

ASSIGNMENT-2

Probability & Random Variables

1

Merugu Balavardhan
BT22BTECH11010

Question

A die is thrown twice. What is the probability that

- 1) 5 will not come up either time?
- 2) 5 will come up at least once?

Solution

Let X be the random variable representing the number of 5's that come up in two die throws. Then X follows a binomial distribution as follows : 2

Parameters	Value	Description
n	2	Number of trials in an Experiment
p	1/6	Probability of Success
q	5/6	Probability of Failure

TABLE 2

The PMF of X is given by:

$$\Pr(X = k) = {}^nC_k \cdot p^k \cdot q^{n-k} \quad (1)$$

$$= {}^2C_k \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{2-k} \quad \forall k = 0, 1, 2 \quad (2)$$

The Cumulative Distribution Function (CDF) of X is given by the probability that X is less than or equal to a given value k, for k = 0, 1, 2.

The CDF of X can be expressed as:

$$F_X(k) = \Pr(X \leq k) \quad (3)$$

$$\therefore F_X(k) = \sum_{i=0}^k {}^nC_i p^i q^{n-i} \quad (4)$$

- 1) To find the probability that 5 will not come up either time, we need to find $\Pr(X = 0)$.

$$\Pr(X = 0) = F_X(0) \quad (5)$$

$$= {}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^2 \quad (6)$$

$$= \frac{25}{36} \quad (7)$$

\therefore The Probability that 5 will not come up either time is $\left(\frac{25}{36}\right)$

- 2) To find the probability that 5 will come up at least once, we need to find $\Pr(X \geq 1)$.

$$\Pr(X \geq 1) = 1 - \Pr(X \leq 0) \quad (8)$$

$$= 1 - F_X(0) \quad (9)$$

$$= 1 - {}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^2 \quad (10)$$

$$= 1 - \frac{25}{36} \quad (11)$$

$$= \frac{11}{36} \quad (12)$$

\therefore The Probability of rolling a 5 at least once is $\left(\frac{11}{36}\right)$