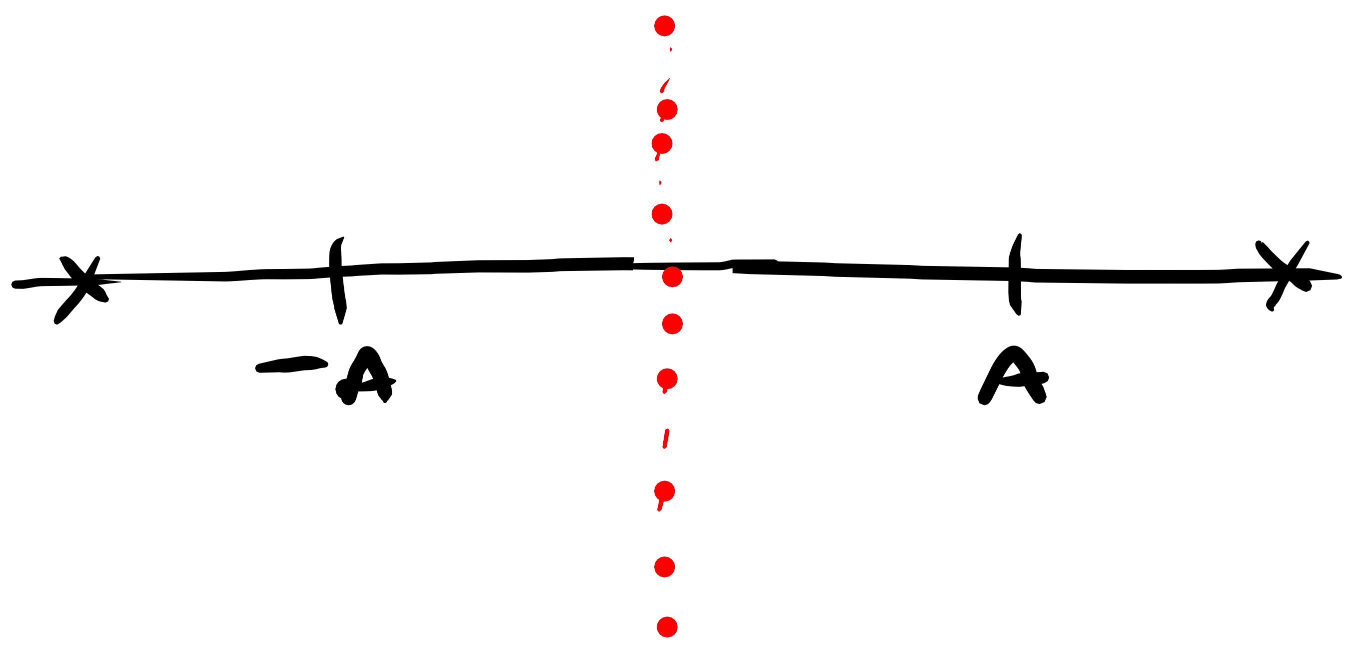
Lec-44, DC, 24-25, Sec A

ML decision rule, since is a min. dist decoding rule, the decision boundaries in the signal space are the midpoints of line joining the pts.



see the decision regions or boundaries from figs. 6.14, 6.15 & 6.16 from the online copy of Madhow's book.

Signal space for pass-band linear modulation

$$s_{b_c,b_s}(t) = b_c\, p(t)\cos(2\pi f_c t) - b_s\, p(t)\sin(2\pi f_c t)$$

$$\varphi_c(t) = p(t)\cos(2\pi f_c t) \quad , \quad \psi_c(t) = \frac{\varphi_c(t)}{\|\varphi_c(t)\|} = \frac{p(t)\cos(2\pi f_c t)}{\|p\|/\sqrt{2}}$$

$$\varphi_s(t) = -p(t)\sin(2\pi f_c t) \; ; \longrightarrow \psi_s(t) = \frac{\varphi_s(t)}{\|\varphi_s(t)\|} = \frac{-p(t)\sin(2\pi f_c t)}{\|p\|/\sqrt{2}}$$

$$s_{b_c,b_s}(t) = b_c\,\varphi_c(t) + b_s\,\varphi_s(t)$$

$$\equiv \begin{bmatrix} b_c \\ b_s \end{bmatrix}$$

$$s_{b_c,b_s}(t) = \frac{\|p\|}{\sqrt{2}} b_c\,\psi_c(t) + \frac{\|p\|}{\sqrt{2}} b_s\,\psi_s(t) = \frac{\|p\|}{\sqrt{2}}\begin{pmatrix} b_c \\ b_s \end{pmatrix}$$

→ M-ary communication vs. binary comm.

