

# **Electronic Differential and Hybrid Powertrain Design for NCSU Formula Hybrid FH.2009**

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## **ABSTRACT**

Research and design were conducted on the 2009 NC State University Formula SAE Hybrid vehicle electric powertrain and differential. The NCSU FH.09 vehicle utilized a series electric hybrid system to create an electronic differential with no mechanical components connecting the two rear wheels. Research and analysis was conducted to govern the movement of the rear wheels to allow maximum traction while allowing instant driver adjustment based on his/her preference. The system mainly consists of two Perm PMG 132 DC permanent magnet motors, IOtech Daqboard 1005 data acquisition, Renesas Technology micro-controller, Hall Effect sensors, and an accelerometer.

## **INTRODUCTION**

### **ELECTRIC HYBRID OVERVIEW**

As the search for alternative forms of energy to power personal transportation needs, the hybrid electric vehicle has been selected by many automotive manufacturers as a viable alternative to the conventional petrol powered vehicle. Currently there are two predominant layouts of hybrid electric vehicles: parallel hybrids and series hybrids. Parallel hybrids, the most common form of hybrid electric vehicles, consist of both an electrical motor as well as an internal combustion engine directly for propulsion. Examples of this hybrid powertrain layout include vehicles such as the Honda Insight, Toyota Prius, and BMW

ActiveHybrid X6. The less common of the two powertrain designs is the series hybrid (Extended Range Hybrid), which does not have an internal combustion engine directly connected to the drive axle. Instead, the internal combustion engine is only used for extending the range of this predominantly electric vehicle when the battery capacity drops. Currently the only production version of this vehicle will be the Chevrolet Volt, set to debut in 2011. In addition to be a series hybrid vehicle, the Volt is also a "Plug-In" Hybrid, meaning that one could plug it in to a power source to charge the electrical accumulator.

While these vehicles do provide much lower fuel consumption, there are also many other innovative concepts that can be implemented on these new hybrid electric vehicles. Since electric motors are very compact, there are a variety of different packaging options. Furthermore, alternative methods of driving the vehicle can be implemented. This paper will go into detail about an alternative to the conventional, mechanical based differential.

## **VEHICLE INFORMATION**

The hybrid powertrain and electronic differential in this report were designed for the 2009 NC State University Formula SAE Hybrid vehicle. Since this a racecar, the system will be susceptible to very extreme circumstances compared to a road car. This is also an excellent test bed for the system because the

chassis is designed to be extremely stiff to withstand the increased roll stiffness, which means that whatever happens with the E-diff will be easily communicated to the driver. Typical vehicle weights for Formula SAE vehicles is approximately 210 kg without the driver. The vehicle uses a typical FSAE layout of the components and drive in that it is a rear engine rear wheel drive. The motors and drivetrain will easily be accommodated in the vehicle since it is built and mounted for the purpose of these electric motors rather than a retrofit.

The NCSU.FH.09 used a somewhat different approach to most other Formula SAE Hybrid vehicles in terms of powertrain layout. The NCSU.FH.09 utilized a series hybrid powertrain to use the Electronic Differential.

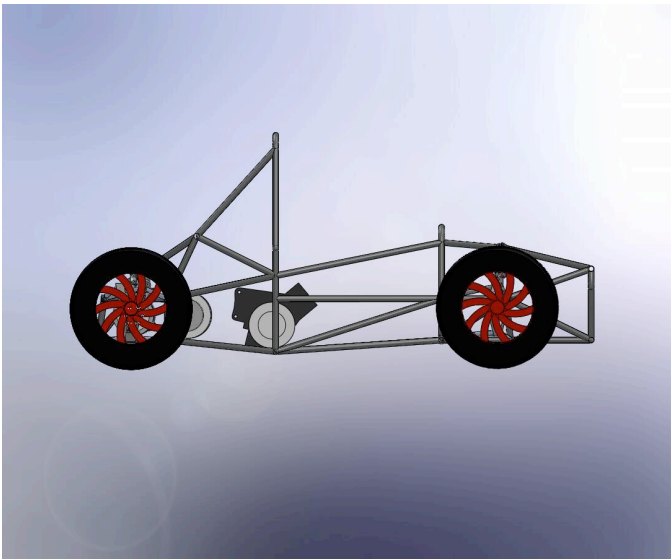


Figure 1: NCSU FH.09 Vehicle

## DIFFERENTIAL INTRODUCTION

There are a wide variety of mechanical differentials available for vehicles today, but they all serve the same purpose of delivering power to the ground. Most differentials fall somewhere between the locked axle or “spool” and the open differential. The modern differential was first created in 1827 for steam traction engines to enhance cornering. For any turn, there needs to be a certain amount of slip between the inside and outside wheels to allow

