Just Another Day with 50 derivatives

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1 Demidovich

1.1 D848

$$y = \frac{(2-x^2)(2-x^3)}{(1-x)^2} = \frac{4-2x^2-2x^3+x^5}{1-2x+x^2}$$

$$(4-2x^2-2x^3+x^5)' = -4x-6x^2+5x^4$$

$$(1-2x+x^2)' = -2+2x$$

$$\left(\frac{4-2x^2-2x^3+x^5}{1-2x+x^2}\right)' = \frac{(-4x-6x^2+5x^4)(1-2x+x^2)-(4-2x^2-2x^3+x^5)(-2+2x)}{(1-x)^4} =$$

$$= \frac{(-4x-6x^2+5x^4)(1-x)+(4-2x^2-2x^3+x^5)*2}{(1-x)^3} = \frac{8-4x-6x^2+2x^3+5x^4-3x^5}{(1-x)^3}$$

1.2 D852

$$y = \frac{1}{x}+\frac{1}{\sqrt{x}}+\frac{1}{x^{1/3}} = x^{-1}+x^{-1/2}+x^{-1/3}$$

$$y' = -x^{-2}-\frac{1}{2}x^{\frac{-3}{2}}-\frac{1}{3}x^{\frac{-4}{3}}$$

1.3 D866

$$y = \sin(\sin(\sin x))$$

$$y' = \cos(\sin(\sin x))\cdot\cos(\sin x)\cdot\cos x$$

1.4 D870

$$y = \frac{\sin x - x\cos x}{\cos x + x\sin x}$$

$$y' = \frac{(\cos x + x\sin x - \cos x)(\cos x + x\sin x) - (\sin x - x\cos x)(-\sin x + \cos x + \sin x)}{(\cos x + x\sin x)^2}$$

$$y' = \frac{x\sin x(\cos x + x\sin x) - x\cos x(\sin x - x\cos x)}{(\cos x + x\sin x)^2}$$

$$y' = \frac{x^2}{(\cos x + x\sin x)^2}$$

1.5 D879

$$y = \left(\frac{1-x^2}{2}\sin(x) - \frac{(1-x)^2}{2}\cos(x)\right)e^{-x}$$

$$(e^{-x})' = -e^{-x}$$

$$\left(\frac{1}{2}\cdot\big((1-x^2)\sin x - (1-2x+x^2)\cos x\big)\right)' =$$

$$= \frac{1}{2}\cdot\big(-2x\sin x + (1-x^2)\cos x - (-2+2x)\cos x + (1-2x+x^2)\sin x\big) =$$

$$= \frac{1}{2}\cdot\big((1-4x+x^2)\sin x - (x^2-3+2x)\cos x\big)$$

$$y' = \left(\frac{1}{2}\cdot\big((1-4x+x^2)\sin x - (x^2-3+2x)\cos x\big)\right)\cdot e^{-x} - \left(\frac{1}{2}\cdot\big((1-x^2)\sin x - (x-1)^2\cos x\big)\right)e^{-x} =$$

$$= \frac{e^{-x}}{2}\cdot\big((1-4x+x^2-1+x^2)\sin x + (-x^2+3-2x+x^2-2x+1)\cos x\big) =$$

$$= \frac{e^{-x}}{2}\cdot\big((2x^2-4x)\sin x + (-4x+4)\cos x\big)$$

$$= e^{-x}((x^2-2x)\sin x + (2-2x)\cos x)$$

1.6 D881

$$y = \frac{\ln 3\sin x + \cos x}{3^x}$$

$$y' = \frac{(\ln 3\cos x - \sin x)3^x - (\ln 3\sin x + \cos x)3^x\ln 3}{9^x}$$

$$y' = \frac{\ln 3\cos x - \sin x - \ln^2 3\sin x - \ln 3\cos x}{3^x}$$

$$y' = -\frac{\sin x(1+\ln^2 3)}{3^x}$$

1.7 D884

$$y = \left(\frac{a}{b}\right)^x \left(\frac{b}{x}\right)^a \left(\frac{x}{a}\right)^b and (a > 0, b > 0)$$

$$y = \frac{b^a}{a^b} \left(\frac{a}{b}\right)^x x^{b-a}$$

$$y' = \frac{b^a}{a^b} \left(\left(\frac{a}{b}\right)^x \ln\frac{a}{b} \cdot x^{b-a} + \left(\frac{a}{b}\right)^x \cdot (b-a) x^{b-a-1}\right)$$

$$y' = \ln\left(\frac{a}{b}\right) a^{x-b} b^{a-x} x^{b-a} + a^{x-b} b^{a-x+1} x^{b-a-1} - a^{x-b+1} b^{a-x} x^{b-a-1}$$

