

法拉第波实验

法拉第波（英语：Faraday waves 或 Faraday ripples）是在气液分界面上，由于流体不稳定性（法拉第不稳定性）产生的一种非线性驻波，当其振荡频率超过临界值时，气液分界面就会不稳定，可用于液基微小物体聚集器，法拉第波得名自麦可·法拉第（Michael Faraday）。

若将一层液体放置在垂直振荡的活塞顶部，则会出现以一半驱动频率振荡的驻波模式，可以给出某些不稳定性的标准[3]。这涉及参数共振的问题。驻波可能以条纹、紧密堆积的六边形、甚至正方形或准周期的型式出现。在可发出铃铛声般的葡萄酒酒杯内，葡萄酒表面通常可以观察到细条纹的法拉第波。

星形的对称性（即分支的数量）与容器的形状和大小无关，并且可以根据振动的幅度和频率进行更改。我们表明，可以设想三个重力波之间的非线性共振耦合来触发观察到的对称性破坏，尽管在最终的周期性状态中肯定会发生更复杂的相互作用。

固定频率8Hz，增大振幅，圆形波，离心centrifugal和向心centripetal。进一步增大振幅，出现五边形图案

1.熟悉试验设备

2.学习faraday波原理，线性水动力方程推导，Floquet理论求解

亚谐振：振动圆频率=2*波圆频率，图案会旋转 π/N

两个振动加速度幅值的阈值：a1:Faraday threshold a2:disorder threshold

欧拉方程 (z 方向) $\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + (g - f \cos wt)$ 无粘
 $(u, v, w) = (\frac{\partial \xi}{\partial x}, \frac{\partial \xi}{\partial y}, \frac{\partial \xi}{\partial z})$ 代入并对方程积分 线性化并略去高阶项和乘积项

则有 $\frac{p}{\rho} + \frac{\partial \phi}{\partial t} - (g - f \cos wt) z = 0$ (类似水头方程) —①

设自由液面 $\xi = \xi(x, y, t)$

自由液面条件 $\frac{D}{Dt}(z - \xi) = \frac{\partial \xi}{\partial t} + u \frac{\partial \xi}{\partial x} + v \frac{\partial \xi}{\partial y} - w = 0$ —②

壁面 $\frac{\partial \xi}{\partial n} = 0$ 底面 $\frac{\partial \xi}{\partial z} = 0$ ($z = h$)

对于小波，略去高阶项 $\Rightarrow \frac{\partial \xi}{\partial t} = w = \frac{\partial \phi}{\partial z}$ ($z = 0$) —③

表面张力 $p = \gamma(\sigma_1 + \sigma_2)$ γ - 表面张力系数 $(\sigma_1 + \sigma_2) = (\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2})$ 弯曲率

代入①有 $\frac{p}{\rho} (\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}) + \frac{\partial \phi}{\partial t} \Big|_{z=0} - (g - f \cos wt) \xi = 0$ —④

$\frac{\partial \phi}{\partial n} = 0 \Rightarrow \frac{\partial^2 \phi}{\partial z \partial n} = \frac{\partial^2 \xi}{\partial n \partial t} = 0$ (由③式) $\Rightarrow \frac{\partial \xi}{\partial n} = \int \frac{\partial^2 \xi}{\partial n \partial t} dt = \frac{\partial \xi}{\partial n} \Big|_{t=0}$ ($= 0$ 令 $t=0$)

对①应用 $\frac{\partial}{\partial n}$ 有 $\frac{\partial}{\partial n} (\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}) = 0 \Rightarrow \xi, \frac{\partial \xi}{\partial x}, \frac{\partial \xi}{\partial y}$ 均满足固面条件

由 3 个固面条件 $\Rightarrow \xi, \frac{\partial \xi}{\partial x}, \frac{\partial \xi}{\partial y}$ 均可进行将特征函数(级数)展开

特征函数满足 $\begin{cases} \frac{\partial^2 \xi}{\partial n} S_m(x, y) = 0 & (x, y) \text{ on } C \text{ (边界)} \\ (\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + k_m^2) S_m(x, y) = 0 & (x, y) \text{ 在 } C \text{ 内部} \end{cases}$ k_m - 特征值

时间空间分离: $\xi(x, y, t) = \sum_0^\infty a_m(t) S_m(x, y)$ —⑤.