the vehicle to turn. In a locked axle, the inside tire causes a significant amount of drag in order to create the slip. A benefit of using a spool is that it delivers an infinite torque bias enabling the vehicle to move even in low traction situations. A good approximation for the amount of tire slip induced during low speed turning for a locked axle is Equation 1 below where  $R_O$  is the turn radius for the outside tire,  $R_I$  is the turn radius for the inside tire, and  $V_{diff}$  is difference in apparent road velocity.

Equation 1 
$$V_{diff} = \frac{R_O}{R_I} - 1$$

On the other side of the spectrum there is the open differential. The open differential solves the problem of low speed drag of the inside wheel by allowing the sum of the inside and outside wheel rotations to equal a constant. The inside wheel is allowed to reduce in speed to cover the shorter distance; however, it comes at a cost. The amount of torque propelling the vehicle is only as much as the wheel with the least traction.

The most common solution to this problem is by combining the open differential and locked axle to a certain degree. In most high performance applications, there is some amount of torque bias the differential is able to operate under before it resumes to acting as an open differential. This has been accomplished multiple ways including a mechanical based torque bias; limited slip differential, such as the Quaife and Torsen differentials and clutch based LSD. The actual mechanics of each system are very different and each has their own advantages and disadvantages.



Figure 2: Torsen Differential Internals

The electronic differential in this paper will use the electric motors to develop a variable differential action without any mechanical linkage combining the two rear wheels.

## MAIN SECTION

### POWERTRAIN DESIGN

#### Powertrain Layout

The first step in developing the electronic differential was selecting a suitable powertrain layout to accommodate the electronic differential. The NCSU.FH.09 used a somewhat different approach to most other Formula SAE Hybrid vehicles in terms of powertrain layout. The NCSU.FH.09 utilized a series hybrid powertrain to reduce complexity of the Electronic Differential. The series hybrid layout also allows the vehicle to run without much fluctuation in engine speed of the internal combustion engine hence reducing fuel consumption.

There are many locations on the vehicle that could possibly work due to the compactness of electric motors. While the motors could be simply placed in line with the rear wheels or even inside the wheels, this route was not taken since it compromise cornering ability of the vehicle. In-wheel motors would drastically increase unsprung mass by as much as 300% and both placing the motors inline with the

wheels and mounting them inside the wheel would further increase the weight bias towards the rear. To attempt to achieve close to a 50/50 weight balance between the front and the rear, the electric motors have to be moved as far forward as possible (close to the internal combustion engine). Another factor to take into account is the moment of inertia imposed by the electric motors. The moment of inertia about the x-axis of the vehicle (body roll) is given below by Equation 2.

$$\text{Equation 2} \quad I_{xx} = m(y^2 + z^2)$$

To reduce the moment of inertia (about the x-axis), the center of gravity of the motors will be placed as close as possible to the centerline of the vehicle. The final layout of the electric motors can be seen in Figure 3.

