Electric Current & Resistance

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Applied Physics

Definition(Basic Concept)

Electric Current: (related to motion of charged particle)

"Motion of Electric charge form ELECTRIC CURRENT"

Or

Flow of electric charges in a conductor is called "Electric Current"

Charge Carriers & Types of Charges

- **Question arises** : In which form do these charges travel or propagate?
- It is necessary for charge to travel that it should be carried by a Particle/Material, without which charge can not be travelled.
- So.. if we say charge travels that means **material is travelling**.
- In the language of Electric Current we say those moving material/particle as a **CHARGE CARRIERS**.
- Charge carriers may be **positive or negative.**

Charge carriers in different materials

In Solids(Metals)

- Charge carriers in metals are Electrons,

In liquids

- In liquids Cat-ions and An-ions perform the task of charge carriers as positive ions and negative ions,

In Gases

- In gases(at very low pressure & at very high potential, charges will be able to move) electrons and positive ions are charge carriers

Movement of Charge Carriers

- **Fundamental concept for the movement of charges**

charges will never start moving from any point or end up at any point**….These Charges always move in a chain.**

- This chain forms a **closed path** for its movement.
- **Electric Circuits forms a closed path for**

the charges to move and produce Electric Current.

Direction of Current

It is conventional to assign to **the current the same direction as the flow of +ve charge**.

In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, **the direction of the current is opposite the direction of flow of electrons.**

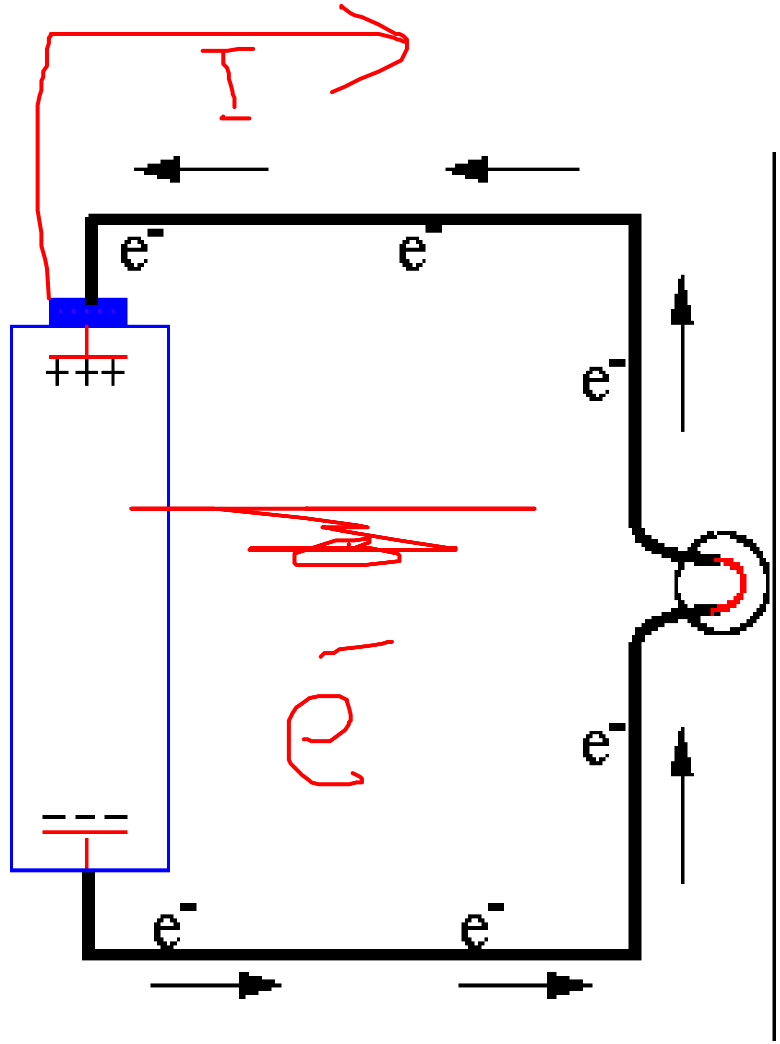
However, if we are considering ***a beam of positively*** **charged protons** in an accelerator, the current is in the **direction of motion of the protons**.

