1. $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & -3 \\ 0 & 1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$k=1 \quad r_{11} = \|a_1\| = \sqrt{3} \quad q_1 = \frac{a_1}{r_{11}} = \frac{1}{\sqrt{3}} (1,1,1,0)^T$

$k=2 \quad r_{12} = q_1^T a_2 = \sqrt{3} \quad q_2 = a_2 - r_{12} q_1 = (-1,1,0,1)^T$

$r_{22} = \|q_2\| = \sqrt{3} \quad q_2 = \frac{q_2}{r_{22}} = \frac{1}{\sqrt{3}} (-1,1,0,1)^T$

$k=3 \quad r_{13} = q_1^T a_3 = -\sqrt{3} \quad r_{23} = q_2^T a_3 = \sqrt{3}$

$q_3 = a_3 - r_{13} q_1 - r_{23} q_2 = (1,1,-2,0)^T$

$r_{33} = \|q_3\| = \sqrt{6} \quad q_3 = \frac{q_3}{r_{33}} = \frac{1}{\sqrt{6}} (1,1,-2,0)^T$

$$Q = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 0 & 1/\sqrt{3} & 0 \end{pmatrix} \quad R = \begin{pmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{pmatrix}$$

$$Rx = Q^T b$$

$$\begin{pmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{3} & 1/\sqrt{3} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{3} \\ 1/\sqrt{3} \\ 0 \end{pmatrix}$$

$$\therefore x = \begin{pmatrix} 2/3 \\ 1/3 \\ 0 \end{pmatrix}$$

4. $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 10^{-3} \\ 10^{-3} & 0 & 0 \end{pmatrix}$

$u_1 = \frac{x_1}{\|x_1\|} = x_1 = (1, 0, 10^{-3})^T$

$u_2 = x_2 - u_1^T x_2 u_1 = (0, 0, -10^{-3})^T \quad u_2 = \frac{u_2}{\|u_2\|} = (0, 0, -1)^T$

$u_3 = x_3 - u_1^T x_3 u_1 - u_2^T x_3 u_2 = (0, 10^{-3}, -10^{-3})^T \quad u_3 = \frac{u_3}{\|u_3\|} = (0, 0.707, -0.707)^T$

$u_1 = \begin{pmatrix} 1 \\ 0 \\ 10^{-3} \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad u_3 = \begin{pmatrix} 0 \\ 0.707 \\ -0.707 \end{pmatrix}$

$k=1 \quad u_1 = \frac{x_1}{\|x_1\|} = (1, 0, 10^{-3})^T \quad u_2 = x_2 \quad u_3 = x_3$

$k=2 \quad u_2 = u_2 - u_1^T u_2 u_1 = (0, 0, -10^{-3})^T \quad u_3 = u_3 - u_1^T u_3 u_1 = (0, 10^{-3}, -10^{-3})^T$

$u_2 = \frac{u_2}{\|u_2\|} = (0, 0, -1)^T \quad u_3 = \frac{u_3}{\|u_3\|} = (0, 0.707, -0.707)^T$

$k=3 \quad u_3 = u_3 - u_2^T u_3 u_2 = (0, 0.707, 0)^T$

$u_3 = \frac{u_3}{\|u_3\|} = (0, 1, 0)^T$

$u_1 = \begin{pmatrix} 1 \\ 0 \\ 10^{-3} \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad u_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}^T$

3. $U = \begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{1+i}{\sqrt{6}} \\ \frac{i}{\sqrt{3}} & \frac{-2i}{\sqrt{6}} \end{pmatrix} \quad U^* = \begin{pmatrix} \frac{1-i}{\sqrt{3}} & \frac{-i}{\sqrt{3}} \\ \frac{1-i}{\sqrt{6}} & \frac{2i}{\sqrt{6}} \end{pmatrix}$

$UU^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

∴ 是酉矩阵

4. $x = (\frac{1}{3}\ -\frac{2}{3}\ -\frac{2}{3})^T$

$u = x - \|x\| e_1 = -\frac{2}{3}(1, 1, 1)^T$

$R = I - \frac{2uu^T}{u^Tu}$

$= \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$

5. $R = I - 2uu^T \quad \|u\| = 1$

$Rx = (I - 2uu^T)x = x - 2uu^Tx = x$

$\therefore uu^Tx = 0 \quad (u^Tx)u = 0$

$\because u \neq 0$

$\therefore u^Tx = 0$

$\therefore x \perp u$ 正交