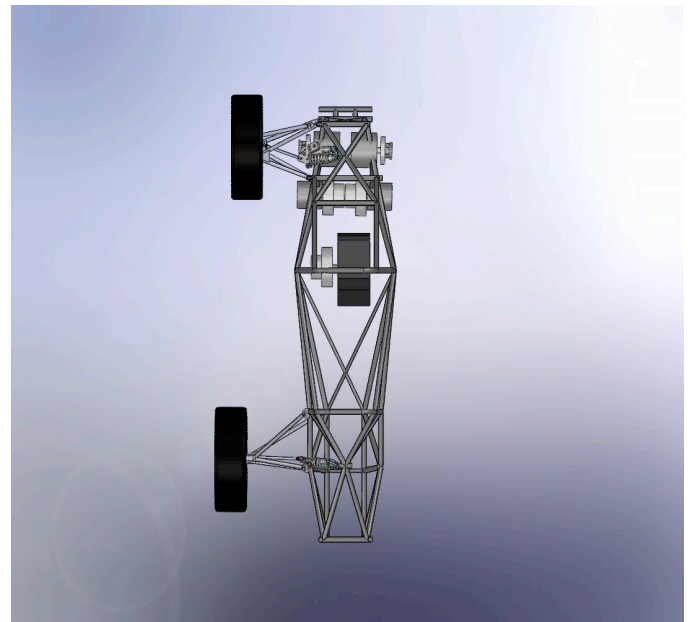


Figure 3: Powertrain and Drivetrain Organization

#### Electric Motor Selection

As the project progressed to the motor selection process, there were a number of factors that were taken into account including:

1. Weight
2. Size
3. Operating speed
4. Power output
5. Efficiency
6. Cost

Since the Formula SAE competitions usually emphasize the handling and accelerating characteristics of the vehicle, weight and power output were especially important. The capability of electric motors to produce peak torque at very low motor speed enhances their ability to accelerate a vehicle quickly. After examining the different motors available, the permanent magnet motors made by Perm Motor were exceptionally small, light, and efficient compared to the rest of the competition. The “pancake” style motors (shown in Figure 4) developed by Perm seem to fit the tight packaging restraints better than others available. Furthermore, the slim design of the pancake motors allow them to be placed closer to the centerline of the vehicle hence reducing the moment of inertia under cornering.



Figure 4: Perm PMG 132

Perm Motor’s permanent magnet motors also are exceptionally efficient. Perm Motor’s AC synchronous motors boasted as much as 93% efficiency while the more cost effective and popular DC Permanent Magnet motor, PMG 132, has an efficiency of up to 90%. Table 1 shows a comparison between the water-cooled AC Motor (PMS 120W), air cooled AC motor (PMS 120L), and the DC permanent magnet motor (PMG 132). These three motors have comparable amounts of torque and power output. The values at the table are for a 72-80V system, which is governed by the preselected batteries. The water cooled

synchronous AC motor produces the best power to mass ratio at .769 kW/kg followed by the DC motor at .656 kW/kg; however, there will be extra weight for the water cooled motor for additional components such as a radiator, water pump, and fluid. The AC motors also have a clear advantage over the DC motor in that they are more efficient and have a 72.4% increase in maximum speed. This higher maximum speed would allow the vehicle to use a larger gear ratio, providing more torque while still being able to reach the maximum speed of the racecourse of approximately 100 km/h.

Table 1: Electric Motor Comparison

|                                | PMS 120W | PMS 120L | PMG 132 |
|--------------------------------|----------|----------|---------|
| Weight (kg)                    | 14.3kg   | 12.3     | 11      |
| Rated Power (kW)               | 11.0     | 7.0      | 7.22    |
| Peak Torque, Pulse Torque (Nm) | 40, 44.6 | 40,45    | 38,38   |
| Efficiency                     | 92%      | 92%      | 90%     |
| Max. Speed (RPM)               | 6000     | 6000     | 3480    |

In the end, the decision was made to use two PMG 132 motors due to their higher availability and lower price. The AC motors would take over 2 months to build before receiving while the PMG 132 motors could be received in less than a week. A CVT will be used to accommodate for the reduced motor speed.

ELECTRONIC DIFFERENTIAL DESIGN

Using the two rear motors to operate independently yields several benefits that

would be difficult if not impossible to duplicate with a mechanical differential. The E-diff would be able to change the amount of torque bias and wheel speed instantly in relation from inputs from the steering angle, throttle position (driver controlled), and wheel speeds. Instead of simply reacting to a loss of traction or an increase in resistance to the inside wheel to change the wheel speeds, the E-diff would anticipate the corner based on the steering angle. Another goal of the E-diff is to provide quick adjustability for different drivers. By changing the locking action with a press of a button, the car could change from slight oversteer to slight understeer based on driver preference.

