

Large Language Models (LLMs) ⇄

References ⇄

- Recurrent neural networks [GBC16; Chapter 10]
- Transformers [VSP+17]
- Neural Networks: Zero to Hero by Andrej Karpathy

[GBC16] I. Goodfellow, Y. Bengio and A. Courville. *Deep Learning* (MIT Press, 2016). Accessed on Aug 28, 2024.

[VSP+17] A. Vaswani, N. Shazeer, N. Parmar *et al.* *Attention Is All You Need*. In: *Advances in Neural Information Processing Systems*, Vol. 30 (Curran Associates, Inc., 2017). Accessed on Oct 11, 2024.

Autoregressive Models ⇔

Given a sequence of n_{ctx} past vectors $\mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}} \in \mathbb{R}^n$, "predict" the next ones. Key idea : *receding horizon*:

$$\begin{aligned} p(\mathbf{x}_0, \mathbf{x}_1 | \mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}}) \\ &= p(\mathbf{x}_0 | \mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}}) p(\mathbf{x}_1 | \mathbf{x}_0, \mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}+1}, \mathbf{x}_{-n_{\text{ctx}}}) \\ &\approx p(\mathbf{x}_0 | \mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}}) p(\mathbf{x}_1 | \mathbf{x}_0, \mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}+1}) \end{aligned}$$

- **Model** : Probability of next vector $\hat{p}(\mathbf{x}_0 | X)$ where X concatenates $\mathbf{x}_{-1}, \dots, \mathbf{x}_{-n_{\text{ctx}}}$.
- **Loss** : Cross-entropy : $\mathcal{L}_{\hat{p}}(X) \triangleq H(p, \hat{p}) = -\mathbf{E}_p[\log(\hat{p})] = -\sum_{x_0} p(\mathbf{x}_0 | X) \log(\hat{p}(\mathbf{x}_0 | X))$
- Particular case for $\hat{p}(\mathbf{x}_0 | X) = \delta_y$: $\mathcal{L}_{\hat{p}}(X) = -\log(\hat{p}(y | X))$

What about Language Models ? ⇔

Given "past text", predict the "following text". How to turn text into vectors of \mathbb{R}^n ?

Text to vectors : step 1 → tokenization ⇄

Why not encode each letter ? ⇄

- **Idea** : Turn each letter into its one-hot encoding in \mathbb{R}^{26} .
- **Issue** : The "past text" only has n_{ctx} characters so n_{ctx} must be **large** but transformers have a complexity **quadratic** in n_{ctx} !
- **Practical details** : Text is encoded with UTF-8 so each character is encoded into 1 to 4 bytes. We encode each byte to a vector in \mathbb{R}^{256} but care must be taken not to generate invalid UTF-8.

Why not encode each word ? ⇄

- **Idea** : Turn each word into its one-hot encoding in \mathbb{R}^n . The value of n is the number of words. Depending on the language (source):

Language	French	English	Dutch	German
n	408,078	350,000	350,000	200,000

- **Issue** : The value of n is **too large**. We cannot trust the words of languages to be a tokenization that optimally compresses text for our dataset.

Byte Pair Encoding ⇄

Byte Pair Encoding algorithm [SHB16] greedily merges the most frequent pair of tokens over the dataset into a new token. Most used implementations are SentencePiece [KR18] and tiktoken (play with it [here](#)). For instance, on [this example](#), the pair ('a', 'a') is the most frequent so we substitute it by a new token, say 'Z' :

```
► Dict(('b', 'd') ⇒ 1, ('a', 'b') ⇒ 2, ('d', 'a') ⇒ 1, ('b', 'a') ⇒ 1, ('a', 'c') ⇒ 1, ('  
1 pair_stats("aaabdaaabc")
```

```
iter_1 = ► BPE("ZabdZabac", Dict(('a', 'a') ⇒ 'Z'))
```

```
1 iter_1 = new_token("aaabdaaabc")
```

```
iter_2 = ► BPE("ZYdZYac", Dict(('a', 'b') ⇒ 'Y', ('a', 'a') ⇒ 'Z'))
```

```
1 iter_2 = new_token(iter_1)
```

Note that the new tokens can also be part of the most frequency pair!

```
iter_3 = ► BPE("XdXac", Dict(('a', 'b') ⇒ 'Y', ('Z', 'Y') ⇒ 'X', ('a', 'a') ⇒ 'Z'))
```

```
1 iter_3 = new_token(iter_2)
```

[SHB16] R. Sennrich, B. Haddow and A. Birch. *Neural Machine Translation of Rare Words with Subword Units* (Jun 2016), [arXiv:1508.07909](#). Accessed on Oct 23, 2024.

[KR18] T. Kudo and J. Richardson. *SentencePiece: A Simple and Language Independent Subword Tokenizer and Detokenizer for Neural Text Processing* (Aug 2018), [arXiv:1808.06226](#). Accessed on Oct 23, 2024.

Increasing length of "past text" ↔

Challenging tradeoff: Encode text to **increase** length of "past text" while keeping n_{ctx} and n **small** enough.

Length of "past text" increases with vocabulary size n_{voc} and context window n_{ctx} .

Name	Ref	n_{voc}	n_{ctx}	Tokenizer
GPT-2	[RWCL19]	<u>50k</u>	1024	tiktoken
GPT-3	[BMRS20]	50k	2048	tiktoken
GPT-3.5		<u>100k</u>	4096	tiktoken
GPT-4		<u>100k</u>	32k	tiktoken
GPT-4o		<u>200k</u>	128k	tiktoken
Gemini-1	[TABA24]	256k	10M	SentencePiece
Gemini-1.5	[TGLB24]	256k	10M	SentencePiece
Gemma	[TMHD24]	256k	8192	SentencePiece
Gemma-2	[TRPS24]	256k	8192	SentencePiece
Llama-2	[TMSA23]	<u>32k</u>	<u>4k</u>	SentencePiece
Llama-3		128k	<u>8k</u>	tiktoken
Llama-3.1		128k	<u>128k</u>	tiktoken
Llama-3.2		128k	<u>128k</u>	tiktoken
MegaByte	[YSFA23]	256	8192	Bytes

[TGL+24] G. Team, P. Georgiev, V. I. Lei *et al.* Gemini 1.5: Unlocking Multimodal Understanding across Millions of Tokens of Context (Aug 2024), [arXiv:2403.05530](#). Accessed on Nov 11, 2024.

