Контрольная работа №2

Поворот в общем виде

$ax^2 + 2bxy + cx + dy^2 + ey + f = 0$

$$\begin{cases} x = x'\cos\varphi - y'\sin\varphi \\ y = x'\sin\varphi + y'\cos\varphi \end{cases}$$

$a(x'^2\cos^2\varphi - x'y'\sin 2\varphi + y'^2\sin^2\varphi) +$
$+ b(x'^2\sin 2\varphi - y'^2\sin 2\varphi + x'y'\cos 2\varphi) +$
$+ c(x'\cos\varphi - y'\sin\varphi) + d(x'^2\sin^2\varphi + x'y'\sin 2\varphi +$
$+ y'^2\cos^2\varphi) + e(x'\sin\varphi + y'\cos\varphi) + f = 0$

$x'^2(a\cos^2\varphi + b\sin 2\varphi + d\sin^2\varphi) +$
$+ x'y'(\underline{-a\sin 2\varphi + 2b\cos 2\varphi + d\sin 2\varphi}) + \; {=}0$
$+ y'^2(a\sin^2\varphi - b\sin 2\varphi + d\cos^2\varphi) +$
$+ x'(c\cos\varphi + e\sin\varphi) + y'(e\cos\varphi - c\sin\varphi) +$
$+ f = 0$ * Частный случай: $\sin 2\varphi = \sqrt{3}\cos 2\varphi$

$\underline{\varphi = \frac{\pi}{6}}$

$(a-d)\sin 2\varphi = 2b\cos 2\varphi$

Пар. перенос в общем виде без $x'y'$:

$ax'^2 + cx' + dy'^2 + ey' + f = 0$

$$\begin{cases} x' = x'' + \alpha \\ y' = y'' + \beta \end{cases}$$

$a(x''^2 + 2x''\alpha + \alpha^2) + c(x'' + \alpha) +$
$+ d(y''^2 + 2y''\beta + \beta^2) + e(y'' + \beta) + f = 0$

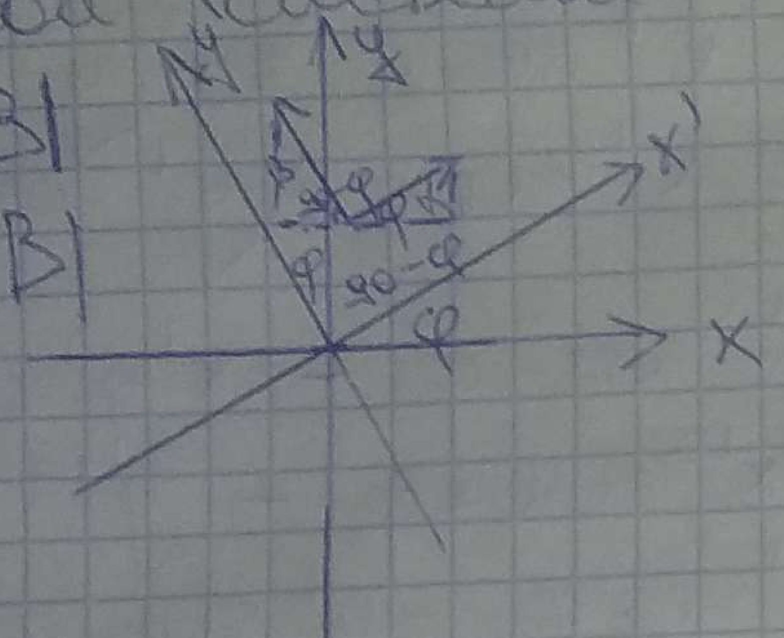
$ax''^2 + dy''^2 + x''(\underbrace{2a\alpha + c}_{=0}) + y''(\underbrace{2d\beta + e}_{=0}) +$
$+ a\alpha^2 + d\beta^2 + c\alpha + e\beta + f = 0$

$$\begin{cases} 2a\alpha + c = 0 \\ 2\beta d + e = 0 \end{cases} \Rightarrow \begin{cases} \alpha = -\dfrac{c}{2a} \\ \beta = -\dfrac{e}{2d} \end{cases}$$

* Центр канонической системы

$\Delta x = \pm\cos\varphi\,|\alpha| \pm \sin\varphi\,|\beta|$

$\Delta y = \pm\sin\varphi\,|\alpha| \pm \cos\varphi\,|\beta|$



① $-3x - 8y = 3$

$\vec{n}(-3; -8)$

② $L_1 \begin{cases} x = 10 + 3t \\ y = -4 - 2t \end{cases}$ $L_2 \begin{cases} x = -14 + 3t \\ y = -12 - 2t \end{cases}$ $L_1 \parallel L_2$

$\dfrac{x-10}{3} = \dfrac{y+4}{-2}$ $\dfrac{x+14}{3} = \dfrac{y+12}{-2}$

③ $L: \dfrac{x+3}{-2} = \dfrac{y-1}{-2} = \dfrac{z-4}{-1}$ $\cancel{M(3, 4, -1)}$