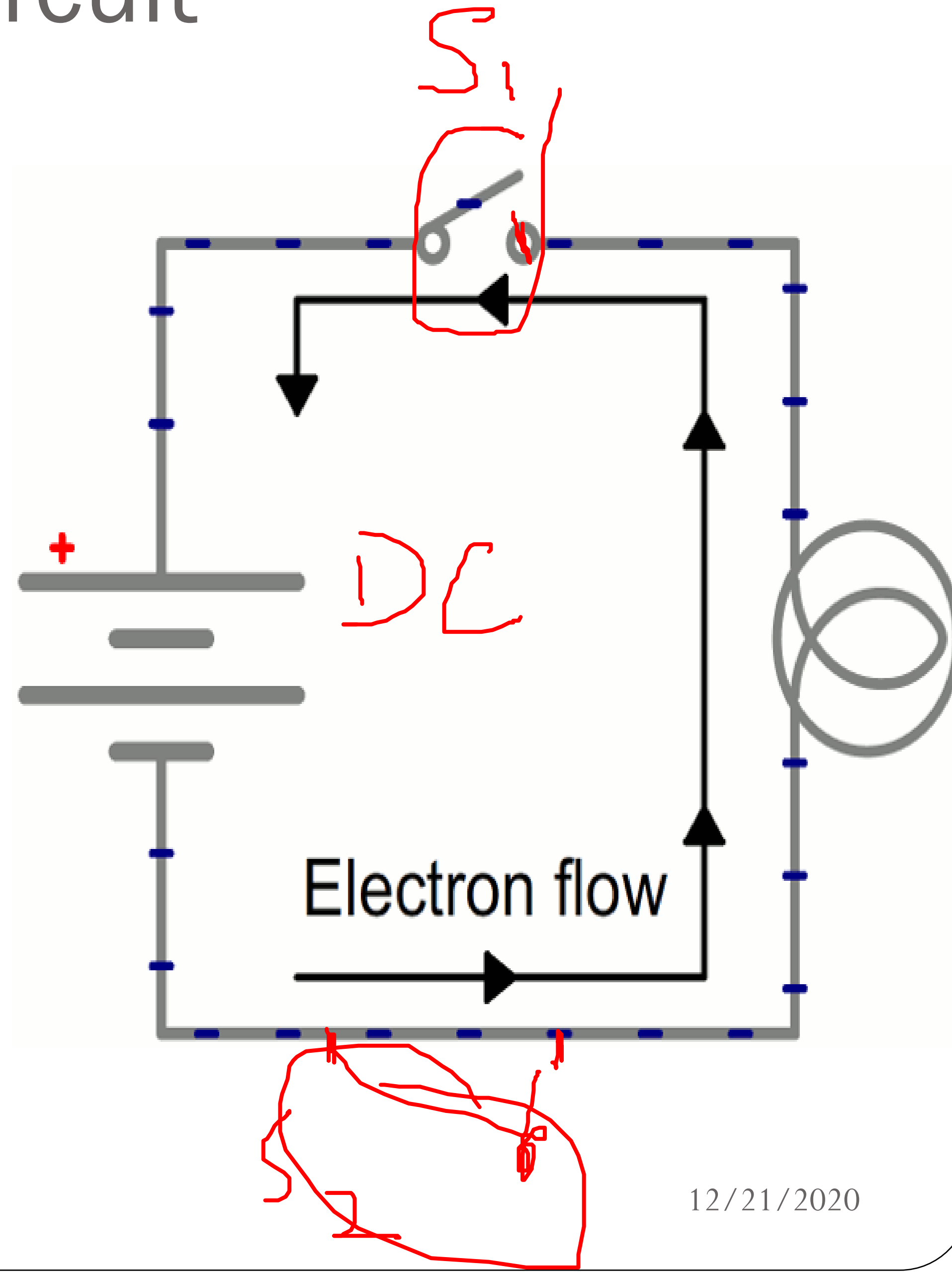
In some cases—such as those involving ***gases and electrolytes***, for instance—the current is the result of the flow **of both positive and negative charges**.

If the ends of the conducting wire are connected to a battery, **all points on the loop are not at the same potential**. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The **electric field exerts forces** on the conduction electrons in the wire, causing them to move around the loop and thus **creating a current**.