### System Layout

The E-Diff uses a fairly simple layout of modifying the power sent to each motor. While there is an onboard PC on the NCSU FH.09, the actual throttle modification occurs using a Renesas Technology micro-controller so that in the event of a computer failure, the vehicle will remain drivable. The pedal actuated throttle control uses a high precision 5k $\Omega$  potentiometer. Based on the voltage drop across the potentiometer, the microcontroller divides the signal into two voltage signals with each going to the respective motor controllers. The actual calculation of the amount of throttle sent to each motor is determined on the microcontroller from the IOtech DaqBoard 1005 data acquisition system on the onboard PC. The throttle is modified from rear wheel and front wheel speeds as well as steering angle from the data acquisition system. Both the steering angle and the wheel speeds are determined by Hall Effect sensors. Instead of measuring the amount of rotation of the steering column, the steering angle is calculated by the movement of the rack, making it much easier to determine the movement with minimum added components. Figure 5 shows the schematic of the system.

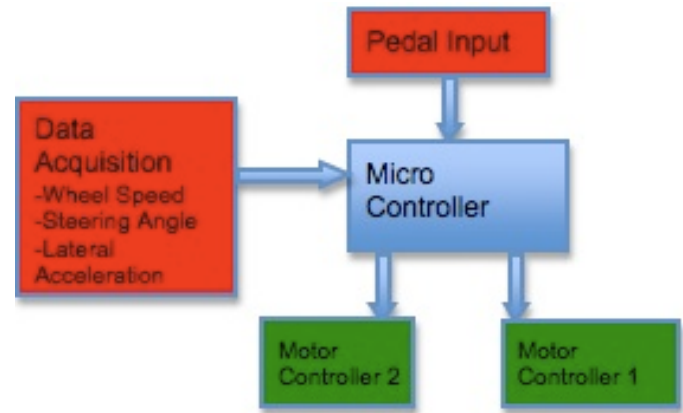


Figure 5: Throttle Control Layout

### Steady State Steering Analysis

It is of the utmost importance to conduct steering analysis of the vehicle. Without conducting steering analysis, it is impossible to calibrate the electronic differential to the steering angle.

#### Assumptions

Steady state of course implies that the system is undergoing constant velocity as it revolves about some turn center. This results in assuming:

- No lateral load transfer
- No longitudinal load transfer
- No roll or pitch
- Linear range tires
- No chassis or suspension compliance

#### Analysis

Steady state cornering analysis was carried out to find out the theoretical wheel speeds of each of the rear wheels and to determine a relationship between the wheel angle and turning radius. The vehicle velocity was first resolved into polar coordinates to determine the wheel speeds. Since the assumption is that the vehicle is maintaining a specific course about some turning radius,  $r$ , the velocity and acceleration in the radial direction,  $r$  and  $\dot{r}$



respectively, is zero. Based on the system layout, Figure 5, our knowns are the turn radius ( $r$ ), lateral acceleration ( $a_n$ ), and rear track ( $t_r$ ). The equation for velocity in the direction of theta,  $V_\theta$ , for the inside and outside wheel is governed by in equations 3 and 4.

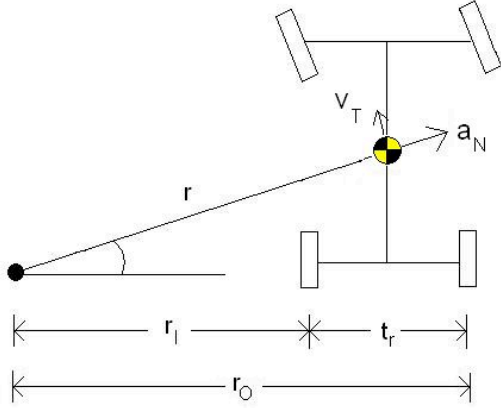


Figure 6: Turning Schematic

Equation 3 
$$V_{T,i} = \left( r - \frac{t_r}{2} \right) \dot{\theta}$$

Equation 4 
$$V_{T,o} = \left( r + \frac{t_r}{2} \right) \dot{\theta}$$

Next, the acceleration in the normal direction to the tangential,  $a_N$ , was examined to determine  $\dot{\theta}$ .