$\cancel{\vec{p}\{-3; 1; 4\}}$

$\vec{s}\{-2; -2; -1\}$

$\vec{n}\{x\ y\ z\}$

$\vec{s}\cdot\vec{n} = 0$

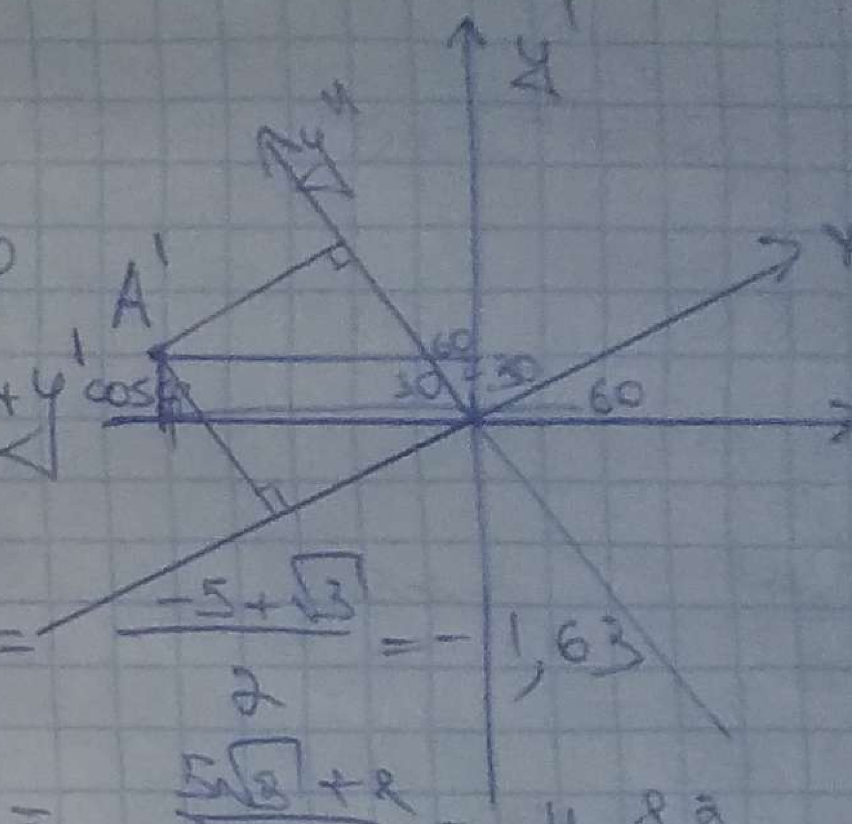
$-2x - 2y - z = 0$

$x = 1 \quad y = 1 \quad z = -4$

$\vec{n}\{1\ 1\ 4\}$

④ $A(-2, -2)$ $\vec{O}(-3; 3)$

$A'(-5; 1)$



$x'' = x'\cos\varphi + y'\sin\varphi$

$y'' = -x'\sin\varphi + y'\cos\varphi$

$x'' = -5\cdot\dfrac{1}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{-5+\sqrt{3}}{2} = -1{,}63$

$y'' = 5\cdot\dfrac{\sqrt{3}}{2} + \dfrac{1}{2} = \dfrac{5\sqrt{3}+1}{2} = 4{,}83$

⑤ $F_1(-14, -1)$ $F_2(10, -1)$ $x = \dfrac{145}{12}$

$2c = 24$ $O(-2; -1)$

$c = 12$ $d = \dfrac{169}{12}$

$\varepsilon = \dfrac{c}{a}$ $d = \dfrac{a}{\varepsilon} = \dfrac{a^2}{c} \Rightarrow a = \sqrt{169} = 13$

$a^2 - b^2 = c^2$

$b^2 = \sqrt{a^2 - c^2} = \sqrt{13^2 - 12^2} = 5$

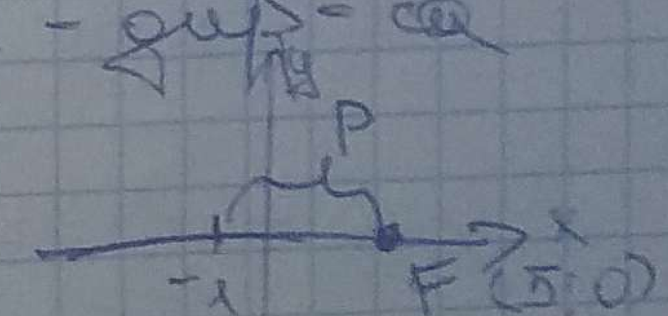
Канон: $\dfrac{x^2}{13^2} + \dfrac{y^2}{5^2} = 1$ Общий: $\dfrac{x^2}{13^2} + \dfrac{(y+1)^2}{5^2} = 1$