[TAB+24] G. Team, R. Anil, S. Borgeaud *et al.* Gemini: A Family of Highly Capable Multimodal Models (Jun 2024), [arXiv:2312.11805](#). Accessed on Nov 11, 2024.

[TMH+24] G. Team, T. Mesnard, C. Hardin *et al.* Gemma: Open Models Based on Gemini Research and Technology (Apr 2024), [arXiv:2403.08295](#). Accessed on Nov 11, 2024.

[TRP+24] G. Team, M. Riviere, S. Pathak *et al.* Gemma 2: Improving Open Language Models at a Practical Size (Oct 2024), [arXiv:2408.00118](#). Accessed on Nov 11, 2024.

[SHB16] R. Sennrich, B. Haddow and A. Birch. Neural Machine Translation of Rare Words with Subword Units (Jun 2016), [arXiv:1508.07909](#). Accessed on Oct 23, 2024.

[RWC+19] A. Radford, J. Wu, R. Child *et al.* Language Models Are Unsupervised Multitask Learners (2019). Accessed on Oct 23, 2024.

[BMR+20] T. B. Brown, B. Mann, N. Ryder *et al.* Language Models Are Few-Shot Learners (Jul 2020), [arXiv:2005.14165](#). Accessed on Nov 11, 2024.

[TMS+23] H. Touvron, L. Martin, K. Stone *et al.* Llama 2: Open Foundation and Fine-Tuned Chat Models (Jul 2023),

[arXiv:2307.09288](#). Accessed on Oct 23, 2024.

[YSF+23] L. Yu, D. Simig, C. Flaherty *et al.* MEGABYTE: Predicting Million-byte Sequences with Multiscale Transformers (May 2023), [arXiv:2305.07185](#). Accessed on Oct 23, 2024.

Text to vectors : step 2 → embedding ⇔

Consider one-hot encoding with vocabulary size n_{voc} and a *bigram model*

$$\hat{p}(x_0|x_{-1}) = \text{softmax}(W_d \tanh(\cdots \tanh(W_1 x_{-1}) \cdots))$$

The matrix W_d has n_{voc} rows and W_1 has n_{voc} columns → issue if n_{voc} is large

Embedding : Use vectors $c_1, \dots, c_{n_{\text{voc}}} \in \mathbb{R}^{d_{\text{emb}}}$ with *embedding size* (aka *hidden size*) $d_{\text{emb}} \ll n_{\text{voc}}$.

Equivalently, we still use one-hot encoding but we add an encoder $C \in \mathbb{R}^{d_{\text{emb}} \times n_{\text{voc}}}$ and decoder $D \in \mathbb{R}^{n_{\text{voc}} \times d_{\text{emb}}}$

$$\hat{p}(x_0|x_{-1}) = \text{softmax}(DW_d \tanh(\cdots \tanh(W_1 C x_{-1}) \cdots))$$

► What difference do you expect with respect to the previous model ?

Forcing $D = C^T$ appears to work well in practice [PW17], this is what is used in [VSP+17].

[PW17] O. Press and L. Wolf. Using the Output Embedding to Improve Language Models (Feb 2017), [arXiv:1608.05859](https://arxiv.org/abs/1608.05859). Accessed on Nov 11, 2024.

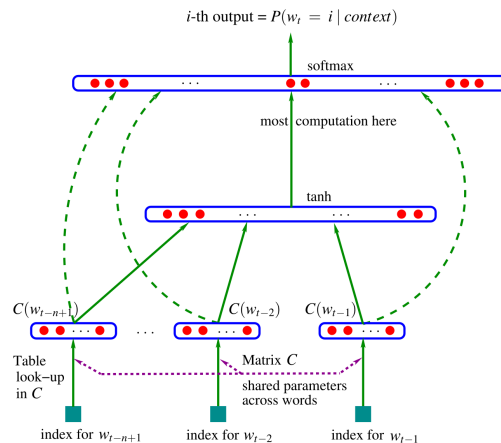
[VSP+17] A. Vaswani, N. Shazeer, N. Parmar *et al.* Attention Is All You Need. In: *Advances in Neural Information Processing Systems*, Vol. 30 (Curran Associates, Inc., 2017). Accessed on Oct 11, 2024.

Shared embedding ⇌

With $n_{\text{ctx}} > 1$, the encoder C is shared by all tokens: See for instance the network below taken from [BDVoo; Figure 1], the first popular application of neural nets for languages:

$$\hat{p}(x_0 | x_{-1}, \dots, x_{-n_{\text{ctx}}}) =$$

$$\text{softmax}(W_2 \tanh(W_1 \begin{bmatrix} Cx_{-1} \\ \vdots \\ Cx_{-n_{\text{ctx}}} \end{bmatrix}))$$



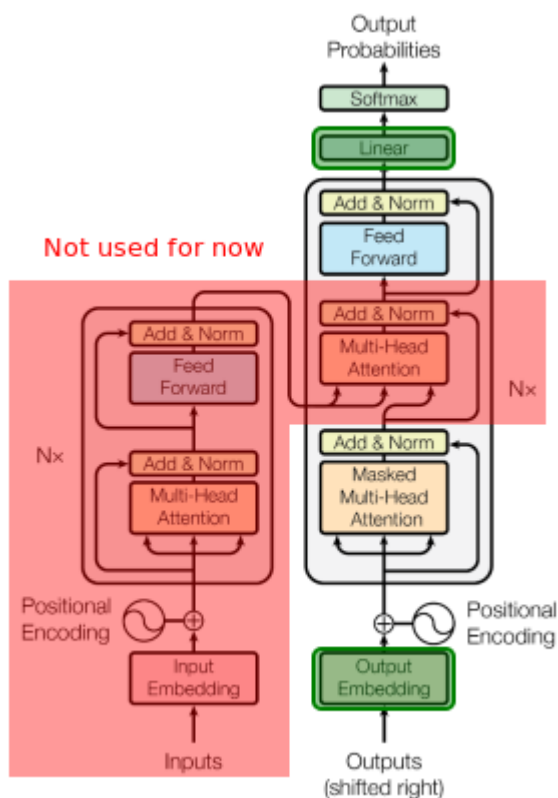
► What are the number of columns of W_1 and number of rows of W_2 now ?

