1 GENERAL THEORY

1.1 DETERMINANT IN SOME BASIS

1.1.1 p-linear form on E.

The $\varphi : E^p \to \mathbb{K}$ application is said to be *p-linear* if and only if each of its partial functions is linear.

1.1.2 Alternated, antisymetric form

We will suppose that φ is a p-linear form on E.

- Definitions

  φ is **alternated** if φ is null on any system of p vectors which contains at least two equal vectors. It is said to be **antisymetric** if, when the S' system is the S system but with two permutated vectors we have : $\varphi(S') = -\varphi(S)$.

- Properties

  1. φ alternated $\implies$ φ antisymetric. true reciprocal if car $\mathbb{K} \neq 2$
  2. φ antisymetric $\iff$ $\forall \sigma \in S_p$, $\forall (x_1, \dots, x_p) \in E^p, \varphi(x_{\sigma(1)}, \dots, x_{\sigma(p)}) = \epsilon(\sigma).\varphi(x_1, \dots, x_p)$.
  3. φ alternated $\iff$ $\forall (x_1, \dots, x_p) \in E^p, ((x_1, \dots, x_p)$ linearly dependant $\implies$ $\varphi(x_1, \dots, x_p) = 0)$.

1.1.3 Theorem :

let E be a vector space of dimension n, and B be a basis of E. There exists a unique n-linear alternated form φ on E which evaluates to 1 on the B basis. By definition, φ = det B. Moreover, we deduce the following formula.

$$\forall x \in E, det_B(x_1, \dots, x_n) = \sum_{\sigma \in \mathcal{S}} \epsilon(\sigma).\varphi_{\sigma(1)}(x_1) \dots \varphi_{\sigma(n)}(x_n). \tag{1}$$

where $(\varphi_1, \dots, \varphi_n)$ is the dual basis of $(e_1, \dots, e_n)$.