

Q18 #10

$$\textcircled{e} \left[\begin{array}{ccccc|c} x & -x & -2 & 0 & -x & 39 \\ -2 & 3 & 5 & x & -x & -39 \\ x & -x & -x & 0 & -x & 39 \\ 3 & -5 & -7 & -x & x & 54 \\ -6 & 9 & 13 & 2 & -x & -126 \end{array} \right] \sim$$

$$\sim \left[\begin{array}{ccccc|c} x & -x & -2 & 0 & x & 39 \\ 0 & x & x & x & -x & 39 \\ 0 & 0 & x & 0 & 0 & -6 \\ 0 & -2 & -x & -x & 4 & -63 \\ 0 & 3 & x & 2 & -7 & 108 \end{array} \right] \sim$$

$$\sim \left[\begin{array}{ccccc|c} x & -x & 0 & 0 & x & 27 \\ 0 & x & 0 & x & -x & 45 \\ 0 & 0 & x & 0 & 0 & -6 \\ 0 & 0 & 0 & x & 2 & 2x \\ 0 & 0 & 0 & \cancel{x} & \cancel{4} & \cancel{2x} \\ & & & 0 & -2 & 0 \end{array} \right]$$

$$x_5 = 0$$

$$x_4 = 2x$$

$$x_3 = -6$$

$$x_2 = 24$$

$$x_1 = 5x$$

Order: $[5x, 24, -6, 2x, 0]$

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 & 1 & -3 & 1 & -64 \\ 0 & 1 & 1 & -1 & 1 & -40 \\ -1 & 0 & 0 & 1 & -1 & 18 \\ 2 & 1 & 2 & -5 & 3 & -122 \\ -2 & 0 & -1 & 4 & -2 & 82 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & -3 & 1 & -64 \\ 0 & 1 & 1 & -1 & 1 & -40 \\ 0 & 0 & 1 & -2 & 0 & -46 \\ 0 & 1 & 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & -2 & 0 & -46 \end{bmatrix} \\
 & \sim \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -64 \\ 0 & 1 & 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & -2 & 0 & -46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{cases} x_1 + x_3 - 3x_4 + x_5 = -64 \\ x_2 + x_4 + x_5 = 6 \\ x_3 - 2x_4 = -46 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 03: & \begin{cases} x_1 + x_3 = 3x_4 - x_5 \\ x_2 = -x_4 - x_5 \\ x_3 = 2x_4 \end{cases} \\
 & \begin{matrix} x_4 = 0 & x_5 = 1 \\ x_5 = 0 & x_4 = 1 \end{matrix} \quad \begin{pmatrix} -1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 2 & 1 & 0 \end{pmatrix}^T = \begin{matrix} \underline{\underline{X}}_1 \\ \underline{\underline{X}}_2 \end{matrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{He } 03: & \quad x_4 = x_5 = 0 : (-18 \ 6 \ -46 \ 0 \ 0)^T \\
 & \quad z = \begin{bmatrix} -18 \\ 6 \\ -46 \\ 0 \\ 0 \end{bmatrix} + C_1 \begin{bmatrix} -1 \\ -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 & \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ -1 & 1 & -2 & -2 & 1 & -17 \\ -1 & 1 & 1 & 4 & -10 & 38 \\ -3 & -3 & -8 & -7 & 15 & -38 \\ -3 & 5 & 14 & 12 & -26 & 55 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ 0 & 1 & 1 & 1 & -1 & 38 \\ 0 & 0 & 1 & 1 & -1 & -24 \\ 0 & 0 & 1 & 1 & -1 & 52 \\ 0 & 0 & 1 & 1 & -1 & -80 \end{bmatrix} \\
 & \sim \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ 0 & 1 & 1 & 1 & -1 & 38 \\ 0 & 0 & 1 & 1 & -1 & -24 \\ 0 & 0 & 1 & 1 & -1 & 52 \\ 0 & 0 & 1 & 1 & -1 & -80 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ 0 & 1 & 1 & 1 & -1 & 38 \\ 0 & 0 & 1 & 1 & -1 & -24 \\ 0 & 0 & 1 & 1 & -1 & 52 \\ 0 & 0 & 1 & 1 & -1 & -80 \end{bmatrix}
 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ 0 & 1 & 1 & 1 & -1 & 38 \\ 0 & 0 & 1 & 1 & -1 & -24 \\ 0 & 0 & 1 & 1 & -1 & 52 \\ 0 & 0 & 1 & 1 & -1 & -80 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -4 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$$03: \quad \begin{cases} x_1 - 2x_2 - 4x_3 - 3x_4 = -9x_5 \\ x_2 + x_3 = 4x_5 \\ x_3 + x_4 = x_5 \\ x_4 = -3x_5 \end{cases}$$

