Applications of Newton's Laws of Motion

Muhammad Adeel

The Runaway Car

Suppose a car is released from rest at the top of the incline, and the distance from the front edge of the car to the bottom is *d*. *How long does it take the front edge to reach the bottom, and what is its speed just as it gets there?*

Because a_x = constant,

we can apply

$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$,

to analyze the car's motion

replace displacement by $x_f - x_i = d$

Put, $v_{xi} = 0$, we get

$$d = \frac{1}{2}a_x t^2$$

(1) $\sum F_x = mg\sin\theta = ma_x$

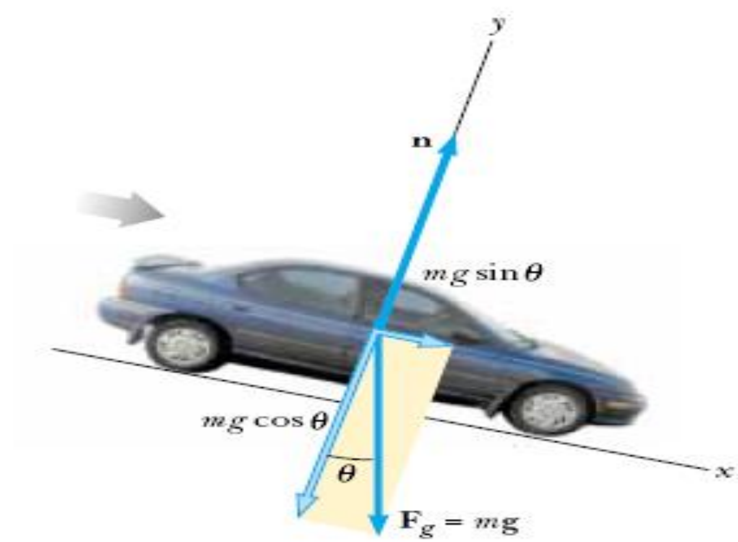
(2) $\sum F_y = n - mg\cos\theta = 0$

Solving (1) for a_x $\boxed{a_x = g\sin\theta}$

put the value of a_x in $d = \frac{1}{2}a_x t^2$ and find time, as follows

$$t = \sqrt{\frac{2d}{a_x}}$$

$$t = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$



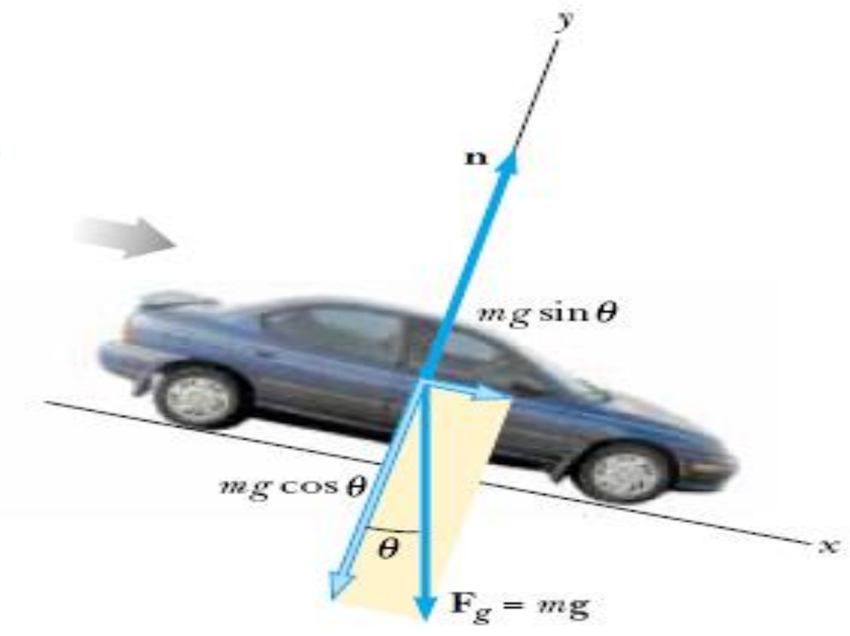
Calculation for Final Velocity

