Assignment # 02

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Exercise 7.2

Q# 11 $\int (\ln x)^2 dx$

$= \int (\ln x)^2 \times 1 \, dx$

let $u = (\ln x)^2$; $dv = 1$

$\therefore \int u \, dv = uv - \int v \, du$ $\int dv = x + C$ $\boxed{v = x}$

$\therefore \frac{du}{dx} = \frac{2\ln(x)}{x} \Rightarrow du = \frac{2\ln(x)}{x} dx$

$\int (\ln x)^2 = (\ln x)^2 (x) - \int x \frac{2\ln(x)}{x} dx$

$= x(\ln x)^2 - 2\int \ln(x)\, dx$ — ①

Again applying by parts

$= \int (\ln x)\, dx$ $\boxed{u = \ln(x)}$, $\boxed{dv = 1}$

$\frac{du}{dx} = \frac{1}{x}$ $\int dv = x$

$du = \frac{dx}{x}$ $\boxed{v = x}$

$\int \ln(x) = \ln(x)(x) - \int x \frac{dx}{x}$

$= x\ln(x) - x + C$

Putting in ①

$① \Rightarrow \int (\ln x)^2 = x(\ln x)^2 - 2[x\ln(x) - x]$

$\int (\ln x)^2 = x(\ln x)^2 - 2x\ln(x) + 2x + C$

Ans.

Q#11.

$\int \cos^{-1}(2x)\,dx$

Solution

$\Rightarrow$ let $u = \cos^{-1} 2x$ & $dv = 1$

$\Rightarrow \int \cos^{-1}(2x)\,dv =$

$$\int dv = \int 1$$
$$\boxed{v = x}$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

$$du = \frac{2}{\sqrt{1-4x^2}}\,dx$$

$$\int \cos^{-1}(2x)\,dx = \cos^{-1}(2x)x - \int \frac{x\,2}{\sqrt{1-4x^2}}\,dx$$

$$= \cos^{-1}(2x)x - 2\int \frac{x\,dx}{\sqrt{1-4x^2}}$$

$$= \cos^{-1}(2x)(x) - \left(\frac{1}{4}\right)\int \frac{8x\,dx}{\sqrt{1-4x^2}}$$

let $y = 1-4x^2$

$$\frac{dy}{dx} = -8x$$

$$-dy = 8x\,dx$$

$$= x\cos^{-1}(2x) - \left(\frac{-1}{4}\right)\int \frac{dy}{\sqrt{y}}$$

$$= x\cos^{-1}(2x) + \frac{1}{4}\left\{\frac{\sqrt{y}}{1/2}\right.$$

$$= \int \cos^{-1}(2x)\,dx = x\cos^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + C$$

Ans

$$\int \cos(\ln x)\,dx$$

Solution

let $u = \cos(\ln x)$, $dv = 1\,dx$

$\frac{du}{dx} = \frac{\sin(\ln x)}{x}$ $\qquad \int dv = \int dx$

$du = \frac{\sin(\ln x)}{x}dx$ $\qquad \boxed{v = x}$

let $I = \int \cos(\ln x)dx = \cos(\ln x)x - \int \cancel{x}\left(\frac{\sin(\ln x)}{\cancel{x}}\right)$

$= x\cos(\ln x) - \int \sin\ln(x)$

let $u = \sin\ln(x)$, $dv = 1\,dx$

$\frac{du}{dx} = \frac{-\cos\ln(x)}{x}$, $\boxed{v = x}$

$= \int \sin\ln(x)\,dx = \sin\ln(x)x - \int \cancel{x}\left(-\frac{\cos\ln(x)}{\cancel{x}}dx\right)$

$= x\sin\ln(x) + \int \cos\ln(x)$

Put I in ①

① $\Rightarrow I = x\cos(\ln x) - [x\sin\ln(x) + \int \cos\ln(x)]$

$I = \int \cos(\ln x) dx$

$I = x\cos(\ln x) - x\sin\ln(x) - I$

$2I = x[\cos(\ln x) - \sin\ln(x)]$

$I = \frac{x[\cos(\ln x) - \sin\ln(x)]}{2}$

$\int \cos(\ln x) dx = \frac{x[\cos(\ln x) - \sin\ln(x)]}{2} + C$

Ans.

