Lec-33, DC, 24-25, Sec A

Problem :- find $H(f)$ to maximize η for a given $G(f)$.

We know from Schwarz Inequality that

$$\left| \int_{-\infty}^{\infty} \phi_1(x)\phi_2(x)\,dx \right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx$$

with equality iff $\phi_1(x) = k\,\phi_2^*(x)$

$k \hookrightarrow$ arbitrary constant

Assume $\phi_1(x) = H(f)$, $x \rightarrow f$

$\phi_2(x) = G(f)\, e^{j2\pi f T}$, $x \rightarrow f$

$$\left| \int_{-\infty}^{\infty} H(f)\,G(f)\,e^{j2\pi f T} df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \times \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$\eta \le \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df \quad (\text{U.B. on } \eta)$$

η is maximized if $H(f)$ is chosen so that

$$\eta = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df \text{ holds.}$$

$$\triangleq \eta_{max} \qquad \Rightarrow \quad H_{opt}(f) = k\, G^*(f)\, e^{-j2\pi f T}$$

$$h_{opt}(t) = k \int_{-\infty}^{\infty} G^*(f)\, e^{-j2\pi f T}\, e^{j2\pi f t}\, df$$

$$= k \int_{-\infty}^{\infty} G^*(f)\, e^{-j2\pi f(T-t)}\, df = k \int_{-\infty}^{\infty} G(-f)\, e^{-j2\pi f(T-t)}\, df$$

$$= k\, g(T-t)$$

For a real valued signal $g(t)$, $G(f) = G^*(-f)$

$\hookrightarrow$ pulse signal

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$$

8 → -5

exp. given is $$\int_{-\infty}^{\infty} G(-f)\, e^{-j2\pi f(T-t)}\, df$$

$f \to -u$

$df \to -du$

$$-\int_{\infty}^{-\infty} G(u)\, e^{j2\pi u(T-t)}\, du \rightarrow \int_{-\infty}^{\infty} G(u)\, e^{j2\pi u(T-t)}\, du = g(t')\Big|_{t'=T-t}$$

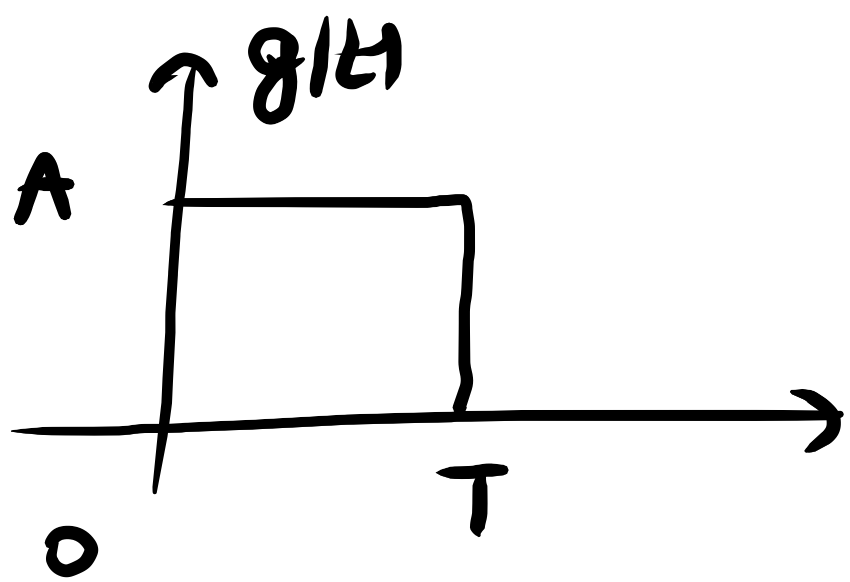
so, $h_{opt}(t) = k\, g(T-t)$

Impulse response of the optimum filter, except for scaling factor k, is a time reversed & delayed version of the i/p sig. $g(t)$ i.e., it is matched to the I/p signal. An LTI filter defined in this way is called a MF.

