Team Control Number

10751

Problem Chosen

B

2020

HiMCM

Summary Sheet

Summary summary.

Contents

1 Introduction

The decrease in biodiversity due to the extinction of endangered plants and animals is a serious problem. To avoid this, people need to spend money on protecting the plants and animals.

However, in some places like Florida, greatly needing biodiversity conservation, people do not have enough money to protect all of the imperiled species there. They need to find out a proper scheme of funding and protecting to make full use of the funds to get as much benefit as possible.

Different projects of conservation have different cost, benefit, and required time, as shown in the table of threatened plants data (abbreviated as TPD in the following parts of the paper). It is to be determined which projects are selected in the optimal plan according to these factors. After choosing the projects, when to start the projects should also be determined to balance the funds spent over time as possible. To make it clear what is going to be done, the problem is restated mathematically in Section 3. The model and the method to derive the results are to be explained in Section 5.

We will write a non-technical memo in Section 8 to give our proposal according to our model.

2 Variables

To make our model concise and straight forward, a list of factors are defined as following, as shown in Table 1.

Table 1: List of symbols

Symbol	Domain	Unit	Definition
I	$\mathbb{N}$	year	time spanning from the beginning of the first project to the end of the last one
T_x	$\mathbb{N}$	year	total time to finish project x
n	$(0, I] \cap \mathbb{Z}$	year	the index of years
L_n	$(0, I] \cap \ L_n$	year	the funds left at the end of n^{th} year
Z	$\mathbb{N}$		set that includes all the index of chosen project
k	$\mathbb{R}^+$	dollar	initial funds
α	$\mathbb{R}^+$	dollar/year	funds raised each year
σ	$\mathbb{R}^+$	dollar	total funds raised
x	$[0, 48) \cap \mathbb{Z}$		the index of projects
B_x	$[0, \infty)$	osu[1]	benefit of project x
U_x	$[0, 1]$		a measure of the uniqueness of the species of project x
S_x	$[0, 1]$		feasibility of success of project x
E_x^m	$[0, \infty)$	dollar	cost of project x m_x years after its start
$\mathbb{E}$	$[0, \infty)$	dollar	total cost of the whole plan
e_x	$[0, \infty)$	dollar	total cost of project x
C_n	$[0, \infty)$	dollar	total cost of all the projects in n^th year
F_n	$[0, \infty)$	dollar	total fund after fundraising
ε_x	$[0, \infty)$	osu	effective benefit of a project made by the product of B_x, U_x, S_x
ζ	$\mathbb{R}^+$	osu/(year · dollar)	set that includes all indexes of projects that are at their first year in n^{th} year
A_n			set that includes all indexes of projects that are at their first year in the nth year

3 Restatement of the problem

There is a list of plant species to be protected.

The conservation of each species lasts several years, with different per-year costs during the process. Our goal is, given a limited funding, to find out the best plan of conserving the species.

[1]The unit osu is invented by us to represent the unit of benefit.

4 General assumptions

Assumption 1. *Funds are provided once per year, depicted by f_n.*

Assumption 2. *Conservation cannot be paused.*

Assumption 3. *Different conservations can be done simultanously and do not affect each other.*

Assumption 4. *Extra funds' last year can be saved for this year.*

Assumption 5. *The conservation plan should not change when it is ongoing.*

5 The model

5.1 Relationship of different variable

Suppose the funds raised each year is a constant α, and the total fund raised σ is given. Then σ can be expressed in terms of α:

$$\sigma = \sum_{u=1}^{I} \alpha = I\alpha \tag{1}$$

$$\alpha = \frac{\sigma}{I} \tag{2}$$

The total cost of a single project x and e_x can be expressed as:

$$e_x = \sum_{v=1}^{T_x} E_x^v, (x \in Q_i \subset \mathbb{Z}) \tag{3}$$

We also can get the total cost of the whole plan:

$$\mathbb{E} = \sum_{x} e_x, (x \in Q_i \subset \mathbb{Z}) \tag{4}$$

From (3), replace e_x with $\sum_{n=1}^{N} E_x^v$

$$\mathbb{E} = \sum_{x} \sum_{v=1}^{T_x} E_x^v, (x \in Q_i \subset \mathbb{Z}) \tag{5}$$

5.2 Evaluation of various project

The effective benefit of a certain project x, ε_x can be expressed as

$$\varepsilon_x = B_x \cdot U_x \cdot S_x \tag{6}$$

We use effective benefit ε_x instead of the benefit provided in the table of Threatened Plants Data (TPD) because benefit in the TPD ignores the fact that it relies on the uniqueness of the plant species and feasibility of success. If U_x equals to 0 (there are lots of other plant species can fill in its position), there is no need to preserve it as the benefit can completely brought by other plant species. Likewise, if S_x equals to 0 (no probability to succeed at all), even highest benefit remains useless. However, in terms of ε_x expression, it becomes 0 when U_x or S_x or both equal to 0, indicating no need for protecting this kind of plant.

As can be seen in the TPD, both U_x and S_x are in the interval $[0, 1]$. When either or both of these variables get close to 1 (high probability and uniqueness), the benefit B_x of a certain project can be fully displayed:

$$\varepsilon_x = B_x \cdot U_x \cdot S_x \approx B_x \cdot 1 \cdot 1 = B_x \tag{7}$$

Whether a plan is successful not only depends on how many species it saves or benefit it brings, but also relies on the expense and time spanning to finish it. Another variable ζ needs to be determined to further evaluate the plan. It can be expressed as:

$$\zeta = \frac{\sum_x (B_x \cdot U_x \cdot S_x)}{\mathbb{E} \cdot I} (x \in Q_i \subset \mathbb{Z}) \tag{8}$$

By using formula (4), we can further expand it:

$$\zeta = \frac{\sum_x (B_x \cdot U_x \cdot S_x)}{(\sum_x e_x) \cdot I} = \frac{\sum_x (B_x \cdot U_x \cdot S_x)}{\sum_x \sum_{v=1}^{T_x} E_x^v \cdot I}, (x \in Q_i \subset \mathbb{Z}) \tag{9}$$

This equation is related to the time and the cost. When the total cost or the time span is too large, the denominator $e_x \cdot T_x$ would decrease significantly, making ζ a small figure. Lower number indicates less efficiency. ζ of the optimal plans would be greater than those of other plans.