Using $v_{xf}^{2} = v_{xi}^{2} + 2a_x(x_f - x_i)$

with $v_{xi} = 0, \quad x_f - x_i = d$

$$v_{xf}^{2} = 2a_x d$$

$$v_{xf} = \boxed{\sqrt{2gd\sin\theta}}$$



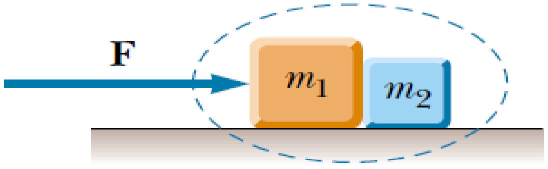
Conclusion

We see that the time t needed to reach the bottom and the speed v_{xf}, are independent of the car's mass.

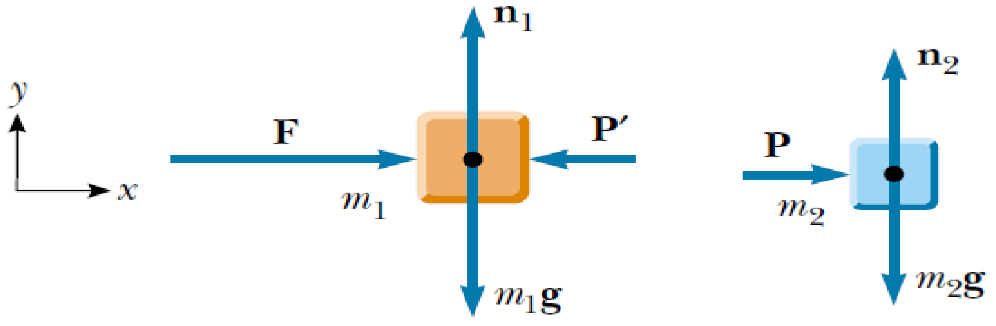
One Block Pushes Another

Two blocks of masses m_1 and m_2 are placed in contact with each other on a frictionless horizontal surface. A constant horizontal force $\mathbf{F}$ is applied to the block of mass m_1. (a) Determine the magnitude of the acceleration of the two-block system.

(b) Determine the magnitude of the contact force between the two blocks.



(a)



(b) (c)

(a) Determine the magnitude of the acceleration of the two-block system.

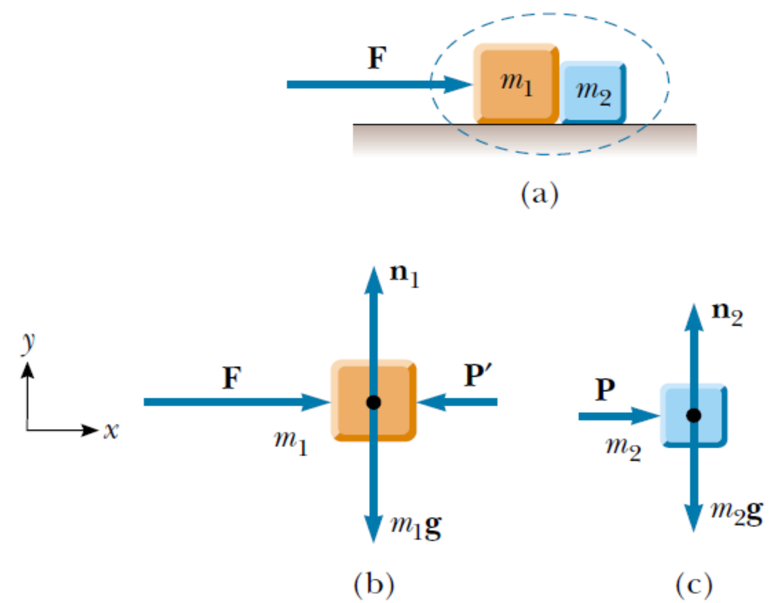
Solution

we know that both blocks must experience the same acceleration because they remain in contact with each other.