1.8 D886

$$y = \lg^3 x^3$$

$$y' = 3 \lg^2 x^3 \cdot \frac{1}{x^3 \ln 10} \cdot 3x^2$$

$$y' = \frac{9x^2 \lg^2 x^3 \lg e}{x^3}$$

1.9 D888

$$y = \ln(\ln^2(\ln^3 x))$$

$$y' = \frac{1}{\ln^2(\ln^3 x)} \cdot 2\ln(\ln^3 x) \frac{1}{\ln^3 x} \cdot 3\ln^2 x \frac{1}{x}$$

$$y' = \frac{6}{\ln(\ln^3 x) \cdot \ln x \cdot x}$$

1.10 D891

$$y = \frac{1}{4(1+x^4)} + \frac{1}{4} \ln\frac{x^4}{1+x^4}$$

$$y' = \frac{1}{4} \cdot \left(-\frac{4x^3}{(1+x^4)^2} + \frac{1+x^4}{x^4} \frac{4x^3(1+x^4) - x^4 \cdot 4x^3)}{(1+x^4)^2}\right)$$

$$y' = -\frac{x^3}{(1+x^4)^2} + \frac{1}{x(1+x^4)}$$

$$y' = \frac{1}{x(1+x^4)^2}$$

1.11 D895

$$y = \ln(x + \sqrt{x^2+1})$$

$$y' = \frac{1}{x + \sqrt{x^2+1}} \cdot (1 + \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x)$$

$$y' = \frac{1 + \frac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

1.12 D900

$$y = \frac{2+3x^2}{x^4}\sqrt{1-x^2} + 3\ln\frac{1+\sqrt{1-x^2}}{x}$$

$$\left(\frac{2+3x^2}{x^4}\right)' = \frac{6x\cdot x^4-(2+3x^2)\cdot 4x^3}{x^8} = \frac{6x^2-8-12x^2}{x^5} = \frac{-8-6x^2}{x^5}$$

$$\left(\sqrt{1-x^2}\right)' = \frac{1}{2\sqrt{1-x^2}}\cdot -2x = \frac{-x}{\sqrt{1-x^2}}$$

$$y' = \frac{-8-6x^2}{x^5}\cdot\sqrt{1-x^2} + \frac{2+3x^2}{x^4}\cdot\frac{-x}{\sqrt{1-x^2}} + 3\cdot\frac{x}{1+\sqrt{1-x^2}}\cdot\frac{\frac{-x}{\sqrt{1-x^2}}\cdot x-(1+\sqrt{1-x^2})\cdot 1}{x^2}$$

$$y' = \frac{(-8-6x^2)\cdot\sqrt{1-x^2}}{x^5} - \frac{2+3x^2}{x^3\cdot\sqrt{1-x^2}} + 3\cdot\frac{\frac{-x^2}{\sqrt{1-x^2}}-(1+\sqrt{1-x^2})}{x\cdot(1+\sqrt{1-x^2})}$$

$$y' = \frac{(-8-6x^2)\cdot\sqrt{1-x^2}}{x^5} - \frac{2+3x^2}{x^3\cdot\sqrt{1-x^2}} + 3\cdot\frac{\frac{-x^2-\sqrt{1-x^2}-1+x^2}{\sqrt{1-x^2}}}{x\cdot(1+\sqrt{1-x^2})}$$

$$y' = \frac{(-8-6x^2)\cdot\sqrt{1-x^2}}{x^5} - \frac{2+3x^2}{x^3\cdot\sqrt{1-x^2}} - \frac{3}{x\cdot\sqrt{1-x^2}}$$

$$y' = \frac{-8-6x^2+8x^2+6x^4-2x^2-3x^4-3x^4}{x^5\cdot\sqrt{1-x^2}}$$

$$y' = -\frac{8}{x^5\cdot\sqrt{1-x^2}}$$

1.13 D902

$$y = \ln\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$y' = \frac{1}{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\frac{1}{\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\frac{1}{2}$$

$$y' = \frac{1}{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cdot\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\cdot 2}$$

$$y' = \frac{1}{\sin\left(x+\frac{\pi}{2}\right)} = \frac{1}{\cos x}$$

1.14 D905

$$y = -\frac{\cos x}{2\sin^2 x} + \ln\sqrt{\frac{1+\cos x}{\sin x}}$$

$$y' = \frac{\sin x \cdot 2\sin^2 x + \cos x \cdot 4\sin x \cdot \cos x}{4\sin^4 x} + \frac{\sqrt{\sin x}}{\sqrt{1+\cos x}} \cdot \frac{1}{2} \cdot \frac{\sqrt{\sin x}}{\sqrt{1+\cos x}} \cdot \frac{-\sin x \cdot \sin x - (1+\cos x)\cdot\cos x}{\sin^2 x}$$

$$y' = \frac{2+2\cos^2 x}{4\sin^3 x} - \frac{1+\cos x}{\sin x \cdot 2(1+\cos x)}$$

$$y' = \frac{1+\cos^2 x}{2\sin^3 x} - \frac{1}{\sin x \cdot 2}$$

$$y' = \frac{1+\cos^2 x - \sin^2 x}{2\sin^3 x} = \frac{\cos^2 x}{\sin^3 x}$$

1.15 D906

$$y = \ln\frac{b + a\cos x + \sqrt{b^2-a^2}\sin x}{a + b\cos x}$$

$$y = \ln\frac{1 + \frac{a}{b}\cos x + \frac{\sqrt{b^2-a^2}}{b}\sin x}{\frac{a}{b} + \cos x}$$

$$\textit{Let}\ \ \frac{a}{b} = k = \sin\phi\ \ \textit{then}\ \ \frac{\sqrt{b^2-a^2}}{b} = \cos\phi$$

$$y = \ln\frac{1+\sin\phi\cos x + \cos\phi\sin x}{k+\cos x} = \ln\frac{1+\sin(\phi+x)}{k+\cos x}$$

