# Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu$ = 45 minutes and $\sigma$ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676  (ANS)
   C. 0.5
   D. 0.6987

   ANS:

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   In [10]: 1-stats.norm.cdf(x=50,loc=45,scale=8)
   Out[10]: 0.26598552904870054

   In [ ]:
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2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu$ = 38 and Standard deviation $\sigma$ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
      ANS:

      ```
      Out[10]: 0.26598552904870054

      In [11]: 1-stats.norm.cdf(x=44,loc=38,scale=6)>stats.norm.cdf(x=44,loc=38,scale=6)-stats.norm.cdf(x=38,loc=38,scale=6)
      Out[11]: False
      ```

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
      ANS:

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      In [13]: stats.norm.cdf(x=30,loc=38,scale=6)*400
      Out[13]: 36.484487890347154

      In [ ]:
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3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 $X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.
   ANS:
      Both X1, X2 follows normal distribution and X1,X2 ~ N($\mu$, $\sigma^2$) and are identical

      $$2X1 = 2 N(\mu, \sigma^2) = N(2\mu, 2\sigma^2)$$
      $$X1 + X2 = N(\mu, \sigma^2) + N(\mu, \sigma^2) = N(2\mu, 2\sigma^2)$$

      Hence, there is no significance between 2X1 and X1+X2

4. Let X ~ N(100, 20²). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5 (ANS)
    E. 90.1, 109.9

    ANS:

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In [14]:  stats.norm.ppf(0.995)
Out[14]:  2.5758293035489004

In [ ]:  |
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    Z score for 99% confidence
  X=20Z+100
    A = -(20*2.576) +100 = 48.5
    B = (20*2.576) +100 = 151.5

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1$ ~ N(5, 3²) and $Profit_2$ ~ N(7, 4²) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    ANS:

        $13.41 = Rs 603.68

    B. Specify the 5th percentile of profit (in Rupees) for the company
    ANS:

        p = $10.59 ≈ Rs 476.33

    C. Which of the two divisions has a larger probability of making a loss in a given year?
    ANS:
        The larger probability of making a loss in a given year is the first division.