Lec-29, DC, 24-25, Sec A

H.W. Prove the last note of lec 28.

→ Euclidean dist. b/w pts. represented by the signal vectors $\bar{s}_i$ & $\bar{s}_k$ is $\to d_{ik}$

$$d_{ik}^2 = \|\bar{s}_i - \bar{s}_k\|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2 =$$

↳ note that it is a scalar.

$$= \int_0^T [s_i(t) - s_k(t)]^2 dt$$

$(2,3)$ $(1,-1)$

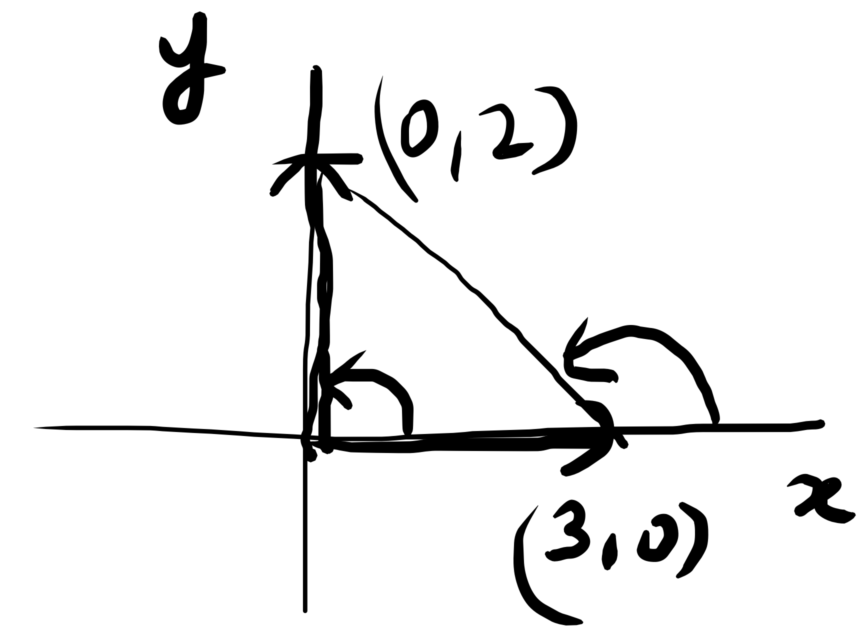
$$\sqrt{(2-1)^2 + (3+1)^2}$$

→ cosine of angle θ_{ik} = $\dfrac{\text{inner product of two vec.}}{\text{Prod. of their individual norms.}}$

$$\cos\theta_{ik} = \frac{\bar{s}_i^T \bar{s}_k}{\|\bar{s}_i\| \, \|\bar{s}_k\|} \quad -(1)$$

. Two Vectors $\bar{s}_i$ & $\bar{s}_k$ are thus orthogonal to each other if their inner prod. (dot prod.) $\bar{s}_i^T \bar{s}_k = 0$, in which case $\theta_{ik} = 90^\circ$

θ_{ik}:- angle b/w the two Vectors.



<u>Schwarz Inequality</u> :- (SI)

Consider any pair of energy signals $s_1(t)$ & $s_2(t)$. The SI states that

$$\left(\int_{-\infty}^{\infty} s_1(t)s_2(t)\,dt\right)^2 \le \left(\int_{-\infty}^{\infty} s_1^2(t)\,dt\right)\left(\int_{-\infty}^{\infty} s_2^2(t)\,dt\right)$$

equality holds iff $s_2(t) = C\,s_1(t)$, where C is any constant.

Proof:- Let $s_1(t)$ & $s_2(t)$ be expressed in terms of the pair of orthonormal basis functions $\Phi_1(t)$ & $\Phi_2(t)$ as follows.

$$s_1(t) = s_{11}\,\Phi_1(t) + s_{12}\,\Phi_2(t)$$

$$s_2(t) = s_{21}\,\Phi_1(t) + s_{22}\,\Phi_2(t)$$

where $\int_{-\infty}^{\infty} \Phi_i^2(t)\,dt = 1$, $i=1,2$ & $\int_{-\infty}^{\infty} \Phi_1(t)\Phi_2(t)\,dt = 0$

Use ① & prove SI.

$$\cos\theta = \frac{\bar{s}_1^T \bar{s}_2}{\|\bar{s}_1\| \, \|\bar{s}_2\|}$$

where $\bar{s}_1 = \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix}$ &

$\bar{s}_2 = \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix}$

$$\frac{s_{11}s_{21} + s_{12}s_{22}}{\sqrt{s_{11}^2 + s_{12}^2} \times \sqrt{s_{21}^2 + s_{22}^2}}$$

$$= \frac{\int_{-\infty}^{\infty} s_1(t)\, s_2(t)\, dt}{\sqrt{\int_{-\infty}^{\infty} s_1^2(t)\, dt} \; \sqrt{\int_{-\infty}^{\infty} s_2^2(t)\, dt}}$$

$$s_{11} = \int_{-\infty}^{\infty} s_1(t)\, \phi_1(t)\, dt$$

$$s_{12} = \int_{-\infty}^{\infty} s_1(t)\, \phi_2(t)\, dt$$

H.W.