$$y' = \frac{k+\cos x}{1+\sin(\phi+x)} \cdot \frac{\cos(\phi+x)\cdot(k+\cos x) + (1+\sin(\phi+x))\cdot\sin x}{(k+\cos x)^2}$$

$$y' = \frac{\cos(\phi+x)\cdot(\sin\phi+\cos x) + (1+\sin(\phi+x))\cdot\sin x}{(1+\sin(\phi+x))\cdot(\sin\phi+\cos x)}$$

$$y' = \frac{\cos(\phi+x)\sin\phi + \cos(\phi+x)\cos x + \sin x + \sin(\phi+x)\sin x}{\sin\phi + \sin(\phi+x)\sin\phi + \cos x + \cos x\sin(\phi+x)}$$

$$y' = \frac{\cos\phi + \sin x + \cos(\phi+x)\sin\phi}{(1+\sin(\phi+x))\cdot(\sin\phi+\cos x)}$$

Возможно, эту производную можно представить покрасивее, но я без понятия как, поэтому так и оставлю

1.16 D907

$$y = \frac{1}{x}(\ln^3 x + 3\ln^2 x + 6\ln x + 6)$$

$$y' = -\frac{\ln^3 x + 3\ln^2 x + 6\ln x + 6}{x^2} + \left(\frac{3\ln^2 x}{x} + \frac{6\ln x}{x} + \frac{6}{x}\right) \cdot \frac{1}{x}$$

$$y' = \frac{3\ln^2 x + 6\ln x + 6 - \ln^3 x - 3\ln^2 x - 6\ln x - 6}{x^2}$$

$$y' = \frac{-\ln^3 x}{x^2}$$

1.17 D909

$$y = \frac{3}{2}(1 - \sqrt[3]{1+x^2}) + 3\ln(1 + \sqrt[3]{1+x^2})$$

$$y' = \frac{3}{2} \cdot \left(-\frac{1}{3}\left(1+x^2\right)^{\frac{-2}{3}} \cdot 2x\right) + 3\frac{1}{1+\sqrt[3]{1+x^2}} \cdot \left(\frac{1}{3}\left(1+x^2\right)^{\frac{-2}{3}} \cdot 2x\right)$$

$$y' = -(1+x^2)^{\frac{-2}{3}} \cdot x + \frac{\left(1+x^2\right)^{\frac{-2}{3}} \cdot 2x}{1+\sqrt[3]{1+x^2}}$$

$$y' = \left(\left(1+x^2\right)^{\frac{-2}{3}} \cdot x\right)\left(\frac{1-\sqrt[3]{1+x^2}}{1+\sqrt[3]{1+x^2}}\right)$$

$$y' = \frac{x}{(1+x^2)^{\frac{2}{3}}} \cdot \frac{1-\sqrt[3]{1+x^2}}{1+\sqrt[3]{1+x^2}}$$

$$y' = \frac{x - x\sqrt[3]{1+x^2}}{(1+x^2)^{\frac{2}{3}} + (1+x^2)}$$

1.18 D914

$$y = \arccos\frac{1-x}{\sqrt{2}}$$

$$y' = -\frac{1}{\sqrt{1-\frac{(1-x)^2}{2}}} \cdot \left(-\frac{1}{\sqrt{2}}\right)$$

$$y' = \frac{1}{\sqrt{2-(1-x)^2}}$$

$$y' = \frac{1}{\sqrt{-x^2+2x+1}}$$

1.19 D919

$$y = x \arcsin \sqrt{\frac{x}{1+x}} + \arctan \sqrt{x} - \sqrt{x}$$

$$\left(\arcsin \sqrt{\frac{x}{1+x}}\right)' = \frac{1}{\sqrt{1-\frac{x}{1+x}}} \cdot \frac{\sqrt{1+x}}{2\sqrt{x}} \cdot \frac{1+x-x}{(1+x)^2} = \frac{1}{\sqrt{\frac{1}{1+x}}} \cdot \frac{\sqrt{1+x}}{2\sqrt{x}} \cdot \frac{1}{(1+x)^2} = \sqrt{1+x} \cdot \frac{\sqrt{1+x}}{2\sqrt{x}} \cdot \frac{1}{(1+x)^2}$$

$$\left(\arcsin \sqrt{\frac{x}{1+x}}\right)' = \frac{1}{2\sqrt{x}(1+x)}$$

$$\left(\arctan \sqrt{x}\right)' = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \frac{x}{2\sqrt{x}(1+x)} + \arcsin \sqrt{\frac{x}{1+x}} + \frac{1}{(1+x) \cdot 2\sqrt{x}} - \frac{1}{2\sqrt{x}}$$

$$y' = \arcsin \sqrt{\frac{x}{1+x}} = \arctan \sqrt{x}$$

1.20 D921

$$y = \arcsin(\sin x)$$

$$y' = \frac{\cos x}{\sqrt{1-\sin^2 x}} = \frac{\cos x}{|\cos x|} = 1 \;\; if \cos x > 0 \;\; and \;\; -1 \;\; if \cos x < 0$$

