Electric Flux and Gauss's Law

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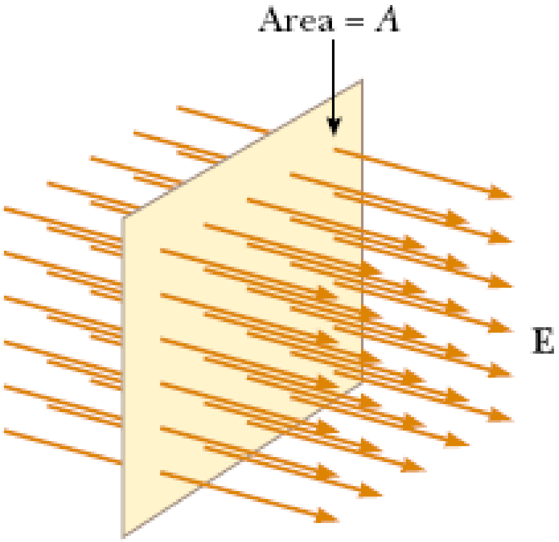
Electric Flux

This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the electric flux φ_E .

$$\Phi_E = EA$$

From the SI units of E and A, we see that Φ_E has units of newton-meters squared per coulomb $(\mathrm{N \cdot m^2/C})$. **Electric flux is proportional to the number of electric field lines penetrating some surface.**



Electric Flux through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of $+1.00\ \mu C$ at its center?

Solution The magnitude of the electric field 1.00 m from this charge is found using Equation 23.9:

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{1.00 \times 10^{-6}\ \mathrm{C}}{(1.00\ \mathrm{m})^2}$$

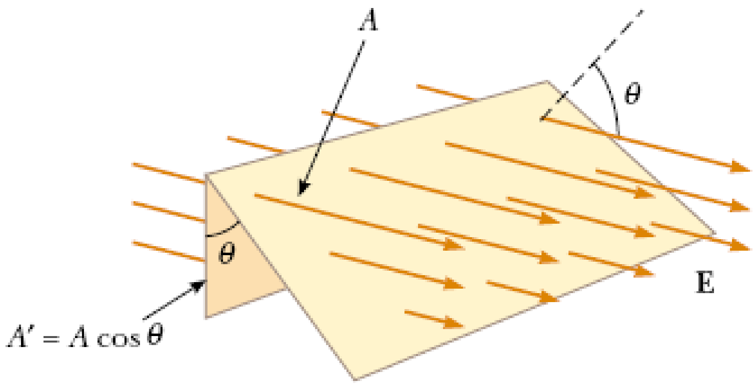
$$= 8.99 \times 10^3\ \mathrm{N/C}$$

The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\ \mathrm{m^2}$) is thus

$$\Phi_E = EA = (8.99 \times 10^3\ \mathrm{N/C})(12.6\ \mathrm{m^2})$$

$$= \boxed{1.13 \times 10^5\ \mathrm{N \cdot m^2/C}}$$

When the surface is not perpendicular to the field



$$\Phi_E = EA' = EA\cos\theta$$

Flux is maximum when surface is perpendicular to the field i.e. θ=0, and Flux is zero when surface is parallel to the field i.e. θ=90˚.

