**INDIAN INSTITUTE OF INFORMATION TECHNOLOGY NAGPUR**

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# Case Study - 3

A Brief Introduction on Shannon's Information Theory

Reference: [1] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J., vol. 27, pp. 379-423, 623-656, July-Oct. 1948.
[2] C.E. Shannon, Communication in the presence of noise, Proc. IRE, 37 (1949), 10-21.
[3] S. Verd´u, Fifty years of Shannon theory, IEEE Transactions On Information Theory, 44 (1998), 2057-2078

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Abstract:

Shannon's Information Theory is a mathematical framework for understanding the fundamental concepts of information, communication, and data transmission. It is based on the idea that information can be quantified and measured in terms of its entropy or uncertainty.

Communication is the exchange of information. A sentence or sentences in English can be viewed as a sequence of letters (‘a’, ‘b’, ‘c’,.. . )and symbols (‘,’,‘.’,‘ ’,. . . ). So, we can just think of sentences conveying different meaning as different sequences.

Consider K sequences() in total, and all sequences appear equally likely. Assume they encode different messages. Regardless of specific messages they encode, we regard them as having the same amount of information. Let’s just employ the number of bits needed to encode a sequence to count the amount of information a sequence encodes (or can provide). It only depends on the probability distribution of these letters/information present in the sequence which is given by entropy. The entropy could be given by the formula :

$$\sum_{i=1}^{n} p_i \times \log_2 \frac{1}{p_i}.$$

It is clearly the expectation (i.e., average in the sense of probability) of the quantity log2(1/pi) associated with the letter xi, for 1 ≤i≤n. This matches the term “average” so that we can define the amount of information a letter xi with probability pi has to be log2(1/pi) bits. In this definition of information, we observe that if a letter has a higher probability it has less information and vice versa.

In a communication system, we have three basic ingredients: the source, the destination, and the media between them. We call the media the (communication) channel. A channel could be in any form. It could be physical wires, cables, open environments in wireless communication, antennas, and certain combinations of them. This theory also introduces the concept of channel capacity, which defines the maximum rate at which information can be transmitted through a communication channel with a given level of noise or interference in a fixed time duration, e.g., per second, or per symbol(time).

Given a channel and a set A of letters (or symbols) that can be transmitted via the channel. For an errorless channel, the maximum amount of information that can be received at the destination equals the maximum amount of information that can be generated at the source.

$$C = \max_{X} H(X), \text{ per symbol,}$$

where X ranges over all possible distributions on A.

A channel with error means that the source generated a letter $X_i \in A$ and transmitted it to the destination via the channel, with some unpredictable error, the received letter at the destination may be $X_j$. Assume statistically, $X_j$ is received with probability $p(X_j|X_i)$ when $X_i$ is transmitted probabilities are called transit probabilities of the channel.

One of the key insights of Shannon's Information Theory is that redundancy in communication can actually improve the reliability of the transmission. This is because redundant information can be used to correct errors that may occur during transmission. The theory has had a significant impact on a wide range of fields, including digital communication, computer science, cryptography, and statistics. It has also played a major role in the development of modern technologies such as the internet, wireless communication, and digital data storage.

Overall, Shannon's Information Theory provides a powerful and flexible framework for understanding and optimizing the transmission of information in a wide range of contexts.