1.21 D928

$$y = \arcsin \frac{1-x^2}{1+x^2}$$

$$y' = \frac{1}{\sqrt{\frac{(1+x^2)^2-(1-x^2)^2}{(1+x^2)^2}}} \cdot \frac{-2x \cdot (1+x^2) - (1-x^2) \cdot 2x}{(1+x^2)^2}$$

$$y' = \frac{|1+x^2|}{\sqrt{(1+x^2-1+x^2)(1+x^2+1-x^2)}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$y' = \frac{|1+x^2|}{|2x|} \cdot \frac{-4x}{(1+x^2)^2} = -\frac{2|x|}{x+x^3}$$

1.22 D940

$$y = \frac{\arcsin x}{\sqrt{1-x^2}} + \frac{1}{2} \ln \frac{1-x}{1+x}$$

$$y' = \frac{1 - \arcsin x \cdot \frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} + \frac{1}{2} \cdot \frac{1+x}{1-x} \cdot \frac{-1-x-1+x}{(1+x)^2}$$

$$y' = \frac{\sqrt{1-x^2} + x \arcsin x}{(1-x^2)^{\frac{3}{2}}} - \frac{1}{1-x^2} = \frac{x \arcsin x}{(1-x^2)^{\frac{3}{2}}}$$

1.23 D943

$$y = \ln \frac{1-\sqrt[3]{x}}{\sqrt{1+\sqrt[3]{x}+\sqrt[3]{x^2}}} + \sqrt{3}\arctan\frac{1+2\sqrt[3]{x}}{\sqrt{3}}$$

$$y = \ln \frac{1-x^{1/3}}{(1+x^{1/3}+x^{2/3})^{1/2}} + \sqrt{3}\arctan\frac{1+2x^{1/3}}{\sqrt{3}}$$

$$\left(1-x^{1/3}\right)' = -\frac{1}{3}x^{-2/3} = -\frac{1}{3x^{2/3}}$$

$$(1+x^{1/3}+x^{2/3})' = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-1/3} = \frac{1}{3x^{2/3}} + \frac{2}{3x^{1/3}} = \frac{1+2x^{1/3}}{3x^{2/3}}$$

$$\left((1+x^{1/3}+x^{2/3})^{1/2}\right)' = \frac{1}{2(1+x^{1/3}+x^{2/3})^{1/2}} \cdot \frac{1+2x^{1/3}}{3x^{2/3}} = \frac{1+2x^{1/3}}{6x^{2/3}(1+x^{1/3}+x^{2/3})^{1/2}}$$

$$\left(\ln\frac{1-x^{1/3}}{(1+x^{1/3}+x^{2/3})^{1/2}}\right)' = \frac{(1+x^{1/3}+x^{2/3})^{1/2}}{1-x^{1/3}} \cdot \frac{-\frac{(1+x^{1/3}+x^{2/3})^{1/2}}{3x^{2/3}} - \frac{(1-x^{1/3})(1+2x^{1/3})}{6x^{2/3}(1+x^{1/3}+x^{2/3})^{1/2}}}{1+x^{1/3}+x^{2/3}} =$$

$$= -\frac{\frac{2(1+x^{1/3}+x^{2/3})+(1-x^{1/3})(1+2x^{1/3})}{6x^{2/3}(1+x^{1/3}+x^{2/3})^{1/2}}}{(1-x^{1/3})(1+x^{1/3}+x^{2/3})^{1/2}} = -\frac{2+2x^{1/3}+2x^{2/3}+1-x^{1/3}+2x^{1/3}-2x^{2/3}}{6x^{2/3}(1-x^{1/3})(1+x^{1/3}+x^{2/3})} =$$

$$= -\frac{3+3x^{1/3}}{6x^{2/3}(1-x^{1/3})(1+x^{1/3}+x^{2/3})} = -\frac{1+x^{1/3}}{2x^{2/3}(1-x^{1/3})(1+x^{1/3}+x^{2/3})} =$$

$$= -\frac{1+x^{1/3}}{2x^{2/3}(1+x^{1/3}+x^{2/3}-x^{1/3}-x^{2/3}-x)} = \frac{1+x^{1/3}}{2x^{2/3}(x-1)}$$

$$\left(\sqrt{3}\arctan\frac{1+2x^{1/3}}{\sqrt{3}}\right)' = \sqrt{3} \cdot \frac{3\cdot\frac{2}{3\sqrt{3}}x^{-2/3}}{4(1+x^{2/3}+x^{4/3})} = \frac{1}{2(x^{2/3}+x^{4/3}+x^2)} = \frac{1}{2x^{2/3}(1+x^{2/3}+x^{4/3})}$$

$$y' = \frac{1+x^{1/3}}{2x^{2/3}(x-1)} + \frac{1}{2x^{2/3}(1+x^{2/3}+x^{4/3})} = \frac{1+x^{1/3}+x^{2/3}+x^{1/3}+x^{2/3}+x+x-1}{2x^{2/3}(1+x^{1/3}+x^{2/3})(x-1)} =$$

$$\frac{2x^{1/3}(1+x^{1/3}+x^{2/3})}{2x^{2/3}(1+x^{1/3}+x^{2/3})(x-1)} = \frac{1}{x^{1/3}(x-1)}$$

1.24 D945

$$y = \arctan\frac{a-2x}{2\sqrt{ax-x^2}}$$

$$y' = \frac{4(ax-x^2)}{a^2-4ax+4x^2+4ax-4x^2} \cdot \frac{-4\sqrt{ax-x^2}-(a-2x)\cdot\frac{a-2x}{\sqrt{ax-x^2}}}{4(ax-x^2)}$$

$$y' = \frac{-4ax+4x^2-a^2+4ax-4x^2}{a^2\sqrt{ax-x^2}} = \frac{-a^2}{a^2\sqrt{ax-x^2}} = -\frac{1}{\sqrt{ax-x^2}}$$

