ASSIGNMENT-2
Probability & Random Variables

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Question

A die is thrown twice. What is the probability that

1) 5 will not come up either time?

2) 5 will come up at least once?

Solution

Let X be the random variable representing the number of 5's that come up in two die throws. Then X follows a binomial distribution with n=2 and p=$\left(\frac{1}{6}\right)$, since each die throw is a Bernoulli trial with probability of success p=$\left(\frac{1}{6}\right)$ probability of failure q=$\left(\frac{5}{6}\right)$

The PMF of X is given by:

$$\Pr\left(X = k\right) = {}^{n}C_{k} \cdot p^{k} \cdot q^{n-k} \tag{1}$$

$$= {}^{2}C_{k} \cdot \left(\frac{1}{6}\right)^{k} \cdot \left(\frac{5}{6}\right)^{2-k} \qquad \forall k = 0, 1, 2 \tag{2}$$

1) To find the probability that 5 will not come up either time, we need to find P(X = 0). Using the PMF from (2) :

$$\Pr\left(X = 0\right) = {}^{2}C_{0} \cdot \left(\frac{1}{6}\right)^{0} \cdot \left(\frac{5}{6}\right)^{2} \tag{3}$$

$$= \left(\frac{5}{6}\right)^{2} \tag{4}$$

$$= \left(\frac{25}{36}\right) \tag{5}$$

$\therefore$ The Probability that 5 will not come up either time is $\left(\frac{25}{36}\right)$

2) To find the probability that 5 will come up at least once, we need to find $\Pr(X \geq 1)$. Using the PMF from (2) :

$$\Pr(X \geq 1) = \Pr(X = 1) + \Pr(X = 2) \tag{6}$$

$$= \left[{}^2C_1 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^1\right] + \left[{}^2C_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^0\right] \tag{7}$$

$$= \left(\frac{2 \times 5}{6^2}\right) + \left(\frac{1}{6^2}\right) \tag{8}$$

$$= \left(\frac{11}{36}\right) \tag{9}$$

$\therefore$ The Probability of rolling a 5 at least once is $\left(\frac{11}{36}\right)$