ДЗ #4

(1) $A(-4, -6, 0)$ $\vec{BA}\{-4, -3, 2\}$

$B(0, -3, -2)$ $\vec{BC}\{3, 2, -5\}$

$C(3, -1, -7)$ $|\vec{BA}| = \sqrt{29}$ $|\vec{BC}| = \sqrt{38}$

$\cos\angle ABC = \frac{(\vec{BA}, \vec{BC})}{|\vec{BA}|\,|\vec{BC}|} = \frac{-12-6-10}{\sqrt{29}\,\sqrt{38}} \approx$

$\approx -0{,}84$. Ответ. $-0{,}84$.

(2) $\vec{a}\,(-5, 4, 3)$

$\vec{b}\,(-4, -1, -4)$

$\vec{c}\,(0, -5, -4)$

$$\begin{vmatrix} -5 & 4 & 3 \\ -4 & -1 & 4 \\ 0 & -5 & -4 \end{vmatrix} = 5(-20+12) +$$

$+ 4(5+16) = -40 + 84 = 44 \neq 0 \Rightarrow$

$\vec{a}, \vec{b}, \vec{c}$ – некомпланарны $\Rightarrow$ они могут образовать базис на мн-ве векторов. Ответ 1.

(3) $\vec{x}\{-5, 1\}$ $\vec{e_1} = \{1, 1\}$ $\vec{e_2} = \{0, 1\}$

$\begin{cases} -5 = \alpha \\ 1 = \alpha + \beta \end{cases} \Rightarrow \begin{cases} \alpha = -5 \\ \beta = 6 \end{cases}$ Ответ. $\{-5, 6\}$

(4) $\vec{a}\{-1, 2\}$, $\vec{b}\{3, -4\}$ $|\vec{e_1}| = 4$, $|\vec{e_2}| = 2$, $\alpha = (\vec{e_1}, \vec{e_2}) = \frac{5\pi}{6}$ $(\vec{a}\vec{b})$ – ?

$(\vec{a}\vec{b}) = (-\vec{e_1} + 2\vec{e_2})(3\vec{e_1} - 4\vec{e_2}) =$

$= -3|\vec{e_1}|^2 - 8|\vec{e_2}|^2 + 10(\vec{e_1}\vec{e_2}) =$

$= -48 - 32 - 40\sqrt{3} = -80 - 40\sqrt{3} \approx -149{,}28$

$(\vec{e_1}\vec{e_2}) = |\vec{e_1}| \cdot |\vec{e_2}| \cdot \cos\alpha = 4 \cdot 2 \cdot \cos\frac{5\pi}{6} =$

$= -4\sqrt{3}$ Ответ. $-149{,}28$

(5) $Пр_{\vec{a}}(4\vec{a} + 2\vec{b})$? $|\vec{a}| = 5$, $|\vec{b}| = 2\sqrt{2}$

$\angle(\vec{a}, \vec{b}) = \frac{\pi}{4}$

$Пр_{\vec{a}}(4\vec{a} + 2\vec{b}) = \frac{(\vec{a}, 4\vec{a} + 2\vec{b})}{|\vec{a}|} =$

$= \frac{4|\vec{a}|^2 + (\vec{a}, 2\vec{b})}{|\vec{a}|} = \frac{4 \cdot 25 + 5 \cdot 4\sqrt{2} \cdot \frac{\sqrt{2}}{2}}{5} =$

$= 24$ Ответ 24

6) $A(-4, -2, -4)$ $\quad \overrightarrow{AB}(-4, -5, 4)$

$B(-8, -7, 0)$ $\quad \overrightarrow{AC}(-2, -4, 14)$

$C(-6, -6, 10)$ $\quad \overrightarrow{AD}(-2, -2, -2)$

$D(-6, -4, -6)$

$$\begin{vmatrix} -4 & -2 & -4 \\ -8 & -7 & 0 \\ -6 & -6 & 10 \end{vmatrix} = \begin{vmatrix} -4 & 2 & -4 \\ -8 & 1 & 0 \\ -6 & 0 & 10 \end{vmatrix} =$$