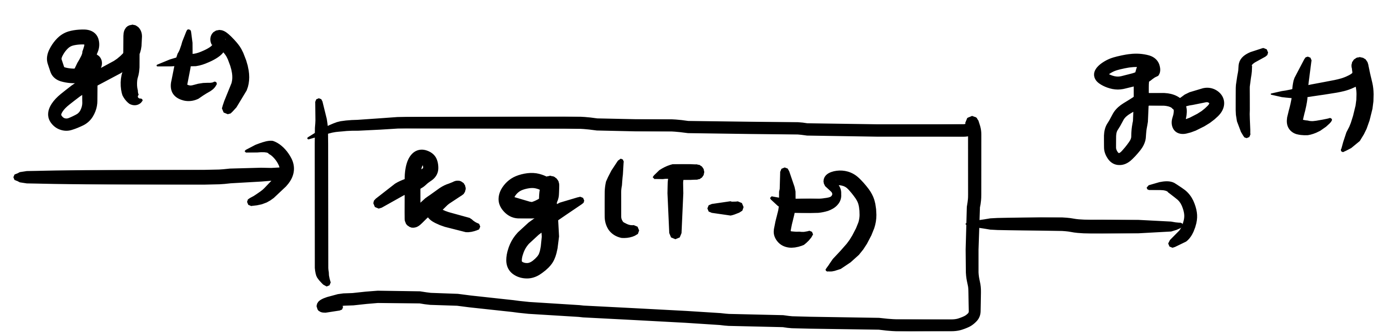
\# no assumption on the statistics of the channel noise $w(t)$, only stationary, white, zeromean & PSD $N_0/2$

→ not necessarily Gaussian →

→ ex.



find o/p from the matched filter $kg(T-t)$.



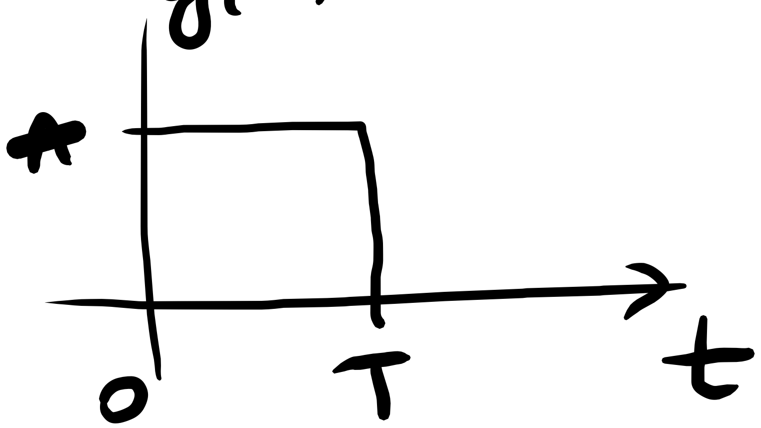
$g(T-t)$

$g(-t) = x(t)$

$g(-(t-T))$

$g(-t+T)$

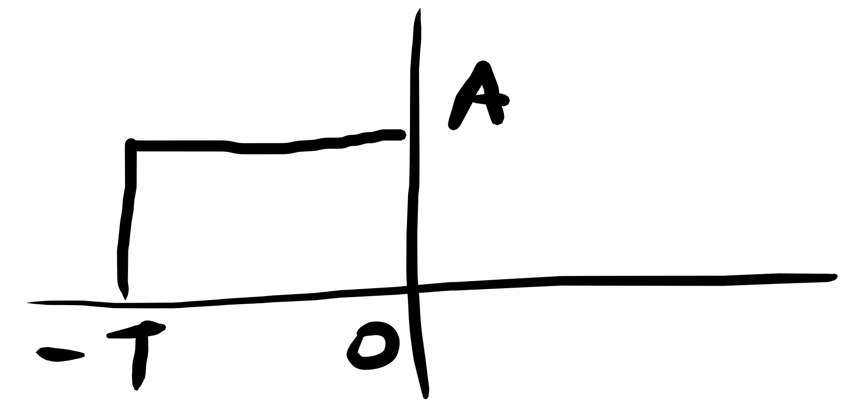
$g(t)$



draw $g(T-t)$

(in next class)

$g(t+T)$



$g(-t+T)$

or $g(-(t+T))$