National University of Computer & Emerging Sciences, Karachi
Spring-2020 CS-Department
Final Examination

29th June 2020, 9:00 am – 12:30 pm

Course Code: MT 224	Course Name: Differential Equations (Cal II)
Instructor Name /Ms. Fareeha Sultan, Ms. Amber Shaikh, and Ms. Javeria Iftikhar	
Student Roll No:	Section No:

Instructions:

- Solve all questions by your own handwriting and then scan and save as pdf.
- Read each question completely before answering it. There are 8 questions and 3 pages.
- All the answers must be solved according to the sequence given in the question paper.

Time: 180 minutes. **Total Marks**: 100

****Important:** Use the last non zero digit of your id, where ever **"a"** is given in the equations.
For example:

- If your id is 19k-2036, use 6 in place of a.
- If your id is 19k-2030, use 3
- If your id is 19k-2300, use 3
- If the id is 19k-2000, use 2

Q1.

a) Use the substitution $u = xy$ and take its derivative to solve the given differential equation for "y" {**HINT:** Use Product Rule for differentiation of 'u'}. [5]

$$y + x\frac{dy}{dx} = e^{x(1+y)}$$

b) A body at a temperature of $50^0\,F$ is placed outdoors where the temperature is 100^0F. If after 5 minutes the temperature of the body is 60^0F, find [5]

i. How long it will take the body to reach a temperature of 75^0F and
ii. The temperature of the body after 20 minutes.

Q2.

a) Solve:

$$y' - \frac{3}{x}y = x^4 y^{1/3} \quad ; \quad y(1) = 0$$ [6]

b) Solve:

$$(x\ Cosx)dx - (1 - 6y^5)dy = 0$$ [4]

Q3.

a) Find Laplace transform of

$f(t) = e^{-t}Sin(\boldsymbol{a}t + t)$ [8]

b) Find Inverse Laplace Transform of

$\frac{6s+3}{s^4+4s^2+3}$ [7]

Q4.

a) Solve the given differential equation by method of undetermined coefficients. [10]

$4y''' - 4y'' - 3y' = 2t + 5 - e^{\boldsymbol{a}t},$
$y(0) = y'(0) = y''(0) = 0$

b) Solve the given differential equations by Variation of parameters method. [10]

$$y'' + 4y' + 3y = \frac{9x^2}{e^{\boldsymbol{a}x}}$$

Q5.

a) Solve the following differential equation by any method. [10]

$x^2y'' - 5xy' + 8y = x^6 + x^{\boldsymbol{a}},\ y\left(\frac{1}{2}\right) = 0, y'\left(\frac{1}{2}\right) = 0;$

"a" cannot be equal to 2. Please use a= 3 if you are getting a=2 only for this question

b) Find a second solution y_2 of given differential equation by formula derived from reduction of order method. [5]

$3x^2y'' + 6xy' + y = 0; \qquad y_1 = x^{-1/2}\, Cos(\frac{\sqrt{3}}{6} lnx)$

Q6. Use sigma notation to write the Taylor series. [5]

$f(x) = e^{\boldsymbol{a}x}; \quad x_0 = ln3$

Q7.

a) Find all the possible second order partial derivatives of $f(x, y)$ at $P(\boldsymbol{a},\ 1)$. [5]

$f(x, y) = x^3\, e^y + y^3 cos\sqrt{x}$

b) Let $r = \boldsymbol{a}x^2y + y^2 + yz^2,\quad x = \rho sin\varphi cos\theta,\ \ y = \rho sin\varphi sin\theta,\ \ z = \rho cos\varphi,$ use appropriate chain rule to find $\frac{\partial r}{\partial \rho}, \frac{\partial r}{\partial \theta},\ and\ \frac{\partial r}{\partial \varphi}$. [6]

Q8.

a) Locate all relative maxima, minima, and saddle points, if any. [7]

$f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

b) Evaluate the iterated integrals. [7]

$\int_{a}^{3}\int_{0}^{2} axye^{x^2y}dx\, dy$

*****GOOD LUCK*****