$\int_{-1}^{1} \ln(x+2) dx.$

Solution.

Let $u = \ln(x+2)$, $dv = dx$

$\frac{du}{dx} = 1 \cdot \frac{1}{x+2}$ $\quad v = x$

$du = \frac{1}{x+2} dx$

$\int_{-1}^{1} \ln(x+2) = \ln(x+2)(x) - \int_{-1}^{1} x \frac{dx}{x+2}$

$$= \left[\ln(x+2)(x)\right]_{-1}^{1} - \int_{-1}^{1} \frac{x+2-2}{x+2}$$

$$= \left[\ln(3)(1) - \ln(1)(-1)\right] - \int_{-1}^{1} 1 - \frac{2}{x+}$$

$$= (1.0986 - 0) - \left[x\right]_{-1}^{1} + \left[2\ln(x+\right.$$

$$= \cancel{1.0986}$$

$$= \ln(3) - 2 + 2\ln(3) - 2\ln(1)$$

$$\int_{-1}^{1} \ln(x+2)\,dx = 1.29583$$

Ans

Q#50

$\int x^3 \sqrt{2x}$

Solu

Q#5

$\int x^3 \sqrt{2x+1}\, dx$

Solution.

$x^3 \cdot \frac{2}{3}(2x+1)^{3/2} - \frac{4}{5}x^2 \cdot (2x+1)^{5/2}$

$+ \frac{48x}{105}(2x+1)^{7/2} - \frac{48}{105}(2x+1)^{9/2}$

x^3	$\sqrt{2x+1}$
$3x^2$	$\frac{2}{3}(2x+1)^{3/2}$
$6x$	$\frac{2}{3}\times\frac{2}{5}(2x+1)^{5/2}$
6	$\frac{4}{15}\times\frac{2}{7}(2x+1)^{7/2}$
0	$\frac{8}{105}(2x+1)^{9/2}$

$\frac{2x^3}{3}(2x+1)^{3/2} - \frac{4x^2}{5}(2x+1)^{5/2}$

$\frac{16x}{35}(2x+1)^{7/2} - \frac{16}{35}(2x+1)^{9/2} + C$

Ans.

Q#6 Use reduction formulas

$\int x^2 e^{3x}\, dx.$

$\int x^n e^x\, dx = x^n e^x - n \int x^{n-1} e^x\, dx$

here $n = 2$ & $x = 3x$

$\int x^2 e^{3x}\, dx = x^2 e^{3x} - 2\int x\, e^{3x}\, dx.$

let $u = x$ & $dv = e^{3x}$

$\frac{du}{dx} = 1$ $\quad dv = \frac{e^{3x}}{3}$

$\boxed{du = dx}$

$\Rightarrow \int x e^{3x} = \frac{x e^{3x}}{3} - \int \frac{e^{3x}}{3} dx$

$= \frac{x e^{3x}}{3} - \frac{1}{3}\left(\frac{e^{3x}}{3}\right)$

$= \frac{x e^{3x}}{3} - \frac{1}{9} e^{3x}$

$= \frac{e^{3x}}{3}\left[x - \frac{1}{3}\right]$

$\int x e^{3x} = \frac{e^{3x}}{9}\left(3x - 1\right)$

$\Rightarrow \int x^2 e^{3x} dx = x^2 e^{3x} - 2\left[\frac{e^{3x}}{9}(3x-1)\right]$

$= x^2 e^{3x} - \frac{2e^{3x}}{9}(3x-1) + C$

$\Rightarrow \int x^2 e^{3x} dx = e^{3x}\left[x^2 - \frac{2}{9}(3x-1)\right] + C$

Ans

Q. ns

$\int$

Sol

let

$\int \frac{dx}{1+ \dots}$

Exercise 7.4

Q#5

$$\int \frac{dx}{(4+x^2)^2}$$

Solution:

let $x = a\tan\theta$ — (1)

