$

$$= k \int_{-\infty}^{\infty} x(\tau)\, g(T_b - t + \tau)\, d\tau .$$

$$x(\tau) = -A + w(\tau)$$

$$0 \le \tau \le T_b$$

$$y(T_b) = k \int_{-\infty}^{\infty} x(\tau)\, g(\tau)\, d\tau$$

$$g(\tau) = A$$

$$0 \le \tau \le T_b$$

$$= k \int_{0}^{T_b} A(-A + w(\tau))\, d\tau = k\left(-A^2 T_b + A \int_{0}^{T_b} w(\tau)\, d\tau\right)$$

Assume $kAT_b = 1$,

$$y(T_b) \triangleq y = -A + \frac{1}{T_b} \underbrace{\int_{0}^{T_b} w(\tau)\, d\tau}_{\text{Random Variable}}$$

y is Gaussian distributed with a mean of $-A$ & variance $\rightarrow$

You have assumed $W(t)$ as a zero mean R.P. with PSD $N_0/2$

ex- $x \sim \mathcal{N}(-3,4)$

$y = 2x+9$

$y \sim \mathcal{N}(3,16)$

$$y+A = \int_0^{T_b} w(\tau)\,d\tau$$

$$\sigma_y^2 = E[(y-\mu_y)^2]$$

$$= E[(y+A)^2]$$

$$= \frac{1}{T_b^2} E\left[\int_0^{T_b}\int_0^{T_b} w(t)w(u)\,dt\,du\right]$$

$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} E[w(t)w(u)]\,dt\,du$$

$W(t) \rightarrow$ Gaussian Random process.

$W(t_1) \rightarrow$ Gaussian Random variable.

$$\int_{t_1}^{t_2} w(t)\,dt$$

$W(t_1), W(t_1+\Delta), W(t_1+2\Delta)\ldots$

there are jointly Gaussian RVs.

Adding $\rightarrow$ Gaussian RV.

$$R_W(t,u) = E[W(t)W(u)] = \frac{N_0}{2}\delta(t-u)$$

$$\sigma_y^2 = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b}\delta(t-u)\,dt\,du \cdot \frac{N_0}{2}$$

$$= \frac{N_0}{2T_b} \quad (\text{show})$$

$$\frac{1}{T_b^2}\int_0^{T_b}\left[\underbrace{\int_0^{T_b}\delta(t-u)\,dt}_{1}\right]du$$

(as $w \in [0, T_b]$)

$$= \frac{T_b}{T_b^2} = \frac{1}{T_b}. \quad y \sim \mathcal{N}\left(-A, \frac{N_0}{2T_b}\right)$$

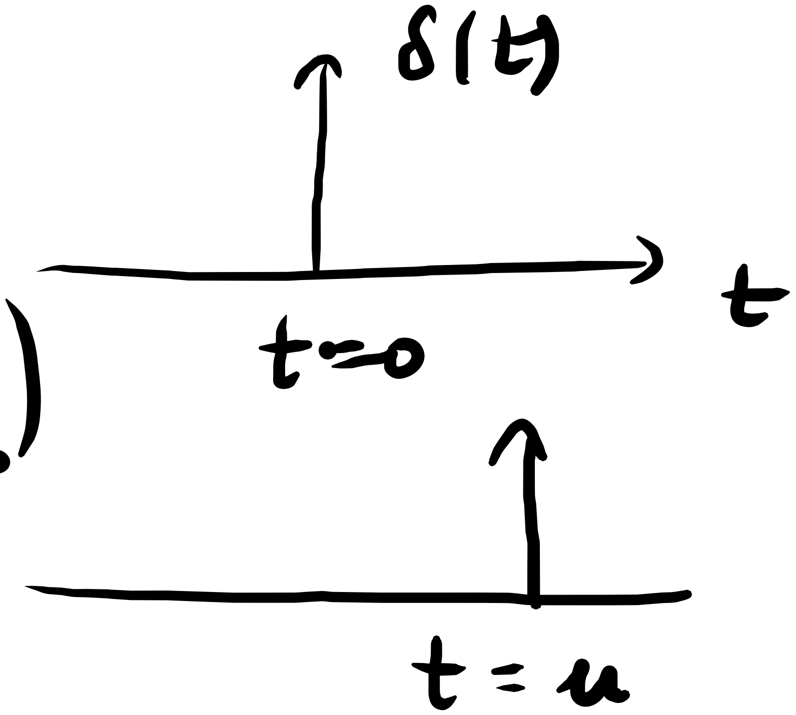
$$f_Y(y|0) = \frac{1}{\sqrt{\pi N_0/T_b}}\, e^{-(y+A)^2/(N_0/T_b)}$$

White process.

$$R_W(\tau) = \delta(\tau)$$

$$E[W(t)W(t+\tau)] = R_W(\tau)$$
$$= \delta(\tau)\frac{N_0}{2}$$

$$E[W(t_1)W(t_2)] = (?)$$
$$\delta(t_1-t_2)\frac{N_0}{2}$$



$f_{Y|}(y|0)$ is the conditional PDF of the RV Y, given '0' is sent.