⑥ $-x^2 + 8x - 3y^2 + 6y + 1 = 0$

$-(x^2 - 8x$ $\alpha = 4 \;\; 8/2$ $\beta = 1 \;\; 6/6$

⑦ $F(5, 0)$ $x = -1$ – дир-са

$P = 6$



$y^2 = 2px$

Канон: $y^2 = 12x$ Общий: $y^2 = 12(x-2)$

8) $A(15\sqrt{2}, 60)$ $\quad y = \pm 4x$

$$\begin{cases} \frac{b}{a} = 4 \Rightarrow b = 4a \Rightarrow b^2 = 16a^2 \\ \frac{450}{a^2} - \frac{3600}{b^2} = 1 \end{cases}$$

$$\frac{450}{a^2} - \frac{3600}{16a^2} = \frac{3600}{16a^2} = \frac{225}{a^2} = 1$$

$a = 15 \quad b = 60$

$$\frac{x^2}{15^2} - \frac{y^2}{60^2} = 1$$

9) $y = -\sqrt{4 - 2x^2 - z^2}$

$$\begin{cases} y^2 = 4 - 2x^2 - z^2 \\ y \le 0 \\ 4 - 2x^2 - z^2 \ge 0 \end{cases} \Rightarrow \frac{2x^2}{4} + \frac{y^2}{4} + \frac{z^2}{4} = 1$$

эллипсоид

10) $11x^2 - 18\sqrt{3}\,xy - 7y^2 = 0$

$$18 \sin 2\varphi = -18\sqrt{3} \cos 2\varphi$$

$$\sin 2\varphi + \sqrt{3} \cos 2\varphi = 0$$

$$\sin 2\varphi \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cos 2\varphi = 0$$

$$\sin\left(2\varphi + \frac{\pi}{3}\right) = 0$$

$$2\varphi + \frac{\pi}{3} = \pi n$$

$$\varphi = \frac{\pi}{2} n - \frac{\pi}{6}$$

$n = 1: \quad \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} = \underline{60}$

sin

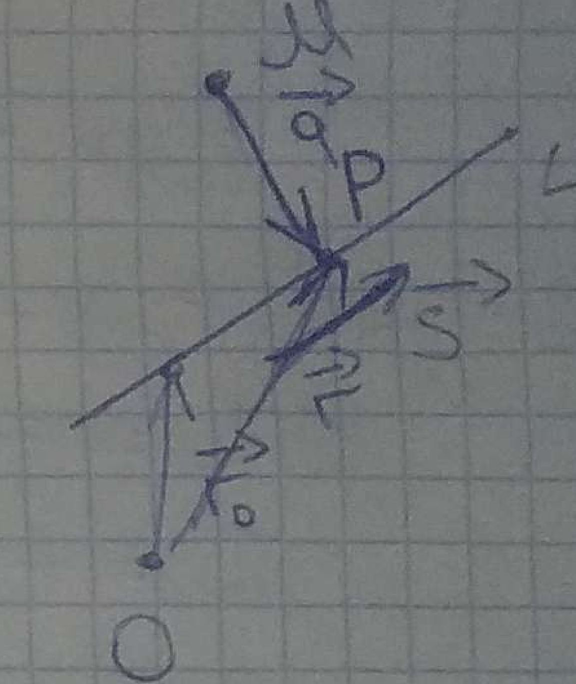
cos

2) $\frac{x+3}{-2} = \frac{y-1}{-2} = \frac{z-4}{-1}$ $\quad M(3, 4, -1)$

$\vec{s}\{-2;\ -2;\ -1\}$

$\vec{r_0}\{-3;\ 1;\ 4\}$

$\vec{q}\{x\ \ y\ \ z\}$



~~$x + 3 = y - 1 = 2z - 8$~~

~~$\begin{cases} x + 3 = y - 1 \\ y - 1 = 2z - 8 \end{cases}$~~

это правильно ?

$P - M = \vec{q} = \{x_p - 3,\ y_p - 4,\ z_p + 1\}$

$\vec{r} = \vec{r_0} + \vec{s}t$

$\{x_p,\ y_p,\ z_p\} = \{-3 - 2t,\ 1 - 2t,\ 4 - t\}$

$\vec{s} \cdot \vec{q} = 0$ т.к. $\vec{s} \perp \vec{q}$

$(x_p - 3)\cdot(+2) + (y_p - 4)\cdot(+2) + (z_p + 1)\cdot(+1) = 0$

$2x_p - 6 + 2y_p - 8 + z_p + 1 = 0$

$2x_p + 2y_p + z_p - 13 = 0$

~~$\begin{cases} \frac{x}{2} + \frac{3}{2} = \frac{y}{2} - \frac{1}{2} \\ \frac{y}{2} - \frac{1}{2} = 4 - z \end{cases}$~~

~~$y - 1 = 4 - z$~~

$$\begin{cases} x - y + 2 = 0 \\ y + z - 5 = 0 \end{cases}$$

$\cancel{(-3-2t-3)\,2}$

$2x_p + 2y_p + z_p - 13 = 0$

$-6 - 4t + 2 - 4t + 4 - t - 13 = 0$

$9t = -13$

$t = -\frac{13}{9}$

$x_p = -3 + \frac{26}{9} \quad y_p = 2 + \frac{26}{9} \quad z_p = 4 + \frac{13}{9}$

$\vec{a} = \left\{ -6 + \frac{26}{9};\ -3 + \frac{26}{9};\ 5 + \frac{13}{9} \right\}$