ASSIGNMENT-2
Probability & Random Variables

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Question

A die is thrown twice. What is the probability that

1) 5 will not come up either time?
2) 5 will come up at least once?

Solution

Let X be the random variable representing the number of 5's that come up in two die throws. Then X follows a binomial distribution as followes : 2

Parameters	Value	Description
n	2	Number of trials in an Experiment
p	1/6	Probability of Success
q	5/6	Probability of Failure

TABLE 2

The PMF of X is given by:

$$\Pr(X = k) = {}^{n}C_{k} \cdot p^{k} \cdot q^{n-k} \tag{1}$$

$$= {}^{2}C_{k}\left(\frac{1}{6}\right)^{k}\left(\frac{5}{6}\right)^{2-k} \qquad \forall k = 0, 1, 2 \tag{2}$$

The Cumulative Distribution Function (CDF) of X is given by the probability that X is less than or equal to a given value k, for k = 0, 1, 2.

The CDF of X can be expressed as:

$$F_X(k) = \Pr(X \le k) \tag{3}$$

$$F_X(k) = \begin{cases} \Pr(X \leq 0) & \text{if } k = 0, \\ \Pr(X \leq 1) & \text{if } k = 1, \\ \Pr(X \leq 2) & \text{if } k = 2, \end{cases} \tag{4}$$

$$F_X(k) = \begin{cases} \Pr(X = 0) & \text{if } k = 0, \\ \Pr(X = 0) + \Pr(X = 1) & \text{if } k = 1, \\ \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) & \text{if } k = 2, \end{cases} \tag{5}$$

$$F_X(k) = \begin{cases} {}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^2 & \text{if } k = 0, \\ {}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^2 + {}^2C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^1 & \text{if } k = 1, \\ {}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^2 + {}^2C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^1 + {}^2C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^0 & \text{if } k = 2, \end{cases} \tag{6}$$

$$F_X(k) = \begin{cases} \frac{25}{36} & \text{if } k = 0, \\ \frac{35}{36} & \text{if } k = 1, \\ 1 & \text{if } k = 2, \end{cases} \tag{7}$$

1) To find the probability that 5 will not come up either time, we need to find $\Pr(X = 0)$.

$$\Pr(X = 0) = F_X(0) \tag{8}$$

$$= \frac{25}{36} \tag{9}$$

$\therefore$ The Probability that 5 will not come up either time is $\left(\frac{25}{36}\right)$

2) To find the probability that 5 will come up at least once, we need to find $\Pr(X \geq 1)$.

$$\Pr(X \geq 1) = 1 - \Pr(X \leq 0) \tag{10}$$

$$= 1 - F_X(0) \tag{11}$$

$$= 1 - \frac{25}{36} \tag{12}$$

$$= \frac{11}{36} \tag{13}$$

$\therefore$ The Probability of rolling a 5 at least once is $\left(\frac{11}{36}\right)$