CSC 225 Assignment 2:
Runtime Analysis and Proofs

Due date:

The submission deadline is 11:59pm on Wednesday, October 1st, 2025.

How to hand it in:

Submit an **image** or **.pdf** of each question to the Assignment 2 Crowdmark page.

Exercises:

1. Determine the number of assignment (A) and comparison (C) operations in the following:

   a. **Algorithm** Q1a(n)

      $result \leftarrow 1$
      for $i \leftarrow 1$ to n^2 **do**
         for $j \leftarrow 1$ to i **do**
            $temp \leftarrow i + j$
            $result \leftarrow result + temp$
         end
      end
      return $result$

   b. **Algorithm** Q1b(n)

      $result \leftarrow 1$
      for $i \leftarrow 1$ to n **do**
         $j \leftarrow 1$
         while $j \leq n$ **do**
            $result \leftarrow result + 1$
            $j \leftarrow j * 2$
         end
      end
      return $result$

      HINT: Assume n is a power of 2

2. Solve the following recurrence relations, given an integer $n \geq 1$, through substitution:

a. $T(n) = \begin{cases} 2, & n = 1 \\ 2T(n-1) + 3, & n \geq 2 \end{cases}$

b. $T(n) = \begin{cases} 1, & n = 1 \\ T(n-1) + 4n, & n \geq 2 \end{cases}$

3. Proof by induction

   a. Prove the following statement using induction:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = (n(n+1)(2n+1))/6, \text{ for any positive integer } n \geq 1.$$

b. Find a formula for $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \cdots + \frac{1}{n(n+1)}$ by examining what the expression produces for small values of n. Then, provide a proof by induction to prove your result. Assume $n \geq 1$.

4. Use a *loop invariant* to prove that the algorithm below **returns** the minimum value in the array.

Algorithm Q4(A, n)
    Input: An array A containing n integers.
    Output: The element from A with the minimum value.

```
min ← A[0]
i ← 1
while i < n do
    if A[i] < min then
        min ← A[i]
    end
    i ← i + 1
end
return min
```

5. Prove each of the following using the definition of Big-Oh. You must provide constants c and n_0 to satisfy the definition of Big-Oh as defined in class.

a. $(n+3)^5$ is $O(n^5)$

b. $\frac{n^5+2n^3-n^2+4}{n^3+1}$ is $O(n^2)$

c. Find the smallest integer x such that $5n^3 + 3n^2 log n$ is $O(n^x)$

6. Order the following functions by their big-Oh notation. Group together (for example, by underlining) functions that are big-Theta of one another (no justification needed). Note: $\log n = \log_2 n$ unless otherwise stated.

$3n \log n$	2^{18}	5^n	$11n$	$2^{\log n}$
$4n^3$	$99n^{0.5}$	$7n^{8/3}$	$1/n$	5^{n^2}
$n^2 \log n$	$\sqrt{2n}$	$n^{0.01}$	2^{2^n}	17