1.25 D950

$$y = x \arctan x - \frac{1}{2}\ln(1+x^2) - \frac{1}{2}\arctan^2 x$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(x\arctan x)' = \arctan x + \frac{x}{1+x^2}$$

$$\left(\frac{1}{2}\ln(1+x^2)\right)' = \frac{2x}{2(1+x^2)} = \frac{x}{1+x^2}$$

$$\left(\frac{1}{2}\arctan^2 x\right)' = \frac{1}{2}\cdot 2\arctan x \cdot \frac{1}{1+x^2} = \frac{\arctan x}{1+x^2}$$

$$y' = \arctan x + \frac{x}{1+x^2} - \frac{x}{1+x^2} - \frac{\arctan x}{1+x^2} = \frac{\arctan x + x^2\arctan x - \arctan x}{1+x^2}$$

$$y' = \frac{x^2 \arctan x}{1+x^2}$$

1.26 D953

$$y = \arcsin\left(\frac{\sin a \sin x}{1 - \cos a \cos x}\right)$$

$$y' = \frac{1-\cos a\cos x}{\sqrt{(1-\cos a\cos x)^2 - \sin^2 a \sin^2 x}} \cdot \frac{\sin a\cos x\cdot(1-\cos a\cos x) - \sin a\sin x\cdot\cos a\sin x}{(1-\cos a\cos x)^2}$$

$$y' = \frac{\sin a\cos x - \sin a\cos a\cos^2 x - \sin a\cos a\sin^2 x}{\sqrt{(1-\cos a\cos x)^2 - \sin^2 a\sin^2 x}\cdot(1-\cos a\cos x)} = \frac{\sin a(\cos x - \cos a)}{\sqrt{(1-\cos a\cos x)^2 - \sin^2 a\sin^2 x}\cdot(1-\cos a\cos x)}$$

$$(1-\cos a\cos x)^2 - \sin^2 a\sin^2 x = 1 - 2\cos a\cos x + \cos^2 a\cos^2 x - (1-\cos^2 a)(1-\cos^2 x) =$$

$$= 1 - 2\cos a\cos x + \cos^2 a\cos^2 x - (1 - \cos^2 a - \cos^2 x + \cos^2 a\cos^2 x) =$$

$$= 1 - 2\cos a\cos x + \cos^2 a\cos^2 x - 1 + \cos^2 a + \cos^2 x - \cos^2 a\cos^2 x =$$

$$= -2\cos a\cos x + \cos^2 a + \cos^2 x = (\cos x - \cos a)^2$$

$$y' = \frac{\sin a(\cos x - \cos a)}{|\cos x - \cos a|\cdot(1-\cos a\cos x)}$$

1.27 D954

$$y = \frac{1}{4\sqrt{3}} \ln \frac{\sqrt{x^2+2} - x\sqrt{3}}{\sqrt{x^2+2} + x\sqrt{3}} + \frac{1}{2} \arctan \frac{\sqrt{x^2+2}}{x}$$

$$\left(\frac{1}{4\sqrt{3}} \ln \frac{\sqrt{x^2+2} - x\sqrt{3}}{\sqrt{x^2+2} + x\sqrt{3}} \right)' =$$

$$= \frac{1}{4\sqrt{3}} \frac{\sqrt{x^2+2} + x\sqrt{3}}{\sqrt{x^2+2} - x\sqrt{3}} \frac{\left(\frac{x}{\sqrt{x^2+2}} - \sqrt{3} \right)(\sqrt{x^2+2} + x\sqrt{3}) - \left(\frac{x}{\sqrt{x^2+2}} + \sqrt{3} \right)(\sqrt{x^2+2} - x\sqrt{3})}{(\sqrt{x^2+2} + x\sqrt{3})^2} =$$

$$= \frac{1}{4\sqrt{3}} \cdot \frac{x + \frac{x^2\sqrt{3}}{\sqrt{x^2+2}} - \sqrt{3(x^2+2)} - 3x - x + \frac{x^2\sqrt{3}}{\sqrt{x^2+2}} - \sqrt{3(x^2+2)} + 3x}{x^2 + 2 - 3x^2} =$$

$$= -\frac{\frac{2x^2\sqrt{3} - 2\sqrt{3}(x^2+2)}{\sqrt{x^2+2}}}{8\sqrt{3}(x^2-1)} = -\frac{-2}{4(x^2-1)\sqrt{x^2+2}} = \frac{1}{2(x^2-1)\sqrt{x^2+2}}$$

$$\left(\frac{1}{2} \arctan \frac{\sqrt{x^2+2}}{x} \right)' =$$

$$= \frac{1}{2} \cdot \frac{x^2}{2x^2+2} \cdot \frac{\frac{x^2}{\sqrt{x^2+2}} - \sqrt{x^2+2}}{x^2} = \frac{-2}{4(x^2+1)\sqrt{x^2+2}} = -\frac{1}{2(x^2+1)\sqrt{x^2+2}}$$