[BDVoo] Y. Bengio, R. Ducharme and P. Vincent. [A Neural Probabilistic Language Model](#). In: *Advances in Neural Information Processing Systems*, Vol. 13 (MIT Press, 2000). Accessed on Oct 11, 2024.

Embedding sizes in LLMs ⇄

See the table below for the size of embeddings of large language models:

Name	Num params	Ref	n_{voc}	d_{emb}
GPT-2	1.5B	[RWCL19]	<u>50k</u>	<u>768</u>
Gemma	2B	[TMHD24]	256k	2048
Gemma	7B	[TMHD24]	256k	3072
Gemma-2	27B	[TRPS24]	256k	4608
Gemma-2	2B	[TRPS24]	256k	2304
Gemma-2	9B	[TRPS24]	256k	3584
Llama-2	7B	[TMSA23]	<u>32k</u>	<u>4096</u>
base		[VSPU17]	37k	512
big		[VSPU17]	37k	1024



[TMH+24] G. Team, T. Mesnard, C. Hardin *et al.* [Gemma: Open Models Based on Gemini Research and Technology](#) (Apr 2024), [arXiv:2403.08295](#). Accessed on Nov 11, 2024.

[TRP+24] G. Team, M. Riviere, S. Pathak *et al.* [Gemma 2: Improving Open Language Models at a Practical Size](#) (Oct 2024), [arXiv:2408.00118](#). Accessed on Nov 11, 2024.

[RWC+19] A. Radford, J. Wu, R. Child *et al.* [Language Models Are Unsupervised Multitask Learners](#) (2019). Accessed on Oct 23, 2024.

[TMS+23] H. Touvron, L. Martin, K. Stone *et al.* [Llama 2: Open Foundation and Fine-Tuned Chat Models](#) (Jul 2023), [arXiv:2307.09288](#). Accessed on Oct 23, 2024.

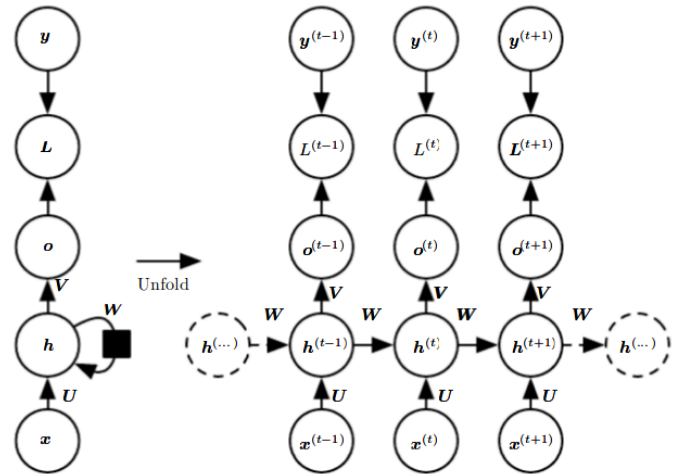
Recurrent neural networks (RNN) ⇌

$$\begin{aligned}h^{(t+1)} &= \tanh(Wh^{(t)} + Ux^{(t+1)} + b) \\o^{(t)} &= Vh^{(t)} + c \\ \hat{y}^{(t)} &= \text{softmax}(o^{(t)})\end{aligned}$$

Illustrated on the right [GBC16; Figure 10.3].

RNNs as language model showcased in [MKB+10].

Issue: Training time and space complexity is proportional to n_{ctx} and **cannot parallelize** to speed up.



[MKB+10] T. Mikolov, M. Karafiát, L. Burget *et al.* [Recurrent Neural Network Based Language Model](#). In: [Proc. Interspeech 2010](#) (2010); pp. 1045–1048. Accessed on Nov 11, 2024.

[GBC16] I. Goodfellow, Y. Bengio and A. Courville. [Deep Learning](#) (MIT Press, 2016). Accessed on Aug 28, 2024.

Extensions of RNNs ⇄

It's difficult to model long-term dependencies as their gradient either vanishes or explodes exponentially (think of the power method) [GBC16; Section 10.7]

Gated extensions attempting to solve this issue [GBC16; Section 10.10]:

- Long short-term memory (LSTM) [Gra14]
- Gated recurrent unit (GRU) [CvMBB14]

Recently, Mamba suggests a solution to the complexity issue [GD24]. As it scales better with n_{ctx} , it is even suggested to get rid of the tokenizer : [WGYR24].

[CvMBB14] K. Cho, B. van Merriënboer, D. Bahdanau and Y. Bengio. *On the Properties of Neural Machine Translation: Encoder-Decoder Approaches* (Oct 2014), [arXiv:1409.1259](#). Accessed on Nov 11, 2024.

[Gra14] A. Graves. *Generating Sequences With Recurrent Neural Networks* (Jun 2014), [arXiv:1308.0850](#). Accessed on Nov 11, 2024.

[GBC16] I. Goodfellow, Y. Bengio and A. Courville. *Deep Learning* (MIT Press, 2016). Accessed on Aug 28, 2024.

[GD24] A. Gu and T. Dao. *Mamba: Linear-Time Sequence Modeling with Selective State Spaces* (May 2024), [arXiv:2312.00752](#). Accessed on Nov 11, 2024.

[WGYR24] J. Wang, T. Gangavarapu, J. N. Yan and A. M. Rush. *MambaByte: Token-free Selective State Space Model* (Aug 2024), [arXiv:2401.13660](#). Accessed on Nov 11, 2024.

