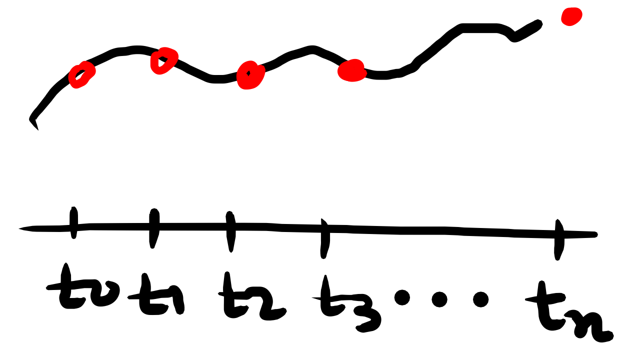
Lec-5, CT303, 24-25, SecA

With instantaneous sampling,

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s) \quad -③$$

sampled signal

$g(t)$

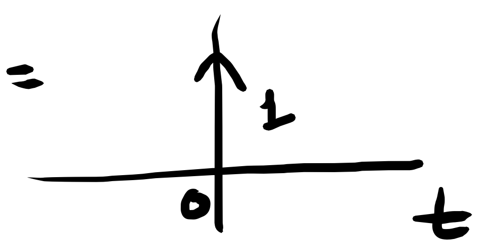


$t_n - t_{n-1} = T_s \quad \forall n$

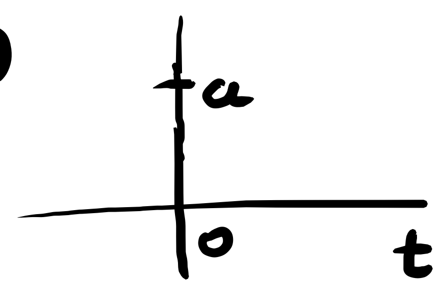
$g(t) \longrightarrow g_\delta(t)$

↳ due to instantaneous sampling.

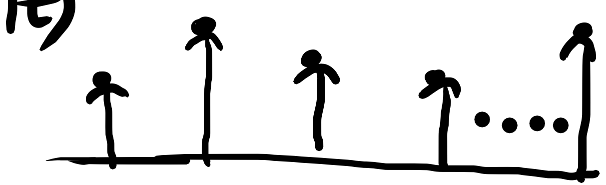
$\delta(t) =$



$a\,\delta(t)$



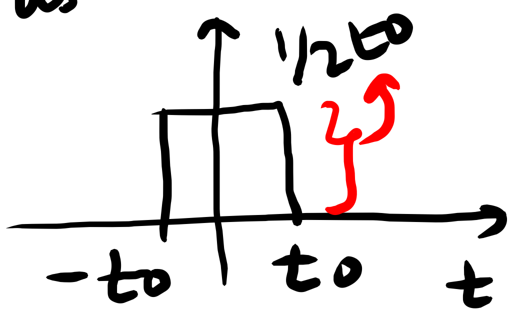
$g_\delta(t)$

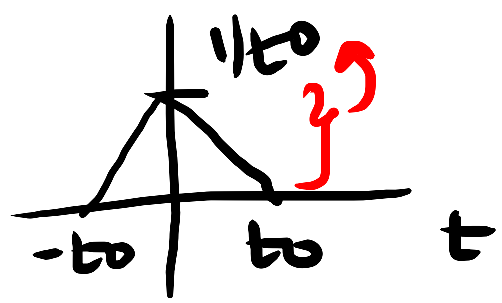


$\delta(t)$

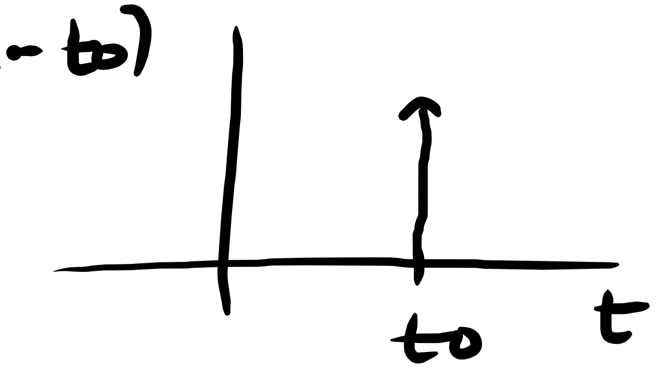
Dirac-Delta function

as $t_0 \to 0$





$\delta(t-t_0)$



This is like weighting (individually) the elements of a periodic seq. of delta functions spaced T_s sec. apart by seq. of numbers $\{g(nT_s)\}$

$g_\delta(t)$:— 'Ideal' sampled signal (SS)

Insta. sampling is leading to an ideal SS

$$g(t)\,\delta(t-nt_0) = \begin{cases} g(nT_0)\,\delta(t-nT_0) & ✓ \\ g(nT_0) & ✗ \end{cases}$$

We should know that $g(nT_0)$ exists only for $t=nT_0$ & not for all t.

