A Physically Based Explicit J–V Model of a Solar Cell for Simple Design Calculations

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***Abstract*—An accurate J–V equation of a practical solar cell is inherently implicit, which necessitates iterative calculations to determine the fill factor and the peak power point. We propose a simple explicit power-law J–V model that is applicable to a wide variety of solar cells. The model allows an easy prediction of the entire J–V curve, peak power point, and fill factor from four simple measurements of the bias points corresponding to V_{oc}, $\sim$0.6 V_{oc}, J_{sc}, and $\sim$0.6 J_{sc}, where V_{oc} is the open-circuit voltage and J_{sc} is the short-circuit current density. The model also provides a closed-form description of the J–V curve, peak power point, and fill factor in terms of physical parameters of the single exponential model.**

***Index Terms*—Curve fitting, fill factor, J–V model, parasitic resistances, solar cell.**

I. Introduction

The current-density–voltage (J–V) equation of solar cells acquires an implicit form with the inclusion of parasitic phenomena such as resistances and space-charge-limited current [1]–[6]. This implicit form calls for iterative calculations to compute the peak power point (J_p, V_p) and the fill factor FF in terms of physical parameters. Existing efforts to derive an explicit solar-cell J–V model, which yields closed-form solutions for (J_p, V_p) point and FF, have achieved limited success. All these efforts have been based on the single exponential J–V model. One effort [2] has employed Lambert W-function; however, this mathematical formulation is fairly complex. Another work [3] has employed a function of the form $J = (V_{oc} - V)/[(V_{oc}/J_{sc}) - CV + BV^2]$; in this case, the constants B and C could not be expressed simply in terms of physical parameters and were numerically computed from two selected bias points on the J–V curve. Yet, another approach [4] has worked for dark J–V characteristics but could not be extended to illuminated characteristics. Considering the aforementioned difficulties, instead of deriving an explicit solar-cell J–V model, some works [5], [6] have focused on the derivation of approximate closed-form solutions for (J_p, V_p) and FF alone.

Once a cell is fabricated, it is important to determine its (J_p, V_p) point and FF. Generally, these parameters are estimated from a detailed measurement of the J–V curve at numerous points [7].

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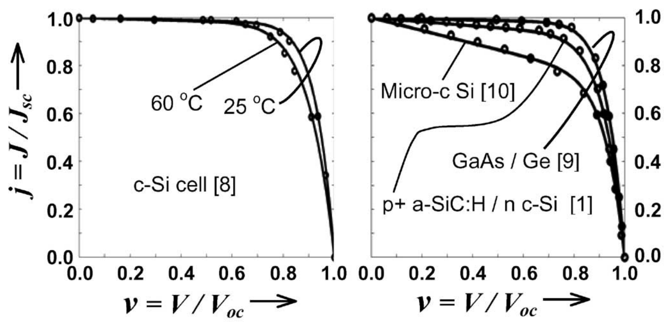


Solar Cell structure	V_{oc} (V)	J_{sc} (mA-cm^{-2})	m	γ	FF	
					Model	Expt
c-Si (25°C)	0.586	35.3	14.0	0.974	0.754	0.74
c-Si (60°C)	0.483	35.6	10.1	0.970	0.700	0.69
Micro-c Si	0.515	21.8	12.9	0.732	0.595	0.58
a-SiC/c-Si	0.548	27.1	12.1	0.915	0.697	0.65
GaAs / Ge	1.012	27.6	14.5	0.977	0.760	0.76

Fig. 1. (Lines) Model fit to (points) experimental data of various solar cells reported in literature. The table gives the m and γ values extracted from measurement and used in calculations of the J–V curve, ν_p, and FF.

In view of the previous discussion, derivation of a simple explicit J–V model, which yields closed-form solutions for the (J_p, V_p) point and FF and allows prediction of these parameters from a few measurements, continues to be an important research problem. We present a new approach to address this problem. Our model and the extraction of its parameters including the (J_p, V_p) point and FF from measurements are described in Section II. The role of our model in simplifying the derivation of cell characteristics from physical parameters is brought out in Section III. Conclusions appear in Section IV.