Numerical dictionary ⇔

What would a numerical dictionary look like ? Consider keys $k_i \in \mathbb{R}^{d_k}$ and values $v_i \in \mathbb{R}^{d_v}$. Given a query $q \in \mathbb{R}^{d_k}$,

```
dict = ▶ Dict([1, 0] ⇒ [1, 1], [0, 1] ⇒ [-1, 1])
```

```
1 dict = Dict([1, 0] => [1, 1], [0, 1] => [-1, 1])
```

```
▶ [1, 1]
```

```
1 dict[[1, 0]]
```

```
numerical_lookup (generic function with 1 method)
```

```
1 function numerical_lookup(dict, query)
2     _, i = findmax([dot(query, key) for key in keys(dict)])
3     return collect(values(dict))[i]
4 end
```

```
▶ [1, 1]
```

```
1 numerical_lookup(dict, [0.8, 0.2])
```

Attention head ⇔

Attention head provides a differentiable numerical dictionary [BCB16]

$$\alpha = \text{softmax}(\langle q, k_1 \rangle, \dots, \langle q, k_{n_{\text{ctx}}} \rangle) \quad \text{Attention}(q, k, v) = \sum_{i=1}^{n_{\text{ctx}}} \alpha_i v_i$$

softmax (generic function with 1 method)

```
1 function softmax(x)
2   y = exp.(x)
3   return y / sum(y)
4 end
```

softmax_lookup (generic function with 1 method)

```
1 function softmax_lookup(dict, query)
2   ks = keys(dict)
3   α = softmax([dot(query, key) for key in keys(dict)])
4   @show α
5   return sum(α * value for (α, value) in zip(α, values(dict)))
6 end
```

► [0.291313, 1.0]

```
1 softmax_lookup(dict, [0.8, 0.2])
```

```
> α = [0.6456563062257954, 0.3543436937742045]
```

[BCB16] D. Bahdanau, K. Cho and Y. Bengio. [*Neural Machine Translation by Jointly Learning to Align and Translate*](#) (May 2016), [arXiv:1409.0473](#). Accessed on Oct 23, 2024.

Matrix form of attention ⇄

$$Q = [q_1 \quad \cdots \quad q_{n_{\text{ctx}}}] \quad K = [k_1 \quad \cdots \quad k_{n_{\text{ctx}}}] \quad K^\top Q = \begin{bmatrix} \langle k_1, q_1 \rangle & \cdots & \langle k_1, q_{n_{\text{ctx}}} \rangle \\ \vdots & \ddots & \vdots \\ \langle k_{n_{\text{ctx}}}, q_1 \rangle & \cdots & \langle k_{n_{\text{ctx}}}, q_{n_{\text{ctx}}} \rangle \end{bmatrix}$$

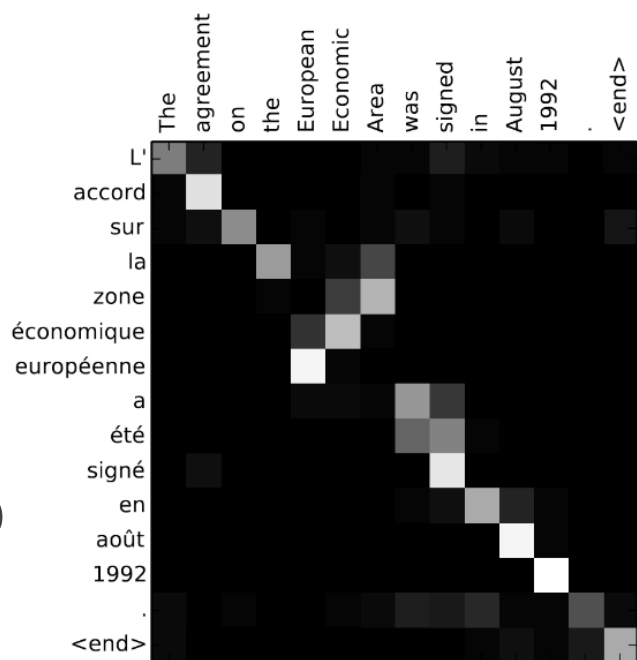
softmax is then applied to each **column**:

$$\text{softmax}(K^\top Q / \sqrt{d_k})$$

Division by $\sqrt{d_k}$ scales the input of softmax to preferable regions [VSP+17; Secton 3.2.1].

Illustrated on the right from [BCB16; Figure 3(a)].

$$\text{Attention}(V, K, Q) = V \text{softmax}(K^\top Q / \sqrt{d_k})$$



[BCB16] D. Bahdanau, K. Cho and Y. Bengio. *Neural Machine Translation by Jointly Learning to Align and Translate* (May 2016), [arXiv:1409.0473](https://arxiv.org/abs/1409.0473). Accessed on Oct 23, 2024.

[VSP+17] A. Vaswani, N. Shazeer, N. Parmar *et al.* *Attention Is All You Need*. In: *Advances in Neural Information Processing Systems*, Vol. 30 (Curran Associates, Inc., 2017). Accessed on Oct 11, 2024.

Masked Attention \Leftrightarrow

💡 **Key idea** In the model for $\hat{p}(x_0|x_{-1}, \dots, x_{-n_{\text{ctx}}})$, incorporate sub-models

Mask prevent \hat{p} to look input the future:

$$\begin{array}{c} \bar{p}(x_0|x_{-1}, \dots, x_{-n_{\text{ctx}}}) \\ \bar{p}(x_{-1}|x_{-2}, \dots, x_{-n_{\text{ctx}}}) \\ \vdots \\ \bar{p}(x_{-n_{\text{ctx}}+1}|x_{-n_{\text{ctx}}}). \end{array}$$

$$M = \begin{bmatrix} 0 & 0 & \dots & 0 \\ -\infty & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\infty & \dots & -\infty & 0 \end{bmatrix}$$

$$\text{Masked-Attention}(V, K, Q) = V \text{softmax}(M + K^\top Q / \sqrt{d_k})$$

Multi-Head Attention \Leftrightarrow

Heads focus on different aspects. Their outputs are **combined** with $W^O \in \mathbb{R}^{d_{\text{emb}} \times h d_v}$:

$$\text{head}_j = \text{Attention}(W_j^V V, W_j^K K, W_j^Q Q)$$

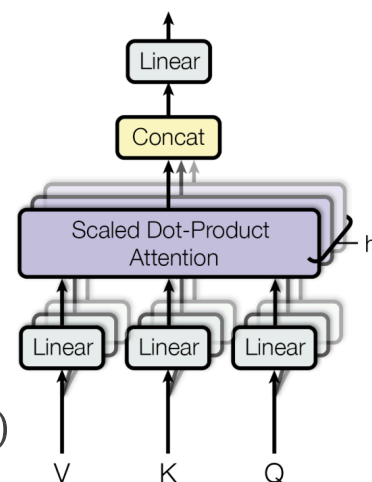
$$\text{MultiHead}(V, K, Q) = W^O \text{vcat}(\text{head}_1, \dots, \text{head}_h)$$

See [VSP+17; Figure 2] on the right.

