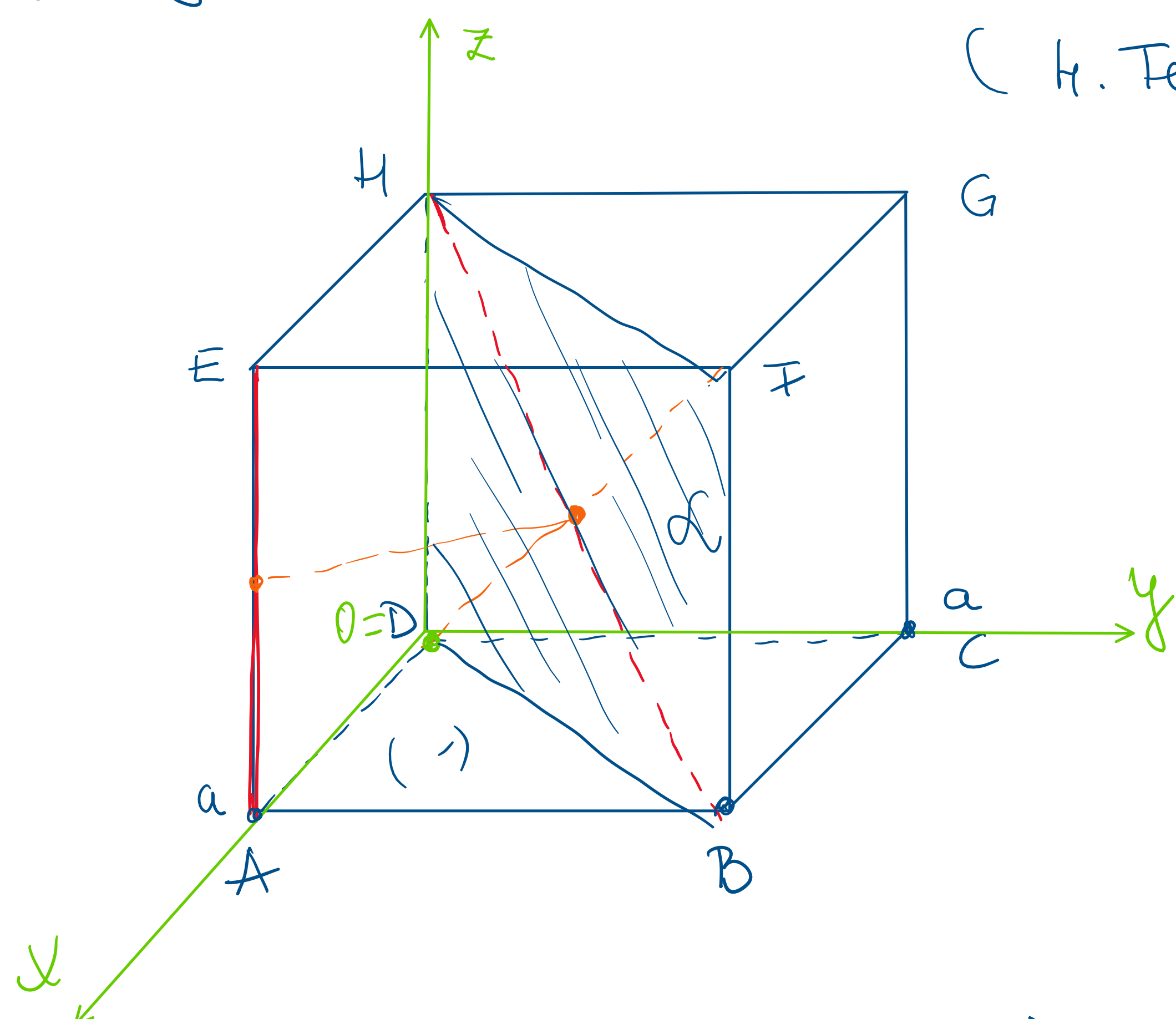
