Lec-19, DC, 24.25, Sec A

⇒ We can improve upon this scheme by estimating (prediction) the value of the kth sample $m[k]$ from knowledge of several previous sample values.

⇒ If our prediction is worth its salt, predicted value $\hat{m}[k]$ will be close to $m[k]$ & thus difference (prediction error) $d[k]$ will be even smaller than the diff. b/w the successive samples.

↗ This scheme is called DPCM. The previous case is a special case, where $\hat{m}[k] = m[k-1]$.

→ Consider, for example, a signal $m(t)$ which has derivatives of all orders at t. Using the Taylor series for this signal, we can express

$$m(t+T_s) = m(t) + T_s m'(t) + \frac{T_s^2}{2!} m''(t) + \frac{T_s^3}{3!} m'''(t) + \cdots$$

$$\approx m(t) + T_s m'(t) \quad \text{for small } T_s.$$

Shows that from the knowledge of the signal & its derivative at instant t, we can predict a future value at $t+T_s$.

Let the kth sample of $m(t) \rightarrow m[k]$ i.e.,

$m(kT_s) = m(k)$ & $m(kT_s \pm T_s) = m\{k \pm 1\}$ & so on.

$$m'(kT_s) \approx \frac{m(kT_s) - m(kT_s - T_s)}{T_s}$$

$m(t)$

$\downarrow$

$m'(t)$

$m'(kT_s)$

We obtain

$$m[k+1] \approx m[k] + T_s \left\{ \frac{m[k] - m[k-1]}{T_s} \right\}$$

$$= 2m[k] - m[k-1] \quad -(1)$$

This shows that we can find a crude prediction of the $(k+1)^{th}$ sample from the two previous samples.

→ approx. in (1) can be improved if we add more terms in the series. To determine the

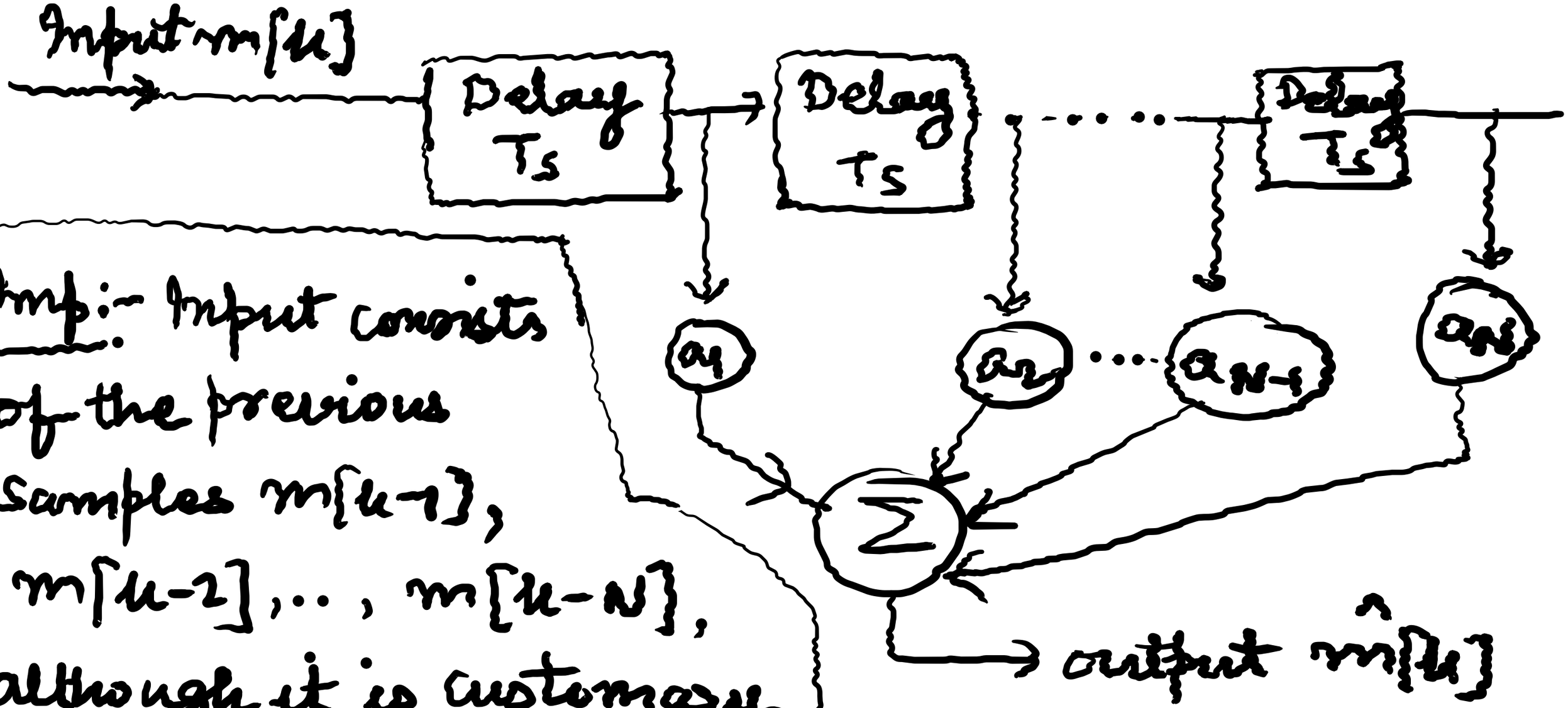
higher order derivatives in the series, we require more samples in the past.

$$m[k] \approx \underbrace{a_1 m[k-1] + a_2 m[k-2] + \cdots + a_N m[k-N]}_{(2)}$$

$\hat{m}[k]$

predicted value of $m[k]$

eq. (2) is the equation for N^{th} order predictor. Larger N would result in better predictors in general.



Imp:- Input consists of the previous samples $m[k-1]$, $m[k-2]$, ..., $m[k-N]$, although it is customary to say that the input is $m[k]$ & the o/p $\hat{m}[k]$

above is a linear predictor. It is basically a transversal filter (a tapped delay line) where the tap gains are set = the prediction coeffs. $(a_1, a_2 \cdots a_{N-1})$

We have a very simple procedure for predictor design. In a more sophisticated approach — MMSE (min. mean squared error) for determining a_j's is used which comes in terms of statistical correlation b/w various samples.

<u>DPCM structure:-</u>

We Tx, $d[k]$ $(= m[k] - \hat{m}[k])$ instead of $m[k]$. At the Rx, generate $\hat{m}[k]$ from past sample values to which $d[k]$ is added to generate $m[k]$.