$x = 2\tan\theta$ — (1)

$$\frac{dx}{d\theta} = 2\sec^2\theta$$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{dx}{(4+x^2)^2} = \int \frac{2\sec^2\theta\, d\theta}{(4+(2\tan\theta)^2)^2}$$

$$= 2\int \frac{\sec^2\theta\, d\theta}{(4+4\tan^2\theta)^2}$$

$$= 2\int \frac{\sec^2\theta\, d\theta}{\{4(1+\tan^2\theta)\}^2}$$

$$= \frac{2}{16}\int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^2}$$

$$= \frac{1}{8}\int \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

$$= \frac{1}{8}\int \frac{1}{\sec^2\theta}\, d\theta$$

$$= \frac{1}{8}\int \cos^2\theta\, d\theta$$

$$\cos^2\theta = \int \frac{1+\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\Rightarrow \int \frac{dx}{(4+x^2)^2} = \frac{1}{8}\int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{1}{16}\int d\theta + \frac{1}{16}\int \cos 2\theta \, d\theta$$

$$= \frac{1}{16}\theta + \frac{1}{16}\int \frac{\sin 2\theta}{2}\, d\theta$$

$$= \frac{1}{16}\theta + \frac{1}{32}\sin 2\theta \, d\theta$$

$$= \frac{1}{16}\theta + \frac{1}{32}(2\sin\theta\cos\theta)$$

$$= \frac{1}{16}\theta + \frac{1}{16}\sin\theta\cos\theta$$

$\Rightarrow x = 2\tan\theta$ $\qquad$ $Hyp = \sqrt{x^2+4}$

$\tan\theta = \frac{x}{2} \rightarrow \frac{P}{B}$ $\qquad$ $\cos\theta = \frac{2}{\sqrt{x^2+4}}$

$\boxed{\theta = \tan^{-1}\frac{x}{2}}$ $\qquad$ $\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$$\Rightarrow \int \frac{dx}{(4+x^2)^2} = \frac{1}{16}\left(\tan^{-1}\frac{x}{2}\right) + \frac{1}{16}\left(\frac{x}{\sqrt{x^2+4}}\right)\left(\frac{2}{\sqrt{x^2+4}}\right)$$

$$\Rightarrow \int \frac{dx}{(4+x^2)^2} = \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{x}{8(x^2+4)} + C$$

Ans

Q# 10 $\int x^3 \sqrt{5 - x^2}\, dx$

Solution:

$\therefore$ let $x = a \sin\theta$

$x = \sqrt{5} \sin\theta$ — (1)

$\frac{dx}{d\theta} = \sqrt{5} \cos\theta$

$dx = \sqrt{5} \cos\theta\, d\theta$

$\int 25\sqrt{5} \sin^3\theta \sqrt{5 - (\sqrt{5}\sin\theta)^2}\, \sqrt{5}\cos\theta\, d\theta$

$\Rightarrow 25\sqrt{5} \int \sin^3\theta \sqrt{5(1 - \sin^2\theta} \, d\theta \sqrt{5}\cos\theta\, d\theta$

$\Rightarrow 25\sqrt{5} \int \sin^3\theta \sqrt{5} \cos^2\theta\, d\theta$

$25 \times 5 \int \sin^3\theta \cos^2\theta\, d\theta$

let $u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta\, d\theta$

$125 \int u^3 du$

$\frac{125}{4} \cdot u^4$

$\frac{125}{4} \cdot (\sin\theta)^4$

$= \frac{125}{4} \cdot \frac{x^4}{(\sqrt{5})^4}$

$= \frac{125}{4} \frac{x^4}{25}$

$= \frac{5x^4}{4} + C$

Q#18

$\int \frac{3x^3}{\sqrt{x^2-25}} dx$

let $x = a\sec\theta$

$x = 5\sec\theta$ — ①

$\frac{dx}{d\theta} = 5\sec\theta\tan\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$= \int \frac{3x^3}{\sqrt{x^2-25}} dx = \int \frac{3(125)(\sec\theta)^3}{\sqrt{25\sec^2\theta - 25}} \times 5\sec\theta\tan\theta$

$= \frac{125 \times 3 \times 5}{5} \int \frac{(\sec^3\theta)(\sec\theta)\tan\theta}{\sqrt{\tan\theta}} d\theta$

$= 375 \int \sec^4\theta\, d\theta$

$= 375 \int \sec^2\theta(1+\tan^2\theta)\, d\theta$

$= 375 \int \sec^2\theta + \sec^2\theta\tan^2\theta\, d\theta$

$= 375 \int \sec^2\theta \, d\theta + 375 \int \sec^2\theta \tan^2\theta \, d\theta$

