Combinations and Binomial Theorem

1) Given a computer science midterm exam with 12 questions, students choose 8 different questions to answer. (a) How many ways are there to complete the exam? (b) If students are required to answer 5 questions from the first half and 3 from the second half, how many ways can the exam be completed? (c) What if students must answer at least 3 questions from the first half of the exam? (d) Write your solution to (c) using sigma notation

a) $C(12,8)$

b) $\binom{6}{5} * \binom{6}{3}$

c) $\binom{6}{3}\binom{6}{5} + \binom{6}{4}\binom{6}{4} + \binom{6}{5}\binom{6}{3} + \binom{6}{6}\binom{6}{2}$

d) $\sum_{k=3}^{6} \binom{6}{k}\binom{6}{8-k}$

2) Given the word ENGINEERING, (a) how many arrangements of the letters are there? (b) How many of these arrangements have no adjacent E's?

a) $\frac{11!}{3!3!2!2!} = 277{,}200$

b) $\frac{8!}{3!2!2!} * \binom{9}{3} = 141{,}120$

3) Express each of the following using Sigma (summation) notation.

a) 6! + 7! + 8!

$\sum_{k=6}^{8} k!$

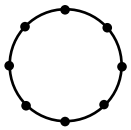
b) 1 + 4 + 9 + 16 + 25 + 36 + 49

$\sum_{k=1}^{7} k^2$

c) $\frac{1}{n} + \frac{2}{n+1} + \frac{3}{n+2} + \ldots + \frac{n+1}{2n}$

$\sum_{k=1}^{n+1} \frac{k}{n+(k-1)}$

4) Given a circle with 8 equally spaced points, we can make shapes by connecting the points within the circle. (a) How many different single triangles can we create (by single I mean we do not want triangles within triangles)? (b) How many different quadrilaterals can we create? (c) How many different polygons can we create by using 3 or more vertices? (d) Write your answer to (c) using Sigma (summation) notation.



a) $\binom{8}{3} = 56$

b) $\binom{8}{4} = 70$

c) $\binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8} = 219$

d) $\sum_{k=3}^{8} \binom{8}{k}$