Simulation Report - The Einstein Effect

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Abstract

This report aims to transcribe the results for 2 simulation related to the gravitational redshift. The first is two Galileo satellites launched in a wrong orbite, the seonc is with the star S2 orbiting the supermassive black hole in the center of our galaxy.

The goal is to compare the theoretical predictions of the Einstein effect with real-world measurement results.

1 Introduction

The Einstein effect, or gravitational redshift, refers to the change in frequency of light or signals due to gravity. When a signal leaves a strong gravitational field, it loses energy, which causes its frequency to decrease.

Imagine throwing a ball upwards. The stronger the gravity, the more energy the ball loses as it rises. Similarly, when light or a signal escapes from a region with strong gravity, it loses energy, resulting in a lower frequency.

In both experiments, we measure how much the frequency of light or signals changes because of gravity, and we compare these measurements to theoretical predictions.

$$z = \frac{\Delta\lambda}{\lambda_{\text{emitted}}} = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} \tag{1}$$

Since both $\lambda_{\text{observed}}$ and λ_{emitted} are in the same units (typically meters), the units cancel out, making z a dimensionless quantity.

In weak gravitational fields, the redshift z can be approximated by:

$$z \approx \frac{GM}{c^2 r} \tag{2}$$

where G is the gravitational constant, M is the mass of the gravitating body, c is the speed of light, and r is the radial distance of the light source from the center of mass. The ratio $\frac{GM}{c^2 r}$ is dimensionless, reinforcing that gravitational redshift has no unit.

2 Simulations using Python

2.1 Galileo satellites experiment

In 2018, two Galileo GPS satellites were accidentally placed into elliptical orbits instead of circular ones. This provided a unique opportunity[DEL] to observe how their signals were affected by the varying gravitational field of Earth as the satellites moved closer to and farther from Earth.

1. Compute the gravitational redshift at the **closest point (perigee)** and the **farthest point (apogee)** of the satellite's orbit.

$$R_{\text{perigee}} = R_{\text{Earth}} + \text{altitude}_{\text{perigee}} \tag{3}$$

$$R_{\text{apogee}} = R_{\text{Earth}} + \text{altitude}_{\text{apogee}} \tag{4}$$

Where R_{Earth} is the radius of Earth, and the altitudes are the respective heights above Earth's surface.[DPS+18]

Parameter	**Value**
Altitude at Perigee	2.32e7
Altitude at Apogee	2.50e7
Distance at Perigee	2.96e7
Distance at Apogee	3.14e7
Gravitational Redshift at Perigee	1.496559606814e-10
Gravitational Redshift at Apogee	1.411739773818e-10

Table 1: Gravitational redshift and associated parameters for the Galileo satellites.

2. Use the gravitational potential formula:

$$U = -\frac{GM}{r} \tag{5}$$

where: - G is the gravitational constant - M is the mass of Earth - r is the distance from the satellite to Earth

3. Calculate the frequency shift using:

$$\frac{\Delta f}{f} = \frac{\Delta U}{c^2} \tag{6}$$

where c is the speed of light.

Orbital Point	**Frequency Redshift**
Perigee	1.50×10^{-10}
Apogee	1.41×10^{-10}

Table 2: Frequency redshift at perigee and apogee of the satellite's orbit.

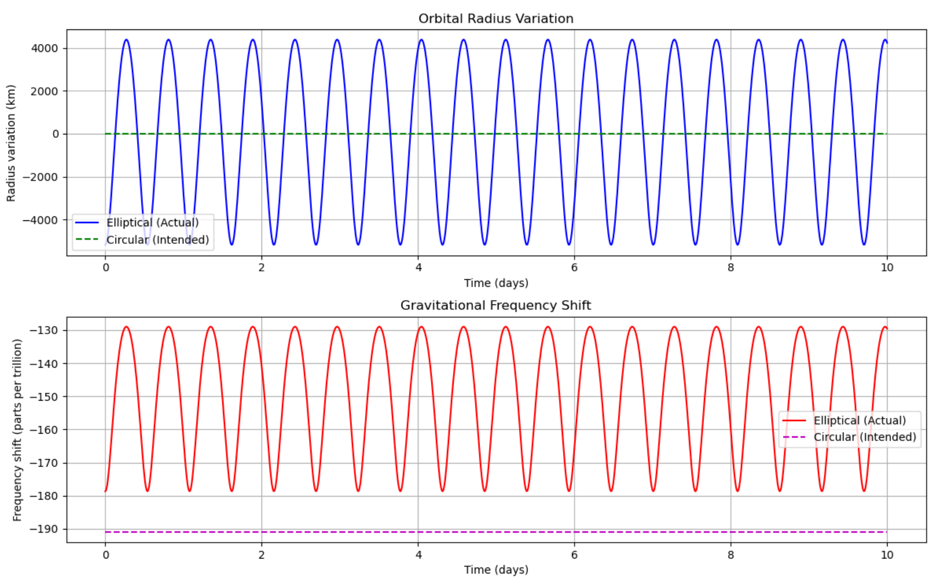

Figure 1: Computed graph showing relation between distance from Earth and gravitational redshift

2.2 Star S2 orbiting the supermassive black hole at the center of our Galaxy

S2 is a star which follows an elliptical orbit around the black hole Sagittarius A*, located at the center of the Milky Way. This black hole has a mass of 4x10^6 times the mass of our Solar's System sun. The trajectory of S2 and its proximity to Sagittarius A* makes this pair a unique way to analyze the gravitational redshift. This phenomenon was best observed in a 2018 experiment, when S2 reached its pericenter; in other words the closest point to the black hole.