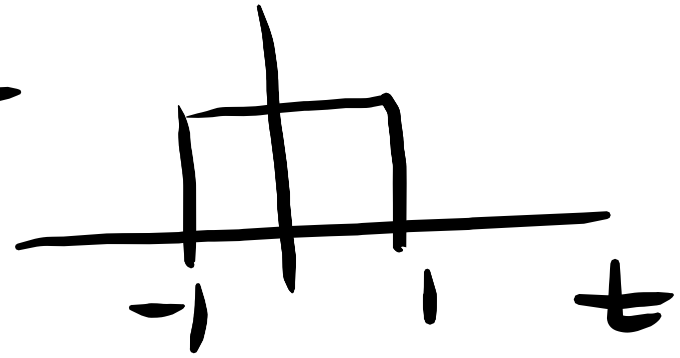
$$\int_{-\infty}^{\infty} g(t)\,\delta(t-nT_0)\,dt = g(nT_0) \quad \text{[sifting prop.]}$$

Now, from Signals & Systems we know

$$g_\delta(t) \xrightarrow{FT} f_s \sum_{m=-\infty}^{\infty} G(f-mf_s) \quad -②$$

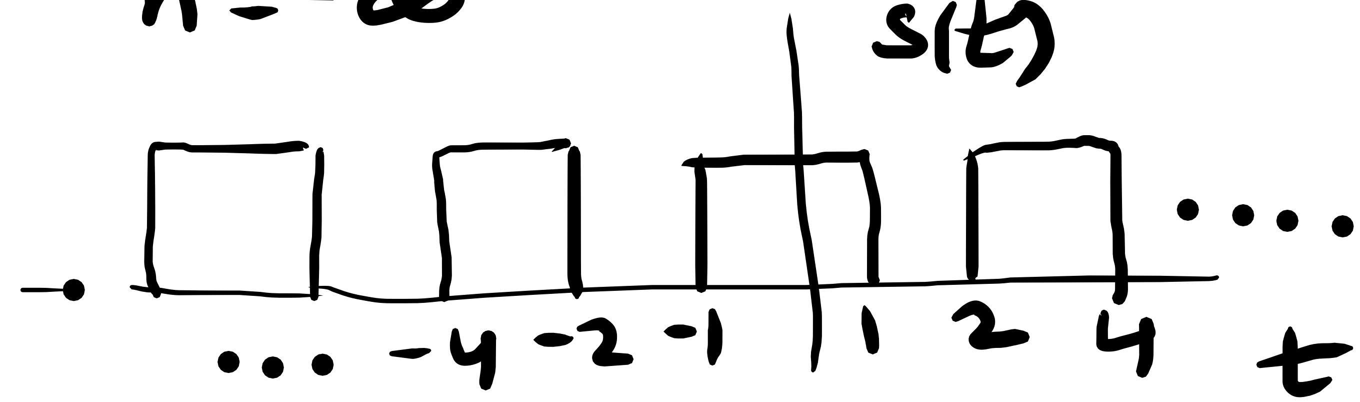
where $g(t) \xrightarrow{FT} G(f)$

$m(t) =$



$$s(t) = \sum_{n=-\infty}^{\infty} m(t-3n)$$

draw $s(t)$



$$\because \sum_{i=-\infty}^{\infty} \delta(t - iT_0) \xrightarrow{FT} \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$$

① ↑

periodic impulse train in time

($1/T_0$ is the FS coefft. of periodic impulse train in ①)

(FT of periodic signal is in terms of FS coefft.) see Oppenheim & Wilsky SS)

how $g_\delta(t)$'s FT is given by eq. ②.

Ans.– $g_\delta(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

FT of $g_\delta(t)$ = $G(f) * FT\{$periodic impulse train in time$\}$

$$= G(f) * \frac{1}{T_s} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_s}\right) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} G\left(f - \frac{m}{T_s}\right)$$

$$G(f) * \delta\left(f - \frac{m}{T_s}\right) = G\left(f - \frac{m}{T_s}\right) \qquad \frac{1}{T_s} = f_s$$

$$\text{FT of } g_\delta(t) = f_s \sum_{m} G(f - m f_s)$$

Another way, take FT of both sides in (3)

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j 2\pi n f T_s} \quad - (4)$$

$$FT\{\delta(t - nT_s)\} = e^{\underbrace{-j 2\pi n f T_s}_{\text{function of } f.}} \qquad FT\{\delta(t)\} = 1$$

The expression in (4) is called DTFT (Discrete time Fourier transform). It may be viewed as a complex Fourier series representation of the periodic freq. func^n $G_\delta(f)$ with seq. of samples $\{g(nT_s)\}$ defining the coef. of exp.

$$G_\delta(f+f_s) \overset{?}{=} G_\delta(f)$$

$$G_\delta(f+f_s) = \sum_n g(nT_s)\, \underbrace{e^{-j2\pi n(f+f_s)T_s}}_{e^{-j2\pi n f T_s}\ \underbrace{e^{-j2\pi n \overbrace{f_s T_s}^{1}}}_{e^{-j2\pi n}}}$$

$$\underbrace{\cos(2\pi n)}_{1} - j\,\underbrace{\sin}_{0}(2\pi n)$$