⑤ 代入 $\frac{\partial^2 \xi}{\partial n} = \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}$, 结合 (x, y) in C 式子有 $\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = -\sum_0^\infty k_m^2 a_m(t) S_m(x, y)$ —⑥

由③ $\dot{\xi} = \int \frac{\partial \xi}{\partial t} dz = \sum_0^\infty \frac{da_m(t)}{dt} \int S_m(x, y) dz$ —⑦ 不妨令 $\int S_m(x, y) dz = \phi_m(x, y)$ —⑧

由连续方程 $\nabla^2 \dot{\xi} = 0 \Rightarrow \sum_0^\infty \frac{da_m(t)}{dt} \nabla^2 \phi_m(x, y) = 0$ 对任一时刻成立.

则 $\nabla^2 \phi_m(x, y) = 0$ —⑨ $\left\{ \begin{array}{l} \text{由⑨和⑧可知(假设)} \\ \phi_m = B \cosh k_m(h-z) S_m(x, y) \end{array} \right.$

由 $\frac{\partial \phi}{\partial z} = 0 \Rightarrow \frac{\partial \phi_m(x, y)}{\partial z} = 0$ —⑩ \downarrow $\phi_m = B \cosh k_m(h-z) S_m(x, y)$ B - 常数

~~代入~~ $\frac{\partial \phi}{\partial t} + g \frac{\partial \phi}{\partial z} = 0$ (线性自由表面条件) 满足 $\nabla^2 \phi_m(x, y, z) = 0$ 可推导.

$$\begin{aligned}
 & \text{有 } \sum_0^{\infty} a''' \phi_m(x, y) + g \sum_0^{\infty} a' S_m(x, y) = 0 \quad \left(\frac{\partial^2}{\partial z^2} \left(\sum_0^{\infty} a' \int S_m(x, y) dz \right) = \sum_0^{\infty} a' S_m(x, y) \right) \\
 & \text{也有 } \sum_0^{\infty} a'' \phi_m(x, y) + g \sum_0^{\infty} a' \frac{\partial \phi_m(x, y)}{\partial z} = 0 \quad \left(\frac{\partial \phi_m(x, y)}{\partial z} \equiv k_m B \sinh(k_m(h-z)) S_m(x, y) \right) \\
 & \Rightarrow \begin{cases} a''' \cdot B \cosh(k_m h - z) + g a' \frac{\partial}{\partial z} = 0 \\ a''' B \cosh(k_m h - z) - g a' B \sinh(k_m h - z) k_m = 0 \end{cases} \Rightarrow B = - \frac{1}{k_m \sinh(k_m h - z)} \Big|_{z=0} \\
 & \Rightarrow \Phi = - \sum_0^{\infty} \frac{d a_m(t)}{dt} \cdot \frac{\cosh(k_m(h-z))}{k_m \sinh(k_m h)} S_m(x, y) \quad (\text{假设 } a_0(t)=0) \quad -⑪ \\
 & \text{把 } ⑤, ⑥, ⑪ \text{ 代入 } ④ \text{ 有 } \sum_1^{\infty} \underbrace{\frac{S_m(x, y)}{k_m \tanh(k_m h)}}_{\text{对 } z \text{ 的项不全为0}} \left[\frac{d^2 a_m(t)}{dt^2} + k_m \tanh(k_m h) \left(\frac{k_m^2 Y}{P} + g - f \cos(wt) \right) a_m \right] = 0
 \end{aligned}$$

$$\text{则有 } \frac{d^2 a_m(t)}{dt^2} + k_m \tanh(k_m h) \left(\frac{k_m^2 Y}{P} + g - f \cos(wt) \right) a_m = 0 \quad -⑫$$

$$\boxed{f'' + \omega^2 x = 0} \text{ 不考虑时间项 } \Rightarrow k_m \tanh(k_m h) \left(\frac{k_m^2 Y}{P} + g - \cancel{f \cos(wt)} \right) = \omega_m^2$$

$$\text{色散关系 } \omega_m^2 = g k_m \tanh(k_m h) + \frac{k_m^3 Y \tanh(k_m h)}{P} \approx g k_m \tanh(k_m h) + k_m^3 \sigma \quad -⑬$$