II. Power-Law Model

Denote the short-circuit current density as J_{sc} and the open-circuit voltage as V_{oc}. Furthermore, let $\nu = V/V_{oc}$ denote the normalized voltage, and let $j = J/J_{sc}$ denote the normalized current density. This normalization enables a compact representation (see Fig. 1) of the J–V measurements of a wide variety of cells in which recombination, tunneling, or space-charge-limited currents may exist in addition to diffusion. This compact representation shows that the following simple explicit power-law function fits the wide variety of J–V measurements accurately

$$j = 1 - (1-\gamma)\nu - \gamma\nu^m. \quad (1)$$

While the linear term of this equation captures the slow fall in current density with voltage near the short-circuit point, the power-law term captures the rapid fall near the open-circuit point. The explicit power-law model (1) has four parameters—γ, m, $V_{\rm oc}$, and $J_{\rm sc}$, as compared to six or more parameters of the single/double exponential and other implicit J–V models [1]–[6].

Closed-form solutions for the (J_p, V_p) point and FF are easily derived from (1). The normalized peak power voltage $\nu_p = V_p/V_{\rm oc}$ is obtained by setting $d(j\nu)/d\nu|_{\nu=\nu_p} = 0$ which yields $1 - 2(1-\gamma)\nu_p - \gamma(m+1)\nu_p^m = 0$. For $\gamma = 1$, this equation has the solution $\nu_p = (m+1)^{-1/m}$, which can be empirically adjusted as

$$\nu_p \approx (m+1)^{-1/m} - 0.05(1-\gamma) \tag{2}$$

to reflect the weak dependence of the accurate ν_p calculations on γ. The unit area load resistance for peak power output is $R_p = (\nu_p/j_p)(V_{\rm oc}/J_{\rm sc})$, and the FF is given by

$$\mathrm{FF} = \nu_p j_p = \nu_p \left[1 - (1-\gamma)\nu_p - \gamma\nu_p^m\right] \tag{3}$$

which increases almost linearly with γ.

As compared to six or more parameters of implicit J–V models [1]–[6], only four parameters need to be extracted to fit our explicit model (1) to measured data and estimate the peak power point and FF. While $V_{\rm oc}$ and $J_{\rm sc}$ are measured directly, m and γ are extracted using two additional simple measurements of j for $\nu = 0.6$ and ν for $j = 0.6$ as follows. For $\nu = 0.6$, $\nu^m \ll 1$ because $m \geq 7$ usually, so that the term $\gamma\nu^m$ can be neglected in the j–ν relation (1). Thus, γ is obtained from the j–ν measurement for this condition as

$$\gamma \approx (j|_{\nu=0.6} - 0.4)/0.6. \tag{4}$$

For $j = 0.6$, the term ν^m plays a significant role in (1), so that, substituting the measured j–ν values for this condition into (1), m can be solved as

$$m = \log\left\lfloor (0.4 - (1-\gamma)\nu|_{j=0.6})\,\gamma^{-1}\right\rfloor / \log \nu|_{j=0.6}. \tag{5}$$

Note that the measurement points should not be close to either the short- or open-circuit point because small measured values are prone to large percentage errors. On the other hand, a point on the sharply turning curve corner also gives erroneous results because the neglect of the term ν^m in the derivation of γ is invalid for this condition. The choice of $\nu = 0.6$ and $j = 0.6$ avoids these problems (other values around 0.6 may also be used, in which case the constants in (4) and (5) should be adjusted appropriately). Using the extracted values of m and γ, we estimate j_p from (1), ν_p from (2), and FF from (3). Thus, our model allows easy estimation of the (J_p, V_p) point and FF from four simple measurements of $V_{\rm oc}$, $J_{\rm sc}$, j for $\nu = 0.6$, and ν for $j = 0.6$.