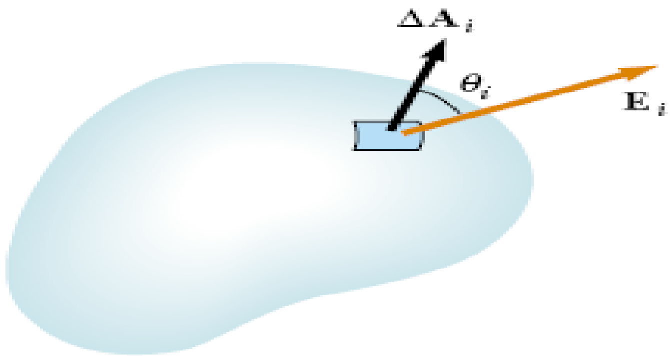
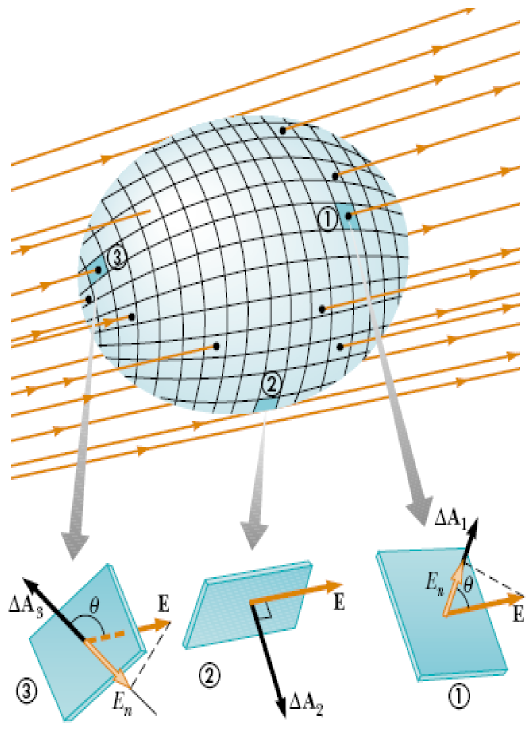


Figure 24.3 A small element of surface area ΔA_i. The electric field makes an angle θ_i with the vector $\Delta\mathbf{A}_i$, defined as being normal to the surface element, and the flux through the element is equal to $E_i\,\Delta A_i\cos\theta_i$.

$$\Phi_E = \lim_{\Delta A_i\to 0}\sum \mathbf{E}_i\cdot\Delta\mathbf{A}_i = \int_{\text{surface}} \mathbf{E}\cdot d\mathbf{A}$$

Flux Through a Closed surface

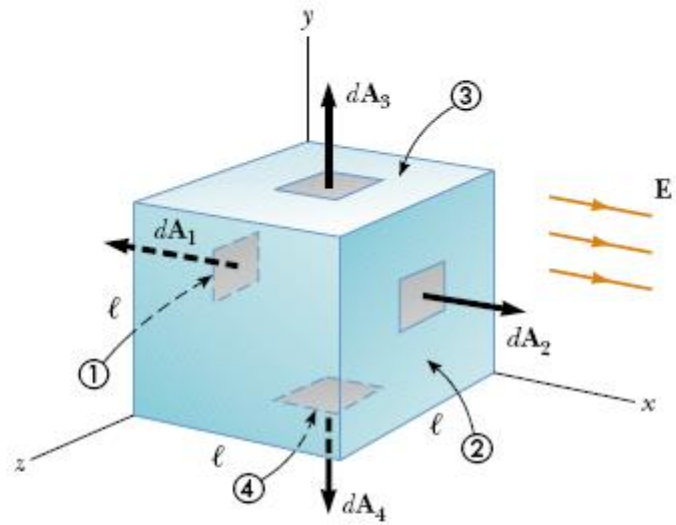
Consider the closed surface in Figure 24.4. The vectors $\Delta \mathbf{A}_i$ point in different directions for the various surface elements, but at each point they are normal to the surface and, by convention, always point outward. At the element labeled ①, the field lines are crossing the surface from the inside to the outside and $\theta < 90°$; hence, the flux $\Delta\Phi_E = \mathbf{E}\cdot\Delta\mathbf{A}_1$ through this element is positive. For element ②, the field lines graze the surface (perpendicular to the vector $\Delta\mathbf{A}_2$); thus, $\theta = 90°$ and the flux is zero. For elements such as ③, where the field lines are crossing the surface from outside to inside, $180° > \theta > 90°$ and the flux is negative because $\cos\theta$ is negative. The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means *the number leaving the surface minus the number entering the surface.* If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative. Using the symbol $\oint$ to represent an integral over a closed surface, we can write the net flux Φ_E through a closed surface as



$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E_n\, dA$$

Flux Through a Cube

Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edge length ℓ, oriented as shown in Figure 24.5.