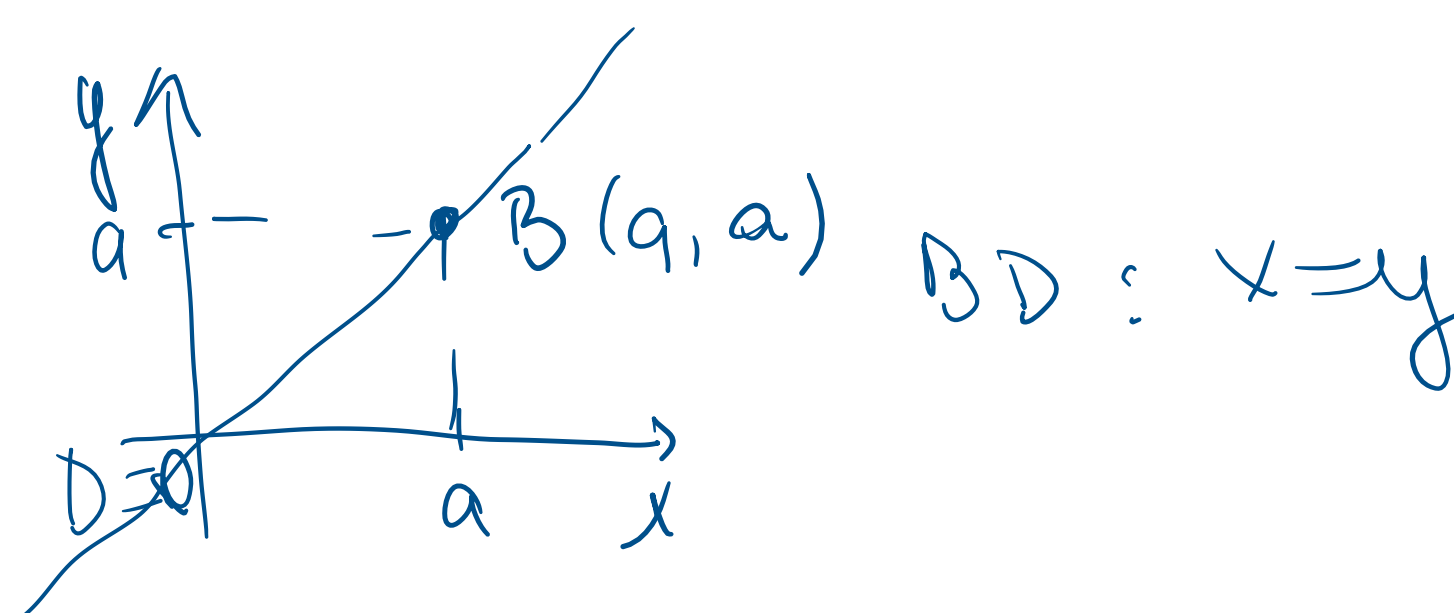