~~$= -4(+6) + 0(-4+16) = 120 - 24 =$~~

~~$= 96 \neq 0 \Rightarrow$~~

$$\begin{vmatrix} -4 & -5 & 4 \\ -2 & -4 & 14 \\ -2 & -2 & -2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 8 \\ 0 & -2 & 16 \\ -2 & -2 & -2 \end{vmatrix} =$$

$= 4 + 22 - 26 - 2(-16+16) = 0 \Rightarrow$

$A, B, C, D \in$ одной пл-ти

$AB = \sqrt{57}$ $\quad AD = \sqrt{12}$

$AC = \sqrt{216}$

$\overrightarrow{AB}(-4, -5, 4)$ $\quad AB = \sqrt{57}$

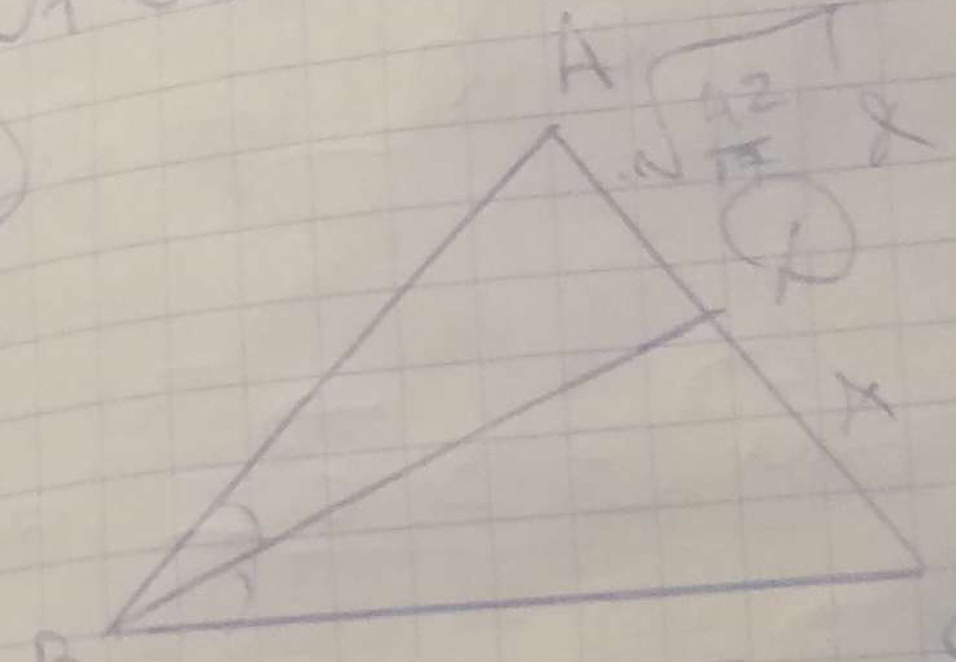
$\overrightarrow{BC}(2, 1, 10)$ $\quad BC = \sqrt{104}$

$\overrightarrow{CD}(0, 2, -16)$ $\quad CD = \sqrt{260}$

$\overrightarrow{AD}(-2, -2, -2)$ $\quad AD = \sqrt{12}$

Ответ: 3

7)



$A(-2, -8, -4)$

$B(2, -3, -3)$

$C(2, -7, -4)$

$D - ?$

По св-ву бис-сы: $\dfrac{AD}{DC} = \dfrac{AB}{BC}$

$\overrightarrow{AB}\{4, 5, 1\}$ $\quad AB = \sqrt{42}$

$\overrightarrow{BC}\{0, -4, -1\}$ $\quad BC = \sqrt{17}$

$\dfrac{AB}{BC} = \sqrt{\dfrac{42}{17}} = \dfrac{AD}{DC}$ $\quad \dfrac{AD}{AC} = \dfrac{\sqrt{\frac{42}{17}}}{\sqrt{\frac{42}{17}} + 1} =$