⑭ 为 Mathieu 方程 \Rightarrow Floquet 理论求解 \Rightarrow ~~无~~ 阶值。

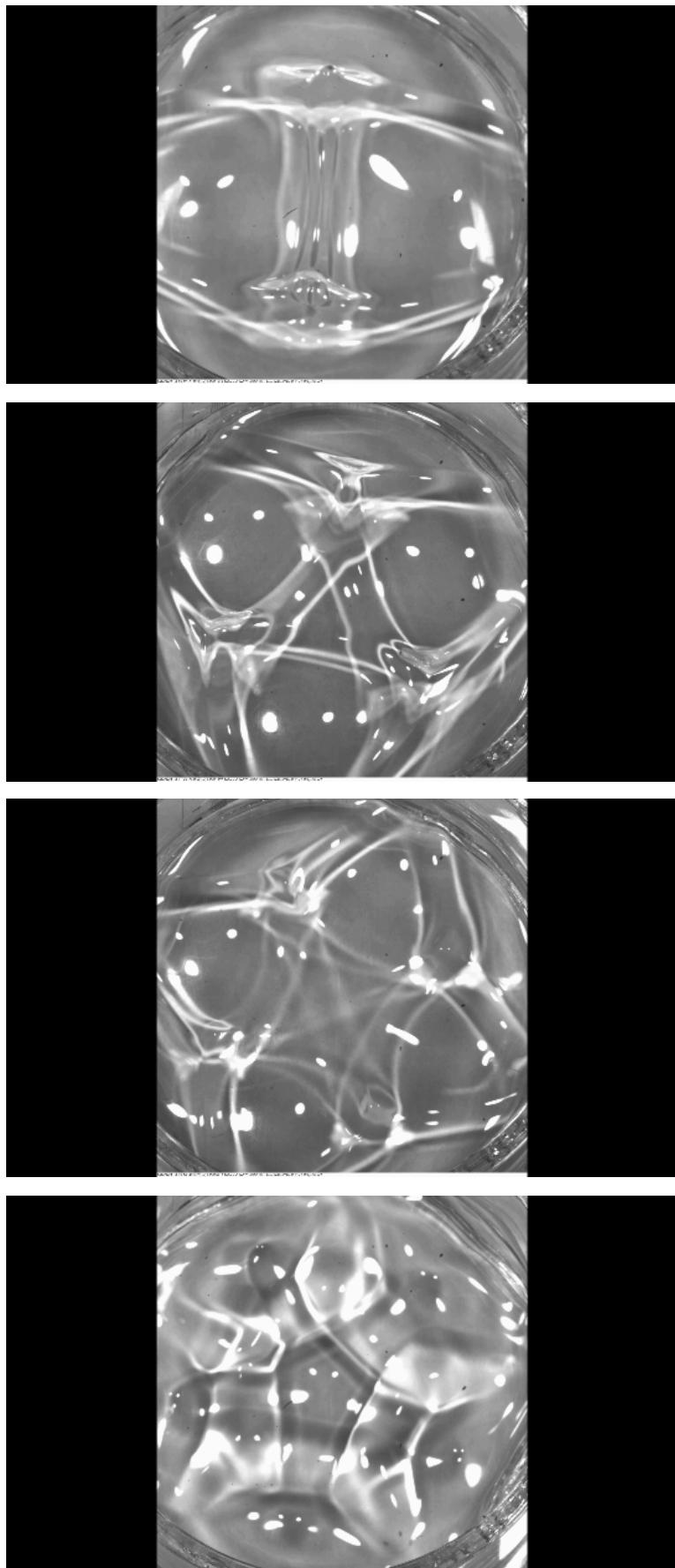
$$\begin{aligned}
 & \phi_m(x, y, z) \text{ 满足 } \nabla^2 \phi_m(x, y, z) = 0 \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \\
 & B \cosh(k_m h - z) \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) + B k_m^2 \cosh(k_m h - z) S_m(x, y) = 0 \\
 & B \cosh(k_m h - z) \underbrace{\left[6 \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + k_m^2 S_m(x, y) \right]}_{=0} = 0
 \end{aligned}$$

