Lec-7, DC, 24_25, Sec A

<u>Sampling theorem</u>:- For strictly BL signals of finite energy.

1. Such a func^n/signal is completely described by values separated in time by $\frac{1}{2W}$ seconds.

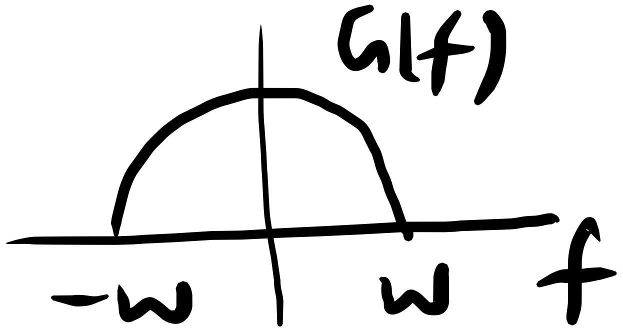
2. (Another form) (equivalent as well) Can be recovered from knowledge of its samples taken at the rate of 2W samples/second.

$f_s = 2W$, $T_s = \frac{1}{2W}$, for any $f_s > 2W$ or $T < \frac{1}{2W}$ the above two conditions are valid as well.

$G(f) = 0, \; |f| \geq W$

Nyquist rate :- 2W samples/sec
Nyquist interval :- 1/2W sec

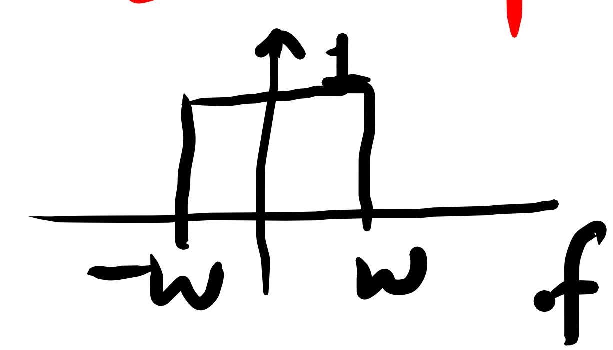
for BL signal of Band width W Hz



range of +ve f for which G(f) is non-zero is called the BW of g(t).

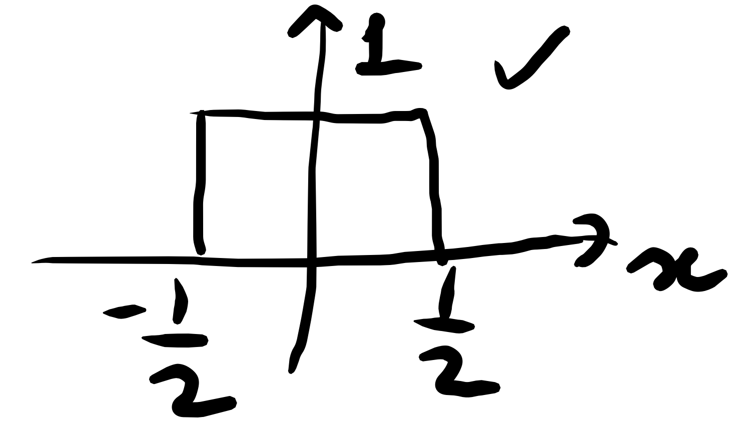
→ Another way to look at interpolation formula :-

Because of a term $f_s G(f)$ in the spectrum, $G_\delta(f)$, of the sampled signal, we can recover g(t) by sending it through an ideal low-pass filter (ILPF) of BW W Hz & gain T_s.



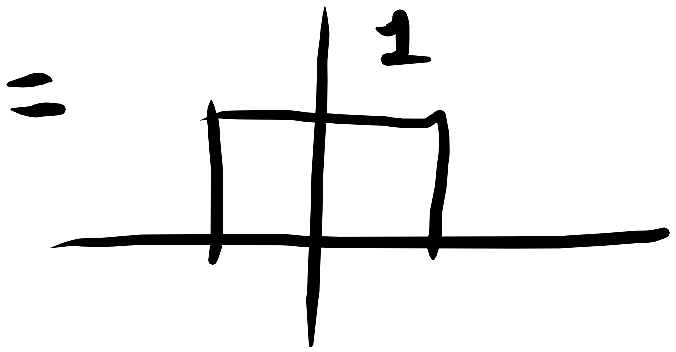
→ The freq. response of an ILPF can be written as -

$H(f) = T_s\, \Pi\left(\frac{f}{2W}\right)$, $\because\ \Pi(x) =$ ✓



$h(t) = 2WT_s\, \mathrm{sinc}(2Wt)$

$\Pi\left(\frac{x}{2y}\right) =$



due to Nyquist sampling rate $2WT_s = 1$, hence

$h(t) = \mathrm{sinc}(2Wt)$

$g_\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow g(t) = g_\delta(t) * h(t)$

$\equiv G_\delta(f) \times H(f)$

Observe that $h(t) = 0$ at all Nyquist sampling instants $(t = \pm n/2W)$ except at $t = 0$.