$\overrightarrow{AC}\{4, 1, 0\}$ $\quad \overrightarrow{AD} = \{4x, x, 0\}$

(7) $D(-2+4\alpha, -8+\alpha, -4) \approx$

$\approx (0{,}44;\ -7{,}39;\ -4)$

(8) $|\vec{a}| = 4, \ |\vec{b}|$, $\alpha = \frac{\pi}{2}$

$(\vec{a}\,\vec{b}) = 0 \qquad |[\vec{a}\,\vec{b}]| = 8$

$|[-4\vec{a} - 2\vec{b},\ 2\vec{a} + 3\vec{b}]| =$

$= |[-4\vec{a}, 2\vec{a}] + [-4\vec{a}, 3\vec{b}] +$

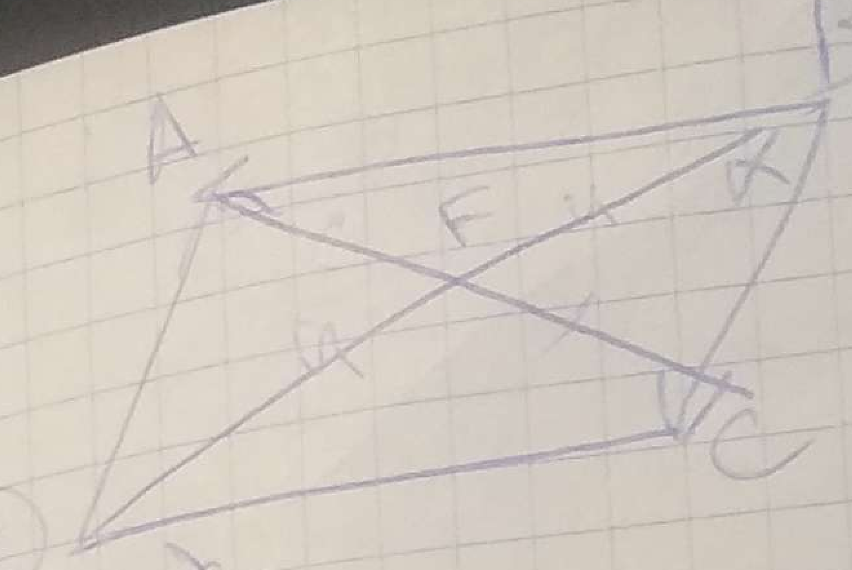
$+ [-2\vec{b}, 2\vec{a}] + [-2\vec{b}, 3\vec{b}]|$

$= |(-12+4)[\vec{a},\vec{b}]| =$

$= 64$ Ответ: 64

(9) $A(1, -3, 3) \quad B(0, 0, -2)$

$C(-2, 0, -6)$



$\vec{BA}\{1, -3, 5\}$

$\vec{BC}\{-2, 0, -4\}$

$|\vec{BA}| = \sqrt{35}$

$|\vec{BC}| = \sqrt{20} = 2\sqrt{5}$

$\cos\alpha = \frac{(\vec{BA}, \vec{BC})}{|\vec{BA}|\cdot|\vec{BC}|} = \frac{-22}{\sqrt{35}\cdot 2\sqrt{5}}$

$\sin\alpha = \sqrt{1 - \frac{22^2}{35\cdot 20}}$

$|[\vec{BA}, \vec{BC}]| = \sqrt{35}\cdot\sqrt{20}\cdot\sqrt{1 - \frac{22^2}{35\cdot 20}} =$