Similarly, in the masked case:

$$\text{head}_j = \text{Masked-Attention}(W_j^V V, W_j^K K, W_j^Q Q)$$

$$\text{Masked-MultiHead}(V, K, Q) = W^O \text{vcat}(\text{head}_1, \dots, \text{head}_h)$$



► Is W^O needed if $h = 1$?

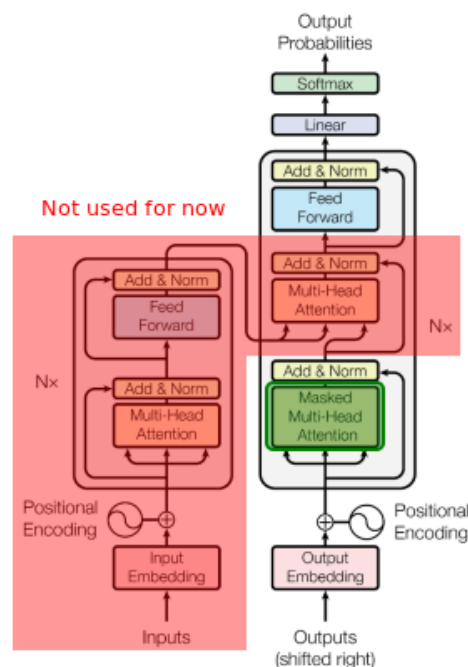
Self-Attention

Self-Attention with embedding C is:

$$\text{Masked-MultiHead}(CX, CX, CX)$$

The embedding vectors CX take then different projections for value, key, query and also for different heads!

$$\text{head}_j = \text{Masked-Attention}(W_j^V CX, W_j^K CX, W_j^Q CX)$$



► Is the order between the tokens taken into account by the model ?

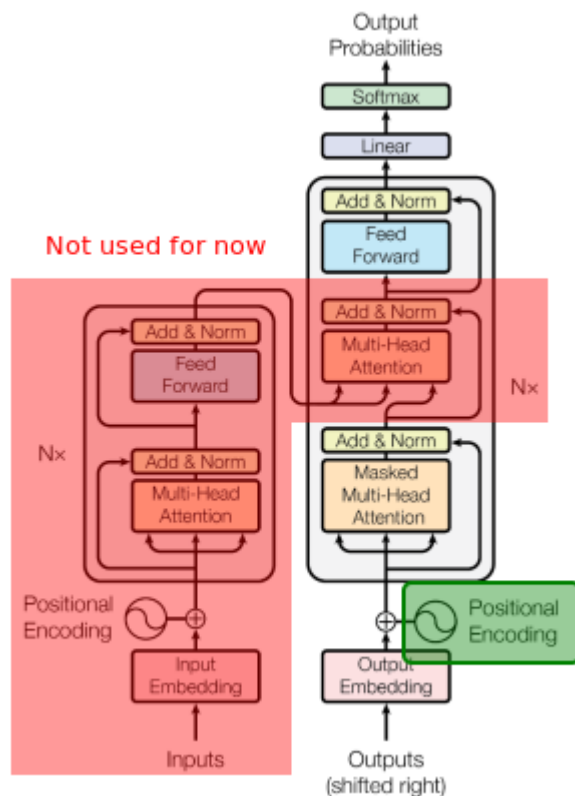
Positional encoding ⇔

Cannot sum $Cx_i + e_i$ with one-hot encoding $e_i \in \mathbb{R}^{n_{\text{ctx}}}$ as the dimension of Cx_i is $\mathbb{R}^{d_{\text{emb}}}$.

So we also add a positional embedding P :
 $Cx_i + Pe_i = Cx_i + p_i$.

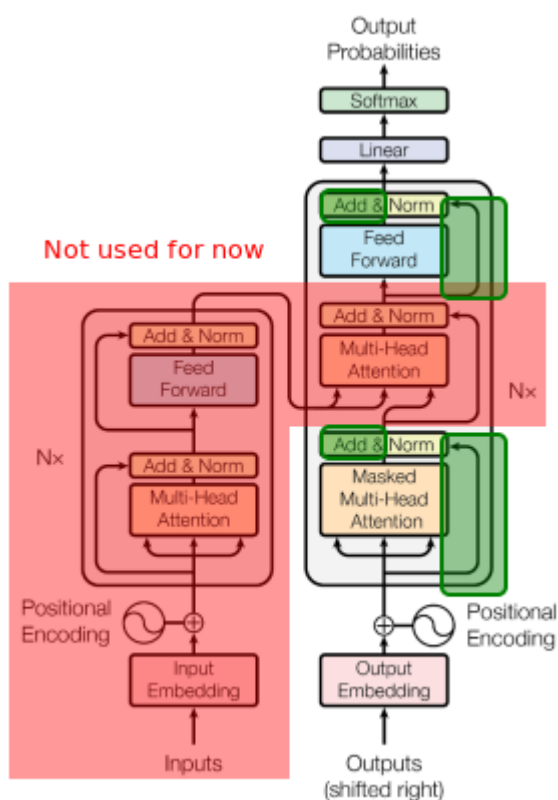
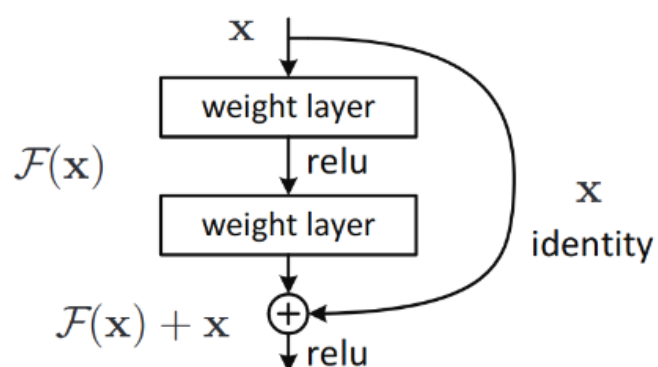
With Self-Attention:

Self-MultiHead($CX + P, CX + P, CX + P$)



Residual connection ⇔

Residual connection [HZRS15]



[HZRS15] K. He, X. Zhang, S. Ren and J. Sun. *Deep Residual Learning for Image Recognition* (Dec 2015), [arXiv:1512.03385](https://arxiv.org/abs/1512.03385). Accessed on Nov 12, 2024.

