1、(1) $[T]_B = ([T(u_1)]_B \quad [T(u_2)]_B \quad [T(u_3)]_B)$

$T(u_1) = (1 \ -1 \ 0)^T \qquad [T(u_1)]_B = (1 \ -1 \ 0)^T$

$T(u_2) = (-1 \ 1 \ -1)^T \qquad [T(u_2)]_B = (-\frac{3}{2} \ \frac{1}{2} \ \frac{1}{2})^T$

$T(u_3) = (0 \ 0 \ 1)^T \qquad [T(u_3)]_B = (\frac{1}{2} \ \frac{1}{2} \ -\frac{1}{2})^T$

$$[T]_B = \begin{pmatrix} 1 & -\frac{3}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$[v]_B = (1 \ 1 \ 0)^T$

(2) $T(v) = (0 \ 0 \ -1)^T$

$[T(v)]_B = (-\frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2})^T$

$$[T]_B [v]_B = \begin{pmatrix} 1 & -\frac{3}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = [T(v)]_B$$

2. $[T]_S = ([T(e_1)]_S \ \cdots \ [T(e_n)]_S)$

$= (T(e_1) \ \cdots \ T(e_n))$

$= (Ae_1 \ \cdots \ Ae_n)$

$= (A_{*1} \ \cdots \ A_{*n})$

$= A$

3. (1) $[A]_B = ([A(u_1)]_B \ [A(u_2)]_B \ [A(u_3)]_B)$

$[A(u_1)]_B = A(u_1) = (1 \ 0 \ 1)^T$

$[A(u_2)]_B = A(u_2) = (2 \ -1 \ 0)^T$

$[A(u_3)]_B = A(u_3) = (-1 \ 0 \ 1)^T$

$$[A]_B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 7 \end{pmatrix}$$

$$[A]_{B'} = \left([A(u_1')]_{B'} \quad [A(u_2')]_{B'} \quad [A(u_3')]_{B'} \right)$$

$A(u_1') = (1\ 0\ 1)^T \quad [A(u_1')]_{B'} = (1\ -1\ 1)^T$

$A(u_2') = (3\ -1\ 1)^T \quad [A(u_2')]_{B'} = (4\ -2\ 1)^T$

$A(u_3') = (2\ -1\ 8)^T \quad [A(u_3')]_{B'} = (3\ -9\ 8)^T$

$$[A]_{B'} = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & -9 \\ 1 & 1 & 8 \end{pmatrix}$$

(2) $Q = [I]_{B'B} = \left([u_1']_B \quad [u_2']_B \quad [u_3']_B \right)$

$= (u_1' \quad u_2' \quad u_3')$

$$\therefore \quad Q = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

4. $T(e_1) = e_1 \quad T(e_2) = e_1 + e_2$

$\therefore \ \forall x = \alpha e_1 + \beta e_2 \in \mathcal{X} = \text{span}\{e_1, e_2\}$

$T(x) = \alpha T(e_1) + \beta T(e_2) = (\alpha+\beta) e_1 + \beta e_2 \in \mathcal{X}$

$[T|_{\mathcal{X}}]_{\{e_1 e_2\}} = \left([T(e_1)]_{\{e_1 e_2\}} \quad [T(e_2)]_{\{e_1 e_2\}} \right)$

$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$