$$y' = \frac{1}{(x^4-1)\sqrt{x^2+2}}$$

1.28 D957

$$y = \arccos(\sin^2 x - \cos^2 x) = \pi - \arccos(\cos 2x)$$

$$y' = \frac{-2\sin 2x}{\sqrt{1 - \cos^2 2x}} = -\frac{2\sin 2x}{|\sin 2x|}$$

1.29 D960

$$y = \arctan e^x - \ln \sqrt{\frac{e^{2x}}{e^{2x}+1}}$$

$$y' = \frac{e^x}{1+e^{2x}} - \sqrt{\frac{e^{2x}+1}{e^{2x}}} \cdot \frac{\sqrt{e^{2x}+1}}{2\sqrt{e^{2x}}} \cdot \frac{2e^{2x} \cdot (e^{2x}+1) - e^{2x} \cdot 2e^{2x}}{(e^{2x}+1)^2}$$

$$y' = \frac{e^x}{1+e^{2x}} - \frac{1}{e^{2x}+1} = \frac{e^x - 1}{e^{2x}+1}$$

1.30 D971

$$y = \frac{b}{a}x + \frac{2\sqrt{a^2-b^2}}{a}\arctan\left(\frac{a-b}{a+b}\,\mathrm{th}\,\frac{x}{2}\right)$$

$$y' = \frac{b}{a}+\frac{2\sqrt{a^2-b^2}}{a}\cdot\frac{\frac{a-b}{(a+b)2\,\mathrm{ch}^2\,\frac{x}{2}}}{1+\left(\frac{a-b}{a+b}\,\mathrm{th}\,\frac{x}{2}\right)^2} = \frac{b}{a}+\frac{2\sqrt{a^2-b^2}\cdot(a-b)}{a(a+b)2\,\mathrm{ch}^2\,\frac{x}{2}\cdot\left(1+\left(\frac{a-b}{a+b}\,\mathrm{th}\,\frac{x}{2}\right)^2\right)}$$

$$y' = \frac{b}{a} + \frac{\sqrt{a^2-b^2}\cdot(a-b)}{a\left((a+b)\,\mathrm{ch}^2\,\frac{x}{2} + \frac{(a-b)^2}{a+b}\cdot\mathrm{sh}^2\,\frac{x}{2}\right)}$$

2 Kudryavtsev

2.1 K13.54

$$y = (a+bx)^a$$

$$y' = ab(a+bx)^{a-1}$$

2.2 K13.56

$$y = \frac{\cos 3}{2}x - \frac{1}{4}\sin(2x+3)$$

$$y' = \frac{\cos 3}{2} - \frac{1}{2}\cos(2x+3)$$

2.3 K13.66

$$y = \frac{\sqrt{x^3}+\sqrt{xa^2}-\sqrt{x^2a}-\sqrt{a^3}}{\sqrt[4]{a^5}+\sqrt[4]{ax^4}-\sqrt[4]{a^4x}-\sqrt[4]{x^5}} = \frac{x^{3/2}+|a|x^{1/2}-a^{1/2}|x|-a^{3/2}}{a^{5/4}+a^{1/4}|x|-|a|x^{1/4}-x^{5/4}}$$

Заметим, что a $>$ 0 и x $>$ 0 по ОДЗ, значит |a| = a, |x| = x:

$$y = \frac{x^{3/2}+ax^{1/2}-a^{1/2}x-a^{3/2}}{a^{5/4}+a^{1/4}x-ax^{1/4}-x^{5/4}}$$

$$y' = \frac{1}{\left(a^{5/4}+a^{1/4}x-ax^{1/4}-x^{5/4}\right)^2}\cdot$$

$$\cdot\left(\frac{3}{2}x^{1/2}+\frac{1}{2}ax^{-1/2}-a^{1/2}\right)\left(a^{5/4}+a^{1/4}x-ax^{1/4}-x^{5/4}\right)-$$

$$-\left(x^{3/2}+ax^{1/2}-a^{1/2}x-a^{3/2}\right)\left(a^{1/4}-\frac{1}{4}ax^{-3/4}-\frac{5}{4}x^{1/4}\right)$$

2.4 K13.68

$$y = \cot x^2 - \frac{1}{3}\tan^3 2x$$

$$y' = -\frac{2x}{\sin^2 x^2} - \frac{1}{3} \cdot 3\tan^2 2x \cdot \frac{1}{\cos^2 2x} \cdot 2$$

$$y' = -\frac{2x}{\sin^2 x^2} - \frac{2\tan^2 2x}{\cos^2 2x}$$

2.5 K13.71

$$y = 2^{\sin 2x}$$

$$y' = 2^{\sin 2x} \cdot \ln 2 \cdot \cos 2x \cdot 2$$

2.6 K13.73

$$y = \frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x}$$

$$y' = \frac{2\sin x\cos x(1+\cot x)+1}{(1+\cot x)^2} + \frac{-2\cos x\sin(1+\tan x)-1}{(1+\tan x)^2}$$

$$y' = \sin 2x\left(\frac{1}{1+\cot x} - \frac{1}{1+\tan x}\right) + \left(\frac{1}{(1+\cot x)^2} - \frac{1}{(1+\tan x)^2}\right)$$

$$y' = \sin 2x \cdot \frac{\sin x - \cos x}{\sin x + \cos x} + \frac{\sin^2 x - \cos^2 x}{(\sin x + \cos x)^2}$$

$$y' = \frac{\sin 2x(\sin^2 x - \cos^2 x) - \cos 2x}{1+\sin 2x}$$

$$y' = -\cos 2x$$

2.7 K13.83

$$y = 10^{x/\log_3 x}$$

$$y' = 10^{x/\log_3 x} \cdot \ln(x/\log_3 x) \cdot \frac{\log_3 x - x \cdot (x\ln 3)^{-1}}{log_3^2 x}$$