Layer normalization ⇔

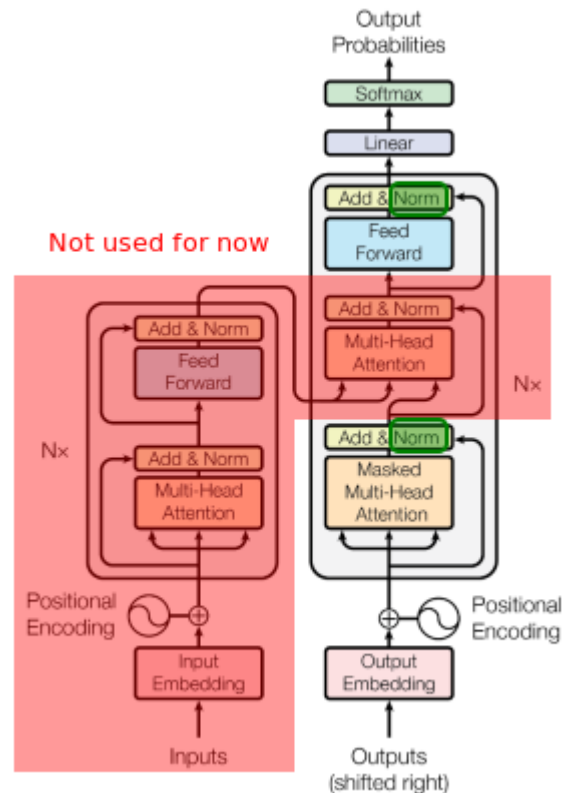
Norm of gradient increases exponentially with depth. Issue for deep neural net. Consider output

$$\begin{bmatrix} y_{1,1} & \dots & y_{1,d_{\text{emb}}} \\ \vdots & \ddots & \vdots \\ y_{d_{\text{batch}},1} & \dots & y_{d_{\text{batch}},d_{\text{emb}}} \end{bmatrix}$$

Normalization : $y_{i,j} \mapsto g(y_{i,j} - \mu_{i,j}) / \sigma_{i,j}$ for gain g , mean μ and standard deviation σ .

- Batch normalization : $\sigma_{i,j} = \sigma_j$ [IS15]
- Layer normalization : $\sigma_{i,j} = \sigma_i$ [BKH16]

Batch norm depends on the batch hence is tricky to implement. Layer normalization is used in [VSP+17].



[IS15] S. Ioffe and C. Szegedy. *Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift* (Mar 2015), [arXiv:1502.03167](https://arxiv.org/abs/1502.03167). Accessed on Nov 12, 2024.

[BKH16] J. L. Ba, J. R. Kiros and G. E. Hinton. *Layer Normalization* (Jul 2016), [arXiv:1607.06450](https://arxiv.org/abs/1607.06450). Accessed on Nov 12, 2024.

[VSP+17] A. Vaswani, N. Shazeer, N. Parmar *et al.* *Attention Is All You Need*. In: *Advances in Neural Information Processing Systems*, Vol. 30 (Curran Associates, Inc., 2017). Accessed on Oct 11, 2024.

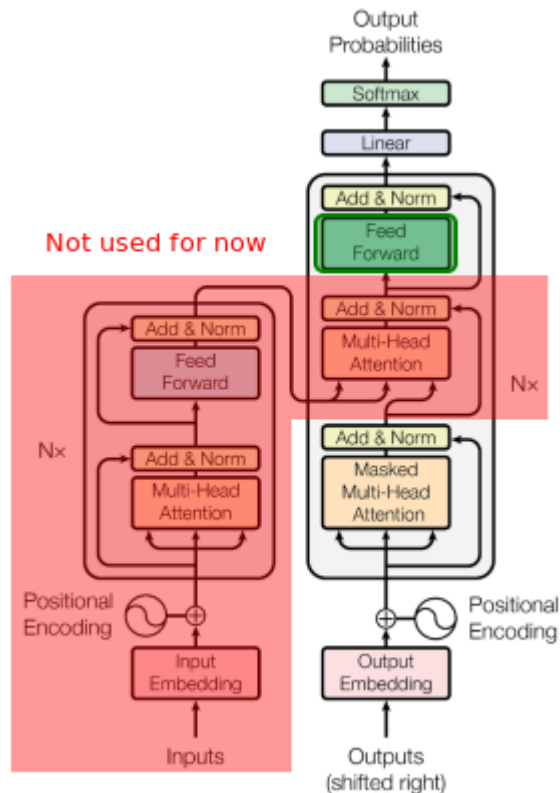
Feed-Forward network ⇌

Different weights $W_1 \in \mathbb{R}^{d_{\text{ff}} \times d_{\text{emb}}}$,
 $W_2 \in \mathbb{R}^{d_{\text{emb}} \times d_{\text{ff}}}$ for each layer:

$$x \mapsto W_2 \max(0, W_1 x + b_1) + b_2$$

Expansion factor $d_{\text{ff}}/d_{\text{emb}}$ is typically 4× like suggested in [VSP+17] (but not for Gemma)

Name	Ref	d_{emb}	d_{ff}
GPT-2	[RWCL19]	<u>768</u>	3072
Gemma	[TMHD24]	2048	32768
Gemma	[TMHD24]	3072	49152
Gemma-2	[TRPS24]	2304	18432
Gemma-2	[TRPS24]	3584	28672
Gemma-2	[TRPS24]	4608	73728
Llama-3			<u>4096</u>
base	[VSPU17]	512	2048
big	[VSPU17]	1024	4096