Határozzuk meg egy  $a$ -élű kocka egyik élének a távolságát egy olyan testetől, amellyel nincs közös pontja.

(4. Feladatlap/33.)



$$\begin{aligned} A(a, 0, 0) \\ B(a, a, 0) \\ C(0, a, 0), D(0, 0, 0) \end{aligned}$$



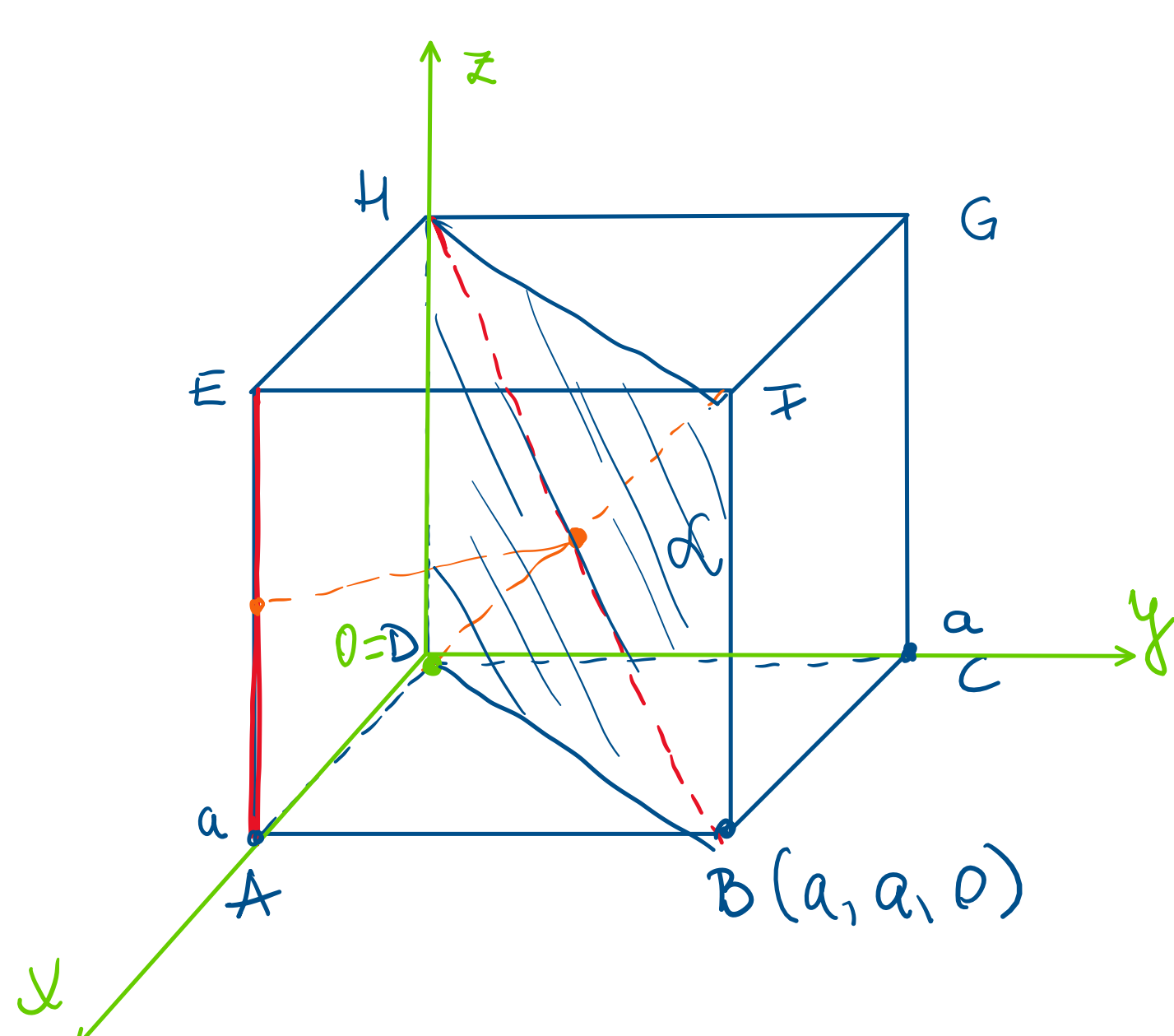
$$E(a, 0, a), H(0, 0, a), G(0, a, a), F(a, a, a)$$

$$EA: \begin{cases} (EAB): y=0 \\ (EAB): x=a \end{cases} \text{ vagy } EA: \frac{x-a}{a-a} = \frac{y-0}{0-0} = \frac{z-0}{a-0}$$

$$EA: \frac{x-a}{0} = \frac{y}{0} = \frac{z}{a} \Rightarrow \begin{cases} x-a=0 \\ y=0 \end{cases} \quad \left| \cdot a \right. \quad \frac{x-a}{0} = \frac{y}{0} = \frac{z}{a}$$

$$HB: \frac{x}{a} = \frac{y}{a} = \frac{z-a}{-a} \quad \left| \cdot a \right. \Rightarrow HB: \frac{x}{1} = \frac{y}{1} = \frac{z-a}{-1}$$

$$\Rightarrow \begin{cases} x=y \\ x+z=a \end{cases}$$



1. lépés:

$\alpha$  nk tartalmazza HB egyenest és  $\parallel EA$ .

$$\Rightarrow \alpha = (HB)$$
 vagyis

$\alpha$  nkot megfigyel:  $\begin{cases} B \text{ pont} \\ HB \text{ irányvektora } (1, 1, -1) \\ EA \text{ } \rightarrow \text{ } (0, 0, 1) \end{cases}$

$$\Rightarrow \alpha: \begin{vmatrix} x-a & y-a & z-0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$\alpha: (x-a) - (y-a) = 0$$

$$\alpha: x-y=0 \quad (\Rightarrow \boxed{x=y})$$

2. lépés:

$\beta$  nk:  $\beta \perp \alpha$  és  $\beta$  tartalmazza EA-t

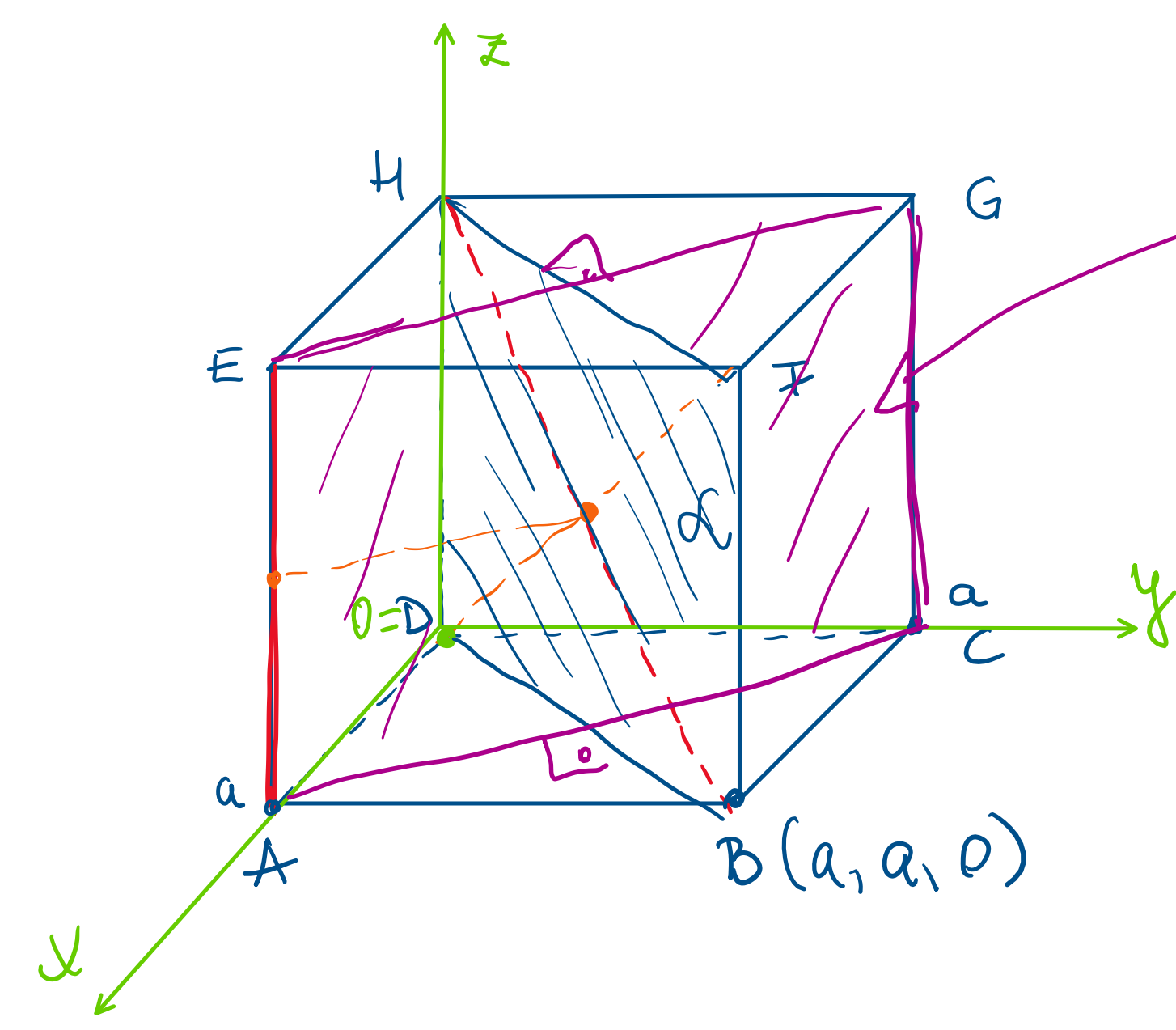
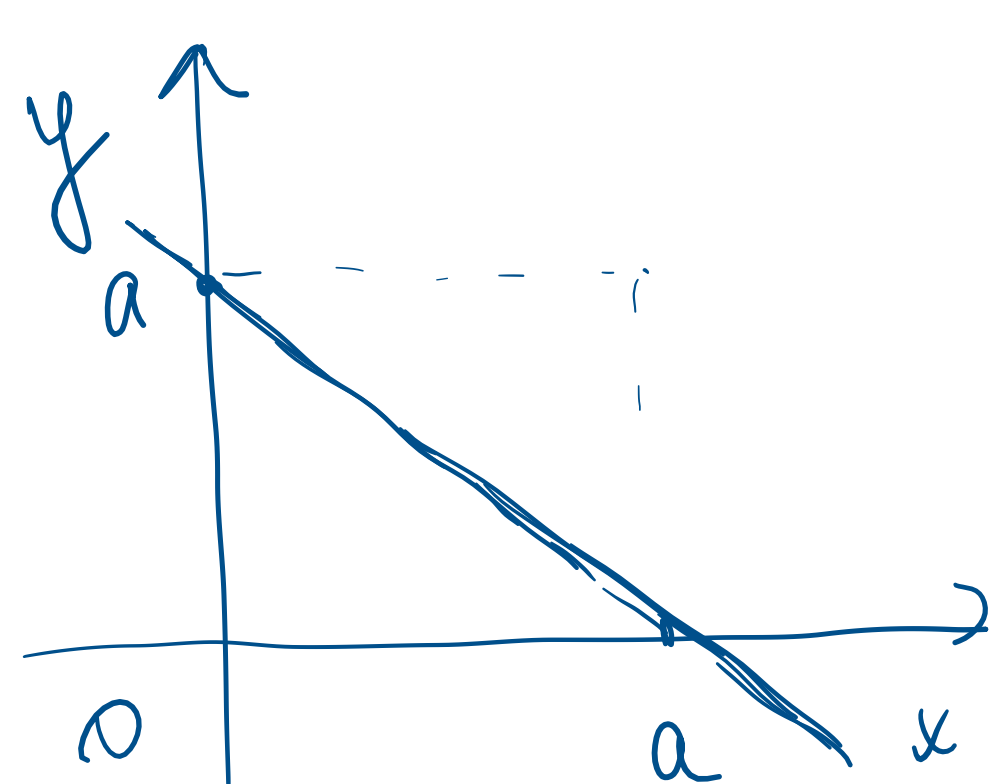
$$\beta \parallel \vec{N}_\alpha(1, -1, 0)$$