It is common to refer to a moving charge (positive or negative) as a mobile charge carrier. For example, ***the mobile charge carriers in a metal are electrons.***

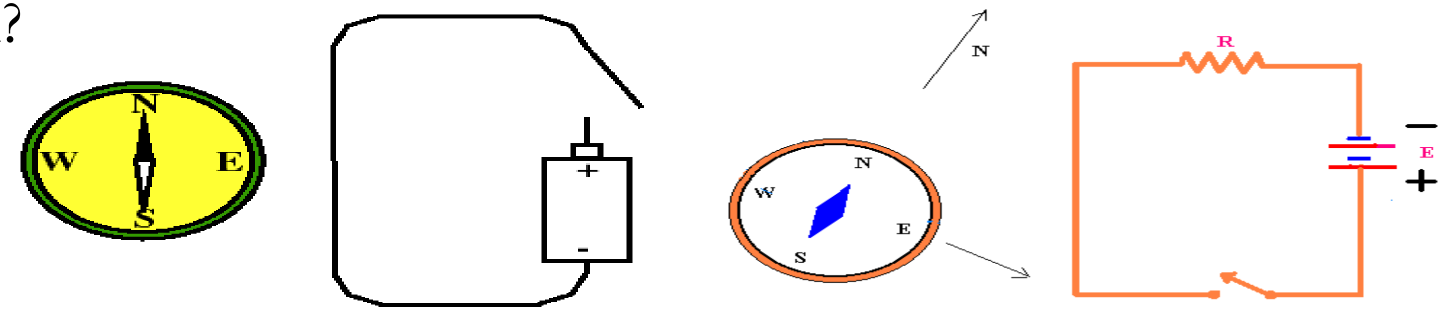
Open & Close Circuit





Measurement of Electric Current

- How do we check whether the current flowing is strong or weak?

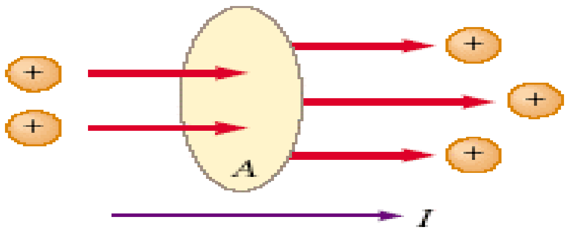


- If one battery shows deflection of 10 degree then two batteries will show deflection of 20 degree and three batteries will show 30 degrees deflection, which means this quantity is measurable.
- In t seconds charges flow in a circuit is Q therefore,
- In 1 sec it will be Q/t , (symbol for current we use I)
- For number of electrons to flow $Ne = Q$

Measurement of Electric Current

Whenever there is a net flow of charge through some region a **current** is said to exist.

Suppose that the charges are moving perpendicular to surface of **area *A***, The current is the rate at which charge flows through this surface.

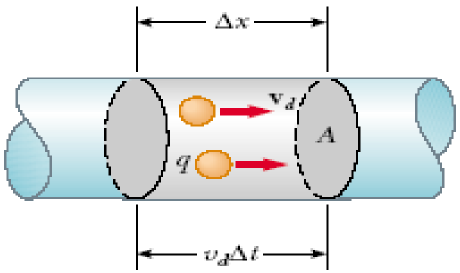


If ***Q*** *is the amount of charge that passes through this area in a time interval t,* the average current $\boldsymbol{I_{av}}$ *is equal to the charge that passes through A per unit time* $I_{avg} = \frac{\Delta Q}{\Delta t}$

If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current *I*** *as the differential limit of average current:*

$$I = \frac{dQ}{dt}, \text{ unit: Coulomb/sec or Ampere}$$

Microscopic Model of Current



We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal.

Consider the current in a conductor of **cross-sectional area** ***A***

The **volume** of a section of the conductor of length ***Δx*** *is* ***AΔx***

The **number** of carriers in the gray section is ***nAΔx*** .