2.8 K13.84

$$y = \frac{2}{\sqrt{31}}\arctan\frac{4x-5}{\sqrt{31}}$$

$$y' = \frac{2}{\sqrt{31}} \cdot \frac{31}{(4x-5)^2+31} \cdot \frac{4}{\sqrt{31}} = \frac{8}{(4x-5)^2+31} = \frac{1}{2x^2-5x+7}$$

2.9 K13.99

$$y = \cos(3\arccos x)$$

$$y' = \sin(3\arccos x) \cdot \frac{3}{\sqrt{1-x^2}}$$

2.10 K13.102

$$y = \ln\left(\frac{x+3}{x+2}\right)^2 - \frac{2x+5}{(x+2)(x+3)} = 2\ln\left(\frac{x+3}{x+2}\right) - \frac{2x+5}{(x+2)(x+3)}$$

$$y' = \frac{2(x+2)}{x+3} \cdot \frac{x+2-x-3}{(x+2)^2} - \frac{2x^2+10x+12-4x^2-20x-25}{(x+2)^2(x+3)^2}$$

$$y' = -\frac{2}{(x+2)(x+3)} + \frac{2x^2+10x+13}{(x+2)^2(x+3)^2}$$

$$y' = \frac{2x^2+10x+13-2x^2-10x-12}{(x+2)^2(x+3)^2}$$

$$y' = \frac{1}{(x+2)^2(x+3)^2}$$

2.11 K13.106

$$y = 3^{\cos^2 x}$$

$$y' = -3^{\cos^2 x} \cdot \ln 3 \cdot \sin 2x$$

2.12 K13.110

$$y = \ln\ln\ln x^2$$

$$y' = \frac{1}{\ln\ln x^2} \cdot \frac{1}{\ln x^2} \cdot \frac{1}{x^2} \cdot 2x = \frac{2}{x\ln x^2 \ln\ln x^2}$$

2.13 K13.115

$$y = e^{\sqrt{\ln(x^2+2x+1)}}$$

$$y' = e^{\sqrt{\ln(x^2+2x+1)}} \cdot \frac{1}{2\sqrt{\ln(x^2+2x+1)}} \cdot \frac{1}{x^2+2x+1} \cdot (2x+2) = \frac{e^{\sqrt{\ln(x^2+2x+1)}}}{\sqrt{\ln(x^2+2x+1)}(x+1)}$$

2.14 K13.116

$$y = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$$

$$y' = \frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}} \cdot \frac{\sqrt{1+\sin x}}{2\sqrt{1-\sin x}} \cdot \frac{-\cos x(1+\sin x) - (1-\sin x)\cos x}{(1+\sin x)^2}$$

$$y' = -\frac{\cos x}{1-\sin^2 x} = \frac{\cos x}{\sin^2 x - 1}$$

2.15 K13.119

$$y = \ln\tan\left(\frac{\pi}{4} + \frac{\sin x}{2}\right)$$

$$y' = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{\sin x}{2}\right)} \cdot \frac{16}{16 + (\pi + 2\sin x)^2} \cdot \frac{1}{2}\cos x$$

$$y' = \frac{8\cos x}{\tan\left(\frac{\pi}{4} + \frac{\sin x}{2}\right) \cdot (16 + (\pi + 2\sin x)^2)}$$

2.16 K13.130

$$y = \operatorname{th} x + \frac{\sqrt{2}}{4}\ln\frac{1 + \sqrt{2}\operatorname{th} x}{1 - \sqrt{2}\operatorname{th} x}$$

$$y' = \frac{1}{\operatorname{ch}^2 x} + \frac{\sqrt{2}}{4} \cdot \frac{1 - \sqrt{2}\operatorname{th} x}{1 + \sqrt{2}\operatorname{th} x} \cdot \frac{\frac{\sqrt{2}}{\operatorname{ch}^2 x}(1 - \sqrt{2}\operatorname{th} x) + \frac{\sqrt{2}}{\operatorname{ch}^2 x}(1 + \sqrt{2}\operatorname{th} x)}{(1 - \sqrt{2}\operatorname{th} x)^2}$$

$$y' = \frac{1}{\operatorname{ch}^2 x} + \frac{1}{\operatorname{ch}^2 x - 2\operatorname{sh}^2 x}$$

2.17 K13.133

$$y = x + \cot x \cdot \ln(1 + \sin x) - \ln\tan(x/2)$$

$$y' = 1 - \frac{\ln(1 + \sin x)}{\sin^2 x} + \cot x \cdot \frac{\cos x}{1 + \sin x} - \frac{1}{2\tan(x/2)\cos^2(x/2)}$$

$$y' = 1 - \frac{\ln(1 + \sin x)}{\sin^2 x} + \frac{\cos^2 x}{\sin x(1 + \sin x)} - \frac{1}{\sin x}$$