3. 尝试拍出论文中很漂亮的 Faraday 波形图，实际经过多种调整，图片差强人意。

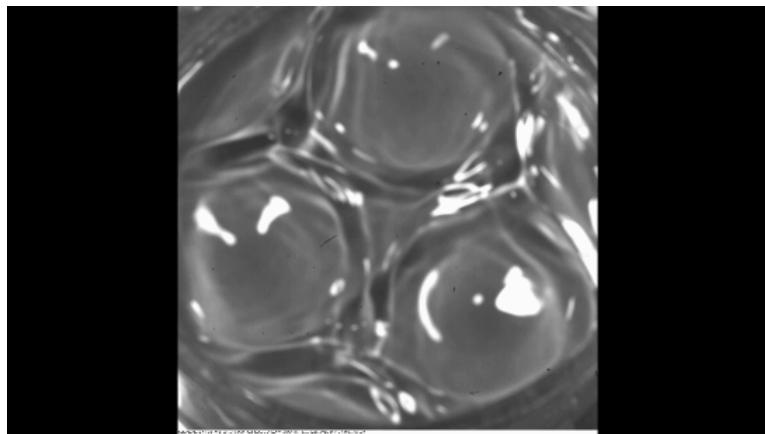
- 纯硅油，到五阶模态前的波形都很清楚，与论文给出的模态图片很接近，但是第六阶（也就是六边形）的波形混乱
- 液体内部的流体运动对拍摄有干扰（李靖老师也有提到），我们想要得到的仅仅是表面波形

为此我们做了一些尝试

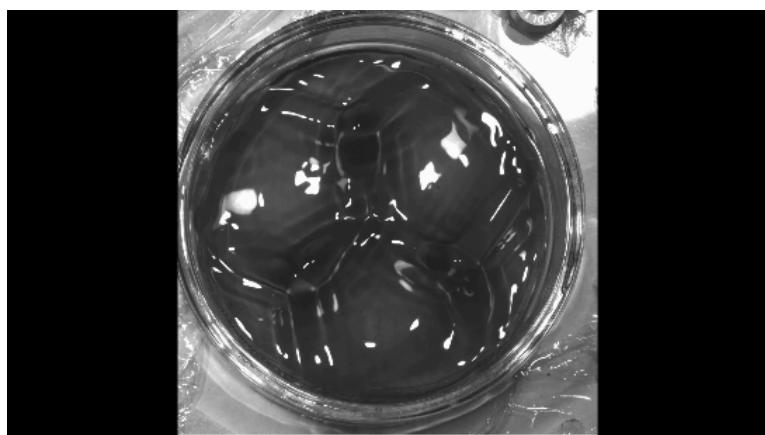
- 在器皿底部垫的滤光片
- 调整拍摄角度和光照角度
- 不透明液体（牛奶），确实只能看到表面波形
- 墨水（含稀释），吸收折射光
- 器皿内部底部形成一层吸光层，我们在硅油中加入一层墨水
- 硅油