$\mathrm{sinc}(2Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt}$

$g(t)$
$g(t - t_0)$
$g(t + t_0)$
$g(at)$
$g(at + t_0)$
$g(a(t - t_0))$

put $t = \pm n/2W$, $\dfrac{\pm \sin(2\pi W n/2W)}{\pm 2\pi W n/2W} = \dfrac{\sin(\pi n)}{\pi n}$

at $n = 0$, 1
$n \neq 0$, 0

→ Each sample in sampled $g(t)$ being an impulse, generates a sinc pulse of height = strength of the sample. Addition of the sinc pulses generated by all the samples results in $g(t)$.

$$\left[g_s(t) = \sum_n g(nT_s)\,\delta(t - nT_s)\right] * h(t)$$

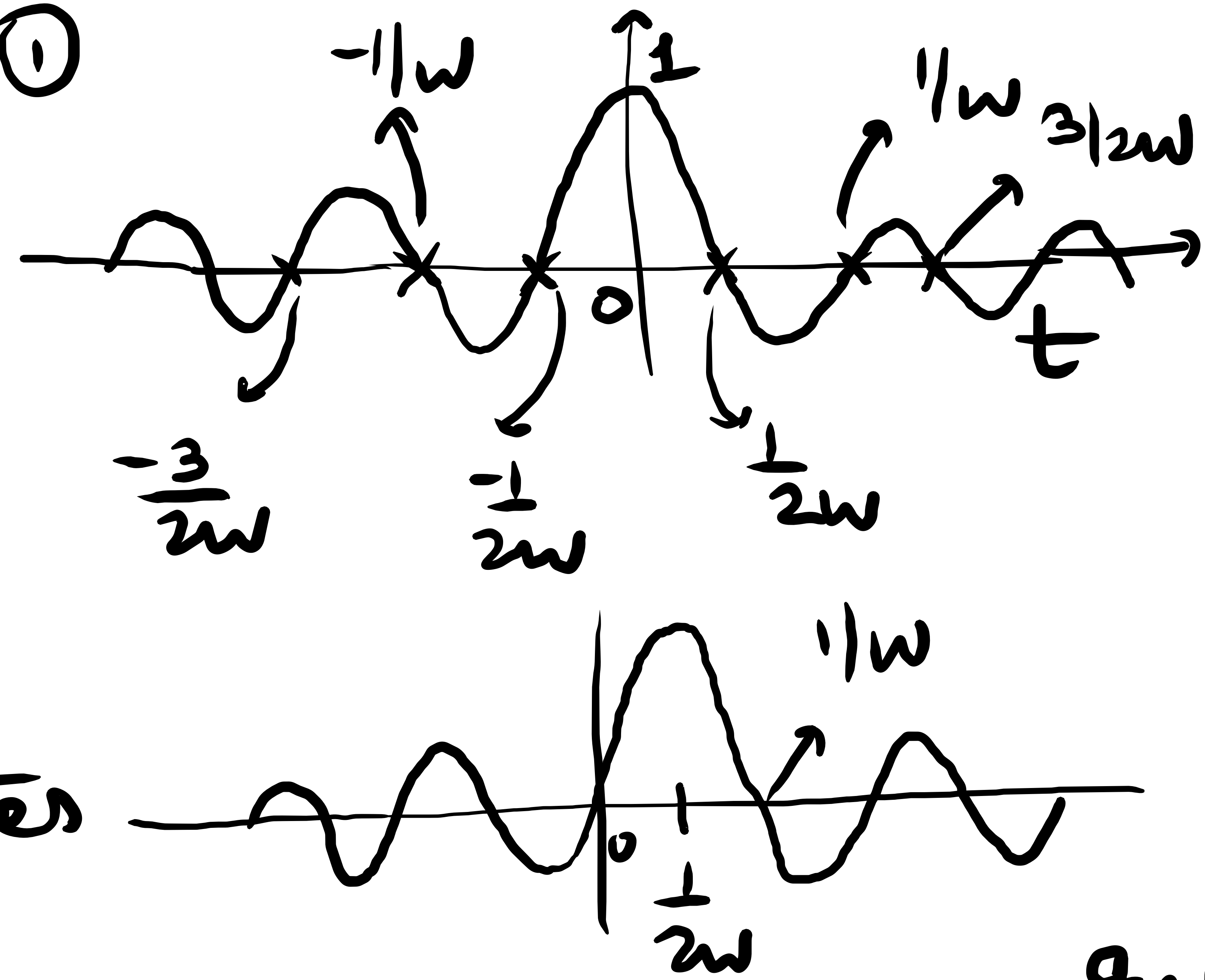