$= \sqrt{35\cdot 20 - 22^2} = \sqrt{216} \approx 14{,}697$

$\frac{224}{\sqrt{216}} \cdot 3 = 12{,}44$

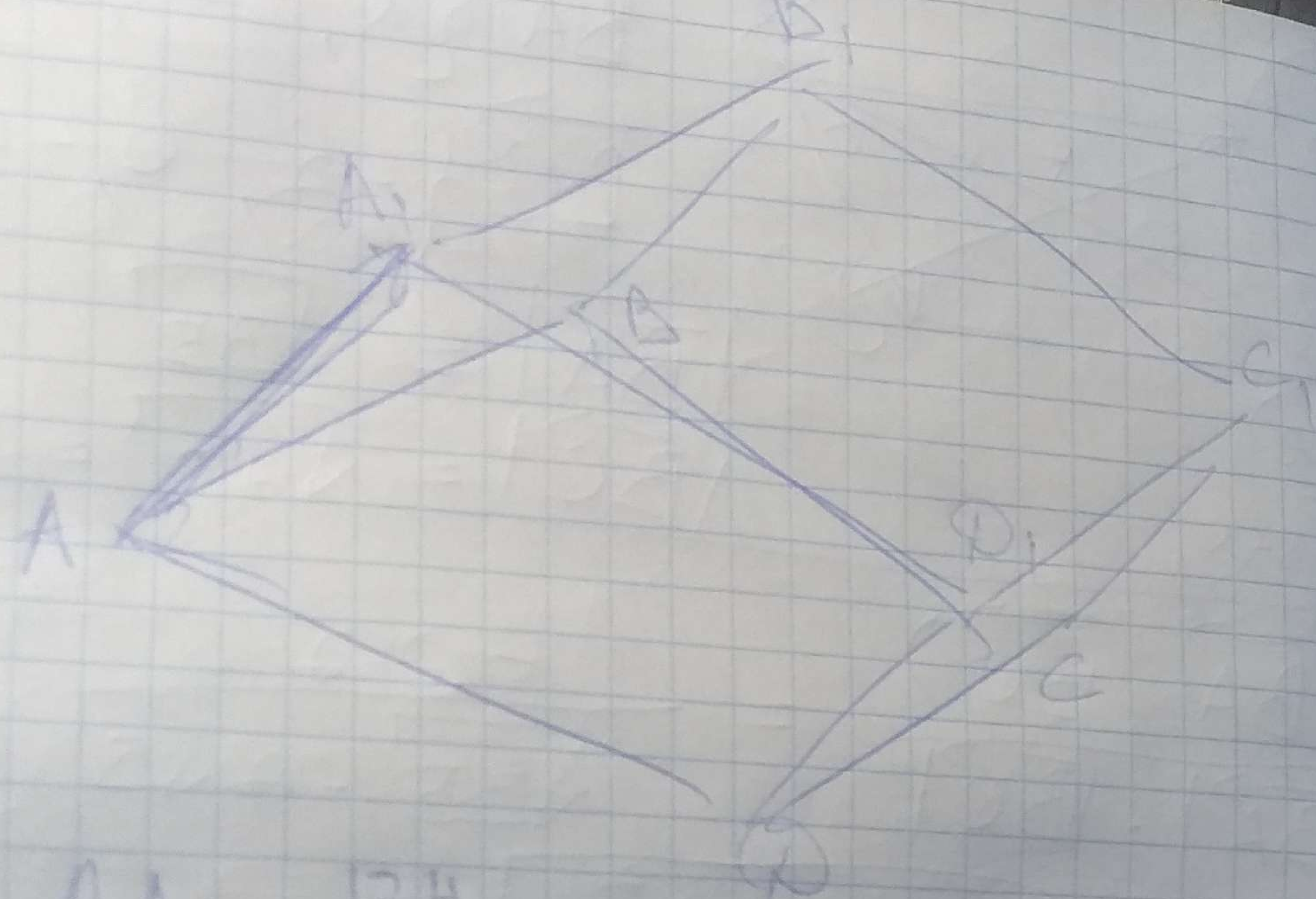
$F(-\frac{1}{2}, -\frac{3}{2}, -\frac{3}{2}) \quad \vec{AD}\{-2, 0, -4\}$

$\vec{CD}\{1, -3, 5\}$

$\vec{BF}\{-\frac{1}{2}, -\frac{3}{2}, \frac{1}{2}\}$

$\vec{BD}\{-1, -3, 1\}$

$D(-1, -3, -1)$



$AA_2 = \frac{124}{\sqrt{216}} = 8.44$

$[\vec{BA}, \vec{BC}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 5 \\ 2 & 0 & -4 \end{vmatrix} =$

$= -12i - \overset{6}{\cancel{14}} j - 6k = \{+12, -\overset{6}{\cancel{14}}, -6\}$

$\frac{\cancel{8.4}\ 124}{\sqrt{216} \cdot \sqrt{216}} = \cancel{6}\,\lambda$

$\vec{AA_2} = -\lambda \cdot [\vec{BA}, \vec{BC}] =$

$= \frac{124}{216} \{+12, -\overset{6}{\cancel{14}}, -6\} =$

$= \{-6.89, +\overset{3.44}{\cancel{8.04}}, +3.44\}$

Ответ. $A_2(-5.89, 0.44, 6.44)$