Using the aforesaid approach, we have extracted m and γ and then calculated the J–V curves, V_p, J_p, and FF for a number of cells whose measured data are available in the literature [1], [8]–[11]. The calculated and measured values are in good agreement, as shown in Fig. 1 and Table I. Note that, in Fig. 1, the agreement exists even with changes in temperatures [8] and for a $p + a - \mathrm{SiC:H/n}$ c-Si cell in which recombination and space-charge-limited currents exist apart from diffusion [1].

TABLE I
COMPARISON BETWEEN MODELED AND MEASURED PEAK POWER POINTS FOR TWO c-Si SOLAR CELLS OF [11]

Device	V_p (V)		J_p (mA-cm^{-2})	
	Model	Expt	Model	Expt
Blue cell	0.428	0.437	23.1	23.1
Grey cell	0.387	0.390	18.5	18.6

III. DERIVATION OF CELL CHARACTERISTICS FROM PHYSICAL PARAMETERS

We now discuss how our model simplifies the calculation of cell characteristics from physical parameters. For this purpose, we employ the implicit single exponential description of the J–V curve in terms of physical parameters given by

$$J = J_{\rm ph} - J_0\left[\exp\left(\frac{V + JR_s}{\eta V_t}\right) - 1\right] - \frac{V + JR_s}{R_{\rm sh}} \tag{6}$$

where J_0 is the dark current density, $J_{\rm ph}$ is the photogenerated current density, V_t is the thermal voltage at temperature T, η is the ideality factor, R_s is the unit-area parasitic series resistance, and $R_{\rm sh}$ is the unit-area parasitic shunt resistance. Model (6) is widely employed because, with empirical adjustments in its physical parameters, it can also fit the results of double exponential and other models employed to account for different conduction mechanisms (e.g., see [1]).

A closed-form description of J in terms of V and the physical parameters is obtained using (1) and the expressions derived later for γ, m, $V_{\rm oc}$, and $J_{\rm sc}$ in terms of physical parameters. Equating the following derivatives of (1) at the short- and open-circuit points

$$\left.\frac{dJ}{dV}\right|_{V=0} = -(1-\gamma)J_{\rm sc}/V_{\rm oc} \tag{7}$$

$$\left.\frac{dJ}{dV}\right|_{J=0} = -(\gamma^{-1} - 1 + m)\gamma J_{\rm sc}/V_{\rm oc} \tag{8}$$

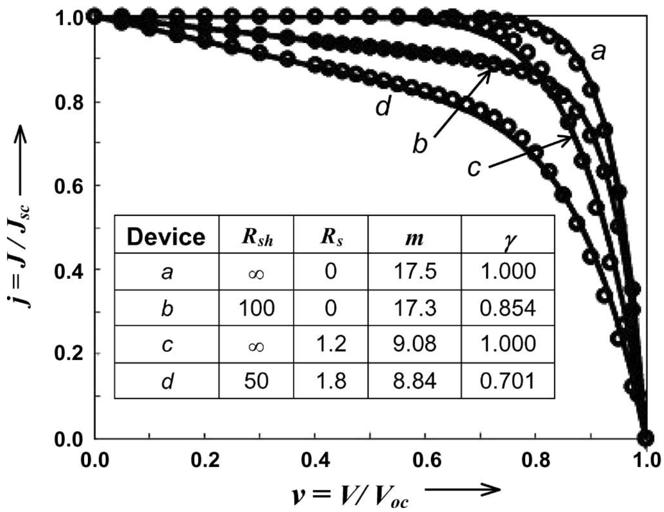
to the corresponding derivatives obtained from (6), i.e.,

$$\left.\frac{dJ}{dV}\right|_{V=0} = \frac{-1}{R_{\rm sh}}\left[\frac{1 + (J_0 R_{\rm sh}/\eta V_t)e^{J_{\rm sc}R_s/\eta V_t}}{1 + (J_0 R_s/\eta V_t)e^{J_{\rm sc}R_s/\eta V_t} + (R_s/R_{\rm sh})}\right] \approx \frac{-1}{R_{\rm sh}} \tag{9}$$

$$\left.\frac{dJ}{dV}\right|_{J=0} = \frac{-J_{\rm sc}}{\eta V_t}\left[\frac{1 - (V_{\rm oc}/J_{\rm sc}R_{\rm sh})}{1 + (J_{\rm sc}R_s/\eta V_t)(1 - V_{\rm oc}/J_{\rm sc}R_{\rm sh})}\right] \tag{10}$$

leads to

$$\gamma \approx 1 - \left(\frac{V_{\rm oc}/J_{\rm sc}}{R_{\rm sh}}\right)$$
$$m \approx \frac{V_{oc}/\eta V_t}{1 + \theta(\gamma J_{\rm sc}R_s/\eta V_t)}. \tag{11}$$