$= 375 (\tan\theta) + 375 \int u^2 \, du$

$= 375 (\tan\theta) + \frac{375}{3} \int u^3$

$= 375 \tan\theta + 125 \tan^3\theta$

$= 125 (3\tan\theta + \tan^3\theta)$

Finding θ

$\sec\theta = \frac{x}{5} \rightarrow \frac{H}{B}$ $\quad P.p = \sqrt{x^2 - 25}$

$\tan\theta = \frac{\sqrt{x^2-25}}{5}$

$\int \frac{3x^3}{\sqrt{x^2-25}} = 125 \left[\frac{3}{5}\sqrt{x^2-25} + \frac{1}{125}(x^2-25)^{3/2} \right]$

$= 75\sqrt{x^2-25} + (x^2-25)^{3/2} + C$

Ans

Q no 17 $\int e^x\sqrt{1-e^{2x}}\,dx$

Solution

let $e^x = \sin\theta$

$\frac{d}{d\theta} e^x = \cos\theta$

$\frac{e^x}{d\theta} = \cos\theta$

$e^x = \cos\theta\, d\theta$

let $u = e^x$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$= \int e^x\sqrt{1-e^{2x}} = \int du\sqrt{1-u^2}$

let $u = a\sin\theta$

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta\, d\theta$

$= \int \cos\theta\, d\theta \sqrt{\cos^2\theta}$

$= \int \cos^2\theta\, d\theta$

$= \int \frac{1+\cos 2\theta}{2}$

$\sin\theta = \frac{u}{1}$

Base $= \sqrt{1-u}$

$\cos\theta = \sqrt{1-}$

$$= \frac{1}{2}\int 1+\cos 2\theta \, d\theta$$

$$= \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta \, d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{2}\frac{\sin 2\theta}{2} d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\frac{2\sin\theta\cos\theta}{2}$$

$$= \frac{1}{2}\theta + \frac{1}{2}(\sin\theta\cos\theta)$$

$$= \frac{1}{2}\sin^{-1}(u) + \frac{1}{2}(u)(\sqrt{1-u^2})$$

Return: e^x

$$\int \sqrt{1-e^{2x}}\,dx = \frac{1}{2}\sin^{-1}(e^x) + \frac{1}{2}(e^x)(\sqrt{1-e^{2x}}) + C$$

Q.26 $\int_0^3 \frac{x^3}{(3+x^2)^{5/2}} dx$

Solution:

let $x = \sqrt{3} \tan\theta \Rightarrow \theta = \tan^{-1}(0) = 0$

$\frac{dx}{d\theta} = \sqrt{3} \sec^2\theta \qquad = \tan^{-1}(3/\sqrt{3})$

$dx = \sqrt{3} \sec^2\theta \, d\theta$

$= \int_0^3 \frac{(\sqrt{3}\tan\theta)^3}{(3+3\tan^2\theta)^{5/2}} \sqrt{3}\sec^2\theta \, d\theta$

$= \int_0^3 \frac{3\times 3}{(3)^{5/2}} \frac{\tan^3\theta \sec^2\theta \, d\theta}{(\sec^2\theta)^{5/2}}$

$= \int_0^{\pi/3} \frac{1}{\sqrt{3}} \frac{\tan^3\theta \cancel{\sec^2\theta}}{\sec^5\theta} d\theta$

$= \frac{1}{\sqrt{3}} \int_0^{\pi/3} \tan^3\theta \cos^3\theta \, d\theta$

$= \frac{1}{\sqrt{3}} \int_0^{\pi/3} \frac{\sin^3\theta}{\cos^3\theta} \cos^3\theta \, d\theta$