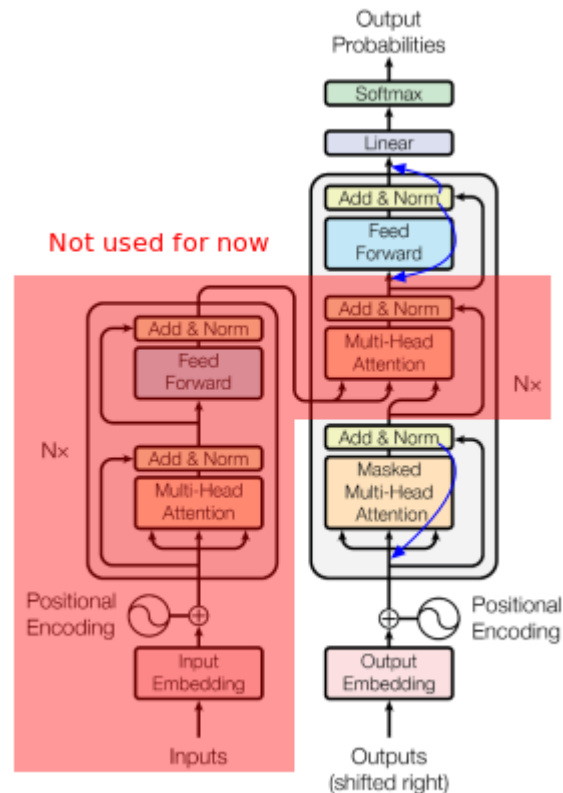


The feed-forward network is implemented **independently** for the output of each query so each query can be processed independently through each **layer**. The next layer allows each queries to then look at the results of the previous layer for **past** (because of the mask) queries.

Transformer variations ⇄

Pre-activation for residual neural networks introduced in [HZRS16] and used in GPT-2 [RWC+19]. See figure on the right.

Rotary Positional Encoding [SLP+23] replaces $W^K(Cx_i + p_i)$ and $W^Q(Cx_i + p_i)$ by $R^i W^K Cx_i$ and $R^i W^Q Cx_i$ where R is a rotation matrix. Advantage : $\langle k_i, q_j \rangle$ contains $R^{i-j} \rightarrow$ **relative** difference of position.



[HZRS16] K. He, X. Zhang, S. Ren and J. Sun. *Identity Mappings in Deep Residual Networks*. In: *Computer Vision – ECCV 2016*, edited by B. Leibe, J. Matas, N. Sebe and M. Welling (Springer International Publishing, Cham, 2016); pp. 630–645.

[RWC+19] A. Radford, J. Wu, R. Child et al. *Language Models Are Unsupervised Multitask Learners* (2019). Accessed on Oct 23, 2024.

[SLP+23] J. Su, Y. Lu, S. Pan et al. *RoFormer: Enhanced Transformer with Rotary Position Embedding* (Nov 2023), [arXiv:2104.09864](https://arxiv.org/abs/2104.09864). Accessed on Nov 12, 2024.

Cost of LLMs ⇄

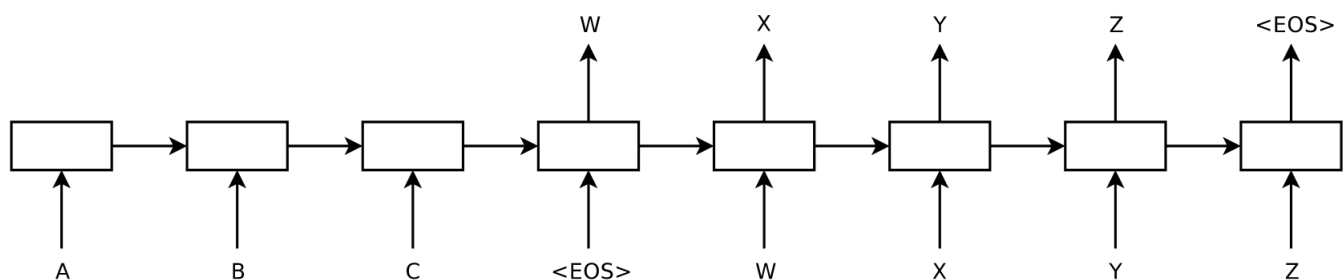
► What is the time complexity of a transformer with respect to d_{emb} , n_{voc} , n_{ctx} , d_{ff} , h and N ?

► How does the number of parameters of transformers compare with [BDV00] or RNNs for large n_{ctx} ?

[BDV00] Y. Bengio, R. Ducharme and P. Vincent. *A Neural Probabilistic Language Model*. In: *Advances in Neural Information Processing Systems*, Vol. 13 (MIT Press, 2000). Accessed on Oct 11, 2024.

Machine translation ⇄

- LSTM **encoder** → **context** → LSTM **decoder** [SVL14]. See [SVL14; Figure 1] below.
- Issue with *encoder bottleneck*. All information has to be summarized in the **context**.



[SVL14] I. Sutskever, O. Vinyals and Q. V. Le. [Sequence to Sequence Learning with Neural Networks](#). In: *Proceedings of the 27th International Conference on Neural Information Processing Systems - Volume 2, NIPS'14* (MIT Press, Cambridge, MA, USA, Dec 2014); pp. 3104–3112. Accessed on Oct 23, 2024.

Cross-Attention ⇌

Cross-Attention between

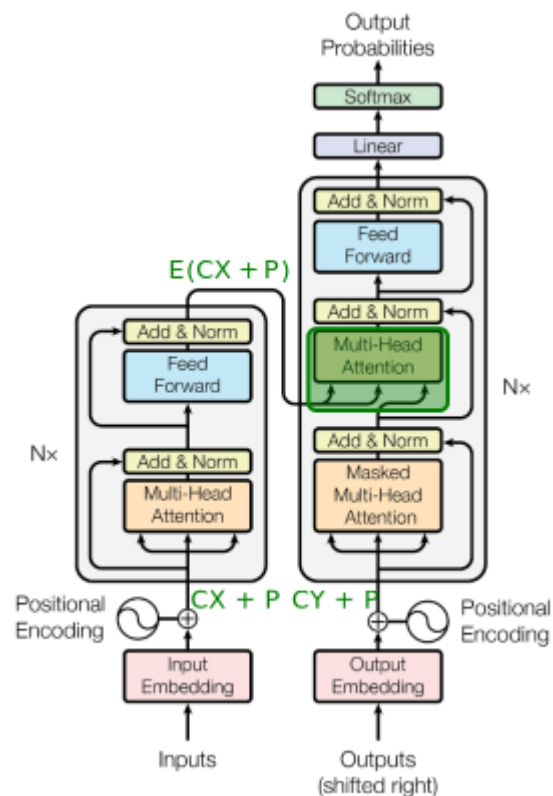
- values and keys $E(CX + P)$ where E is the encoder, and X is the matrix of input tokens
- query Q depending on past output Y and number of layers already applied

$$\text{MultiHead}(E(CX + P), E(CX + P), Q)$$

The embedding vectors CX take then different projections for value, key, query and also for different heads!

