1. $A = \begin{pmatrix} 0 & -20 & -14 \\ 3 & 27 & -4 \\ 4 & 11 & -2 \end{pmatrix}$

$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$ $\quad P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$P_1 = P_{12}P_{23} = \begin{pmatrix} 0 & \frac{3}{5} & \frac{4}{5} \\ -1 & 0 & 0 \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 0 & -20 & -14 \\ 3 & 27 & -4 \\ 4 & 11 & -2 \end{pmatrix}$

$P_1A = \begin{pmatrix} 5 & 23 & -4 \\ 0 & 20 & 14 \\ 0 & -15 & 2 \end{pmatrix}$

$P_{12} = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$ $\quad P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{4}{5} & -\frac{3}{5} \\ 0 & \frac{3}{5} & \frac{4}{5} \end{pmatrix}$

$P_2P_1A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{4}{5} & -\frac{3}{5} \\ 0 & \frac{3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 5 & 23 & -4 \\ 0 & 20 & 14 \\ 0 & -15 & 2 \end{pmatrix}$

$= \begin{pmatrix} 5 & 23 & -14 \\ 0 & 25 & 10 \\ 0 & 0 & 10 \end{pmatrix}$

$$P = P_2P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{4}{5} & -\frac{3}{5} \\ 0 & \frac{3}{5} & \frac{4}{5} \end{pmatrix}\begin{pmatrix} 0 & \frac{3}{5} & \frac{4}{5} \\ -1 & 0 & 0 \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} = \begin{pmatrix} 0 & \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{12}{25} & -\frac{9}{25} \\ -\frac{3}{5} & -\frac{16}{25} & \frac{12}{25} \end{pmatrix}$$

$$T = \begin{pmatrix} 5 & 23 & -14 \\ 0 & 25 & 10 \\ 0 & 0 & 10 \end{pmatrix}$$

$4+4+4$
12

2. $A = \begin{pmatrix} 1 & 19 & -34 \\ -2 & -5 & 20 \\ 2 & 8 & 37 \end{pmatrix}$

$R_1 = I - 2\frac{u_1u_1^T}{u_1^Tu_1}$ $\quad u_1 = Ax_1 - \|Ax\|e_1 = (-2 \ \ -2 \ \ 2)^T$

$$R_1 = I - \frac{1}{6}(-2 \ -2 \ \ 2)^T(-2 \ -2 \ \ 2) = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

$$R_1A = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 19 & -34 \\ -2 & -5 & 20 \\ 2 & 8 & 37 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 15 & 0 \\ 0 & -9 & 54 \\ 0 & 12 & 3 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -9 & 54 \\ 12 & 3 \end{pmatrix}$$

$$\hat{R}_2 = I - 2\frac{u_2 u_2^*}{u_2^* u_2} \qquad u_2 = A_{2*1} + \|A_{2*1}\| e_1 = (6 \;\; 12)^T$$

$$\hat{R}_2 = I - \frac{1}{90}(6 \;\; 12)^T (6 \;\; 12) = \frac{1}{5}\begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix}$$

$$\hat{R}_2 A_2 = \frac{1}{5}\begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix}\begin{pmatrix} -9 & 54 \\ 12 & 3 \end{pmatrix} = \begin{pmatrix} -15 & 30 \\ 0 & -45 \end{pmatrix} \longrightarrow$$

取负，保证对角元是正

$$R_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & \frac{4}{5} \\ 0 & \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$R_2 R_1 A = \begin{pmatrix} 3 & 15 & 0 \\ 0 & 15 & -30 \\ 0 & 0 & 45 \end{pmatrix}$$

$$P = R_2 R_1 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & \frac{4}{5} \\ 0 & \frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ \frac{14}{15} & \frac{1}{3} & -\frac{2}{15} \\ -\frac{2}{15} & \frac{2}{3} & \frac{11}{15} \end{pmatrix}$$

$$Q = P^T = \begin{pmatrix} \frac{1}{3} & \frac{14}{15} & -\frac{2}{15} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{15} & \frac{11}{15} \end{pmatrix} \qquad R = T = \begin{pmatrix} 3 & 15 & 0 \\ 0 & -15 & 30 \\ 0 & 0 & -45 \end{pmatrix}$$

3. $B_\chi = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right\}$ $B_y = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$

(1) $B_\chi \cap B_y = \phi$

$$\mathrm{rank} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} = 3$$

$\therefore$ $B_\chi \cup B_y$ 为 R^3 的基

$\therefore$ χ, y 为 R^3 的补空间

(2) $P = [X \mid 0][X \mid Y]^{-1}$

$$= \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix}$$

$$Q = I - P = \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix} = P \qquad Q^2 = \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix} = Q$$

4、 $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$\quad = B + C$

$B^T = \frac{1}{2}(A^T + A) = B \qquad -C^T = -\frac{1}{2}(A^T - A) = C$

∴ 任意 $A \in R^{n\times n}$ 可以用一个对称矩阵和反对称矩阵表示、

∴ $R^{n\times n} = S + K$，且 $S \cap K = 0$

∴ $R^{n\times n} = S \oplus K$