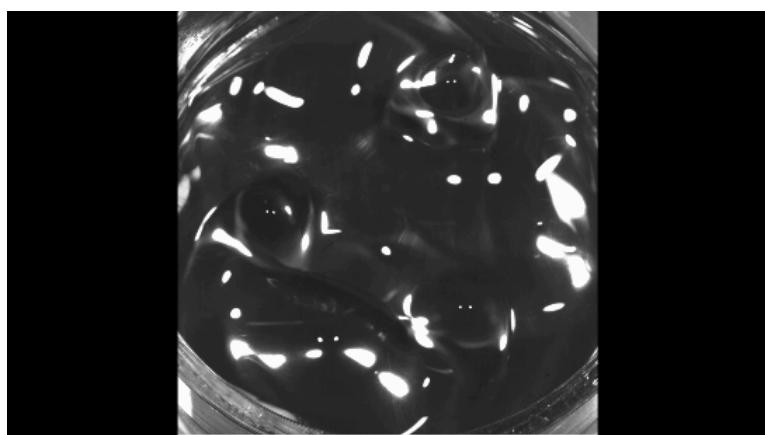
水



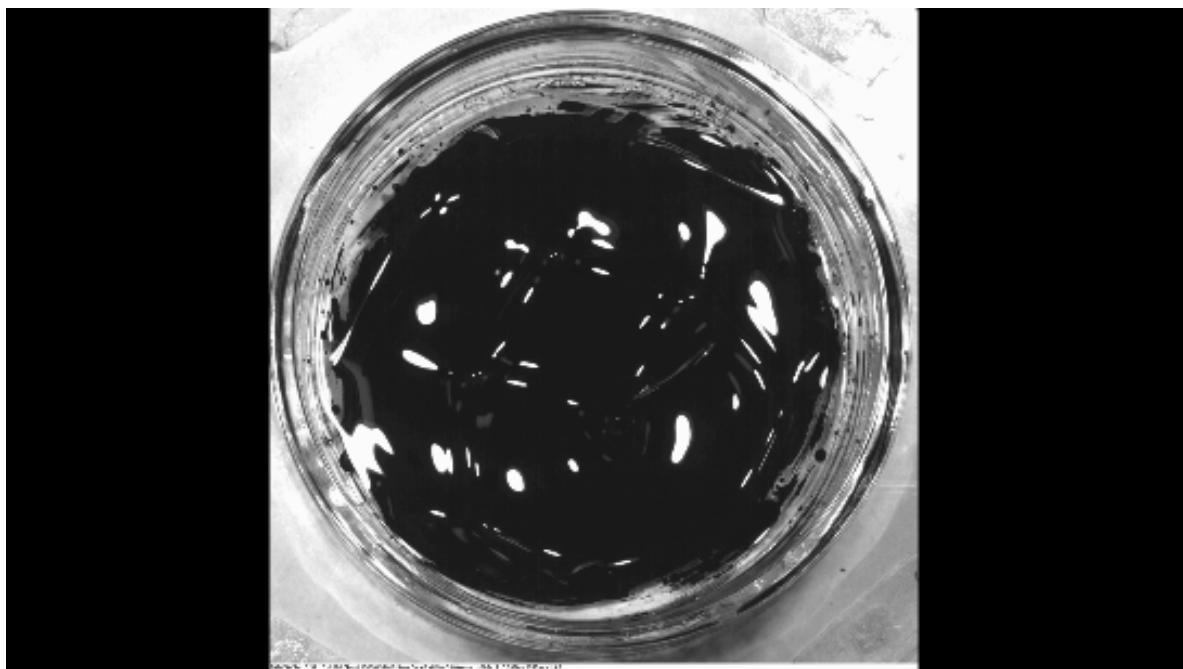
墨水



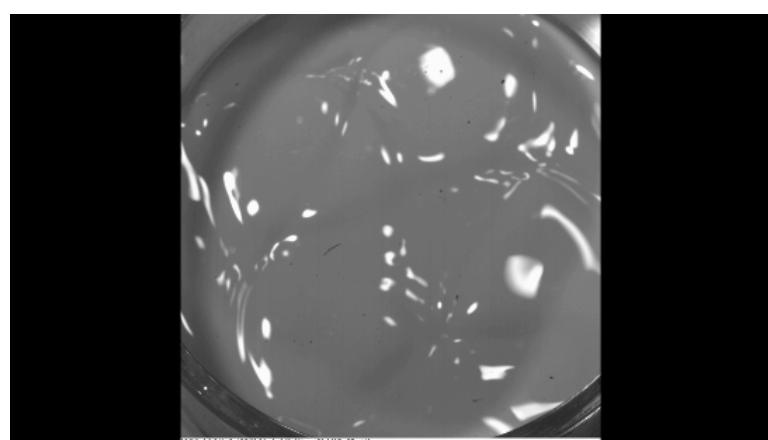