Device	R_{sh}	R_s	m	γ
a	∞	0	17.5	1.000
b	100	0	17.3	0.854
c	∞	1.2	9.08	1.000
d	50	1.8	8.84	0.701

Fig. 2. Normalized J–V curves of silicon solar cells having $\eta = 1.3$, $J_0 = 1\ \mathrm{nA \cdot cm^{-2}}$, $J_{\mathrm{ph}} = 40\ \mathrm{mA \cdot cm^{-2}}$, $T = 300$ K, and parasitic resistances (in ohms–square centimeter) shown in the table inserted in the figure. Lines show the explicit model (1), (11)–(13) using $\theta = 0.6$, whereas points show the implicit model (6).

While deriving the expression for m by equating (8) and (10), the term $(\gamma^{-1} - 1)$ in (8) is ignored, and the term containing R_s in (10) is multiplied by an empirical factor θ whose value will be discussed shortly. These refinements suppress the errors between calculations of the explicit and implicit models, occurring at high values of R_s. As is well known, the expressions for J_{sc} and V_{oc} are derived from (6) as

$$J_{\mathrm{sc}} = J_{\mathrm{ph}} - J_0(e^{J_{\mathrm{sc}}R_s/\eta V_t} - 1) - \frac{J_{\mathrm{sc}}R_s}{R_{\mathrm{sh}}}$$

$$\approx J_{\mathrm{ph}}(1 + R_s/R_{\mathrm{sh}})^{-1} \tag{12}$$

$$V_{\mathrm{oc}} \approx \eta V_t \ln\left[\frac{J_{\mathrm{ph}} - \eta V_t \ln(J_{\mathrm{ph}}/J_0)/R_{\mathrm{sh}}}{J_0}\right] \tag{13}$$

where we assume that $J_0 \ll J_{\mathrm{ph}}$ and $V_{\mathrm{oc}} \approx \eta V_t \ln(J_{\mathrm{ph}}/J_0)$ while writing the numerator of the term in square brackets of (13).

Our aforementioned results are in accordance with Shockley's prediction that the FF of a solar cell devoid of parasitic resistances depends solely on V_{oc} [11]. Indeed, setting $R_s \to 0$ and $R_{\mathrm{sh}} \to \infty$ in (11), we get $\gamma = 1$ and $m = V_{\mathrm{oc}}/\eta V_t$, leading to the expression $\mathrm{FF} = m(1+m)^{-1-1/m}$ from (2) and (3); this FF depends solely on V_{oc}. Fig. 2 shows the J–V calculations using $\theta = 0.6$, starting from the values of R_{sh}, R_s, η, I_0, I_{ph}, and T, for a variety of c-Si cells having $0.78 \leq \nu_p \leq 0.85$ and $0.56 \leq \mathrm{FF} \leq 0.77$. The results of the closed-form power-law model (1), (11)–(13) are seen to be in agreement with the iterative calculations of the single exponential model (6). For the curves shown in Fig. 2, the FF and ν_p calculated iteratively using (6) differ from those calculated using closed-form formulas (2), (3), and (11)–(13) by no more than 1.85% and 1.37%, respectively.

IV. Conclusion

This letter gives a simple explicit J–V model that is useful for the design of a variety of solar cells having different conduction phenomena. The model allows closed-form estimation of the fill factor and peak power point both from simple J–V measurements and from physical parameters. The model also simplifies the prediction of J–V curves from physical parameters of the single exponential model.

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