Big-Oh

For each of the following exercises, determine the value for x that satisfies the expression: $f(n)$ is $O(x)$. Prove each by providing positive constants c and n_0 that satisfy the definition of Big-Oh.

1) $f(n) = 10,000n^2 + 25n$

$$10000n^2 + 25n \le 10000n^2 + 25n^2 \text{ for all } n \ge 1$$
$$\le 10025n^2$$

$\therefore f(n) \in O(n^2)$ with $c = 10025$ and $n_0 = 1$

$$10000n^2 + 25n \le 10001n^2$$
$$25n \le n^2$$
$$25 \le n$$

$\therefore f(n) \in O(n^2)$ with $c = 10001$ and $n_0 = 25$

2) $f(n) = 4n + 20n^4 + 117$

$$4n + 20n^4 + 117 \le 4n^4 + 20n^4 + 117n^4 \text{ for all } n \ge 1$$
$$\le 141n^4$$

$\therefore f(n) \in O(n^4)$ with $c = 141$ and $n_0 = 1$

3) $f(n) = 1083$

$1083 \le (1083)1$ for all $n \ge 0$

$\therefore f(n) \in O(1)$ with $c = 1083$ and $n_0 = 0$

4) $f(n) = 3\log n$

$3\log n \le 4\log n$ for all $n \ge 1$

$\therefore f(n) \in O(\log n)$ with $c = 4$ and $n_0 = 1$

5) $f(n) = 3\log n + \log\log n$

$3\log n + \log\log n \le 4\log n$ for all $n \ge 1$

$\therefore f(n) \in O(\log n)$ with $c = 4$ and $n_0 = 1$

6) $f(n) = 2^{17}$

$2^{17} \le 2^{17}(1)$ for all $n \ge 0$

$\therefore f(n) \in O(1)$ with $c = 2^{17}$ and $n_0 = 0$

7) $f(n) = 2^{\log 2n}$

$2^{\log 2n} = 2n \le 2(n)$ for all $n \ge 1$

$\therefore f(n) \in O(n)$ with $c = 2$ and $n_0 = 1$

8) $f(n) = 1^n$

$1^n = 1$ for all $n \ge 0$

$\therefore f(n) \in O(1)$ with $c = 1$ and $n_0 = 0$

9) $f(n) = 64n^{1/2} - 256\log n + n\log n + 128n$

$$\begin{aligned} & 64n^{1/2} - 256\log n + n\log n + 128n \\ \le\ & 64n^{1/2} + 0 + n\log n + 128n \\ \le\ & 64n\log n + n\log n + 128n\log n \quad \text{for all } n \ge 2 \\ \le\ & 193n\log n \end{aligned}$$

$\therefore f(n) \in O(n\log n)$ with $c = 193$ and $n_0 = 2$

10) $f(n) = \sum_{i=1}^{n} i$

$\sum_{i=1}^{n} i \le \sum_{i=1}^{n} n = n^2$ for all $n \ge 1$

$\therefore f(n) \in O(n^2)$ with $c = 1$ and $n_0 = 1$