The charge Q in this section is ΔQ= number of carriers in section X charge per carrier

$$\Delta Q = (nA\Delta x)q \;, \text{ where } v_d = \frac{\Delta x}{\Delta t} \text{(drift speed)}$$

$$\Delta Q = (nA\, v_d\, \Delta t)q \qquad \Delta x = v_d\, \Delta t$$

So ,

$$I_{avg} = \frac{\Delta Q}{\Delta t} = \frac{(nA\, v_d\, \Delta t)q}{\Delta t} = \boxed{nqA\, v_d}$$

The speed of the charge carriers v_d is an average speed called the **drift speed.**

Current Density(J)

J is represented as current density defined as current per unit area

$$\mathbf{J} = \frac{I}{A} = nq\,\mathbf{v_d} \quad \text{, (unit is A/m}^2\text{)} \qquad \text{where } \mathrm{I} = nq\,A\,\mathrm{v_d}$$

where J has SI units of A/m^2. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current. In general, the current density is a vector quantity:

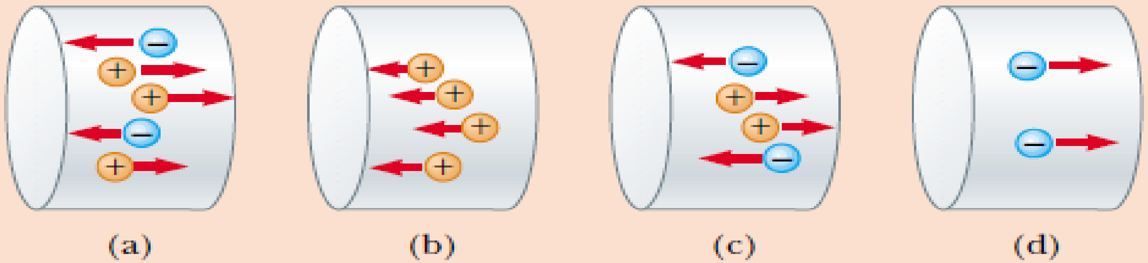
A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor. If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

where the constant of proportionality σ is called the **conductivity**

Brainstorming

Quick Quiz Consider positive and negative charges moving horizontally through the four regions shown in **Figure** Rank the current in these four regions, from lowest to highest.



Charges move through four regions.

d, b = c, a.

The current in part (d) is equivalent to two positive charges moving to the left. Parts **(b) and (c)** each represent four positive charges moving in the same direction because negative charges moving to the left are equivalent to positive charges moving to the right.

The current in part (a) is equivalent to five positive charges moving to the right.

Example 1:

- In a cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?

$$I = \frac{\Delta Q}{\Delta t} \qquad \Delta Q = I\Delta t = (30.0 \times 10^{-6}\ \text{A})(40.0\ \text{s}) = 1.20 \times 10^{-3}\ \text{C}$$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = \boxed{7.50 \times 10^{15}\ \text{electrons}}$$

Example 2

- The quantity of charge q (in coulombs) that has passed through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds.

  (a) What is the instantaneous current through the surface at $t = 1.00$ s?

  (b) What is the value of the current density?

$$q = 4t^3 + 5t + 6$$

$$A = (2.00\ \text{cm}^2)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00 \times 10^{-4}\ \text{m}^2$$

(a) $$I(1.00\ \text{s}) = \left.\frac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.(12t^2 + 5)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$$

(b) $$J = \frac{I}{A} = \frac{17.0\ \text{A}}{2.00 \times 10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$$

Example 3

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31×10^{-6} m^2. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5 \text{ g}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3$$

Because each copper atom contributes one free electron to the current, we have

Example 3(cont'd)

$$n = \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3}\left(\frac{1.00 \times 10^6 \text{ cm}^3}{1 \text{ m}^3}\right)$$

$$= 8.49 \times 10^{28} \text{ electrons/m}^3$$

we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$

$$= \frac{10.0 \text{ C/s}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(3.31 \times 10^{-6} \text{ m}^2)}$$

$$= \boxed{2.22 \times 10^{-4} \text{ m/s}}$$

Ohm's Law & Resistance

It states that : for many materials (including most metals), the ratio of the current density to the electric field is a **constant** $\boldsymbol{\sigma}$ that is independent of the electric field producing the current.

$$\frac{\boldsymbol{J}}{\boldsymbol{E}} = \boldsymbol{\sigma}$$

Materials that obey the above equation are said to follow Ohm's law

Materials that follow Ohm law are **ohmic (E and J are Ohmic)**.
and which do not obey Ohm law are said to be **non- ohmic**
(A bulb is a non-ohmic conductor. Its voltage-current graph does not follow a straight line)

Ohm's law is not a fundamental law of nature but rather an empirical relationship valid only for certain materials

