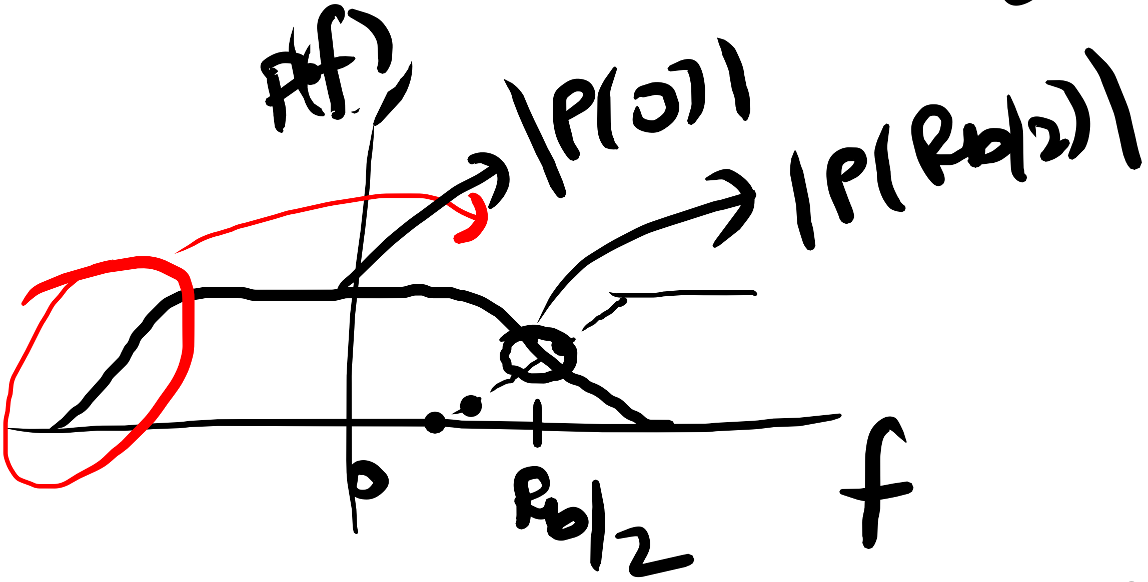
Lec-26, DC, 24-25, Sec A

Note that this requires $|P(R_b/2)| = \frac{1}{2}|P(0)|$



B.W. in Hz of $P(f)$ is $\frac{R_b}{2} + f_x$, where f_x is the B.W. in excess of the minimum B.W. $R_b/2$

$$r = \frac{\text{excess B.W.}}{\text{theoretical min. B.W.}} = \frac{f_x}{R_b/2} = 2 f_x T_b$$

as f_x cannot be larger than $R_b/2$, $0 \le r \le 1$

$f_x = r\frac{R_b}{2}$, so B.W. of $p(t)$ is $\frac{R_b}{2} + r\frac{R_b}{2}$

$= (1+r)\frac{R_b}{2}$. $\quad$ r:- roll-off factor & also expressed as a %.

for ex- if $P(f)$ is a Nyquist first criterion spectrum with B.W. that is 50% higher than the theoretical min., its roll-off factor $\alpha = 0.5$ or 50%.

→ $p(t)$ can be generated as a unit impulse response of a filter with T.F. $P(f)$.

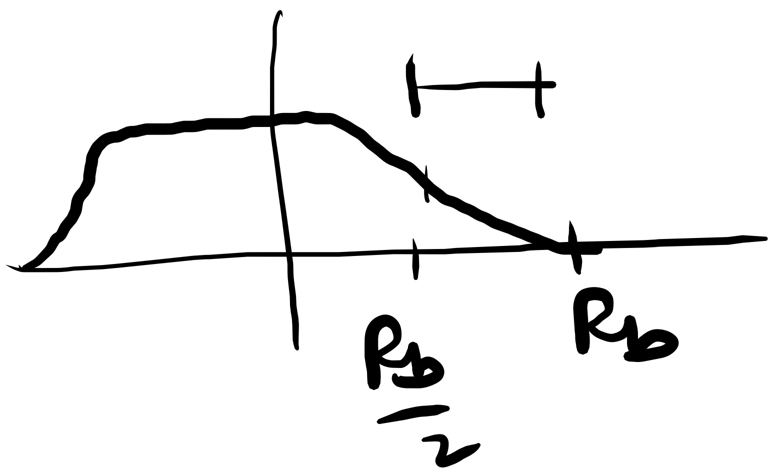
→ But because $P(f) = 0$ over a freq. band. it violates Paley-wiener criterion & is unrealizable.

→ However, roll off is gradual, can be more closely approximated by a practical filter.