滤光纸底部



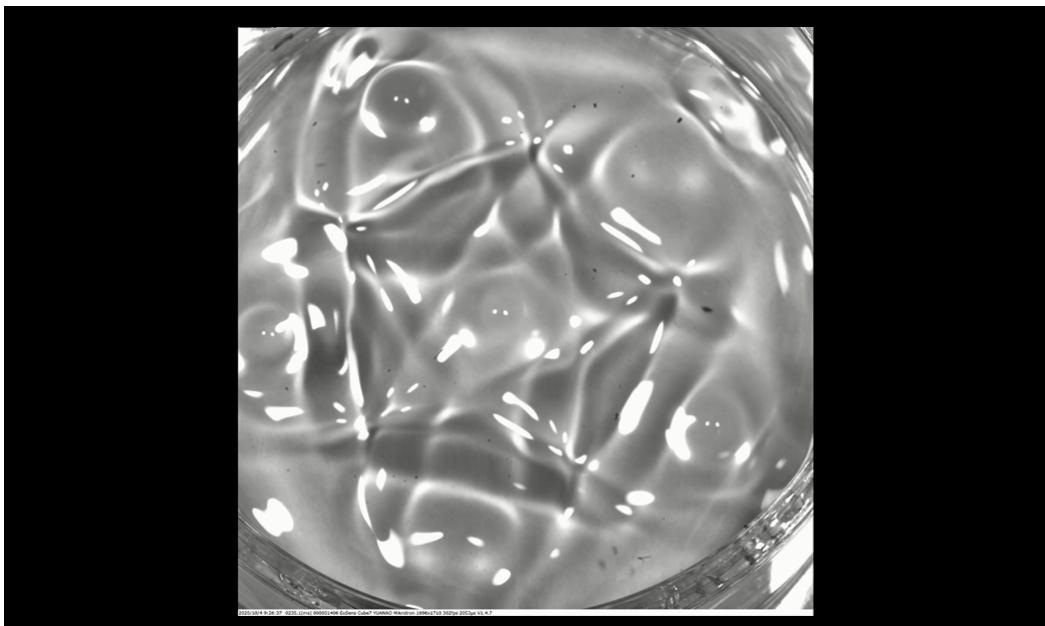
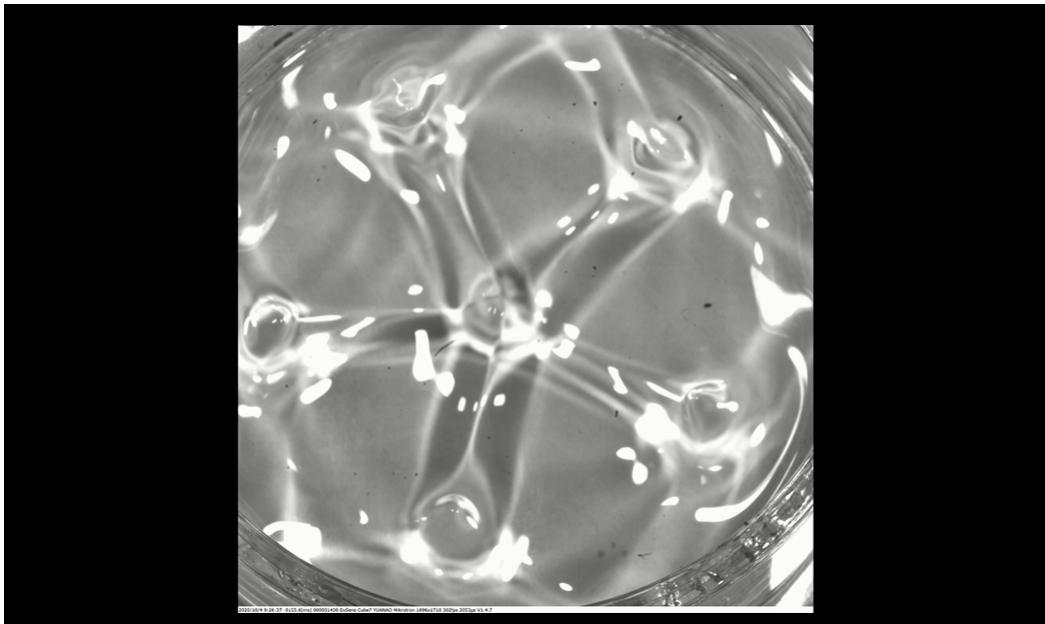
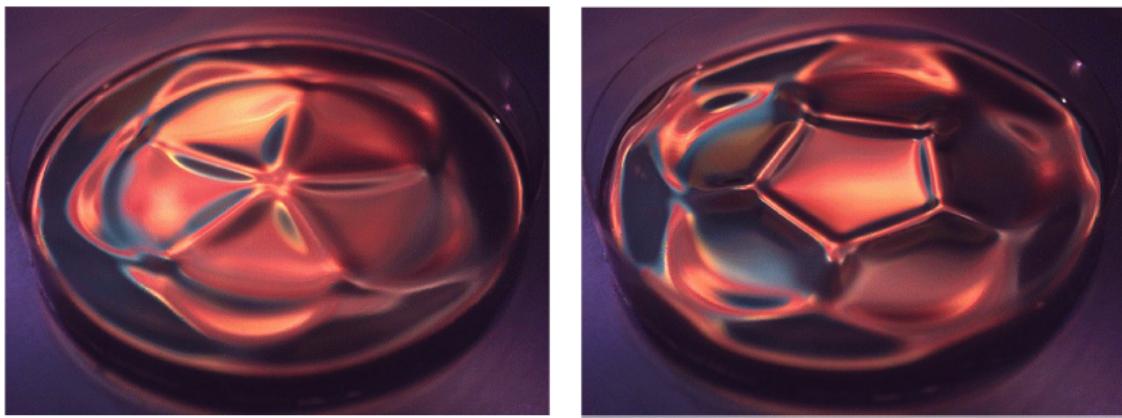
墨水底