2.18 K13.136

$$y = \ln\sqrt{\frac{\sqrt{x^4+1} - \sqrt{2}x}{\sqrt{x^4+1} + \sqrt{2}x}} - \arctan\frac{\sqrt{2}x}{\sqrt{x^4+1}} = \frac{1}{2}\ln\frac{\sqrt{x^4+1} - \sqrt{2}x}{\sqrt{x^4+1} + \sqrt{2}x} - \arctan\frac{\sqrt{2}x}{\sqrt{x^4+1}}$$

$$\left(\frac{1}{2}\ln\frac{\sqrt{x^4+1} - \sqrt{2}x}{\sqrt{x^4+1} + \sqrt{2}x}\right)' =$$

$$= \frac{1}{2} \cdot \frac{\sqrt{x^4+1} + \sqrt{2}x}{\sqrt{x^4+1} - \sqrt{2}x} \cdot \frac{\left(\frac{2x^3}{\sqrt{x^4+1}} - \sqrt{2}\right)(\sqrt{x^4+1} + \sqrt{2}x) - \left(\frac{2x^3}{\sqrt{x^4+1}} + \sqrt{2}\right)(\sqrt{x^4+1} - \sqrt{2}x)}{(\sqrt{x^4+1} + \sqrt{2}x)^2} =$$

$[(a-b)(c+d) - (a+b)(c-d) = ac - bc + ad - bd - ac - bc + ad + bd = 2ad - 2bc]$

$$= \frac{\frac{2\sqrt{2}x^4}{\sqrt{x^4+1}} - \sqrt{2(x^4+1)}}{x^4 + 1 - 2x^2} = \frac{2\sqrt{2}x^4 - \sqrt{2}(x^4+1)}{\sqrt{x^4+1}(x^2-1)^2} = \frac{\sqrt{2}(x^4-1)}{\sqrt{x^4+1}(x^2-1)^2} = \frac{\sqrt{2}(x^2+1)}{\sqrt{x^4+1}(x^2-1)}$$

$$\left(\arctan\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)' = \frac{x^4+1}{(x^2+1)^2}\cdot\frac{\sqrt{2(x^4+1)}-\frac{4x^3\sqrt{2}x}{2\sqrt{x^4+1}}}{x^4+1} = \frac{\sqrt{2(x^4+1)}-\frac{2x^3\sqrt{2}x}{\sqrt{x^4+1}}}{(x^2+1)^2} = \frac{\sqrt{2}(x^4+1)-2\sqrt{2}x^4}{\sqrt{x^4+1}(x^2+1)^2}$$

$$= \frac{\sqrt{2}(1-x^4)}{\sqrt{x^4+1}(x^2+1)^2} = \frac{\sqrt{2}(1-x^2)}{\sqrt{x^4+1}(x^2+1)}$$

$$y' = \frac{\sqrt{2}(x^2+1)}{\sqrt{x^4+1}(x^2-1)} - \frac{\sqrt{2}(1-x^2)}{\sqrt{x^4+1}(x^2+1)} = \frac{2\sqrt{2}(x^4+1)}{\sqrt{x^4+1}(x^4-1)} = \frac{2\sqrt{2}\sqrt{x^4+1}}{x^4-1}$$

2.19 K13.140

$$y = e^x \arcsin\sqrt{\frac{e^x}{e^x+1}} + \arctan\sqrt{e^x} - \sqrt{e^x}$$

$$y' = e^x \arcsin\sqrt{\frac{e^x}{e^x+1}} + e^x\frac{\sqrt{e^x+1}}{\sqrt{e^x+1-e^x}}\cdot\frac{\sqrt{e^x+1}}{2\sqrt{e^x}}\cdot\frac{e^x(e^x+1)-e^{2x}}{(e^x+1)^2} + \frac{1}{1+e^x}\cdot\frac{e^x}{2\sqrt{e^x}} - \frac{e^x}{2\sqrt{e^x}}$$

$$y' = e^x \arcsin\sqrt{\frac{e^x}{e^x+1}} + \frac{\sqrt{e^x}e^x}{2(e^x+1)} + \frac{e^x}{2\sqrt{e^x}}\left(\frac{1-1-e^x}{1+e^x}\right)$$

$$y' = e^x \arcsin\sqrt{\frac{e^x}{e^x+1}} + \frac{\sqrt{e^x}e^x}{2(e^x+1)} - \frac{e^x\sqrt{e^x}}{2(e^x+1)}$$

$$y' = e^x \arcsin\sqrt{\frac{e^x}{e^x+1}}$$

2.20 K13.151

$$y = (\arcsin\sin^2 x)^{\arctan x} = e^{\arctan x(\ln(\arcsin\sin^2 x))}$$

$$(\sin^2 x)' = \sin 2x$$

$$(\arcsin\sin^2 x)' = \frac{\sin 2x}{\sqrt{1-\sin^4 x}}$$

$$(\ln(\arcsin\sin^2 x)))' = \frac{\sin 2x}{\arcsin\sin^2 x\sqrt{1-\sin^4 x}}$$

$$y' = e^{\arctan x(\ln(\arcsin\sin^2 x))}\cdot\left(\frac{\ln(\arcsin\sin^2 x)}{1+x^2} + \frac{\arctan x\sin 2x}{\arcsin\sin^2 x\sqrt{1-\sin^4 x}}\right)$$