$$\text{head}_j = \text{Attention}(W_j^V V, W_j^K K, W_j^Q Q)$$

where $V = K = E(CX + P)$



Utils ⇌

```
1 md"## Utils"
```

```
1 using PlutoUI, DataFrames, PrettyTables, LinearAlgebra, Luxor, LaTeXStrings,  
   MathTeXEngine
```

```
1 import DocumenterCitations, CSV, Logging
```

qa (generic function with 2 methods)

```
1 include("utils.jl")
```

```
biblio =
```

```
► CitationBibliography("/home/runner/work/LINMA2472/LINMA2472/Lectures/biblio.bib", AlphaSt
```

```
1 biblio = load_biblio!()
```

```
① Loading bibliography from `/home/runner/work/LINMA2472/LINMA2472/Lectures/bibli  
o.bib`...
```

```
① Loading completed.
```

```
cite (generic function with 1 method)
```

```
1 cite(args...) = bibcite(biblio, args...)
```

```
bib (generic function with 1 method)
```

```
1 bib(args...) = bibrefs(biblio, args...)
```

```
draw_transformer (generic function with 2 methods)
```

```
1 function draw_transformer(decoder_only = true)
2     scale(0.4, 0.4)
3     Luxor.placeimage(readpng("images/transformer.png"), centered = true)
4     if decoder_only
5         sethue("red")
6         setopacity(0.4)
7         box(Point(-350, -160), Point(320, 20), :fill)
8         box(Point(-350, 20), Point(0, 460), :fill)
9         translate(Point(-170, -190))
10        setopacity(1)
11        fontsize(32)
12        text("Not used for now", halign = :center)
13    end
14 end
```

```
highlight (generic function with 1 method)
```

```
1 function highlight(a, b, c, d)
2     sethue("green")
3     setopacity(0.4)
4     #box(Point(a, b), Point(c, d), :fill)
5     polysmooth(box(Point(a, b), Point(c, d), vertices=true), 10, action = :fill)
6     setopacity(1)
7     polysmooth(box(Point(a, b), Point(c, d), vertices=true), 10, action = :stroke)
8 end
```

```
1 struct BPE
2     text::String
3     pairs::Dict{Tuple{Char,Char},Char}
4 end
```

add_pair (generic function with 1 method)

```
1 function add_pair(bpe::BPE, subs)
2     pairs = copy(bpe.pairs)
3     push!(pairs, subs)
4     return BPE(replace(bpe.text, prod(subs.first) => subs.second), pairs)
5 end
```

pair_stats (generic function with 1 method)

```
1 function pair_stats(text::String)
2     stats = Dict{Tuple{Char,Char},Int}{}
3     for i in eachindex(text)
4         j = nextind(text, i)
5         if j > lastindex(text)
6             break
7         end
8         a = text[i]
9         b = text[j]
10        stats[(a, b)] = get(stats, (a, b), 0) + 1
11    end
12    return stats
13 end
```

substitute (generic function with 1 method)

```
1 function substitute(text::String, pair::Tuple{Char,Char})
2     new_char = min('Z' + 1, minimum(text)) - 1
3     return replace(text, prod(pair.first) => pair.second)
4 end
```

new_token (generic function with 1 method)

```
1 new_token(text::String) = new_token(BPE(text, Dict()))
```

new_token (generic function with 2 methods)

```
1 function new_token(bpe::BPE)
2     stats = pair_stats(bpe.text)
3     pair = findmax(stats)[2]
4     new_char = min('Z' + 1, minimum(bpe.text)) - 1
5     return add_pair(bpe, pair => new_char)
6 end
```

```
llms =
```

	Name	Num params	Ref	``n_text
1	"Gemini-1.5"	missing	"[TGLB24]"	"256k"
2	"Gemini-1"	"1.[8B/3.25B](https://storage.googleapis."	"[TAB24]"	"256k"
3	"Gemma-2"	"27B"	"[TRPS24]"	"256k"
4	"Gemma-2"	"9B"	"[TRPS24]"	"256k"
5	"Gemma-2"	"2B"	"[TRPS24]"	"256k"
6	"Gemma"	"7B"	"[TMHD24]"	"256k"
7	"Gemma"	"2B"	"[TMHD24]"	"256k"
8	"GPT-2"	"1.5B"	"[RWCL19]"	"[50k](https://github"
9	"Llama-2"	"7B"	"[TMSA23]"	"[32k](https://github"
10	"GPT-4o"	missing	missing	"[200k](https://githu"
: more				
19	"big"	missing	"[VSPU17]"	"37k"

```
1 llms = load_llms()
```

load_llms (generic function with 1 method)

```
1 function load_llms()
2   llms = DataFrame(CSV.File("llms.csv"))
3   rename!(llms, "Embedding dimension" => "``d_\\text{emb}``")
4   rename!(llms, "Vocabulary size" => "``n_\\text{voc}``")
5   rename!(llms, "Context window" => "``n_\\text{ctx}``")
6   rename!(llms, "Feed-Forward hidden dimension" => "``d_\\text{ff}``")
7   return llms
8 end
```

```
► ["Name", "Num params", "Ref", "``n_\\text{voc}``, "``d_\\text{emb}``, "``n_\\text{ctx}``"]
```

```
1 names(llms)
```

table (generic function with 1 method)

```
1 function table(df; mandatory_columns = String[], included_columns = nothing)
2   for col in mandatory_columns
3     df = df[(!ismissing).(df[!, col]), :]
4   end
5   if !isnothing(included_columns)
6     df = unique(df[!, included_columns])
7   end
8   Markdown.parse(pretty_table(
9     String,
10    sort(df),
11    backend = Val(:markdown),
12    show_subheader = false,
13    header_decoration = MarkdownDecoration(),
14    allow_markdown_in_cells = true,
15    formatters = (v, _, _) -> ismissing(v) ? "" : v,
16  ))
17 end
```

d =

	Name	B
1	"A"	"C"

```
1 d = DataFrame("Name" => String["A"], "B" => String["C"])
```

	Name	B
1	"A"	"C"
2	"a"	"d"

```
1 push!(d, ["a", "d"])
```