$= \frac{1}{\sqrt{3}} \int_0^{\pi/3} \sin^3\theta \, d\theta$

$= \frac{1}{\sqrt{3}} \int_0^{\pi/3} (1-\cos^2\theta)\sin\theta \, d\theta$

$$\frac{1}{\sqrt{3}}\int_0^{\pi/3} \sin\theta \, d\theta - \frac{1}{\sqrt{3}}\int_0^{\pi/3} \cos^2\theta \sin\theta \, d\theta$$

$$\frac{1}{\sqrt{3}}\left[-\cos\theta\right]_0^{\pi/3} - \frac{1}{3\sqrt{3}}\left[\cos^3\theta\right]_0^{\pi/3}$$

$$\frac{1}{\sqrt{3}}\left[\frac{1}{2} - 1\right] - \frac{1}{3\sqrt{3}}\left[\frac{1}{8} - 1\right]$$

$$\frac{1}{\sqrt{3}}\left[-\frac{1}{2}\right] - \frac{1}{3\sqrt{3}}\left[-\frac{7}{8}\right]$$

$$\frac{1}{2\sqrt{3}} + \frac{7}{24\sqrt{3}}$$

$$\frac{12+7}{24\sqrt{3}}$$

$$\frac{19}{24\sqrt{3}}$$

Q. 37

$$\int \frac{dx}{\sqrt{3+2x-x^2}}$$

$$= -\int \frac{dx}{\sqrt{x^2-2x-3}}$$

$$= -\int \frac{dx}{\sqrt{x^2-2x+1-3-1}}$$

$$= -\int \frac{dx}{\sqrt{(x-1)^2-4}}$$

Let $u = x-1$

$\frac{du}{dx} = 1$

$du = dx$

$$= -\int \frac{du}{\sqrt{u^2-4}}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right) + C$$

$$= -\left[\ln\left(u+\sqrt{u^2-4}\right)\right] + C$$

$$= -\left[\ln\left\{(x-1)+\sqrt{(x-1)^2-4}\right\}\right]+C$$

Ans

Qu. $\int \frac{2x+3}{4x^2+4x+5}\,dx$

Solution:

$$\int \frac{2x+3}{4x^2+4x+1+5-1}\,dx$$

$$\int \frac{2x+3}{(2x+1)^2+4}\,dx$$

let $u = (2x+1)$

$$\frac{du}{dx} = 2$$

$$\frac{du}{2} = dx$$

$$\int \frac{u+2}{u^2+4}\,\frac{du}{2}$$

$$\frac{1}{2}\int \frac{u}{u^2+4}\,du + \frac{2}{2}\int \frac{du}{u^2+4}$$

$$\frac{1}{4}\int \frac{2u\,du}{u^2+4} + \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$$

$$= \frac{1}{4} \ln(u^2+4) + \frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right) + C$$

$$= \frac{1}{4} \ln\{(2x+1)^2+4\} + \frac{1}{2} \tan^{-1}\left(\frac{2x+1}{2}\right) + C$$

Ans

Exercise 7.5 (Partial Fractions)

Q# 4 $\frac{x^2}{(x+2)^3}$

$$\frac{x^2}{(x+2)^3} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} \quad \text{--- (1)}$$

$$\frac{x^2}{(x+2)^3} = \frac{A(x+2)^2 + B(x+2) + C}{(x+2)^3}$$

$x^2 = A(x^2 + 2(2)x + 4) + Bx + 2B + C$

$x^2 = Ax^2 + 4Ax + Bx + 4A + 2B + C$

$\boxed{A = 1}$

$B + 4A = 0$

$\boxed{B = -4}$

$4A + 2B + C = 0$

$C = -4A - 2B$

$C = -4 - 2(-4)$

$C = -4 + 8 = 4$

Putting in (1)

$$\frac{x^2}{(x+2)^3} = \frac{1}{x+2} + \frac{(-4)}{(x+2)^2} + \frac{4}{(x+2)^3}$$

Ans

Q:

$$\frac{1-3x^4}{(x-2)(x^2+1)^2}$$

Solution:

$$\Rightarrow \frac{1-3x^4}{(x-2)(x^2+1)^2} = \frac{A}{x-2} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$$

$$\Rightarrow 1-3x^4 = A(x^2+1)^2 + (Bx+C)(x-2)(x^2+ \dots$$
$$+ (Dx+E)(x-2)$$

$$\Rightarrow 1-3x^4 = A(x^4+2x^2+1) + (Bx+C)(x^3+x-2x \dots$$
$$+ (Dx+E)(x-2)$$

$$\Rightarrow 1-3x^4 = Ax^4+2Ax^2+A+Bx^4-Bx^2-2Bx+C \dots$$
$$-Cx-2C+Dx^2-2Dx+Ex-2E$$