Solution The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (③, ④, and the unnumbered ones) is zero because **E** is perpendicular to $d\mathbf{A}$ on these faces.

The net flux through faces ① and ② is

$$\Phi_E = \int_1 \mathbf{E}\cdot d\mathbf{A} + \int_2 \mathbf{E}\cdot d\mathbf{A}$$

For face ①, **E** is constant and directed inward but $d\mathbf{A}_1$ is directed outward ($\theta = 180°$); thus, the flux through this face is

$$\int_1 \mathbf{E}\cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA = -E\int_1 dA = -EA = -E\ell^2$$

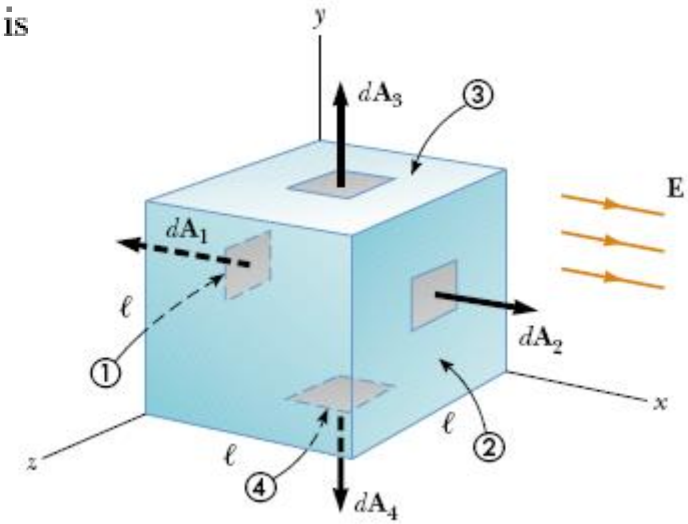
because the area of each face is $A = \ell^2$.

For face ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}_2$ ($\theta = 0°$); hence, the flux through this face is

$$\int_2 \mathbf{E} \cdot d\mathbf{A} = \int_2 E(\cos 0°)\, dA = E \int_2 dA = +EA = E\ell^2$$

Therefore, the net flux over all six faces is

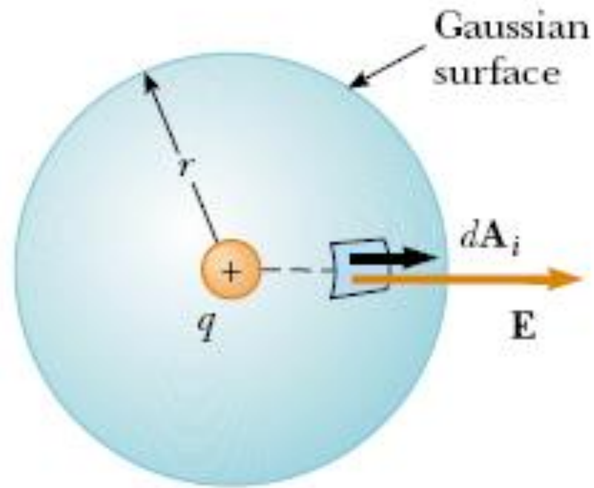
$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$



Gauss's Law

Let us consider a positive point charge q located at the center of a sphere of radius r, as shown in Figure.

$$\mathbf{E} \cdot \Delta \mathbf{A}_i = E \, \Delta A_i$$



the net flux through the Gaussian surface is,

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E \, dA = E \oint dA$$

where we have moved E outside of the integral because, by symmetry, E is constant over the surface and given by $E = k_e q / r^2$. Furthermore, because the surface is spherical, $\oint dA = A = 4\pi r^2$. Hence, the net flux through the gaussian surface is