Recognising that

$$|\cos\theta| \leq 1 \Rightarrow \cos^2\theta \leq 1$$

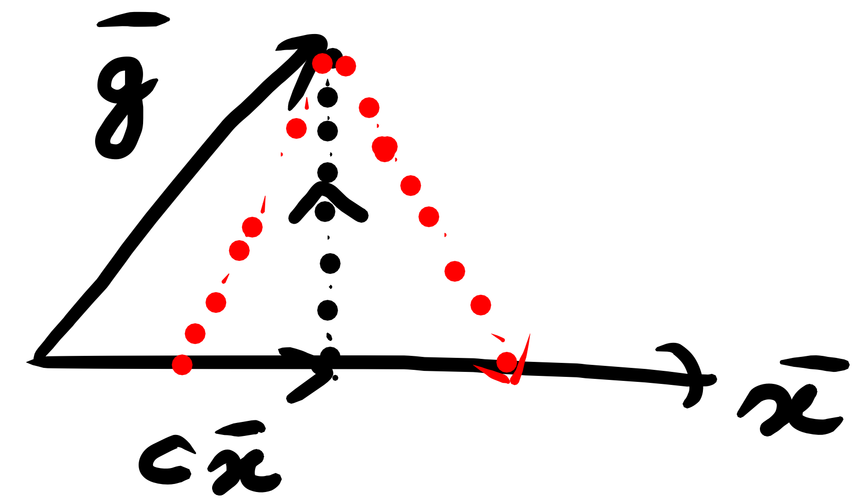
we can show that SI holds

→ we know that $|\cos\theta| = 1$, when $\theta = n\pi$, where $n \in \mathbb{Z}$, so both vectors must lie on a line i.e., we may express them as $\bar{S}_2 = c\bar{S}_1$ or $S_2(t) = c\,S_1(t)$, where c is a constant.

Gram Schmidt Orthogonalization procedure:-

→ We first study component of a vector along another vector.

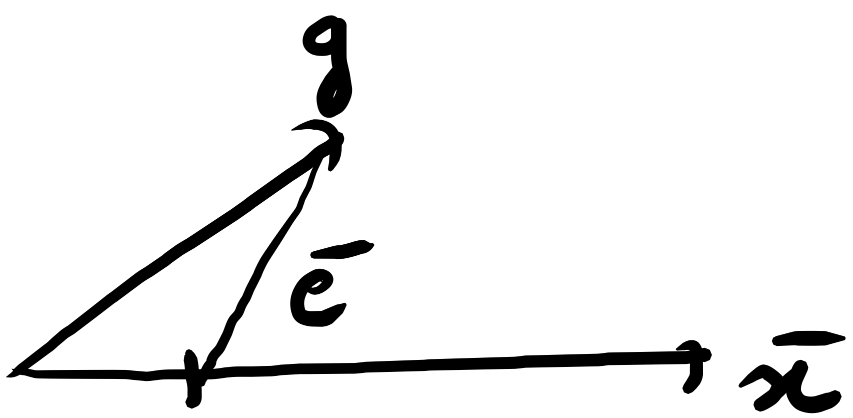
Consider two vectors $\bar{g}$ & $\bar{x}$

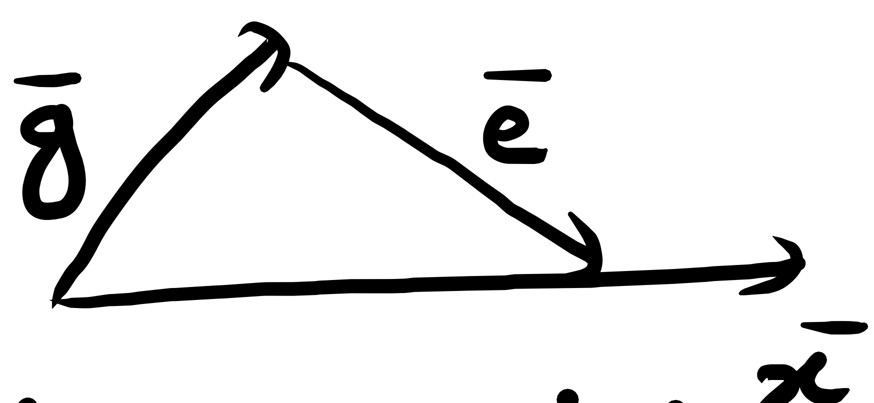


→ We know that component of $\bar{g}$ along $\bar{x}$ be $c\bar{x}$. Geometrically, the component of $\bar{g}$ along $\bar{x}$ is the projection of $\bar{g}$ on $\bar{x}$ & is obtained by drawing a perpendicular from the tip of $\bar{g}$ on the vector $\bar{x}$.

<u>Ques:</u>- What is the mathematical significance of the component of a vector along another vector?

<u>A:</u> $\bar{g} = c\bar{x} + \bar{e}$. Q2. What is the best decomposition?





these two show two of the infinite possibilities. Then Q2 arises.

The concept of optimality depends on what we wish to accomplish by decomposing $\bar{g}$ into two components.