Ohm's Law & Resistance

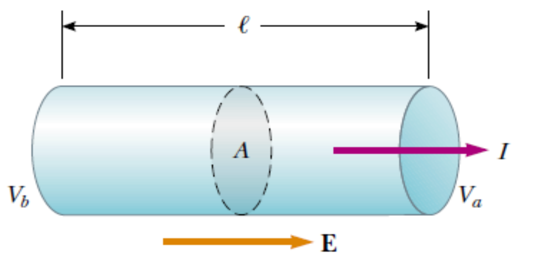
A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the potential difference is related to the field through the relationship[2]

$$\Delta V = E\ell$$

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s} = E\int_0^\ell dx = E\ell$$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$



Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

The quantity $R = \ell/\sigma A$ is called the **resistance** of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$\boxed{R = \frac{\Delta V}{I}}$$

We will use this equation over and over again when studying electric circuits. From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be one **ohm** (Ω):

The inverse of conductivity is **resistivity**[3] ρ:

$$\rho = \frac{1}{\sigma}$$

where ρ has the units ohm-meters $(\Omega \cdot \mathrm{m})$.

Because $R = \ell/\sigma A$,

resistance of a uniform block of material along the length ℓ as

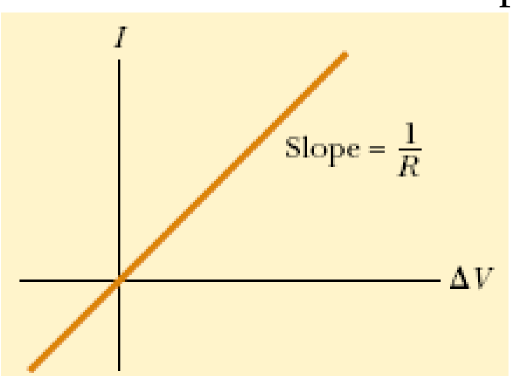
$$R = \rho\,\frac{\ell}{A}$$

Most electric circuits use devices called **resistors** to control the current level

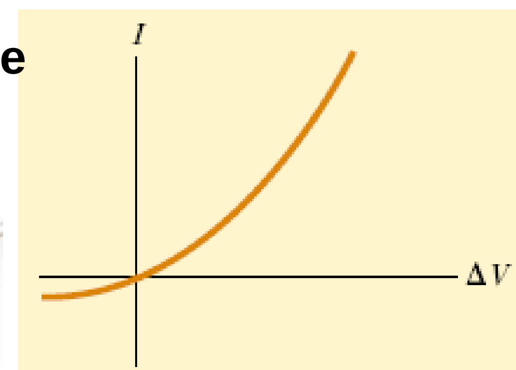
Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.

Current – Potential Difference

I – V Characteristic graph



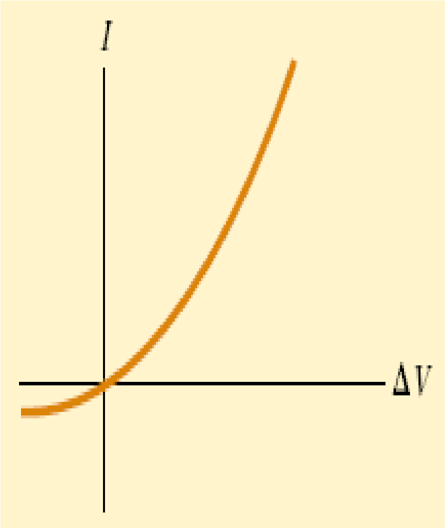
Ohmic conductors form linear graph



Non Ohmic conductors form curved graph

Quick Quiz A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, the resistance of the wire (a) increases (b) decreases (c) remains the same.

Quick Quiz In Figure **below** as the applied voltage increases, the resistance of the diode (a) increases (b) decreases (c) remains the same.



(b). The doubling of the radius causes the area *A* to be four times as large, so Equation $R = \rho \frac{\ell}{A}$ tells us that the resistance decreases.

(b). The slope of the tangent to the graph line at a point is the reciprocal of the resistance at that point. Because the slope is increasing, the resistance is decreasing.

Example 4

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of $2.00 \times 10^{-4}\ \mathrm{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \mathrm{m}$.

