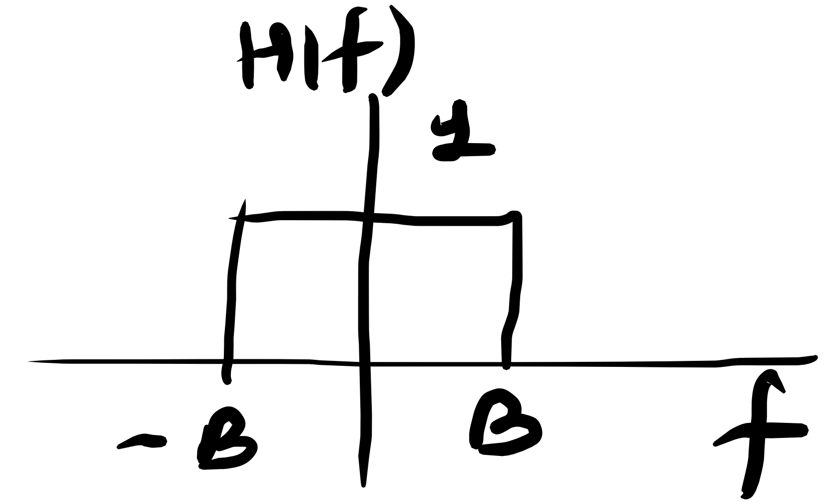
→ WN has ∞ average power - hence not physically realizable. but $S_W(f)$ & $R_W(\tau)$ make it useful in statistical system analysis.

→ Apply a WGN, $w(t)$, of zero mean & PSD $N_0/2$ Watt/Hz to an ideal LPF of B.W. B Hz & pass band mag. resp. of 1.



At output PSD is

$$S_W(f) = \begin{cases} N_0/2, & -B < f < B \\ 0, & |f| > B \end{cases}$$

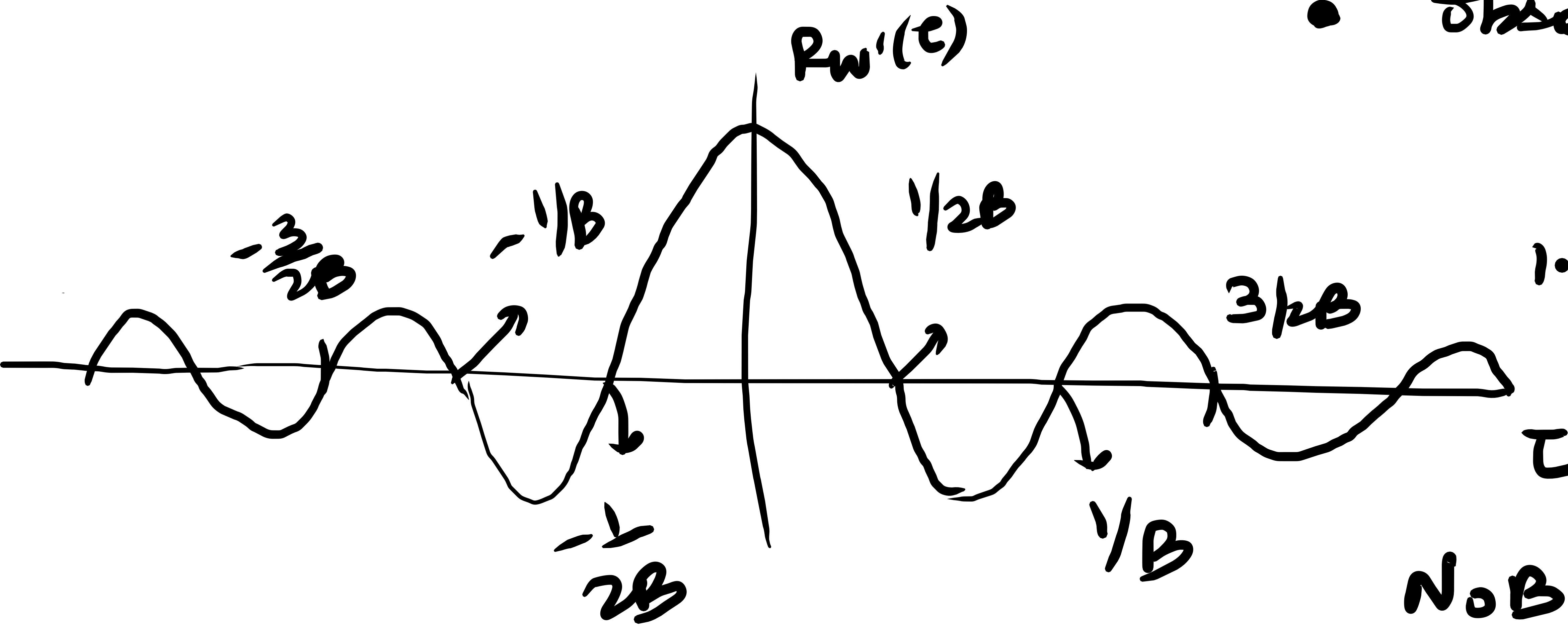
$S_W(f) = \frac{N_0}{2} \;\; \forall f$

$w(t) \longrightarrow \boxed{h(t)} \longrightarrow w'(t)$

Taking the Inverse FT of the PSD of $w'(t)$, we obtain autocorr. as

$$R_{W'}(\tau) = \int_{-B}^{B} \frac{N_0}{2} e^{j2\pi f \tau} \, df$$

$$= N_0 B \, \mathrm{sinc}(2B\tau)$$



$w(t) \rightarrow$

$$R_W(\tau) = \frac{N_0}{2} \delta(\tau)$$

$w'(t)$, will $R_{W'}(\tau)$ stay the same as $R_W(\tau)$?

