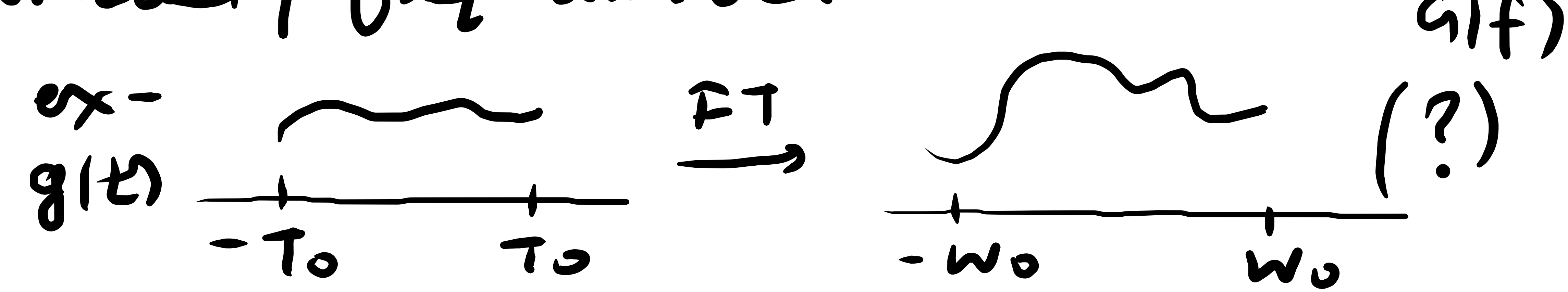
Lec-8, DC, 24-25, Sec A

See fig. 6.2(c) on pg 254 of Lathi & Ding's TB for a pictorial description of reconstruction formula.

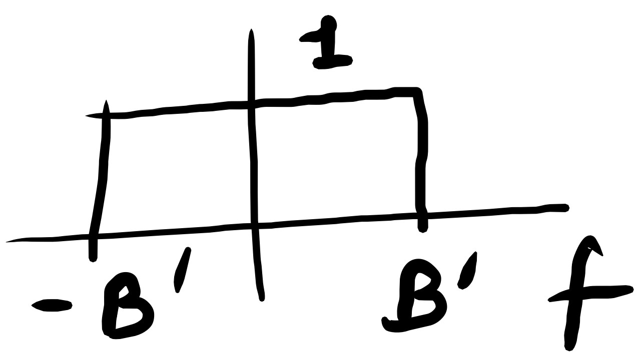
→ Can a signal be both time-limited as well as band-limited / freq. limited.

for ex-



Since $G(f)=0,\ |f|>W_0$ we can write

$$G(f) = G(f)\,\Pi(f/2B')\ ,\ \text{where } B' > W_0$$

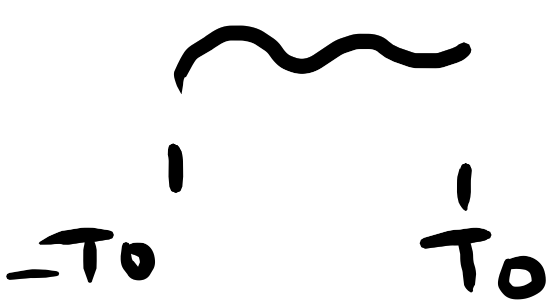


with G(f) as above, can you obtain

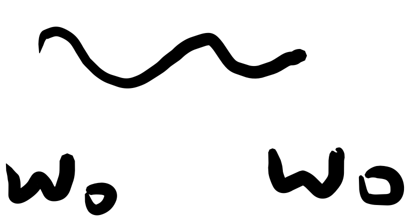
$$g(t) = \mathrm{IFT}[G(f)] = g(t) * \mathrm{IFT}[\Pi(f/2B')]$$

$\mathrm{IFT}\left[\Pi\left(f/2B'\right)\right] = 2B' \operatorname{sinc}(2\pi B' t)$ ∵ See any Standard on signals & sys. or may be Lathi's initial chapters

$g(t) * 2B' \operatorname{sinc}(2\pi B' t)$ is time-limited?



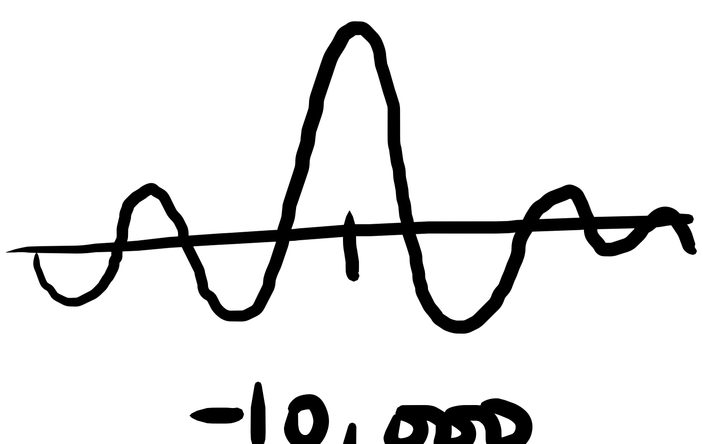
$g(t) \longrightarrow G(f)$



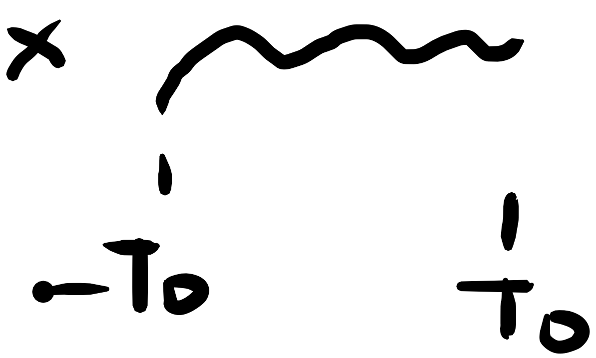
$\Vert$

$\xleftarrow{\mathrm{IFT}} G(f)\, \Pi\left(f/2B'\right)$ $\quad B' > W_0$

$g(t) * 2B' \operatorname{sinc}(2\pi B' t)$ → this convolution results in a signal which is not time-limited due to infinite support of sinc function.



