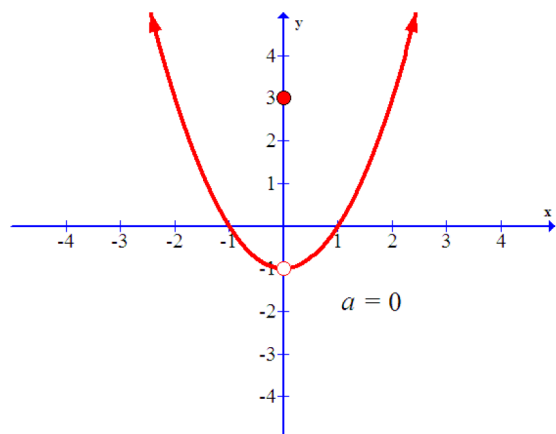


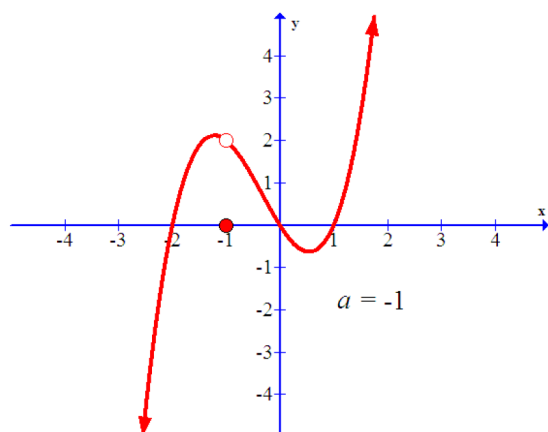
## Exercise Set 2.3: One-Sided Limits

Find  $\lim_{x \rightarrow a^+} f(x)$ ,  $\lim_{x \rightarrow a^-} f(x)$  and  $\lim_{x \rightarrow a} f(x)$ , if they exist.

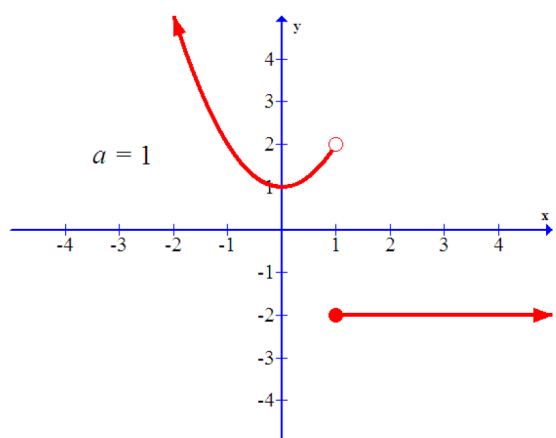
1.



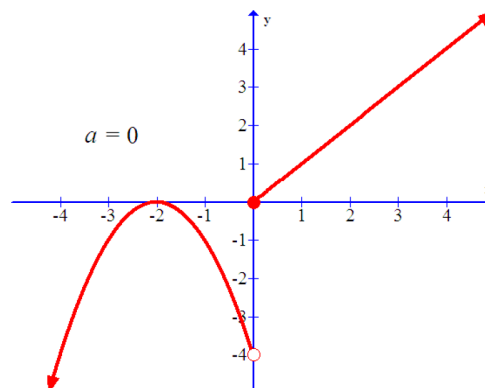
2.



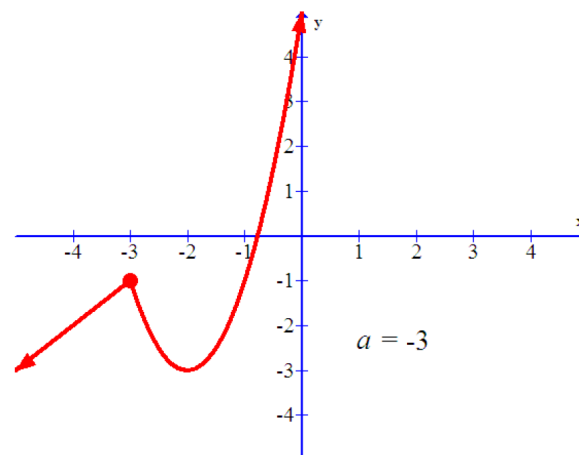
3.