F is the only external horizontal force acting on the system (the two blocks), we have

$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$

(1) $$\boxed{a_x = \frac{F}{m_1 + m_2}}$$



(b) Determine the magnitude of the contact force between the two blocks.

To solve this part of the problem, we must treat each block separately with its own free-body diagram, We denote the contact force by **P**.

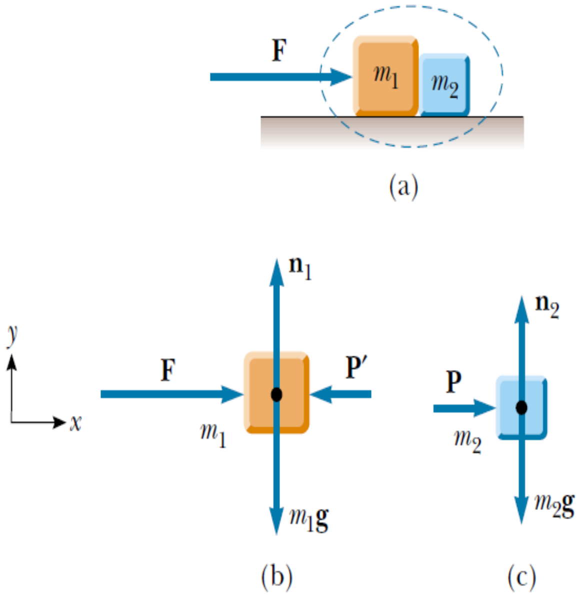
From Figure c, we see that the only horizontal force acting on block 2 is the contact force **P** (the force exerted by block 1 on block 2), which is directed to the right. Applying Newton's second law to block 2 gives

$$(2) \qquad \sum F_x = P = m_2 a_x$$

Substituting into (2) the value of a_x given by (1), we obtain

$$(3) \qquad \boxed{P = m_2 a_x = \left(\frac{m_2}{m_1 + m_2}\right) F}$$

$$P = m_2 a_x = \left(\frac{m_2}{m_1 + m_2}\right) F$$



From this result, we see that the contact force **P** exerted by block 1 on block 2 is *less* than the applied force **F**. This is consistent with the fact that the force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

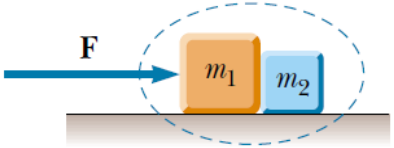
It is instructive to check this expression for P by considering the forces acting on block 1, shown in Figure b. The horizontal forces acting on this block are the applied force **F** to the right and the contact force **P**′ to the left (the force exerted by block 2 on block 1). From Newton's third law, **P**′ is the reaction to **P**, so that $|\mathbf{P}'| = |\mathbf{P}|$. Applying Newton's second law to block 1 produces

$$(4) \qquad \sum F_x = F - P' = F - P = m_1 a_x$$

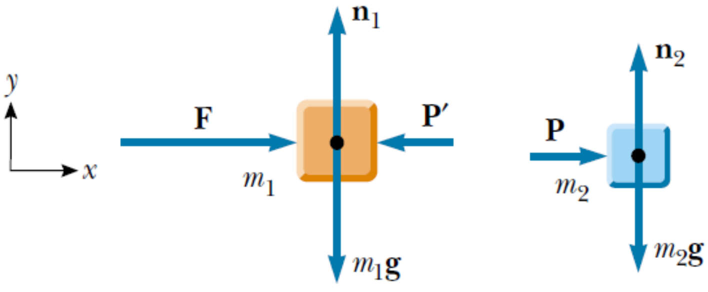
$$(4) \qquad \sum F_x = F - P' = F - P = m_1 a_x$$

Substituting into (4) the value of a_x from (1), we obtain

$$P = F - m_1 a_x = F - \frac{m_1 F}{m_1 + m_2} = \left(\frac{m_2}{m_1 + m_2}\right) F$$



(a)



