(7.) Írjuk fel annak a síknak az egyenletét, amely átmegy a $\begin{cases} P_1: 2x+y-z-2=0 \\ P_2: x-3y+z+1=0 \\ P_3: x+y+z-3=0 \end{cases}$ síkok metszéspontján és párhuzamos a $P_4: x+y+2z=0$ síkkal.

Megoldás:

$P_1 \cap P_2 \cap P_3 = \{A\}$

$A(1,1,1)$

$\alpha = ?$ ú. h. $A \in \alpha$ és $\alpha \parallel P_4: x+y+2z=0$

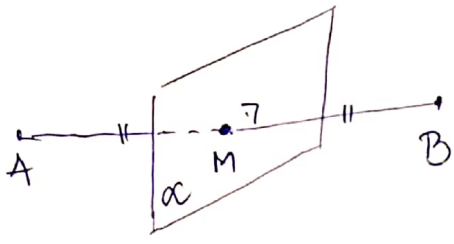
$\alpha \parallel P_4 \Rightarrow \alpha: x+y+2z+D=0$

$A(1,1,1) \in \alpha$

$\Rightarrow 1+1+2+D=0 \Rightarrow D=-4$

$\Rightarrow \alpha: x+y+2z-4=0.$

(8.) $\alpha = ?$ ú. h. α az $[AB]$ szakasz felezőmerőlegos síkja, ahol $A(2,-1,3)$, $B(4,5,-3)$.



$M(3,2,0) \leftarrow \begin{cases} x_M = \frac{x_A+x_B}{2} \\ y_M = \frac{y_A+y_B}{2} \\ z_M = \frac{z_A+z_B}{2} \end{cases}$

$AB \perp \alpha \Rightarrow \overrightarrow{AB} \perp \alpha \Rightarrow \overrightarrow{AB} = \vec{N}_\alpha$

$\overrightarrow{AB}(4-2, 5+1, -3-3) \Leftrightarrow \overrightarrow{AB}(2,6,-6)$

$M \in \alpha$

$\Rightarrow \alpha: A(x-x_0)+B(y-y_0)+C(z-z_0)=0 \Leftrightarrow \alpha: 2\cdot(x-3)+6(y-2)-6\cdot(z-0)=0$

$\alpha: \quad 2x-6+6y-12-6z=0 \;/:2$

$\alpha: \quad x+3y-3z-9=0.$

(9.) $\alpha=?$ ú. h. $d: \begin{cases} 2x+y-z-2=0 \\ x-3y+z+1=0 \end{cases}$ egyenes $\subset \alpha$. és

a) α áthalad az origón

b) $\alpha \parallel Oy$

d) $O(0,0,0) \in \alpha$

$d \quad \subset \alpha.$

1. Módszer:

- a d egyenest átírjuk kanonikus alakra:

$$\begin{cases} 2x+y-z-2=0 \\ x-3y+z+1=0. \end{cases}$$

$x \overset{jó}{=} t$

$$\begin{cases} y-z=2-2t \\ -3y+z=-1-t \end{cases} \Big| \oplus \Rightarrow -2y=1-3t$$

$$y=-\frac{1}{2}+\frac{3}{2}t$$

$$z=-1-t+3y=-1-t-\frac{3}{2}+\frac{9}{2}t$$

$$z=-\frac{5}{2}+\frac{7}{2}t.$$

$$d: \begin{cases} x=t \\ y=\frac{-1}{2}+\frac{3}{2}t \\ z=-\frac{5}{2}+\frac{7}{2}t, \; t\in\mathbb{R} \end{cases}$$

$$(=) \; d: \begin{cases} x=2t \\ y=-\frac{1}{2}+3t \\ z=-\frac{5}{2}+7t \end{cases} \Rightarrow d: (t=) \; \frac{x}{2}=\frac{y+\frac{1}{2}}{3}=\frac{z+\frac{5}{2}}{7}$$

$$\Rightarrow A\left(0,-\frac{1}{2},-\frac{5}{2}\right) \in d., \; \vec{d}\,(2,3,7)$$

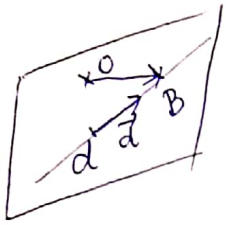
Választhatunk „szebb" pontot is d-nél.

pl.

$$d: \begin{cases} x = 2t \\ y = -\frac{1}{2} + 3t \\ z = -\frac{5}{2} + 7t, \quad t \in \mathbb{R} \end{cases}$$

pl. legyen $t = \frac{1}{2} \Rightarrow x = 1,\ y = \frac{-1}{2} + \frac{3}{2} = 1,\ z = -\frac{5}{2} + \frac{7}{2} = 1.$

$\Rightarrow B(1,1,1) \in d.$



α síkot meghatározza:

* $O(0,0,0)$
* $\overrightarrow{OB}(1,1,1)$
* $\vec{d}(2,3,7)$.

$$\Rightarrow \alpha: \begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 1 & 1 \\ 2 & 3 & 7 \end{vmatrix} = 0$$

$$(=) \alpha: 4x - 5y + z = 0.$$

2. Módszer: Választunk 3 pontot: $\rightarrow O(0,0,0)$

$\hookrightarrow$ 2 pontot d-nél:

pl. $A\left(0, -\frac{1}{2}, -\frac{5}{2}\right)$, $B(1,1,1)$

$$\Rightarrow \alpha = (OAB) (=) \quad \alpha: \begin{vmatrix} x-0 & y-0 & z-0 \\ 0-0 & -\frac{1}{2}-0 & -\frac{5}{2}-0 \\ 1-0 & 1-0 & 1-0 \end{vmatrix} = 0$$

$$(=) \alpha: \begin{vmatrix} x & y & z \\ 0 & -\frac{1}{2} & -\frac{5}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$(=) \alpha: x\left(-\frac{1}{2} + \frac{5}{2}\right) - y \cdot \frac{5}{2} + z \cdot \frac{1}{2} = 0 \quad /\cdot 2$$

$$\alpha: 4x - 5y + z = 0.$$

(13.) $A(1,2,3)$, $B(-2,1,4)$

$\Rightarrow AB: \frac{x-1}{-2-1} = \frac{y-2}{1-2} = \frac{z-3}{4-3}$

$AB: \frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z-3}{1}$.

$AB \cap (xOy) = ?$ $(xOy): z=0 \Rightarrow \frac{x-1}{-3} = \frac{y-2}{-1} = \frac{0-3}{1} = -3$

$\Rightarrow x-1 = 9$; $y-2 = 3$

$x = 10$ $y = 5$

$\Rightarrow M_1(10,5,0)$.

$AB \cap (yOz) = ?$ $(yOz): x=0 \Rightarrow \frac{-1}{-3} = \frac{y-2}{-1} = \frac{z-3}{1}$

$3(y-2) = -1$ $z-3 = \frac{1}{3}$

$y-2 = -\frac{1}{3}$ $z = \frac{10}{3}$

$y = \frac{5}{3}$

$M_2\left(0, \frac{5}{3}, \frac{10}{3}\right)$

$AB \cap (xOz) = ?$ $(xOz): y=0 \Rightarrow \frac{x-1}{-3} = \frac{-2}{-1} = \frac{z-3}{1}$

$x-1 = -6$ $z-3 = 2$

$x = -5$ $z = 5$

$M_3(-5,0,5)$