Solution we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \mathrm{m}) \left(\frac{0.100\ \mathrm{m}}{2.00 \times 10^{-4}\ \mathrm{m}^2} \right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \mathrm{m}) \left(\frac{0.100\ \mathrm{m}}{2.00 \times 10^{-4}\ \mathrm{m}^2} \right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

As you might guess from the large difference in resistivities, the resistances of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

Example 5

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \mathrm{m})^2 = 3.24 \times 10^{-7}\ \mathrm{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\cdot\mathrm{m}$ (see Table 27.1). Thus, we can find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\mathrm{m}}{3.24 \times 10^{-7}\ \mathrm{m}^2} = \boxed{4.6\ \Omega/\mathrm{m}}$$

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω,

$$I = \frac{\Delta V}{R} = \frac{10\ \mathrm{V}}{4.6\ \Omega} = \boxed{2.2\ \mathrm{A}}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

Worksheet (Questions)

Q1. A small sphere that carries a charge q is whirled in a circle at the end of an insulating string. The angular frequency of rotation is ω. What average current does this rotating charge represent?

Q2. The electron beam emerging from a certain high-energy electron accelerator has a circular cross section of radius 1.00 mm.
(a) The beam current is 8.00 μA. Find the current density in the beam, assuming that it is uniform throughout.
(b) The speed of the electrons is so close to the speed of light that their speed can be taken as $c = 3.00 \times 10^8$ m/s with negligible error. Find the electron density in the beam.
(c) How long does it take for Avogadro's number of electrons to emerge from the accelerator?

Worksheet (Questions)

Q3.Calculate the current density in a gold wire at 20°C, if an electric field of 0.740 V/m exists in the wire.

Q4. A lightbulb has a resistance of 240 ohm when operating with a potential difference of 120 V across it. What is the current in the lightbulb?

Q5.A resistor is constructed of a carbon rod that has a uniform cross-sectional area of 5.00 mm^2. When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of

4.00×10^{-3} A. Find,

(a) the resistance of the rod and (b) the rod's length.

Relation of Resistance & Temperature

- **Variation of ρ with temperature** $\rho = \rho_0[1 + \alpha(T - T_0)]$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the **temperature coefficient of resistivity.** we see that the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

Temperature coefficient of resistivity

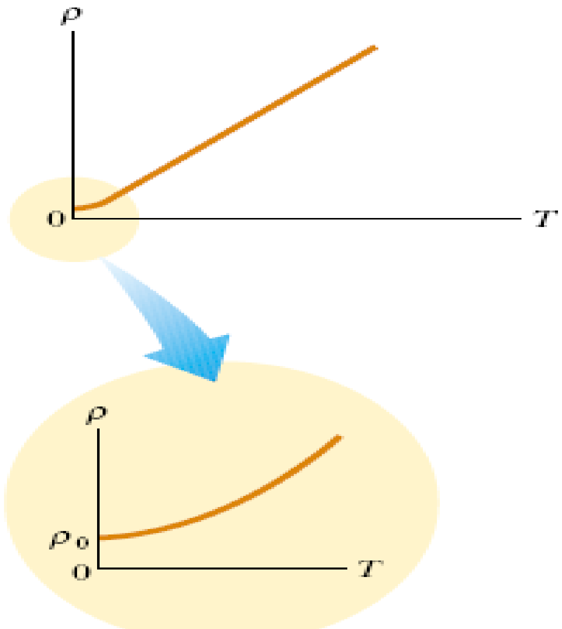
where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

Because **resistance is proportional to resistivity**, we can write the variation of resistance as

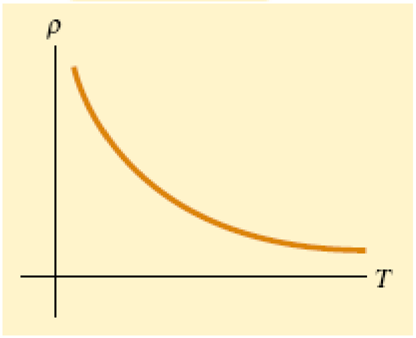
$$R = R_0[1 + \alpha(T - T_0)]$$

Resistivity vs Temperature Graph

Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures and *resistivity increases* with *increasing temperature*. As *T* ***approaches absolute zero*** (inset), the resistivity approaches a **finite value 0** .



Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.

