1. (1) $T(X_{n\times n}) = AX - XA$

$T(X+Y) = A(X+Y) - (X+Y)A = AX - XA + AY - YA$

$= T(X) + T(Y)$

$T(\alpha X) = A\alpha X - \alpha XA = \alpha(AX - XA)$

$= \alpha T(X)$

$\therefore$ 是

(2) $T(A) = A^T$

$T(X+Y) = (X+Y)^T = X^T + Y^T$

$= T(X) + T(Y)$

$T(\alpha X) = (\alpha X)^T = \alpha X^T$

$= \alpha T(X)$

$\therefore$ 是

(3) $T(X_{n\times n}) = \dfrac{X + X^T}{2}$

$T(X+Y) = \frac{1}{2}[X+Y+(X+Y)^T] = \frac{1}{2}(X+X^T+Y+Y^T)$

$= T(X) + T(Y)$

$T(\alpha X) = \frac{1}{2}[\alpha X + (\alpha X)^T] = \frac{\alpha}{2}(X+X^T)$

$= \alpha T(X)$

$\therefore$ 是

(4) $T(X_{n\times 1}) = Ax + b$, $b \neq 0$

$T(x+y) = A(x+y) + b$

$T(x)+T(y) = A(x+y)+2b$

$\therefore$ 不是

2. (1) $[I]_B = \left([I(u_1)]_B \quad [I(u_2)]_B \quad [I(u_3)]_B\right)$

$= \left([u_1]_B \quad [u_2]_B \quad [u_3]_B\right)$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

$[I]_{B'} = [I]_B = I$

$[I]_{BB'} = \left([I(u_1)]_{B'} \quad [I(u_2)]_{B'} \quad [I(u_3)]_{B'}\right)$

$= \begin{pmatrix} -1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$

(2) $[P]_{BB'} = \left([P(u_1)]_{B'} \quad [P(u_2)]_{B'} \quad [P(u_3)]_{B'}\right)$

$p(u_1) = (1\ 0\ 0)^T \quad [p(u_1)]_{B'} = (-1\ 0\ 0)^T$

$p(u_2) = (1\ 1\ 0)^T \quad [p(u_2)]_{B'} = (-1\ 1\ 0)^T$

$p(u_3) = (1\ 1\ 0)^T \quad [p(u_3)]_{B'} = (-1\ 1\ 0)^T$

$[P]_{BB'} = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

3. (1) $[T]_{BB'} = \left([T(e_1)]_{B'} \;\; [T(e_2)]_{B'} \;\; [T(e_3)]_{B'}\right)$

$T(e_1) = (1,0)^T \quad [T(e_1)]_{B'} = (1,0)^T$

$T(e_2) = (1,1)^T \quad [T(e_2)]_{B'} = (1,1)^T$

$T(e_3) = (0,-1)^T \quad [T(e_3)]_{B'} = (0,-1)^T$

$$[T]_{BB'} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$[L]_{B'} = \left([L(e_1)]_{B'} \;\; [L(e_2)]_{B'}\right)$

$L(e_1) = (2,1)^T \quad [L(e_1)]_{B'} = (2,1)^T$

$L(e_2) = (-1,0)^T \quad [L(e_2)]_{B'} = (-1,0)^T$

$$[L]_{B'} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

(2) $T(x,y,z) = (x+y,\, y-z) \quad L(u,v) = (2u-v,\, u)$

$C = LT = [2(x+y)-(y-z),\, x+y]$

$= (2x+y+z,\, x+y)$

$[C]_{BB'} = \left([C(e_1)]_{B'} \;\; [C(e_2)]_{B'} \;\; [C(e_3)]_{B'}\right)$

$C(e_1) = (2,1)^T \quad [C(e_1)]_{B'} = (2,1)^T$

$C(e_2) = (1,1)^T \quad [C(e_2)]_{B'} = (1,1)^T$

$C(e_3) = (1,0)^T \quad [C(e_3)]_{B'} = (1,0)^T$

$$\therefore [C]_{BB'} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$[L]_{B'}[T]_{BB'} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} = [C]_{BB'}$$