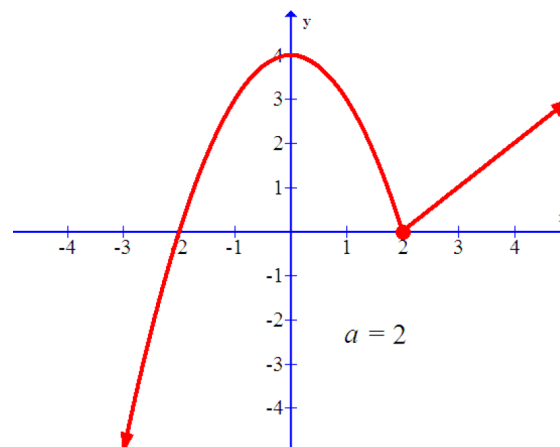
4.



5.

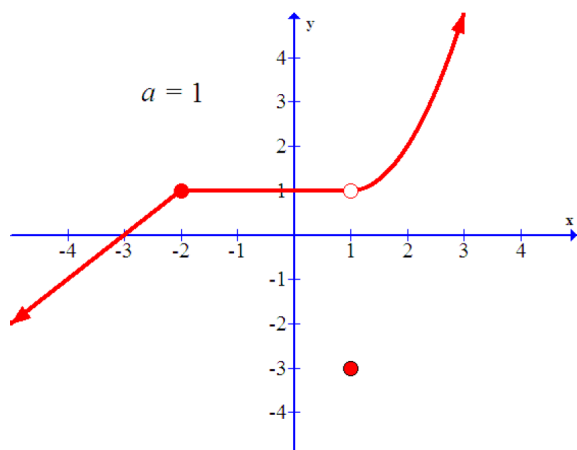


6.

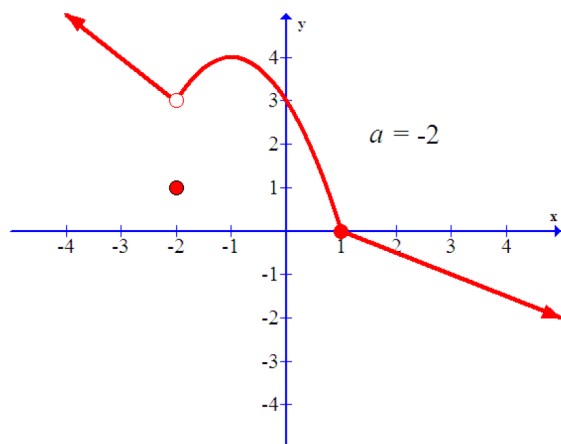


## Exercise Set 2.3: One-Sided Limits

7.



8.



Find  $\lim_{x \rightarrow a^+} f(x)$ ,  $\lim_{x \rightarrow a^-} f(x)$  and  $\lim_{x \rightarrow a} f(x)$ , if they exist.

9.  $f(x) = \begin{cases} 2x-1, & x < 0 \\ x^2+4x-1, & x \geq 0 \end{cases}, a = 0$

10.  $f(x) = \begin{cases} x^2+3, & x < -1 \\ 2x+6, & x \geq -1 \end{cases}, a = -1$

11.  $f(x) = \begin{cases} x+2, & x < 2 \\ 2, & 2 \leq x \leq 6, \\ 14-2x, & x > 6 \end{cases}, a = 6$

12.  $f(x) = \begin{cases} x+2, & x < 2 \\ 2, & 2 \leq x \leq 6, \\ 14-2x, & x > 6 \end{cases}, a = 2$

13.  $f(x) = \begin{cases} x^2-3, & x < 0 \\ -3, & 0 \leq x \leq 4, \\ 2x-5, & x > 4 \end{cases}, a = 0$

14.  $f(x) = \begin{cases} x^2-3, & x < 0 \\ -3, & 0 \leq x \leq 4, \\ 2x-5, & x > 4 \end{cases}, a = 4$

15.  $f(x) = \begin{cases} x^2+x+3, & -4 \leq x < -1 \\ 4-x^2, & -1 \leq x \leq 3 \\ 2x^2-5x+1, & x > 3 \end{cases}, a = -1$

16.  $f(x) = \begin{cases} x^2+x+3, & -4 \leq x < -1 \\ 4-x^2, & -1 \leq x \leq 3 \\ 2x^2-5x+1, & x > 3 \end{cases}, a = 3$

Use a graphing calculator to find  $\lim_{x \rightarrow a^+} f(x)$ ,

$\lim_{x \rightarrow a^-} f(x)$  and  $\lim_{x \rightarrow a} f(x)$ , if they exist.

