Lec-41, DC, 24_25, Sec A

Under each of the hypotheses, you can recover the same $y(t)$.

$$
\left.\begin{aligned}
bc, bs = (1,1)\ (1) \quad & s_{1}(t) + n_1(t) = y(t) \\
(1,-1)\ (2) \quad & s_{2}(t) + n_2(t) = y(t) \\
(-1,-1)\ (3) \quad & s_{3}(t) + n_3(t) = y(t) \\
(-1,1)\ (4) \quad & s_{4}(t) + n_4(t) = y(t)
\end{aligned}\right\} \quad p(y(t)|(1))
$$

The pairs (bc, bs) are obtained from the binary waveform (discrete)

$\underline{00}\,\underline{11}\,\underline{10}\,\underline{11}\,\underline{10}\,\underline{00}\,\underline{00}\,\underline{10}\,\underline{\tilde{0}1}$

There are 4 prob.

(1)	00	P_1
(2)	01	P_2
(3)	10	P_3
(4)	11	P_4

$M = 2$

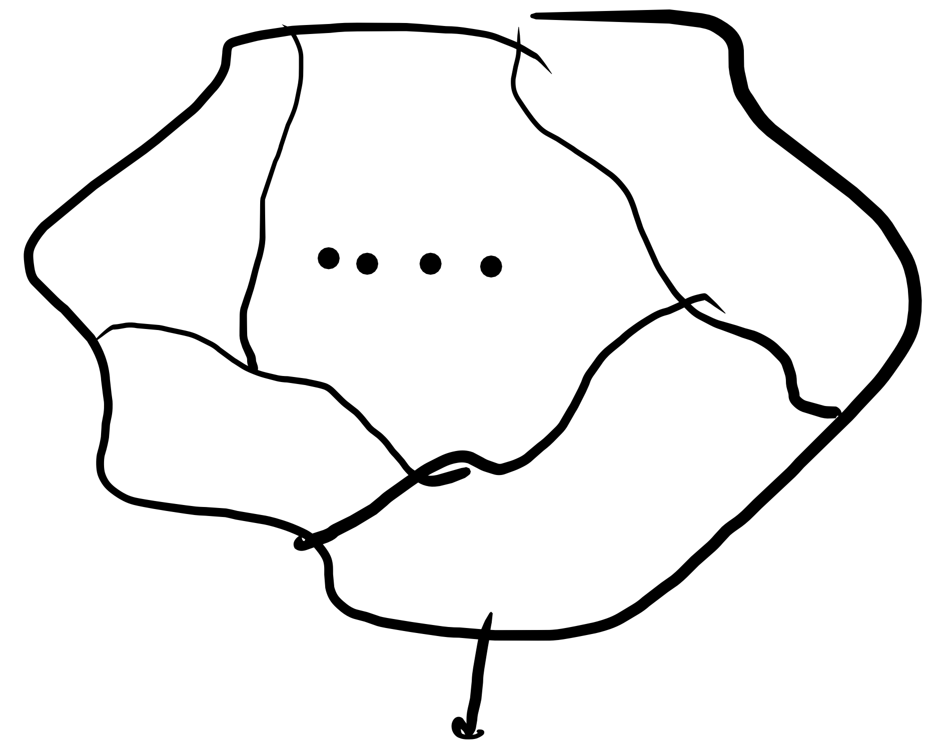
$P_i \rightarrow$ prior prob. of hypotheses.

$\delta(y(t)) \rightarrow \{1,2,\dots M\}$ or $\{0,1,2\dots,M-1\}$

Input

$y(t) \in \Gamma$

Γ



What is Γ_i → If $y(t) \in \Gamma_i$, you say H_i is true

M-disjoint regions such that their union is Γ

$\Gamma_1 \cup \Gamma_2 \dots \cup \Gamma_M = \Gamma$

Continuing ex 6.1.1,

$H_0 = 4/5$

$H_1 = 1/5$

ans:- $y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{4}{3} \ln 16 = 3.6968$

average error prob.

$$P(H_0 \rightarrow \text{true})\, P(e \mid H_0) + P(H_1 \rightarrow \text{true})\, P(e \mid H_1\, \text{true})$$

$$P(\text{error}) = P(H_0 \rightarrow \text{true}, \text{error} \cup H_1 \rightarrow \text{true}, \text{error})$$

$$\text{or} \quad P(\underbrace{H_0 \rightarrow \text{true} \cap \text{error}}_{E_1} \cup \underbrace{H_1 \rightarrow \text{true} \cap \text{error}}_{E_2})$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(\underbrace{E_1 \cap E_2}_{\phi})$$

$$= P(E_1) + P(E_2) = P(H_0 \rightarrow \text{true})\, P(e \mid H_0) + P(H_1 \rightarrow \text{true})\, P(e \mid H_1)$$

In case of binary hypotheses testing, we can define the likelihood ratio.

(ML)

$$P(y|1) \underset{H_0}{\overset{H_1}{\gtrless}} P(y|0) \Rightarrow \frac{P(y|1)}{P(y|0)} \underset{H_0}{\overset{H_1}{\gtrless}} 1$$

$$\triangleq L(y)$$

LRT (likelihood ratio test) – in general form

$$L(y) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

ML: $\gamma = 1$

MAP: $\gamma = \pi_0/\pi_1$

In case of MAP,

$$\pi_1 P(y|1) \underset{H_0}{\overset{H_1}{\gtrless}} \pi_0 P(y|0) \Rightarrow L(y) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\pi_0}{\pi_1} = \gamma$$

ex (solved) $H_0: Y \sim \mathcal{N}(0, v^2)$, $H_1: Y \sim \mathcal{N}(m, v^2)$

let π_0 & π_1 be the prior prob.

find ML & MAP rules using LRT

$\gamma = 1$ $\quad \gamma = \frac{\pi_0}{\pi_1}$

$$L(y) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

$\Rightarrow$?

$$\log_e L(y) \underset{H_0}{\overset{H_1}{\gtrless}} \log_e \gamma$$

$$p(y|0) = \frac{e^{-y^2/2v^2}}{\sqrt{2\pi v^2}}, \quad -\infty \le y < \infty$$

$$p(y|1) = \frac{e^{-(y-m)^2/2v^2}}{\sqrt{2\pi v^2}} \quad ''$$

H.W. derive for $m<0$.

$$y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{m}{2} \quad \text{ML}, (m>0)$$

MAP: $$y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{m}{2} + \frac{v^2}{m} \log\left(\frac{\pi_0}{\pi_1}\right) \quad m>0$$

Practice problem :- ex - 6.1.2

So, we have seen hypotheses testing when Y is a Random variable. What if it is a random process?