$$\Rightarrow 1-3x^4 = x^4(A+B)+x^3(C)+x^2(2A-B+D)$$
$$+x(-2B-2D+E)+A-2C-2E$$

$\Rightarrow A+B=-3$

$\Rightarrow \boxed{C=0}$

$\Rightarrow 2A-B+D=0 \Rightarrow 2A-(-3-A)+D=0$

$\Rightarrow -2B-2D+E=0$ $\qquad D=-3A-3$

$\Rightarrow A-2E=0$

$\Rightarrow -2(-3-A)-2D+\frac{A}{2}=0$

$6+2A-2(\dots$

$6+2A+6A+\dots$

$\frac{5A+A}{2} = -12$

$\frac{17A}{2} = -12$

$17A = -24$

$\boxed{A = -\frac{24}{17}}$

$\boxed{E = -\frac{48}{17}}$

$\boxed{D = \frac{21}{17}}$

$\boxed{B = -\frac{27}{17}}$

$$\frac{1-3x^4}{(x-2)(x^2+1)^2 \dots}$$

$\Rightarrow 6 + 2A - 2(-2A-3) + \frac{A}{2} = 0$

$\Rightarrow 6 + 2A + 4A + 6 + \frac{A}{2} = 0$

$\Rightarrow 6A + \frac{A}{2} = -12$

$\Rightarrow \frac{13A}{2} = -12$

$\Rightarrow 13A = -24$

$\Rightarrow \boxed{A = -\frac{24}{13}}$

$\boxed{E = -\frac{48}{17}}$

$\boxed{D = \frac{21}{13}}$

$\boxed{B = -\frac{27}{13}}$

$$\frac{1-3x^4}{(x-2)(x^2+1)^2} = \frac{-24}{13(x+2)} - \frac{27+}{13(x^2+2)} + \frac{21x-48}{17(x^2+1)^2}$$

$$\Rightarrow \int \frac{5x-5}{3x^2-8x-3}\,dx$$

$$= \frac{1}{3}\int \frac{5x-5}{x^2-\frac{8x}{3}-1}\,dx$$

$$= \frac{5}{3}\int \frac{x-1}{x^2-\frac{8}{3}x-1}\,dx$$

$$L.T = \frac{(M.T)^2}{4\times F.T} = \frac{\frac{64x^2}{9}}{4\times x^2} = \frac{64x^2}{36x^2} = \frac{16}{9}$$

$$\int \frac{x-1}{x^2-\frac{8}{3}x+\frac{16}{9}-1-\frac{16}{9}}$$

$$\int \frac{x-1}{\left(x-\frac{4}{3}\right)^2-\frac{25}{9}}$$

$$\int \frac{x-1}{\left(x-\frac{4}{3}\right)^2-\left(\frac{5}{3}\right)^2}$$

$$= \int \frac{x-1}{\left(x-\frac{4}{3}+\frac{5}{3}\right)\left(x-\frac{4}{3}-\frac{5}{3}\right)}$$

$$\int \frac{x-1}{\left(\frac{3x+1}{3}\right)\left(\frac{3x-9}{3}\right)}$$

$$= 15\int \frac{x-1}{(3x+1)(3x-9)}$$

$$= \frac{15}{3}\int \frac{x-1}{(3x+1)(x-3)} \quad -(1)$$

$$= \frac{x-1}{(3x+1)(x-3)} = \frac{A}{3x+1} + \frac{B}{x-3}$$

$$\Rightarrow x-1 = A(x-3) + B(3x+1)$$

$$= Ax - 3A + 3Bx + B$$

$$\Rightarrow x-1 = x(A+3B) + B - 3A$$

$$A + 3B = 1$$

$$B - 3A = -1$$

$$\frac{B+1}{3} = A$$

$$\frac{B+1}{3} + 3B = 1$$

$$B + 1 + 9B = 3$$

$$10B = 2$$

$$\boxed{B = \frac{1}{5}} \quad \boxed{A = \frac{2}{5}}$$

$$= \frac{15}{3}\int \frac{2}{5(3x+1)} + \frac{1}{5(x-3)}$$

$$= 2\ln(3x+1) + 5\ln(x-3)$$

Date

Exercise 2.6 (u = Tan $\frac{x}{2}$ subs)

Q.5 $\int \frac{dx}{1+\sin x + \cos x}$

Solution:

$$\int \frac{2}{1+u^2} \div 1 + \frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2}$$

$$\int \frac{2}{1+u^2} \div \frac{1+u^2+2u+1-u^2}{(1+u^2)}\,du$$

$$\int \frac{2}{1+u^2} \div \frac{2+2u}{1+u^2}\,du$$

$$\int \frac{2}{2(1+u)}\,du$$

$$\ln(1+u)$$

$$= \ln\left(1+\tan\frac{x}{2}\right) + C$$

Ans.