17.  $f(x) = \begin{cases} e^x-1, & x < 0 \\ x^2+x-1, & x \geq 0 \end{cases}, a = 0$

18.  $f(x) = \begin{cases} 3x^2+5x-2, & x < -1 \\ x^3+4x+1, & x \geq -1 \end{cases}, a = -1$

19.  $f(x) = \begin{cases} -2x^2-x+6, & x < -2 \\ 2-3x-x^3, & x \geq -2 \end{cases}, a = -2$

## Exercise Set 2.3: One-Sided Limits

$$20. f(x) = \begin{cases} 2^x + 2, & x < 2 \\ 2x^2 - 3x + 3, & x \geq 2 \end{cases}, a = 2$$

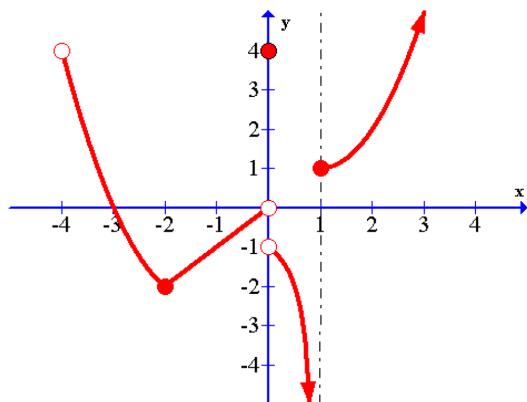
$$21. f(x) = \begin{cases} \sqrt{x+5}, & x < 4 \\ x^2 - 5x + 6, & x \geq 4 \end{cases}, a = 4$$

$$22. f(x) = \begin{cases} \sqrt{2x+5}, & x < 2 \\ \sqrt{20-x^2}, & x \geq 2 \end{cases}, a = 2$$

$$23. f(x) = \begin{cases} \sqrt{x^2 + 3x + 5}, & x < 1 \\ \sqrt{3^x + 1}, & x \geq 1 \end{cases}, a = 1$$

$$24. f(x) = \begin{cases} \sqrt{x^2 + 5x + 4}, & x < -1 \\ \sqrt{2^x + \frac{3}{2}}, & x \geq -1 \end{cases}, a = -1$$

Use the graph of  $f(x)$  to determine if the statements in problems 25 – 32 are true or false.



$$25. \lim_{x \rightarrow -3^+} f(x) \text{ does not exist}$$

$$26. \lim_{x \rightarrow -2^+} f(x) = -1$$

$$27. \lim_{x \rightarrow 0^+} f(x) = -1$$

$$28. \lim_{x \rightarrow 0^-} f(x) = 4$$

$$29. \lim_{x \rightarrow 0} f(x) = 4$$

$$30. \lim_{x \rightarrow 1^-} f(x) \text{ does not exist}$$

$$31. \lim_{x \rightarrow 1^+} f(x) = 1$$

$$32. \lim_{x \rightarrow \infty} f(x) = 3$$

For problems 33 – 44, find the indicated limit.

$$33. \lim_{x \rightarrow 2^+} (3x + 1)$$

$$34. \lim_{x \rightarrow -1^-} (1 - 5x)$$

$$35. \lim_{x \rightarrow 3^+} \left( \frac{x+2}{x-1} \right)$$

$$36. \lim_{x \rightarrow -2^-} \left( \frac{2x+5}{x+4} \right)$$

$$37. \lim_{x \rightarrow -4^+} \left( \frac{3x-1}{x+4} \right)$$

$$38. \lim_{x \rightarrow \left(\frac{1}{2}\right)^-} \left( \frac{x+4}{2x-1} \right)$$

$$39. \lim_{x \rightarrow 1^+} \left( \frac{x+3}{x^2 - 2x + 1} \right)$$

$$40. \lim_{x \rightarrow -1^-} \left( \frac{x^2 - 9}{x^2 + 5x + 4} \right)$$

$$41. \lim_{x \rightarrow 2^+} \sqrt{x-2}$$

$$42. \lim_{x \rightarrow 4^-} \sqrt{4-x}$$

$$43. \lim_{x \rightarrow -1^+} \left( \frac{\sqrt{x+1}}{x^2 + 5} \right)$$

$$44. \lim_{x \rightarrow -4^+} \left( \frac{\sqrt{x+4}}{x^2 - 3} \right)$$