牛奶



对比



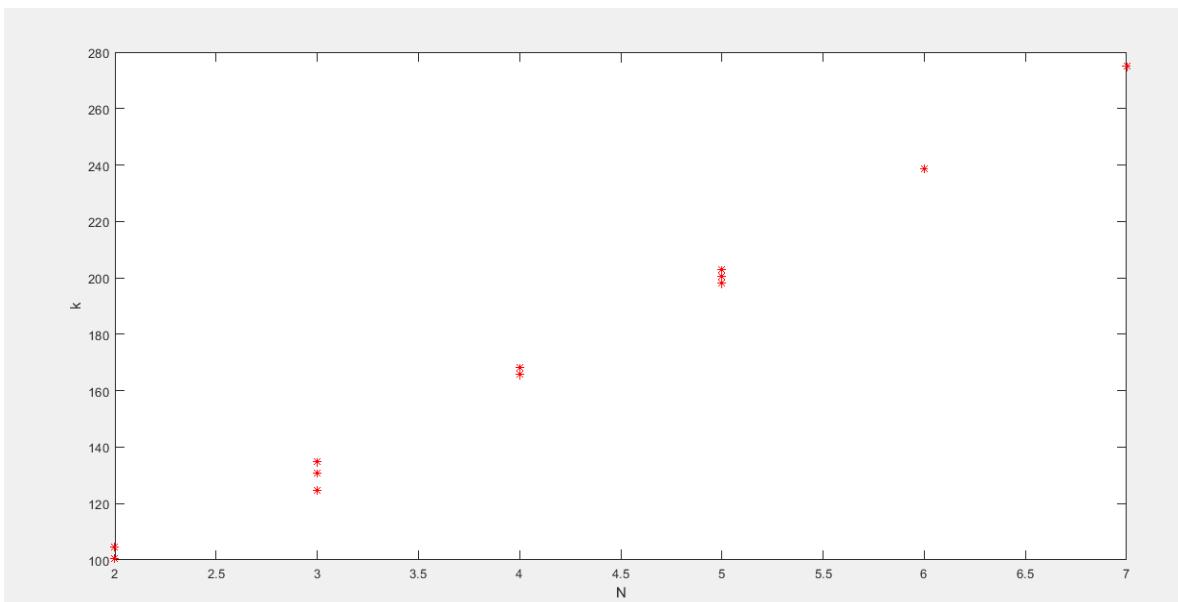
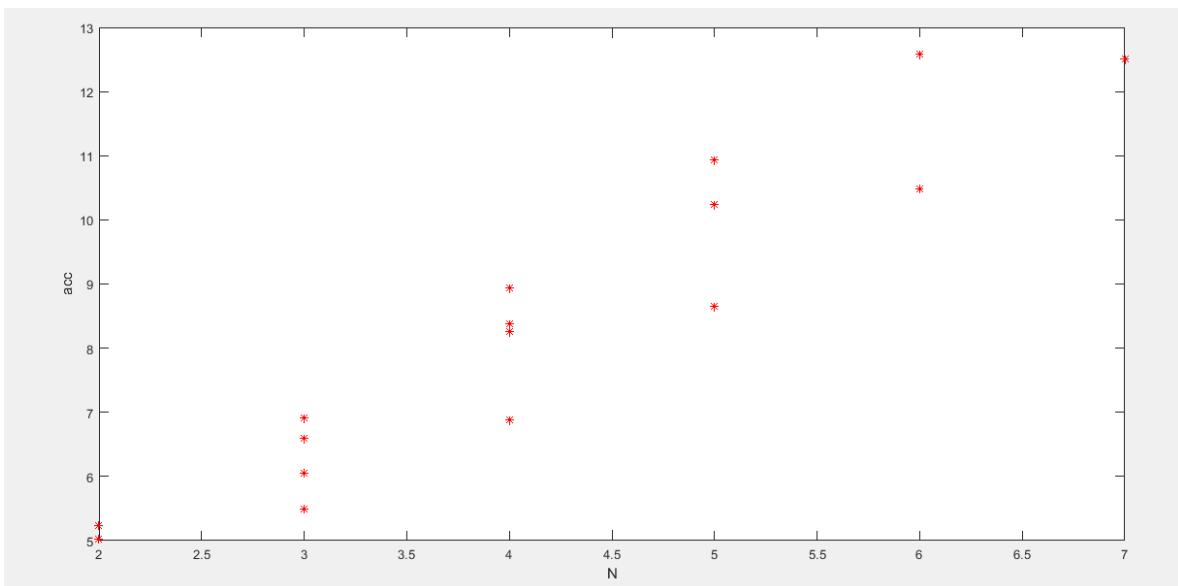
问题