Q.6 $\int \frac{dx}{2+\sin x}$

Solution.

$$\int \frac{2}{1+u^2} \div 2 + \frac{2u}{1+u^2}$$

$$\div \frac{2(1+u^2)+2u}{1+u^2}$$

$$\int \frac{2}{2(u^2+u)}\,du$$

Date

$= \int \frac{1}{u} + \int \frac{1}{u^2} + \int \frac{1}{u}$ [illegible]

$= \left(-\frac{1}{u} \right) + \ln(u)$

$= \frac{x^2}{2} - \frac{2}{x} + \ln\left(\frac{x^2}{2}\right) + C$

Ans

Ques 8: $\int \frac{dx}{4 \sin x - 3 \cos x}$

Solution

$= \int \frac{\frac{2}{1+u^2}}{4\left(\frac{2u}{1+u^2}\right) - 3\left(\frac{1-u^2}{1+u^2}\right)} du$

$= \int \frac{2}{1+u^2} \div \frac{8u - 3 + 3u^2}{1+u^2} du$

$= \int \frac{2}{8u + 3u^2 - 3} du$

$= \frac{2}{4} \int \frac{du}{u} + \frac{2}{3} \int \frac{du}{u^2} - \frac{2}{3} \int du$

$= 2 \cdot \frac{1}{4} \ln(u) + \frac{2}{3}\left(-\frac{1}{u}\right) - \frac{2}{3}(u)$

$= \frac{1}{4} \ln\left(\frac{x}{2}\right) - \frac{2}{3} \cdot \frac{2}{x} - \frac{2}{3}\left(\frac{x}{2}\right)$

Date

$$\ln\left(\frac{x}{3}\right) = \frac{4}{3x} = \frac{x}{3} + C$$

Ans

Q.no.

$$\int \frac{\sin x}{\sin x + \tan x}\, dx$$

$$\int \frac{2u}{1+u^2} \times \frac{2}{1+u^2} = \frac{2u}{1+u^2} + \frac{2u}{1-u^2}$$

$$\frac{4u}{1+u^2} = \frac{2u(1+u^2) + 2u(1-u^2)}{(1-u^2)(1+u^2)}$$

$$\frac{2}{4u(1-u^2)}$$

$$\frac{}{4u\{1-u^2+1+u^2\}}$$

$$\int \frac{1-u^2}{2} du$$

$$1-u^2\, du$$

$$u - \frac{u^3}{3} + C$$

$$- \frac{x^3}{6 \times 3} + C$$

$$-\frac{x^3}{24} + C \quad \text{Ans}$$

Date

Exercise 7.8 (Improper integral)

(1) $\int_{-\infty}^{3} \frac{dx}{x^2+9}$

Solution:

$$\int_{-\infty}^{3} \frac{dx}{x^2+9} = \int \lim_{b \to -\infty} \int_{b}^{3} \frac{dx}{x^2+9}$$

$$= \lim_{b \to -\infty} \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_{b}^{3}$$

$$= \lim_{b \to -\infty} \left[\frac{1}{3} \tan^{-1} \frac{3}{3} - \frac{1}{3} \tan^{-1} \frac{b}{3} \right]$$

= Applying limit

$$= \frac{1}{3} \frac{\pi}{4} - \frac{1}{3} \tan^{-1} \left(\frac{-\infty}{3} \right)$$

$$= \frac{1}{12} \pi - \frac{1}{3} \tan^{-1}(-\infty)$$

$$= \frac{\pi}{12} + \frac{1}{3} \cdot \frac{\pi}{2}$$

$$= \frac{\pi}{12} + \frac{\pi}{6}$$

$$= \frac{3\pi}{12}$$

$$\boxed{= \frac{\pi}{4}}$$

Convergent

Date

Q#13 $\int_{-\infty}^{0} \frac{e^x dx}{3-2e^x}$

Solution

$= \int_{b-\infty}^{0} \frac{e^x dx}{3-2e^x} = \lim_{b \to -\infty} \int_{b}^{0} \frac{e^x dx}{3-2e^x}$