$x \sim \mathcal{N}(\mu, \sigma^2)$

$$P_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

$$-\infty < x < \infty$$

Decision logic:- $y \underset{0}{\overset{1}{\gtrless}} \lambda$

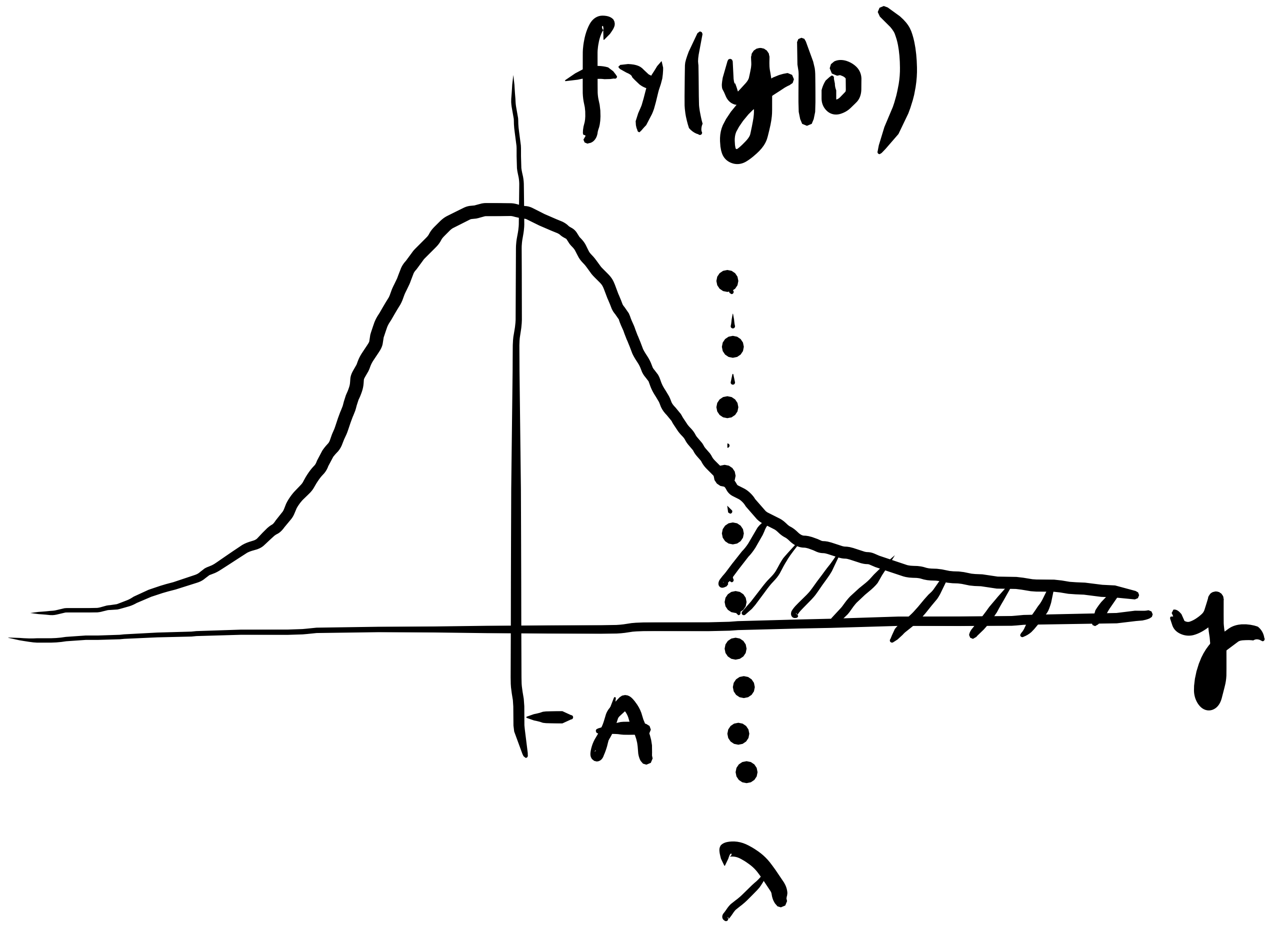
$y = \lambda \rightarrow$ receiver just guesses b/w 0 & 1: such a decision is the same as that obtained by flipping a coin.

types of error:- E_1: 0 Txd $\rightarrow$ 1 detected | chosen

E_2: 1 Txd $\rightarrow$ 0 " | "

$\text{Prob}(E_1) \triangleq P_{10}$, $1 \to T$, $0 \to R|D$ $\qquad$ $\text{Prob}(E_2) \triangleq P_{01}$, $0 \to D|R$, $1 \to T$

$f_Y(y|0)$



$P_{10} \rightarrow$ conditional prob. of error given that symbol '0' was sent.

$$P_{10} = P(y > \lambda \mid \text{'0' is sent})$$

$$= \int_{\lambda}^{\infty} f_Y(y|0)\, dy.$$