- 六阶模态后，比较混乱
- 水平问题
- 表面波形问题（液体内部流动的干扰）
- 黑白高速摄像机CCD?，需要灯光很亮
- 灯光的布置

尝试

- 加色素
- 彩色摄像机和灯光
- 其它液体材料

液高 h (mm)	位移峰 峰值 s (mm)	频率 f (Hz)	模态 N	备注	圆频率	加速度	波频率	波圆频率	波长	波数
5	1.2	11.3- 12.0	3		70.99993	6.049189	5.65	35.49997		
5	1.2	13.2	4		82.93798	8.254449	6.6	41.46899		
6	1	11.8	3		74.14152	5.496966	5.9	37.07076		
6	1	13.2	4		82.93798	6.878708	6.6	41.46899		
6	1	14.8	5		92.99106	8.647338	7.4	46.49553		
6	1	16.3	6		102.4158	10.489	8.15	51.20792		
6	1	17.8	7		111.8406	12.50832	8.9	55.9203		
					0	0	0	0		
6	1.2	10.3	2	不可 高于 17Hz	64.71675	5.02591	5.15	32.35838		
6	1.2	11.8	3		74.14152	6.596359	5.9	37.07076		
6	1.2	13.3	4		83.56629	8.379991	6.65	41.78315		
6	1.2	14.7	5		92.36275	10.23705	7.35	46.18137		
6	1.2	16.3	6		102.4158	12.5868	8.15	51.20792		
					0	0	0	0		
6	1.3	10.1	2	不可 高于 16Hz	63.46012	5.235343	5.05	31.73006		
6	1.3	11.6	3		72.88489	6.905869	5.8	36.44244		
6	1.3	13.2	4		82.93798	8.94232	6.6	41.46899		
6	1.3	14.6	5		91.73443	10.93977	7.3	45.86721		



波形跟倾斜方向有关

波形重构同样是只要表面波形进行重构，滤去折射透射光很重要吧