Equation 6 
$$a_N = \frac{v_T^2}{r}$$

Since  $\dot{\theta} = \frac{v_\theta}{r}$ , Equation 6 can be rewritten as:

Equation 7 
$$a_N = \dot{\theta}^2 r$$

After solving for  $\dot{\theta}$ , the velocity for the inside and outside wheel can be rewritten as:

Equation 9 
$$v_{T,i} = \left( r - \frac{t_r}{2} \right) \sqrt{\frac{a_N}{r}}$$

Equation 10 
$$v_{T,o} = \left( r + \frac{t_r}{2} \right) \sqrt{\frac{a_N}{r}}$$

Equations 9 and 10 are used to determine the theoretical wheel velocity, which the microcontroller will attempt to match by either adding power or reducing power in a loop. The velocities do not have to be exactly the same as in equations 9 and 10, but usually there has to be less than 9% slip (from Equation 1) depending on the tire before traction is lost. When adjusting the different modes of the differential, the amount of slip can be adjusted to help induce a slight understeer or oversteer to the driver's preference.

The next factor that needs to be determined is a relationship between the steering angle and the turn radius. The relationship between steering angle and turn radius, however, is not always constant; it is also dependent on slip angle denoted as  $\alpha$ . Slip angle is a tire dependant factor that changes with a variety of variables and induces the lateral force. Below in Figure 7 is a plot of the relationship between slip angle and lateral force for the predicated vehicle mass per corner of 75 kg with the driver and fuel. The plot also assumes a tire pressure of 0.9 bar and  $-2^\circ$  of camber for the Michelin FSAE radial tires.

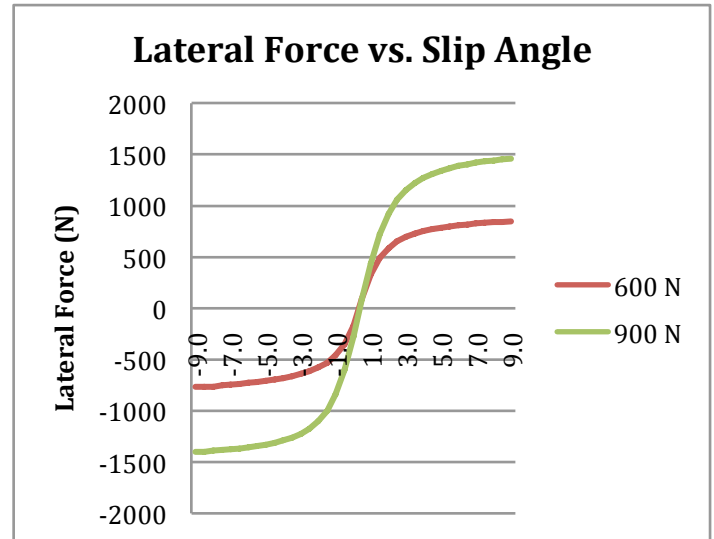


Figure 7: Lateral Force vs. Slip Angle

From the chart, the linear range of the tire occurs between approximately  $-1.5^\circ$  and  $1.5^\circ$ . The equation for the relationship between the steering angle and the Ackerman steering angle is given below in Equation 11 from

Milliken [3] where  $\delta$  is the steering angle,  $\delta_{ack}$  is the Ackerman steering angle,  $\alpha_F$  is the slip angle at the front tires, and  $\alpha_R$  is the slip angle at the rear.

Equation 11 
$$\delta = \delta_{ack} + (-\alpha_F + \alpha_R)$$

Equation 11 can be rewritten as Equation 12 where  $l$  is the wheelbase and  $r$  is the turn radius.

Equation 12 
$$\delta = \frac{l}{r} + (-\alpha_F + \alpha_R)$$

Equation 12 can be further simplified for assuming an understeering, oversteering, or neutral steering vehicle. It is the easiest to compute the steer angle for a neutral steer vehicle since, the slip angle at the front and rear are the same; however, this assumption should not be made until the vehicle's suspension is sorted and determined whether it understeers or oversteers. Whether the vehicle understeers or oversteers, the slip angles at the front and rear wheels can be simplified to a ratio of the slip angle of a neutral steer vehicle, eliminating the need for accelerometer readings at the front and the rear of the vehicle. Equation 12 can be rearranged to solve for the turn radius so that from a given ratio of the steering wheel rotation to the steering angle, the turn radius can be approximated.