$= \lim_{b \to -\infty}$

let $u = 3-2e^x$

$\frac{du}{dx} = 0-2e^x$

$\frac{-du}{2} = e^x dx$

$= \lim_{b \to -\infty} \frac{-1}{2} \int_{b}^{0} \frac{du}{u}$

$= \frac{-1}{2} \lim_{b \to -\infty} \ln(u)$

$= \frac{-1}{2} \lim_{b \to -\infty} \left[\ln(3-2e^x) \right]_{b}^{0}$

$= \frac{-1}{2} \left[\ln[3-2(1)] - \ln(3-2e^{-\infty}) \right]$

$= \frac{-1}{2} \left[\ln(1) - \ln(3-2e^{-\infty}) \right]$

$= \frac{-1}{2} \left[0 - \ln(3) \right]$

$= \boxed{0.5493}$ Convergent

let

$u = 3$

$\frac{du}{dx}$

$\frac{-du}{2}$

Date

Q. 16

$$\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\, dt$$

Let $u = e^{-t}$

$\frac{du}{dt} = e^{-t}$

$du = e^{-t}\, dt$

Solution:

$$\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\, dt = \int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}}\, dt + \int_{0}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\, dt$$

$$= \int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-t}e^{-t}}\, dt + \int_{0}^{+\infty} \frac{e^{-t}}{1+e^{-t}e^{-t}}\, dt$$

$\cancel{\text{let } u = (1+e^{-t}e^{-t})}$

$\cancel{\frac{du}{dt} = 0 + e^{-2t} \cdot -2}$

$\cancel{-\frac{du}{2e^{-t}} = e^{-t}\, dt}$

Let $u = e^{-t}$

$\frac{du}{dt} = e^{-t} \cdot -1$

$-du = e^{-t}\, dt$

$$= \int_{-\infty}^{0} \frac{-du}{1+u^2} + \int_{0}^{+\infty} \frac{-du}{1+u^2}$$

$$= -\left[\ln(1+u^2)\right]_{-\infty}^{0} - \left[\ln(1+u^2)\right]_{0}^{\infty}$$

$$= -\left[(\ln(1) - \ln(-\infty)) - (\ln(1) - \ln(\infty))\right]$$

$$= -\{0-0\} - \{0-\infty\}$$

$= -\infty$ Divergent

Q) 4, iv $\int_0^{\pi/2} \frac{\sec^2 x \, dx}{1 - \tan x}$

Sol

$$= \int_0^{\pi/2} \frac{\sec^2 x \, dx}{1-\tan x} = \lim_{a \to \frac{\pi}{2}} \int_0^a \frac{\sec^2 x \, dx}{1-\tan x}$$

$$= \lim_{a \to \frac{\pi}{2}} - \int_0^a \frac{-\sec^2 x \, dx}{1-\tan x}$$

$$= \lim_{a \to \frac{\pi}{2}} - \left[\ln(1-\tan x) \right]_0^a$$

$$= \lim_{a \to \frac{\pi}{2}} - \left[\ln(1-\tan a) - \ln(1) \right]$$

= Applying limit

$$= -\ln(-\infty) - 0$$

$$= \boxed{\infty}$$

Converging

$$\int_0^1 \frac{dx}{(x-1)^{2/3}}$$

Solution:

$$\int \frac{dx}{(x-1)^{2/3}} = \lim_{a \to 1^-} \int_0^a \frac{dx}{(x-1)^{2/3}}$$

$$= \lim_{a \to 1^-} \int_0^a (x-1)^{-2/3} dx$$

$$= \lim_{a \to 1^-} \left[\frac{(x-1)^{1/3}}{1/3} \right]_0^a$$

$$= \lim_{a \to 1^-} 3 \left[(a-1)^{1/3} - (-1)^{1/3} \right]$$

Applying lim

$$3\{(1-1)^{1/3} + 1\}$$

$$= 3(0+1)$$

$$= \boxed{3}$$

Convergent