$$\quad x_5 = r : (-2 \ 0 \ 4 \ -3 \ 1)^T$$

$$\text{He } 03: \quad x_5 = 0 : (-48 \ 1 \ 37 \ -58 \ 0)$$

$$\begin{aligned}
 & \quad z = \begin{bmatrix} -48 \\ 1 \\ 37 \\ -58 \\ 0 \end{bmatrix} + C_1 \begin{bmatrix} -2 \\ 0 \\ 4 \\ -3 \\ 1 \end{bmatrix} \\
 4) & \quad \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ -1 & 1 & -2 & -2 & 1 & -17 \\ -1 & 1 & 1 & 4 & -10 & 38 \\ -3 & -3 & -8 & -7 & 15 & -38 \\ -3 & 5 & 14 & 12 & -26 & 55 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -4 & -3 & 2 & -45 \\ 0 & 1 & 1 & 1 & -1 & 38 \\ 0 & 0 & 1 & 1 & -1 & -24 \\ 0 & 0 & 1 & 1 & -1 & 52 \\ 0 & 0 & 1 & 1 & -1 & -80 \end{bmatrix} \\
 & \quad \begin{cases} x_2 = x_4 + 2x_5 \\ x_3 = 2x_4 - x_5 \\ x_1 = 2x_2 - 2x_3 + 2x_4 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 & \quad x_4 = 0 \quad x_5 = r : (r \ 2 \ 2 \ 0 \ r)^T \\
 & \quad x_4 = r \quad x_5 = 0 : (2 \ 1 \ 1 \ r \ 0)^T
 \end{aligned}$$

$$\begin{aligned}
 5) & \quad \begin{pmatrix} -4 & -5 & -2 & 2 & 3 \\ -3 & -4 & -4 & 2 & 4 \\ -4 & -4 & -3 & 1 & 5 \\ -4 & -5 & -1 & 2 & 2 \end{pmatrix} \quad \begin{matrix} 2-4: (0 \ 0 \ -2 \ 0 \ 1) \\ 3-2: (0 \ -1 \ 1 \ 1 \ -2) \end{matrix}
 \end{aligned}$$

$$⑥ \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & b^1 \\ -1 & 2 & 2 & -2 & -1 & b^2 \\ 2 & -4 & -2 & 3 & 1 & b^3 \\ -1 & 1 & 0 & 2 & 0 & b^4 \\ 1 & -3 & -2 & 2 & 0 & b^5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & b^1 \\ 0 & 1 & 3 & -2 & 0 & b^2 \\ 0 & 0 & -2 & 7 & 1 & b^3 - 2b^2 \\ 0 & 0 & 1 & 4 & 1 & b^4 + b^2 \\ 0 & -2 & -1 & 2 & -1 & b^5 - b^2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & b^1 \\ 0 & 1 & 3 & -2 & 0 & b^2 \\ 0 & 0 & -2 & 7 & 1 & b^3 - 2b^2 \\ 0 & 0 & 1 & 4 & 1 & b^4 + b^2 \\ 0 & -2 & -1 & 2 & -1 & b^5 - b^2 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & b^1 \\ 0 & 1 & 3 & -2 & 0 & b^2 \\ 0 & 0 & -2 & 7 & 1 & b^3 - 2b^2 \\ 0 & 0 & 1 & 4 & 1 & b^4 + b^2 \\ 0 & 0 & 0 & 0 & 0 & b^5 - b^3 + b^1 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & b^1 \\ 0 & 1 & 3 & -2 & 0 & b^2 \\ 0 & 0 & -2 & 7 & 1 & b^3 - 2b^2 \\ 0 & 0 & 1 & 4 & 1 & b^4 + b^2 \\ 0 & 0 & 0 & 0 & 0 & b^5 - b^3 + b^1 \end{bmatrix}$$

$$b^4 - b^1 \begin{cases} b^1 - 2b^2 - b^3 + b^4 = 0 \\ b^1 - b^3 + b^5 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & -2 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & 1 & 0 \\ 0 & 2 & 0 & -1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & -2 & 0 & 1 & -1 \end{bmatrix} \begin{cases} b_2 = b_3 - b_5 \\ 2b_2 = -b_4 + b_3 \end{cases}$$

$$\begin{cases} 2b_3 = 1 & b_3 = b_4 = 0 \\ 2b_4 = 1 & b_3 = b_5 = 0 \end{cases} \Rightarrow \begin{pmatrix} -1 & 1/2 & 0 & 0 & 1 \\ 0 & -1/2 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{cases} b_3 = 1 & b_4 = b_5 = 0 \end{cases} : \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$