$\times$



A time-limited signal cannot be band-limited & also a band-limited signal " " time-limited

however, a signal can be simultaneously non-time limited & non-band limited.

Solve prob. 6.1-8 from Lathi & Ding

→ All practical signals are time-limited i.e., they are of finite duration or width. Hence, necessarily they are non-BL.

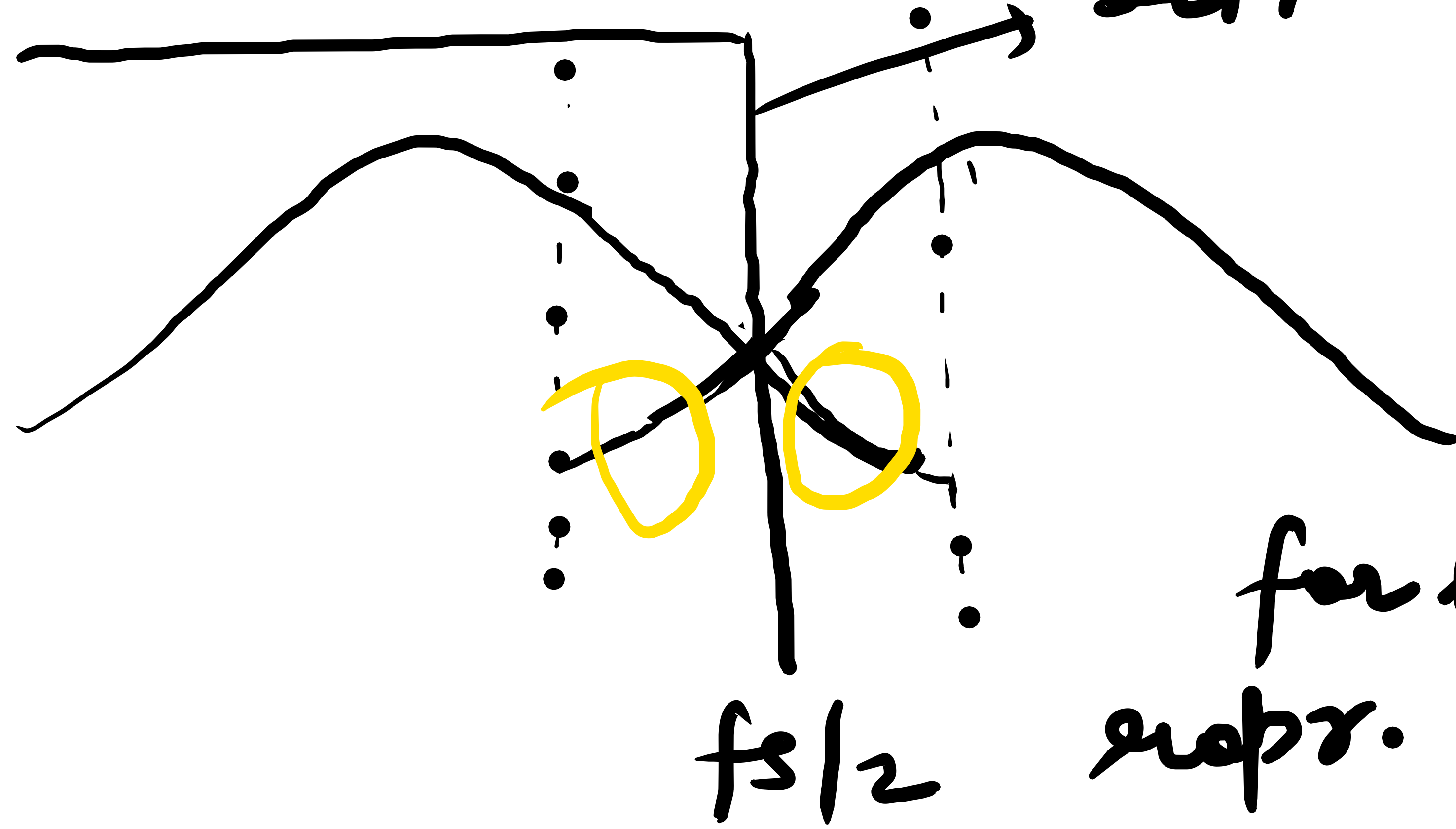
→ So, spectrum (sampled) consists of overlapping cycles of G(f) repeating every fs Hz. (regardless of sampling rate)

→ sampled signal & its spectrum no longer has complete information of spectrum / signal (orig.)

→ If passed through ILPF (sampled signal) then

(1). loss of tail of $G(f)$ beyond $|f| > f_s/2$ Hz

(2) Reappearance of this tail inverted or folded back onto the spectrum.



$\frac{f_s}{2} + f_z$

$\frac{f_s}{2} - f_z$

for better pictorial repr. see fig. 6.8 on Pg. 260 of TB.

→ Folding freq. :- $f_s/2 = 1/2T_s$ Hz

Spectrum may be viewed as if the last tail is folding back onto itself at the folding freq.

→ component of freq. $\frac{f_s}{2} + f_2$ shows up as, or impersonates a component of lower freq $\frac{f_s}{2} - f_2$ in the reconstructed signal.

→ This is known as spectral folding or aliasing

→ Solution :- antialiasing filter, which eliminates the component above $f_s/2$ (folding freq.) from $g(t)$ before sampling.