Equation 13 
$$r = \frac{l}{\delta + \alpha_F - \alpha_R}$$

Based on the readings from the accelerometer from the data acquisition, the lateral force can be computed. From the tire data, the corresponding slip angle can be found. From equations 9, 10, and 13, the wheel speed is theoretical wheel speed is easily computed after finding the ratio of the steering rack movement to the steering angle. From there the differential can be fully calibrated on a skidpad.

## Driver Adjustment

Although the microcontroller uses equations 9, 10, and 13 to compute approximate theoretical wheel speeds on braking and applying power through a turn, some adjustment is made for the driver's preference. The only parameter that will remain the same for the different settings is that maximum power will be delivered relative to throttle position when the steering angle is zero. To change the differential the amount of oversteer induced in the settings, the relative rear wheel velocities will change up to 9% since 9% is the approximately where the tires will begin to lose traction. The inside wheel receives less power to achieve the desired relative velocity so that the driver is controlling the maximum power input. The differential setting is controlled by a button on the steering wheel connected to the microcontroller to cycle through the settings. Furthermore, in the event of loss of traction, the wheel speeds are locked together to place the driver in complete command.

## Comparison to Limited Slip Differentials

The purpose of the electronic differential is to quickly distribute power to the rear wheels, maintaining the maximum possible traction while providing almost instant adjustability to the driver's preference. For quite some time, varying forms of Limited Slip Differentials (LSD) have dominated in the racing and high performance industry. It provides a good balance between the wheel speed differentiation from an open differential while maintaining a limited amount of torque bias to allow the wheels to maintain traction under high torque. The LSD and all of its variants do work well, but lack quick adjustability. Depending on the type of LSD, the torque bias, as well as ramp angles, can be changed, but it requires removing the differential and physically changing mechanical components inside it. Depending on the type and the settings of the LSD, the torque bias ratio can be modified. The "torque bias ratio" is defined as the quotient of the torque delivered by the

axle with more grip divided by the torque delivered to the axle with the lesser grip. Normally, the torque bias ratio is set to no higher than a 4:1 depending on the application. The Electronic Differential however can deliver an infinite torque bias ratio in that it can provide power to only one of the two axles. This provides a large reduction in turn radius. Furthermore, many of the speed sensitive LSDs (usually types depending on fluid friction) tend to have a slight lag between the onset of wheel slip and the detection by the differential.

## CONCLUSION

The selection of motors as well as the electronic differential system provides a unique and advantageous design for the NCSU FH.09. Further testing will have to be conducted to optimize the differential settings and provide a factor for the equations used to govern it to accommodate for weight transfer and acceleration effects. The electronic differential is currently set up to provide maximum control and feedback to the driver by not automatically trying to regain traction. A non-intrusive traction control will be developed further on that will attempt to find a balance between driver control and traction control. Further study of this electronic differential system is recommended to acquire actual testing data.

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## DEFINITIONS, ACRONYMS, ABBREVIATIONS

Ackerman Steering Angle ( $\delta_{ack}$ ): geometric steer angle require for car with wheelbase,  $l$ , and turn radius  $R$ .  $\delta_{ack} = \frac{l}{R}$

Camber: Angle between the tilted wheel plane and the vertical where it is positive if the top of the tire is farther out from the vehicle than the bottom.

CVT: Continuously Variable Transmission

LSD: Limited Slip Differential

Oversteer: slip angle at the front is less than slip angle at rear resulting in the vehicle turning in greater than the Ackerman Steering Angle

Slip Angle ( $\alpha$ ): angle between the normal to the tire and the direction of travel

Torque Bias Ratio: quotient of the torque delivered to the axle with more grip divided by the torque provided to the axle with less grip

Understeer: slip angle at the front is greater than slip angle at rear resulting in the vehicle turning out of the turn.