→ Family of spectra that satisfies Nyquist's 1st criterion is

$$\boxed{\frac{R_b}{2} - f_x < |f| < \frac{R_b}{2} + f_x} \quad (4)$$

$$P(f) = \begin{cases} 1, & \longrightarrow |f| \le \frac{R_b}{2} - f_x \quad (1) \\ \frac{1}{2}\left[1 - \sin \pi\left(\frac{f - R_b/2}{2f_x}\right)\right], & \longrightarrow \left|f - \frac{R_b}{2}\right| < f_x \quad (2) \\ 0, & \longrightarrow |f| > \frac{R_b}{2} + f_x \quad (3) \end{cases}$$



(2) = (4)

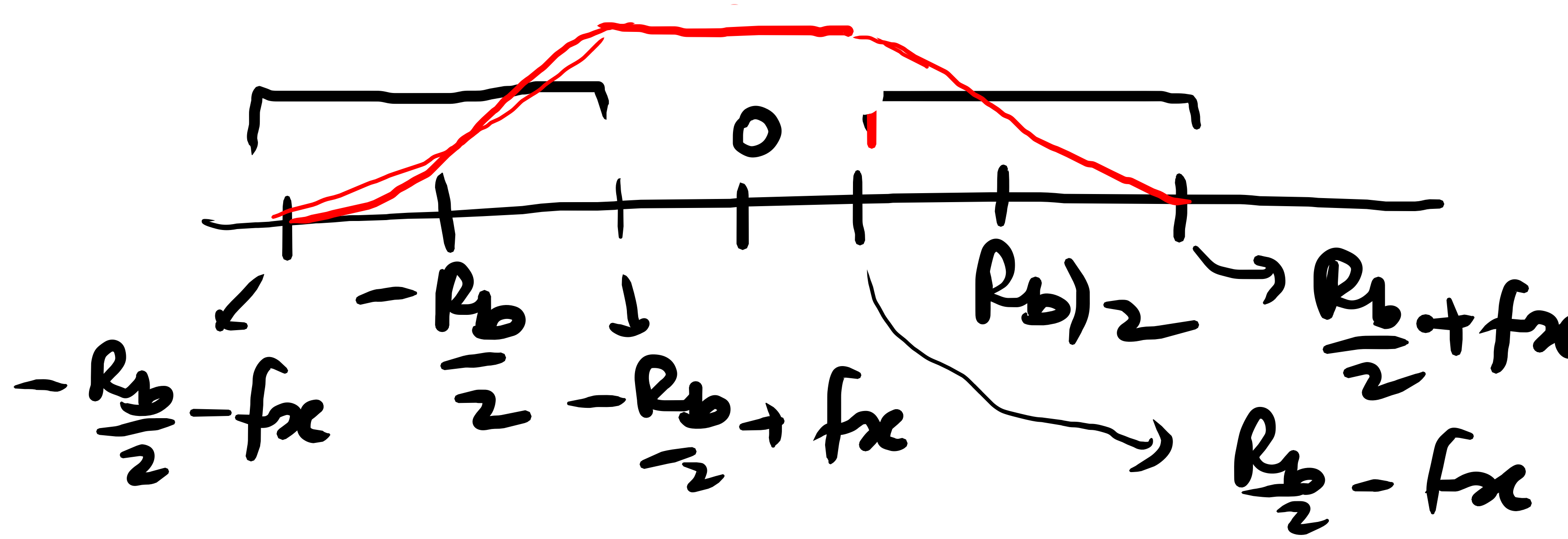
$$\frac{R_b}{2} - f_x < f < \frac{R_b}{2} + f_x$$

$$-\frac{R_b}{2} + f_x > f > -\frac{R_b}{2} - f_x$$

(1) $-\frac{R_b}{2} + f_x \le f \le \frac{R_b}{2} - f_x$

(2) $-f_x + \frac{R_b}{2} < f < f_x + \frac{R_b}{2}$

(3) $f > \frac{R_b}{2} + f_x, \; f < -\frac{R_b}{2} - f_x$



H.W. To see if ③ = ④

for $f_x = \frac{R_b}{2}$ (i.e., $r=1$), we have

$$P(f) = \frac{1}{2}(1+\cos \pi f T_b)\, \Pi(f/2R_b)$$

$$= \cos^2\left(\frac{\pi f T_b}{2}\right) \Pi(f T_b/2)$$

This is known as raised cosine characteristic (RC)

Its Inverse F.T. is

$$p(t) = R_b \frac{\cos(\pi R_b t)}{1-4R_b^2 t^2} \operatorname{sinc}(\pi R_b t)$$

raised cosine pulse.

Q. What happens to $p(t)$ when $t = \frac{1}{2R_b}$? H.W.

Features of the RC pulse

1. B.W. of pulse is R_b Hz.

2. It decays rapidly, as $1/t^3$, as a result, the RC pulse is relatively insensitive to deviations in sampling rate, timing jitter & so on.

3. Value of $p(t)$ is zero not only at all the remaining signaling instants but also at pts. midway b/w all " ". (H.W.)

→ Det. the pulse Tx rate in terms of the Tx B.W. B_T & roll off factor r.

Assume a scheme using Nyquist's 1st criterion

Ans:— $B_T = (1+r)\frac{R_b}{2}$ (we derived)

so $R_b = \frac{2B_T}{1+r}$, as $0 \le r \le 1$, the pulse Tx. rate varies from $2B_T$ to B_T, depending on the choice of r.

A smaller r gives a higher samp. rate, but $p(t)$ decays slowly. For RC pulse, $r=1$, $R_b = B_T$ we achieve 1/2 the theoretical max. rate.