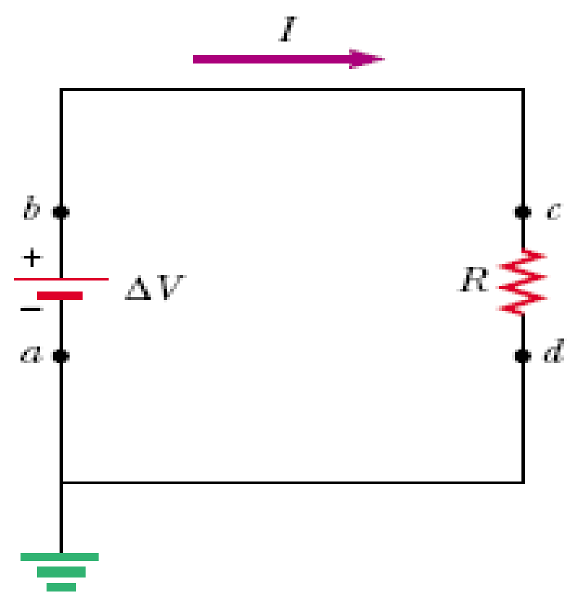


Electrical Energy & Power

If a battery is used to establish an electric current in a conductor, the chemical energy stored in the battery is continuously transformed into kinetic energy of the charge carriers.

The chemical energy stored in the battery is continuously transformed to internal energy associated with the temperature of the conductor.

Consider a simple circuit consisting of a battery whose terminals are connected to a resistor, as shown in Figure:

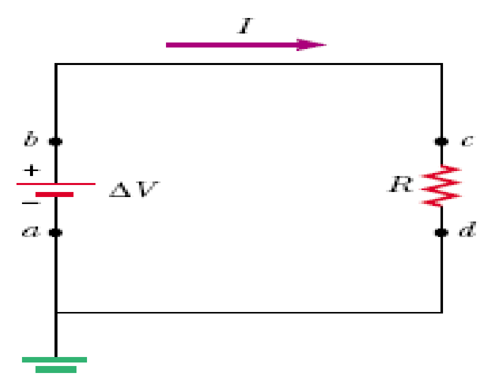


Electrical Energy & Power

Imagine following a positive quantity of charge *Q that is* moving clockwise around the circuit from point *a through the battery and resistor* back to point *a*. As the charge moves from *a to b through the battery, its electric potential energy U increases by an amount ΔV Δ Q (where Δ V is the potential difference between b and a), while the chemical potential energy in the battery decreases by the same* amount.

$$\frac{dU}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\Delta V = I\,\Delta V$$

The charge cannot build up at any point the current is the same everywhere in the circuit.



Electrical Energy & Power

- the rate at which the charge loses energy equals the power delivered to the resistor (which appears as internal energy),

$$\mathcal{P} = I\,\Delta V$$

In this case, the power is supplied to a resistor by a battery.
ΔV = IR for a resistor, we can express the power delivered to the resistor in the alternative form:

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R}$$

When the internal resistance of the battery is neglected, the potential difference between points *a* and *b* in Figure 27.14 is equal to the emf $\mathcal{E}$ of the battery—that is, $\Delta V = V_b - V_a = \mathcal{E}$. This being true, we can state that the current in the circuit is $I = \Delta V/R = \mathcal{E}/R$. Because $\Delta V = \mathcal{E}$, the power supplied by the emf source can be expressed as $\mathcal{P} = I\mathcal{E}$, which equals the power delivered to the resistor, I^2R.

Example 6

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

Solution Solving for ΔT and using the α value for platinum given in Table 27.1, we obtain

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}(°\text{C})^{-1}](50.0\ \Omega)}$$
$$= 137°\text{C}$$

Because $T_0 = 20.0°\text{C}$, we find that T, the temperature of the melting indium sample, is 157°C.

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Example 7

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{8.00\ \Omega} = \boxed{15.0\text{ A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\text{ A})^2(8.00\ \Omega) = 1.80 \times 10^3\text{ W}$$

$$\mathcal{P} = \boxed{1.80\text{ kW}}$$

Worksheet 7(Question)

Q6. What is the required resistance of an immersion heater that increases the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?

Q7. How does the resistance for copper and for silicon change with temperature? Why are the behaviors of these two materials different?

Table 27.1

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] $\alpha[(^\circ C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	