5.3 Function and constraint

In the first year, the total fund F_1 after fundraising is:

$$F_1 = k + \alpha \tag{10}$$

Suppose the all the projects that would be conducted in the first year A_1 is given, their total cost in the first year can be expressed as:

$$C_1 = \sum_{u_1} E_{u_1}^1, (u_1 \in A_1) \tag{11}$$

The total cost in the first year could not exceed the total fund raised. This constraint can be showed as:

$$F_1 = k + \alpha > C_1 = \sum_{u_1} E_{u_1}^1, (u_1 \in A_1) \tag{12}$$

And the left fund L_1 is:

$$L_1 = F_1 - C_1 = k + a - \sum_{u_1} E_{u_1}^1, (u_1 \in A_1) \tag{13}$$

Second year's fund is the combination of the fund left in the first year and the fund raised in the second year:

$$F_2 = L_1 + \alpha \tag{14}$$

Suppose all projects initiated in the second year A_2 is also given, the total cost in the second year C_2 the combination of the cost of projects initiated in the first year and projects initiated in the second year:

$$C_2 = \sum_{u_1} E_{u_1}^2 + \sum_{u_2} E_{u_2}^1, (u_1 \in A_1, u_2 \in A_2) \tag{15}$$

Also, the constraint is that the total fund in second year is greater than the total cost:

$$F_2 = L_1 + \alpha > C_2 x``` \tag{16}$$

The fund left in the second year L_2 is:

$$L_2 = F_2 - C_2 = L_1 + \alpha - C_2 = k + 2\alpha - (\sum_{u_1} E_{u_1}^2 + \sum_{u_2} E_{u_2}^1), (u_1 \in A_1, u_2 \in A_2) \tag{17}$$

Based on the same principal, we can find the total cost of projects in n^{th} year:

$$C_n = \sum_{u_1} E_{u_1}^n + \sum_{u_2} E_{u_2}^{n-1} + \cdots + \sum_{u_n} E_{u_n}^1 = \sum_{i=1}^{n} \sum_{u_i} E_{u_i}^{n-i+1} \tag{18}$$

$$(define \forall n - i + 1 > T_{u_i} : E_u i^{n-i+1} = 0)$$

where $(u_p \in A_p, (p = 1, 2, \cdots, n))$. And the total fund in F_n can also be found:

$$F_n = L_{n-1} + \alpha = F_{n-1} - C_{n-1} + \alpha = F_{n-1} - \sum_{i=1}^{n-1} \sum_{u_i} E_{u_i}^{(n-1)-i+1} + \alpha \tag{19}$$

$$(u_p \in A_p, (p = 1, 2, \cdots, n-1))$$

Through this recursion formula, we can find F_n in terms of F_1:

$$\begin{aligned} F_n &= F_{n-2} - \sum_{i=1}^{n-2}\sum_{u_i} E_{u_i}^{(n-2)-i+1} - \sum_{i=1}^{n-1}\sum_{u_i} E_{u_i}^{(n-1)-i+1} + \alpha + \alpha = F_{n-3} - \cdots + 3\alpha \\ &= F_1 - \sum_{j=1}^{n-1}\sum_{i=1}^{j}\sum_{u_i} E_{u_i}^{j-i+1} + (n-1)\alpha = k + \sum_{j=1}^{n-1}\sum_{i=1}^{j}\sum_{u_i} E_{u_i}^{j-i+1} + n\alpha, \end{aligned} \tag{20}$$

where $u_p \in A_p$, and $p = 1, 2, \cdots, n-1$).

Therefore, by using (1), we can get a series of constraints:

$$F_P > C_p, (p = 1, 2, \cdots, n)$$

$$k + \sum_{j=1}^{p-1}\sum_{i=1}^{j}\sum_{u_i} E_{u_i}^{p-i+1} + p\alpha > \sum_{i=1}^{p}\sum_{u_i} E_{u_i}^{p-i+1}, (p = 1, 2, \cdots, n) \tag{21}$$

$$k + \sum_{j=1}^{p-1}\sum_{i=1}^{j}\sum_{u_i} E_{u_i}^{p-i+1} + p\frac{\sigma}{I} > \sum_{i=1}^{p}\sum_{u_i} E_{u_i}^{p-i+1}, (p = 1, 2, \cdots, n) \tag{22}$$

Under these constraints, in order to give the best plan for plant conservation, we need to find the greatest value of ζ (9):

$$\zeta = \frac{\sum_x (B_x \cdot U_x \cdot S_x)}{(\sum_x \sum_{v=1}^{T_x}) \cdot I}, (x \in Q_i \subset Z = \bigcup_{t=1}^{n} A_t) \tag{23}$$

6 The approach

6.1 Minimize the funds when the time efficiency maximum

7 Pros and cons

Our model have some pros and cons in solving the problem described in 1. The pros are

1. Pro 1.
2. Pro 2.
3. Pro 3.

The cons are

1. Con 1.
2. Con 2.
3. Con 3.

8 The memo

Memo memo.

9 Appendix

References

[1] Ref 1.