TUTORIAL-4

DESIGN & ANALYSIS OF ALGORITHM.

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Master's Theorem

$T(n) = aT(n/b) + f(n)$

$a \geq 1$ & $b > 1$ & $f(n)$ is asymptotically positive

1. If $f(n) < n^c$

   $T(n) = \Theta(n^c)$

$\boxed{c = \log_b a}$

Compare n^c & $f(n)$

2. $f(n) = n^c$

   $T(n) = \Theta(n^c \log n)$

3. $f(n) > n^c$

   $T(n) = \Theta(f(n))$

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1) $T(n) = 3T(n/2) + n^2$

$a = 3, \; b = 2.$

$c = \log_2 3 = 1.5.$

$f(n) > n^c$

$n^2 > n^{1.5}$

$\therefore T(n) = \Theta(n^2)$

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2) $T(n) = 4T(n/2) + n^2$

$a = 4, \; b = 2$

$c = \log_2 4.$

$c = 2.$

$f(n) = n^c$

$n^2 = n^2$

$T(n) = \Theta(n^2 \log n)$

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3) $T(n) = T(n/2) + 2^n.$

$a = 1, \; b = 2$

$c = \log_2 1$

$c = 0 \quad (n^0 = 1)$

$2^n > 1$

$T(n) = \Theta(2^n)$

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4) $T(n) = 2^n T(n/2) + 2^n.$

$a = 2^n, \; b = 2$

$c = \log_2 2^n$

$c = n^2 \qquad n^c = n^n.$

$f(n) < n^c$

$\Rightarrow \Theta(n^n)$

(5) $T(n) = 16T(n/4) + n$

$a = 16, \; b = 4$

$c = \log_4^{16}$

$c = 2$

comp $f(n)$ & n^c

$n < n^2$

$T(n) = O(n^2)$

(6) $T(n) = 2T(n/2) + n\log n.$

$c = \log_2^2 = 1$

$f(n) > n^c$

$n\log n > n$

$T(n) = O(n \log n)$

(7) $T(n) = 2T(n/2) + n/\log n.$

$a = 2, \; b = 2$

$c = \log_2^2 \quad c = 1$

$n/\log n < n^1$

$f(n) < n^c$

$O(n).$

(8) $T(n) = 2T(n/4) + n^{0.51}.$

$a = 2, \; b = 4$

$c = \log_4^2 \quad c = 1/2$

$f(n) = n^c$

$n^{0.5} = n^{0.5}$

$T(n) = O(n^{0.5} \log n)$

(9) $T(n) = 0.5T(n/2) + 1/n.$

$c = \log^{1/2} = -1$

$n^{-1} = n^{-1}$

$T(n) = O(n^{-1} \log n)$

(10) $T(n) = 16T(n/4) + n!$

$a = 16, \; b = 4$

$c = \log_4^{16} \quad c = 2.$

$n! \; n^2$

if $(n > 3) \; n! > n^2$

$O(n!)$

if $(n \leq 3) \; n^2 > n!$

$O(n^2)$

(4) $T(n) = 4T(n/2) + \log n.$

$a = 4, \; b = 2$

$c = \log_2^4, \; c = 2$

$\log n < n^2$

$T(n) = O(n^2)$

(11) $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n}, \; b = 2$

$c = \log_2^{n^{1/2}}$

(13) $T(n) = 3T(n/2) + n$

$a = 3, b = 2$

$c = \log_2 3 = 1.5$

$f(n) < n^c$

$n < n^{1.5}$

$T(n) = O(n^{1.5})$

(14) $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, b = 3$

$c = \log_b^a = 1$

$f(n) = n^{1/2}$

$n^c = n^1$

$f(n) < n^c$

$T(n) = O(n)$

(15) $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$

$c = \log_2 2^2 \Rightarrow 2$

$f(n) < n^c$

$cn < n^2$

$T(n) = O(n^2)$

(16) $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3$

$c = \log_b^a = 1$

$f(n) = n/2$

$n^{c1} > n$

$f(n) < n^c$

$T(n) = n.$

(17) $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4$

$c = \log_4 3$

$f(n) = n\log n$

$n^c = n^{0.7}$

$f(n) > n^c$

$T(n) = n\log n$

(18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6, b = 3$

$c = \log_3 6 = 1.6$

$f(n) > n^c$

$T(n) = O(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n\log n$

$a = 4, b = 2$

$c = \log_2 4 = 2.$

$f(n) = n\log n.$

$n^c = n^2$

$n\log n < n^2$

$\therefore c = O(n^2)$

(20) $T(n) = 64T(n/8) - n^2\log n.$

$a = 64, b = 8$

$c = \log_8 64 \Rightarrow 2$

$f(n) > n^c$

$n^2\log n > n^2$

$Tc = O(n^2 \log n)$

(21) $T(n) = 7T(n/3) + n^2$

$a = 7, \; b = 3$

$c = \log_3^7 = 1.77$

$f(n) > n^c$

$n^2 > n^{1.77}$

$T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \; b = 2$

$\log_2^1 = 0$

$f(n) > n^c$

$n(2 - \cos n) > n^0$

$T(n) = \Theta(n(2 - \cos n))$

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