$\beta \parallel EA$  irányvektora  
 $\beta \parallel (0, 0, 1)$

$$\Rightarrow \beta: \begin{vmatrix} x-a & y-0 & z-0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \beta: -x+a-y=0$$

$$\beta: x+y=a$$

$$\boxed{y=-x+a}$$



$\beta$  nk.

3. lépés:

$\gamma$  nk:  $\begin{cases} \gamma \perp \alpha \Rightarrow \gamma \parallel \vec{N}_\alpha(1, -1, 0) \\ \gamma \text{ tartalmazza HB-t} \end{cases}$   
 $\hookrightarrow B(a, a, 0) \in \gamma$   
 $\gamma \parallel HB(1, 1, -1)$

$$\Rightarrow \gamma: \begin{vmatrix} x-a & y-a & z \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$\gamma: (x-a) + (y-a) - z = 0$$

$$\gamma: x+y+z-2a=0$$

$$z=0 \Rightarrow x+y=2a$$

$$\Rightarrow \text{a közös merőleges: } \begin{cases} \beta: x+y=a \\ \gamma: x+y+z=2a \end{cases}$$

$$\Rightarrow \begin{cases} \beta: x+y=a \\ \gamma: x+y+z=a/2 \end{cases}$$

$$d(AE, HB) = ? \quad \frac{a\sqrt{2}}{2}$$

$$d(AE, HB) = d(AE, \alpha) = d(A, \alpha) = \frac{|a+0|}{\sqrt{1+1}} = \frac{a}{\sqrt{2}} = \frac{a\sqrt{2}}{2}$$

$$\alpha: x+y=0$$

$$A(a, 0, 0)$$