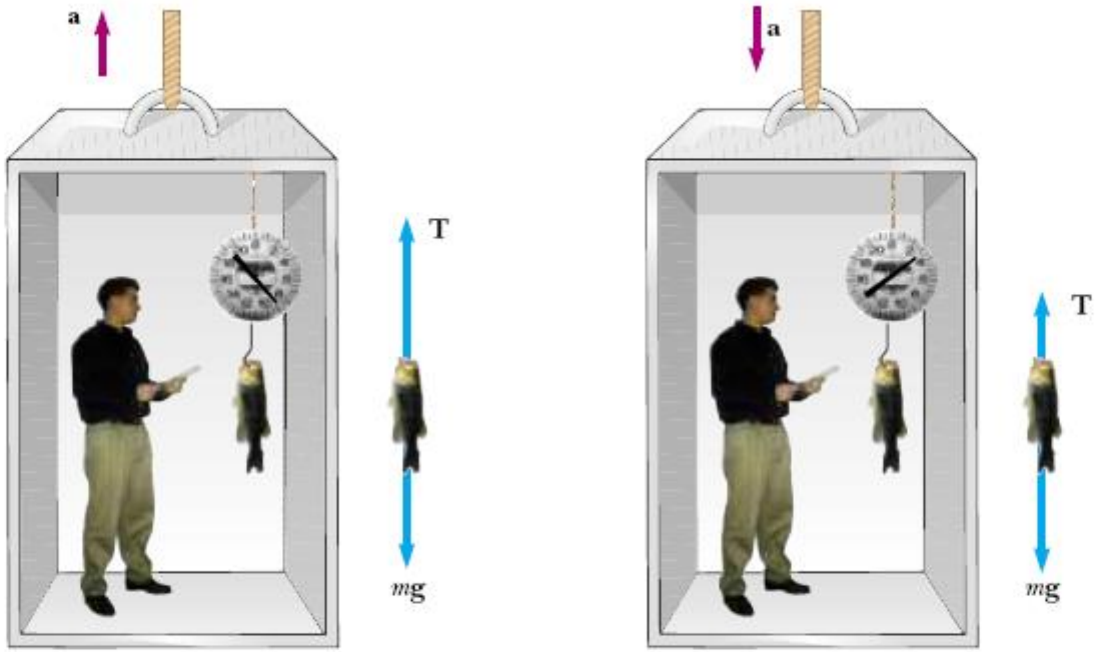
(b) (c)

Task 1

Exercise If $m_1 = 4.00$ kg, $m_2 = 3.00$ kg, and $F = 9.00$ N, find the magnitude of the acceleration of the system and the magnitude of the contact force.

Answer $a_x = 1.29\ \mathrm{m/s^2}$; $P = 3.86$ N.

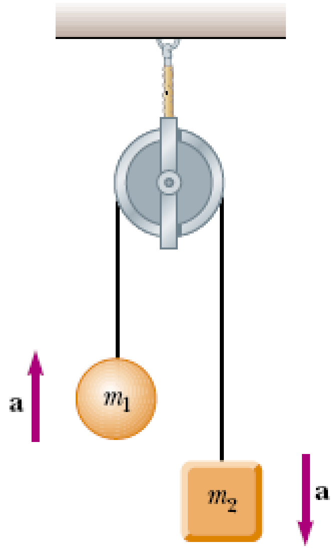
Weighing a fish in an Elevator

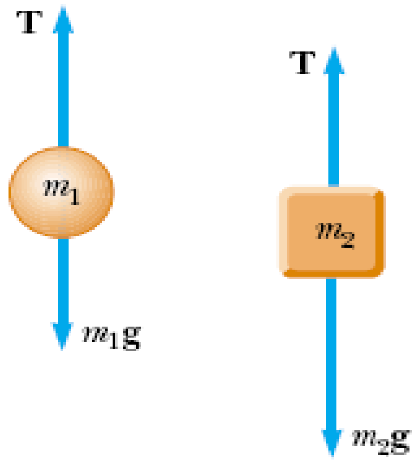


$$\sum F_y = T - mg = ma_y$$

$$T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right)$$

The Atwood Machine





$$\sum F_y = T - m_1 g = m_1 a_y$$

$$\sum F_y = m_2 g - T = m_2 a_y$$

$$a_y = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$