Team Control Number

10751

Problem Chosen

B

2020

HiMCM

Summary Sheet

Summary summary.

Contents

1 Introduction

The decrease in biodiversity due to the extinction of endangered plants and animals is a serious problem. To avoid this, people need to spend money on protecting the plants and animals.

However, in some places like Florida, greatly needing biodiversity conservation, people do not have enough money to protect all of the imperiled species there. They need to find out a proper scheme of funding and protecting to make full use of the funds to get as much benefit as possible.

Different projects of conservation have different cost, benefit, and required time, as shown in the table of threatened plants data (abbreviated as TPD in the following parts of the paper). It is to be determined which projects are selected in the optimal plan according to these factors. After choosing the projects, when to start the projects should also be determined to balance the funds spent over time as possible. To make it clear what is going to be done, the problem is restated mathematically in Section 3. The model and the method to derive the results are to be explained in Section 5.

We will write a non-technical memo in Section 8 to give our proposal according to our model.

2 Variables

To make our model concise and straight forward, a list of symbols are defined as following, as shown in Table 1. The meaning of the symbols are to be explained in detail in the rest of this article.

Table 1: List of symbols

Symbol	Domain	Unit	Definition
x	$\mathbb{N}$		the index of projects
D	$\mathbb{Z}^+$	year	Duration of the plan
d_x	$\mathbb{Z}^+$	year	Duration of project x
n	$\mathbb{N}$	year	Time (index of years)
Z	$\mathscr{P}(\mathbb{N})$		The set of chosen projects
F	$\mathbb{R}^+$	dollar	Total funds
b_x	$[0, \infty)$	osu[1]	Benefit of project x
u_x	$[0, 1]$		Taxonomic uniqueness of project x
s_x	$[0, 1]$		Feasibility of success of project x
$c_{x,n}$	$[0, +\infty)$	dollar	Cost of project x in the nth year
E	$[0, +\infty)$	dollar	Cost of the plan
c_x	$[0, +\infty)$	dollar	Total cost of project x
C_n	$[0, +\infty)$	dollar	Cost of the plan in the nth year
β_x	$[0, +\infty)$	osu	Effective benefit of project x
T	$[0, +\infty)$	osu/year	Time efficiency of the plan

3 Restatement of the problem

There is a list of plant species to be protected.

The conservation of each species lasts several years, with different per-year costs during the process. Our goal is, given a limited funding, to find out the best plan of conserving the species.

4 General assumptions

Assumption 1. *Funds are provided once per year, depicted by f_n.*

Assumption 2. *Conservation cannot be paused.*

Assumption 3. *Different conservations can be done simultanously and do not affect each other.*

Assumption 4. *Extra funds' last year can be saved for this year.*

Assumption 5. *The conservation plan should not change when it is ongoing.*

[1]The unit osu is invented to represent the unit of benefit.

5 The model

5.1 Raw attributes of projects

The word **project** is used to represent a conservation of a certain species of plant. In this article, the symbol x is used to number and denote a project. Note that in this article, all numberings (like x here numbering the projects, and n later numbering the years) start with 0 instead of 1 to meet the convention in computer science.

According to the TPD, there are some **raw datas** attributed to a project x, which are the **unique id**, the **taxonomic uniqueness** u_x, the **feasibility of success** s_x, the **cost** $c_{x,n}$ of the nth year.

To make the concepts more clear and make it easy to use them later, although the meanings of the raw datas are described in the TPD, their meanings should be re-described in a mathematical way.

Definition 1 (benefit)**.** *The* ***benefit*** *of a project is a real number* $b_x \in [0, +\infty)$.

The benefit is an important indicator indicating how imperiled the species is and how easy the project can be done. However, b_x cannot be directly used to measure how much can we benefit from finishing the project. Another derived concept called **effective benefit** should be used instead, as explained in Section 5.3.

Definition 2 (taxonomic uniqueness)**.** *The* ***taxonomic uniqueness*** *of a project is a real number* $u_x \in [0, 1]$.

The taxonomic uniqueness is a measure of how the species is unique from other species. It is stipulated that $u_x = 0$ if there exists a species that is exactly the same as the species conserved in project x, and $u_x = 1$ if the species conserved in project x does not have any similarities with any other species.

Definition 3 (feasibility of success)**.** *The* ***feasibility of success*** *of a project is a real number* $s_x \in [0, 1]$.

The feasibility of success is the probability of the success of the project.

5.2 Constraints to the plan

The word **plan** is used to represent how the projects are going to be executed, including which projects are executed and when should they start.

5.3 Effective benefit

Although b_x is defined for a project, hardly can a project lead to so much benefit finally. A new concept should be defined to describe how much can people actually benefit from finishing the project.

Definition 4. *The* ***effective benefit*** *of a project is a real number* $\beta_x \in [0, +\infty)$ *defined as*

$$\beta_x := b_x u_x s_x. \tag{1}$$

Here is the explanation of why the effective benefit should be defined as Equation 1.

First, consider the effect of u_x. If $u_x = 0$, which means there is a species the same as the conserved species, the conservation is useless because even if the conserved species die out, there are other species taking the place. If $u_x = 1$, which means there are no similar species to the conserved species, the conservation is meaningful and can exert full benefit. From these two cases, it is reasonable to multiply b_x with u_x, which means u_x represents the portion of b_x that can be exerted if the project is successfully finished.

Then, consider the effect of s_x. It is a probability, so the effect should be considered statistically. Imagine there are many parallel universes, in each of which project x is carried out under the same condition. According to the law of large numbers, the portion of successful executions is the probability s_x, exerting the benefit, while the rest does not make any benefit. Then, the mean of all the benefits is $s_x \cdot b_x + \left(1 - s_x\right) \cdot 0 = b_x s_x$.

Combining the two effects above, the formula for effective benefit (Equation 1) can be derived.

From now on, when measuring how much can a project benefit people, β_x is used instead of b_x.

5.4 The goal of the model

6 The approach

6.1 Minimize the funds when the time efficiency maximum

7 Pros and cons

Our model have some pros and cons in solving the problem described in 1. The pros are

1. Pro 1.
2. Pro 2.
3. Pro 3.

The cons are

1. Con 1.
2. Con 2.
3. Con 3.

8 The memo

Memo memo.

9 Appendix

References

[1] Ref 1.