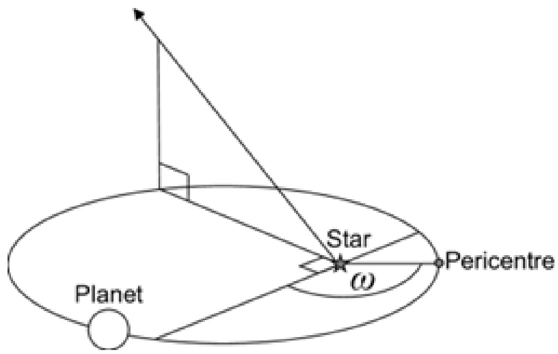

Figure 2: Depiction of the pericenter of an orbit

The key aspects of this experiment are of course S2's orbit around this supermassive black hole [GAA18]. S2 follows an orbit with a period of 16 years around Sagittarius A*, and arrives as close as 120 times the average sun/Earth distance : around 1.8x1013 m.

That proximity is important as it makes the relativistic effects more significant than in other places of the trajectory. That distance adds more relevancy to the experiment, since the presence of higher gravitational fields due to the enormous mass of the black hole apply stronger forces to the star. Those fields being stronger imply that the newtonian physics can't fully explain the behavior of the star, thus making relativistic effects dominate and being the ideal natural test lab, to study the laws of relativistic physics in extreme gravitational conditions.

The second key element of this experiment is the intense gravitational field of Sagittarius A*, which causes the frequency of the light emitted by S2 to shift towards lower frequencies. This effect was actually predicted by Einstein's General Theory of Relativity, as it states that the light escaping a strong gravitational field will have its wavelength increasing. This increase of the wavelength is detected by a shift of S2's emitted light towards the red part of the waves spectrum.

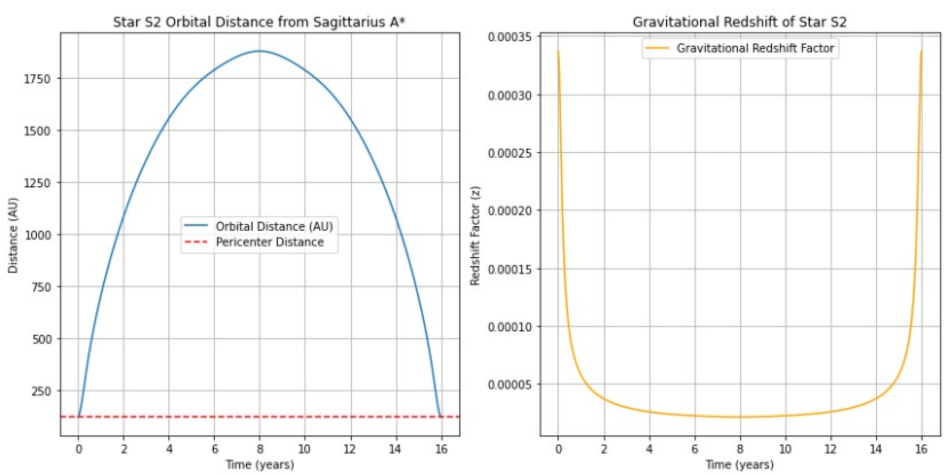

Figure 3: S2 orbiting a supermassive blackhole

The redshift is more intense when S2 is near the pericentre of its orbit (the intersection between the blue and dotted red lines that occurs every 16 years)

References

[DEL] Dr Pacome DELVA. Quand galileo confirme les prédictions d'einstein.

[DPS+18] P. Delva, N. Puchades, E. Schönemann, F. Dilssner, C. Courde, S. Bertone, F. Gonzalez, A. Hees, Ch. Le Poncin-Lafitte, F. Meynadier, R. Prieto-Cerdeira, B. Sohet, J. Ventura-Traveset, and P. Wolf. Gravitational redshift test using eccentric galileo satellites. *Phys. Rev. Lett.*, 121:231101, Dec 2018.

[GAA18] GRAVITY Collaboration, Abuter, R., and Amorim, A. Detection of the gravitational redshift in the orbit of the star s2 near the galactic centre massive black hole. *AA*, 615:L15, 2018.