Baseband

↳ ex- antipodal signalling

On-off "

BPSK - pass band

0101110 0111 binary Tx $p(t)$

0101110 0111 $M=4$

... 4-ary Tx. $p(t)$

Bandwidth efficiency:- ratio of the data rate in bits/sec to the 'effectively' utilized channel B.W.

(ρ)

(bps) R_b :- data rate, utilized B.W. = B (Hz)

$$\rho = \frac{R_b}{B} \text{ bits/sec/Hz.}$$

In case R_b is fixed & known. $T_b = 1/R_b$

In M-ary case, you combine $\log_2 M$ bits together

$$T_S = T_b \cdot \log_2 M \quad , \quad R_S = \frac{1}{T_S} = \frac{R_b}{\log_2 M}$$

Now, for Tx. Infor. at rate of R, mini. B.W. req. is $R/2$. If binary Tx, mini. B.W = $R_b/2$

If M-ary Tx, min B.W = $R_S/2 = \frac{1}{2T_S} = \frac{1}{2T_b \log_2 M} = \frac{R_b}{2\log_2 M}$

$$\rho = \frac{R_b}{R_b} 2\log_2 M = 2\log_2 M \text{ bits/s/Hz.}$$

Another way, band width is given & fixed. say B, then the data rate $= 2B$, i.e., $R_s = 2B$

$$\rho = \frac{R_b}{B} = 2\log_2 M.$$

$$\frac{R_b}{\log_2 M} = 2B$$

$$R_b = 2B\log_2 M$$

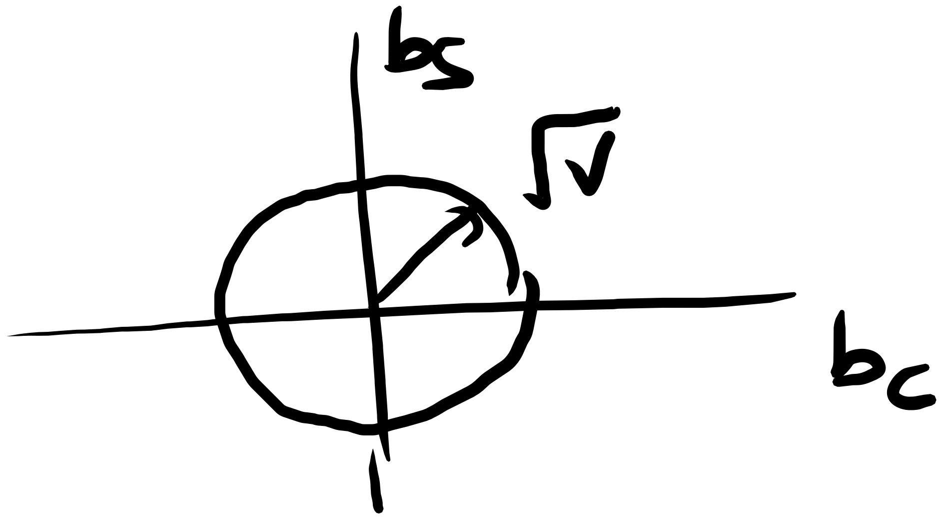
The results show $\rho \uparrow$ with $\uparrow M$,

but what about the error prob. at Rx.?

With the power of Tx, fixed, the constellation points are packed (more packed) in a space as $M\uparrow$, leading to higher & higher error prob.

$s_{bc,bs}(t) \equiv \frac{\|p\|}{\sqrt{2}}\begin{pmatrix} bc \\ bs \end{pmatrix}$, as $\frac{\|p\|}{\sqrt{2}}$ is a constant, we can say energy of constell :- $bc^2 + bs^2$

Fix it (suppose) $= V$, $bc^2 + bs^2 = V$



then ↑ M will lead to more & more points lying on the circle, leading to narrower decision regions & more P_e.

but yes, with ↑ M, you can ↑ V & this way P_e can be ↓.

This is called power-bandwidth tradeoff. i.e; ↓ B req. ↑ power for same P_e.