- Observe from $R_{W'}(\tau)$

1. $R_{W'}(\tau)$ has its max. value of $N_0 B$ at the origin.

$$E[(W(t))^2] = N_0 B$$

& it passes through zero at $\tau = \pm \frac{k}{2B}$, $k = 1, 2, 3 \ldots$ — ②

→ Since $w(t)$ is Gaussian, it follows that the B.L. noise $w'(t)$ at the filter o/p is also Gaussian.

(see proof from any T.B.)

$$E\left[w(t)\, w\left(t \pm \frac{k}{2B}\right)\right] = 0$$

$w(t) \rightarrow$ at a value $w(t_1)$

$w\left(t_1 + \frac{1}{2B}\right)$

→ Sample $w'(t)$ at the rate 2B times per second. From ② above, we see that resulting noise samples are uncorrelated & being Gaussian, they are statistically independent.

34 - -5

→ Let there be a linear T.I.V. filter of IR. $h(t)$ at the receiver / processor at Rx.

$$x(t) = g(t) + w(t) \qquad 0 \leq t \leq T$$

↳ pulse corr. to binary symbol 1 or 0 in a digital system $T = T_b$

↳ this may also corr. to a symbol. $T = T_s$

$0 \rightarrow p_1(t) = p(t)$ $T = T_b$

$1 \rightarrow p_2(t) = -p(t)$

00 – $a_1 p(t)$

01 – $a_2 p(t)$

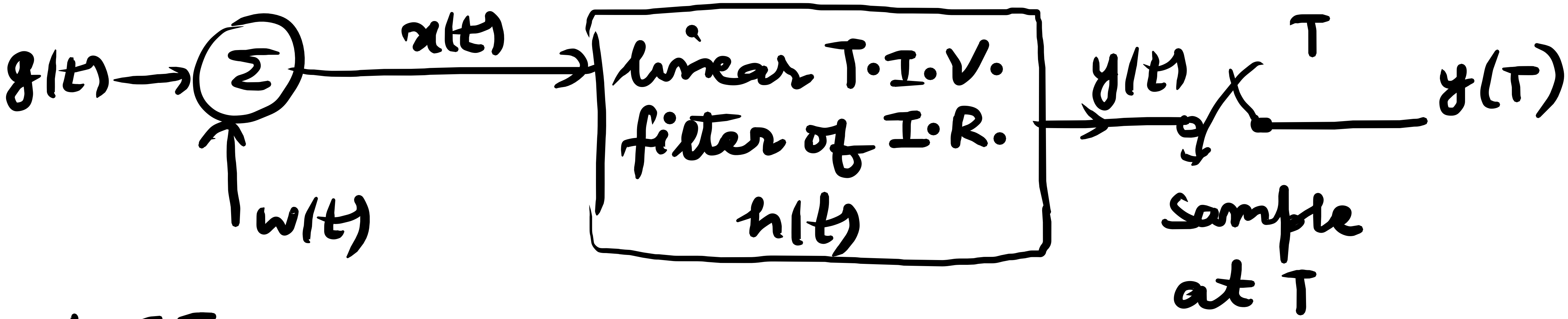
10 – $a_3 p(t)$ $T = T_s = 2T_b$

11 – $a_4 p(t)$

$$x(t) \longrightarrow \boxed{\text{processing}} \longrightarrow a$$

where a is the level used to generate $g(t)$.

$w(t)$:- sample function of a WN process of 0 mean & PSD $N_0/2$.



$0 \le t \le T$

Func. of Rx:- detect the pulse $g(t)$ in an optimum manner, given $x(t)$.

let $y(t) = g_0(t) + n(t)$

we want output signal

$$\left(y(t) = \underbrace{g(t) * h(t)}_{g_0(t)} + \underbrace{w(t) * h(t)}_{n(t)} \right)$$

Component to be

optimize the design of filter so as to minimize the effects of noise at the filter output in some statistical sense.

considerably > output noise comp. This can be ensured by the filter by making the instantaneous power in $g_0(t)\big|_{t=T}$ as large as possible compared with avg. power of o/p noise $n(t)$

$$\eta = \frac{|g_0(T)|^2}{E[n^2(t)]}$$

$|g_0(T)|^2$ → inst. pow. in o/p sig.

$E[n^2(t)]$ → avg. noise pow.

Peak pulse signal power to noise (avg. power) ratio.

$$g_0(t) = \int_{-\infty}^{\infty} H(f)\, G(f)\, e^{j2\pi f t}\, df$$

I.F.T of $H(f)G(f)$

$$|g_o(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT}\,df \right|^2$$

$$S_N(f) = \frac{N_0}{2}|H(f)|^2 \;;\Rightarrow E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\,df$$

H.W. (?)

$$= \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

$$\eta = \frac{\left| \int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT}\,df \right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df}$$