$$= \sum_n g(nT_s)\, h(t - nT_s)$$

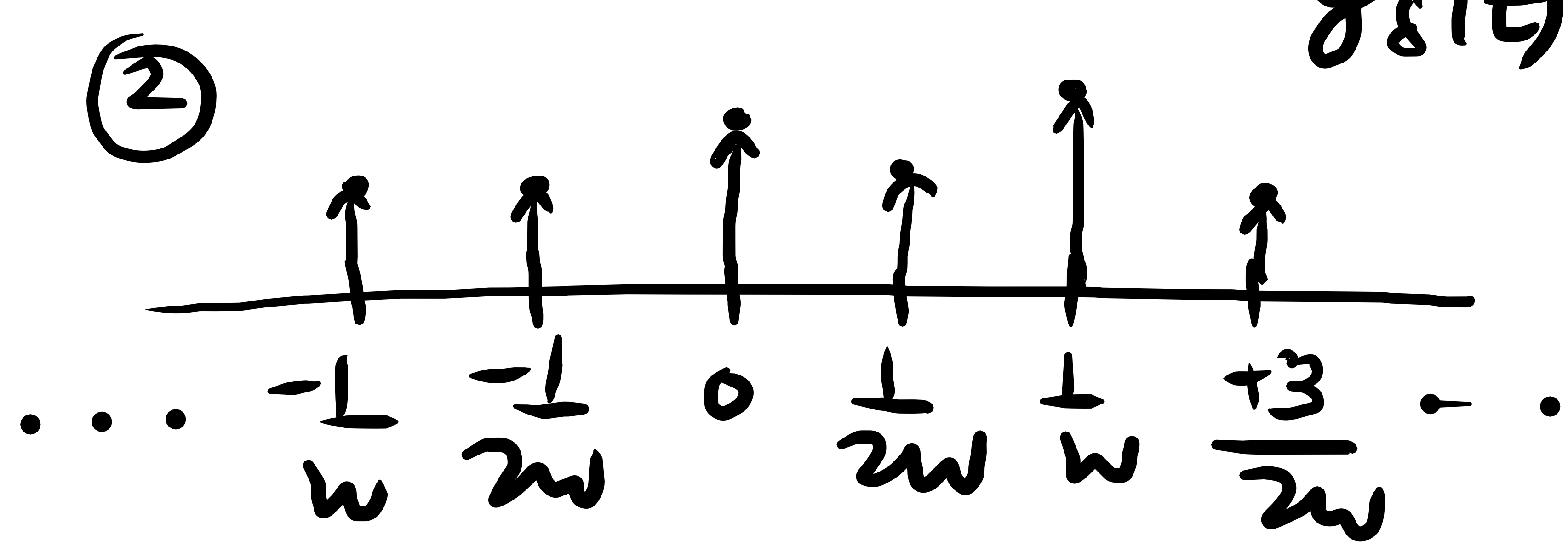
$$h(t - nT_s) = \text{sinc}\left(2W(t - nT_s)\right) = \text{sinc}\,(2Wt - n)$$

sinc(2Wt)

sinc(2Wt-1)

when you multiply ① with ②, which of the sample value contributes to the product?





possibility of $f_s = 2W$:- If the spectrum $G(f)$ has no impulse at the highest freq. W, then the overlap is still zero as long as the sampling rate $\geq$ Nyquist rate.

→ on the other hand, if $G(f)$ contains an impulse at the highest freq. $\pm W$, then the equality must be removed or else overlap will occur.

$f_s > 2W$ Hz