$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

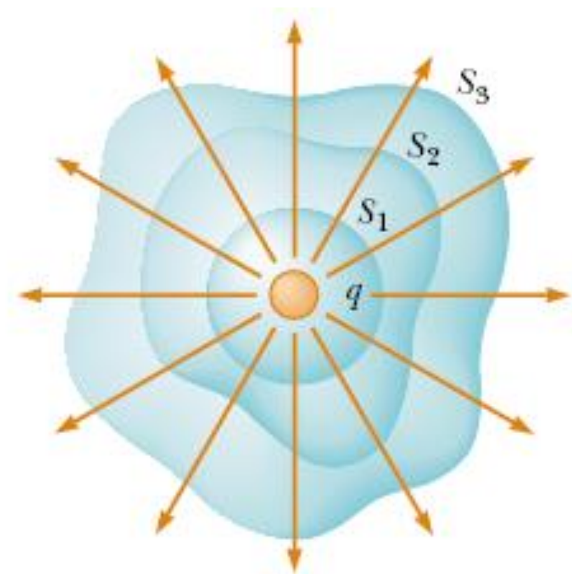
$k_e = 1/4\pi\epsilon_0$, we can write this equation in the form

$$\Phi_E = \frac{q}{\epsilon_0}$$

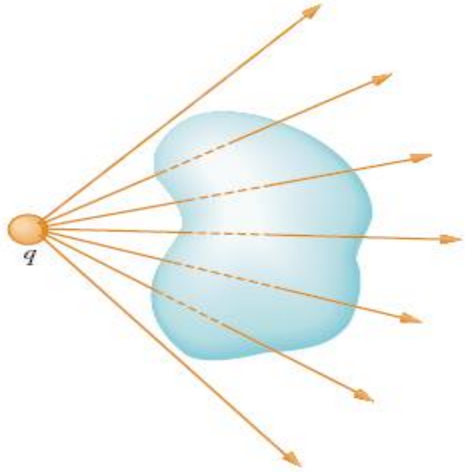
Note from above equation that the net flux through the spherical surface is proportional to he charge inside. The flux is independent of the radius r because the area of the spherical surface is proportional to r^2, whereas the electric field is proportional to $1/r^2$. Thus, in the product of area and electric field, the dependence on r cancels.

Now consider several closed surfaces surrounding a charge q,

Figure shows that the number of lines through S_1 *is equal to* the number of lines through the non spherical surfaces S_2 *and* S_3. *Therefore, we* conclude that the net flux through any closed surface surrounding a point charge q is given by and q/ϵ_0 is independent of the shape of that surface.



Now consider a point charge located outside a closed surface of arbitrary shape, as shown in Figure,



the net electric flux through a closed surface that surrounds no charge is zero.

Let us extend these arguments to two generalized cases: (1) that of many point charges and (2) that of a continuous distribution of charge. We once again use the superposition principle, which states that **the electric field due to many charges is the vector sum of the electric fields produced by the individual charges.** Therefore, we can express the flux through any closed surface as

$$\oint \mathbf{E} \cdot d\mathbf{A} = \oint (\mathbf{E}_1 + \mathbf{E}_2 + \cdots) \cdot d\mathbf{A}$$

Gauss's law, which is a generalization of what we have just described, states that the net flux through *any* closed surface is

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

Flux Due to a Point Charge

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if

(A) the charge is tripled,

(A) The flux through the surface is tripled because flux is proportional to the amount of charge inside the surface.

(B) the radius of the sphere is doubled,

(B) The flux does not change because all electric field lines from the charge pass through the sphere, regardless of its radius.

(C) the surface is changed to a cube,

(C) The flux does not change when the shape of the gaussian surface changes because all electric field lines from the charge pass through the surface, regardless of its shape.