CS302: Design and Analysis of Algorithms
Assignment 01 (Fall 2020)

Due Date: 5th Oct 2020 **Total Marks: 100**

- Write in your own words what is meant by design and analysis of algorithm? What are the key properties of algorithms? Briefly explain [5 Points]
- Show the steps insertion sort uses to sort the following list of integers in the descending order (from the highest to the lowest / biggest to the smallest):

  $$11, 6, 4, 15, 0, 2, 25, 16$$

  Show the value of the key variable, k, at each step. Explain briefly why Time complexity of insertion sort is $O(n^2)$. Use Loop invariant to show its correctness [5 Points]
- Show the steps merge sort uses to sort the following list of integers in the descending order (from the highest to the lowest / biggest to the smallest):

  $$11, 6, 4, 15, 0, 2, 25, 16$$

  . Using Master Theorem, show that the time complexity of merge sort is $O(nlogn)$. Use Loop invariant to show its correctness [5 Points]
- Repeat above for Quick Sort. Use Loop invariant to show its correctness [5 Points]
- Prove $3n^2 - 5n + 6 = O(n^2)$. Determine the values of constant c and n_0 [5 Points]
- Prove $5n^3 log_2 n + 8n^2 = O(n^3 log_2 n)$. Determine the values of constant c and n_0 [5 Points]
- Watch the video lecture on Big O, Big Ω and Big Θ notation from http://www.youtube.com/watch?v=6Ol2JbwoJp0 Write the summary of the lecture in your words [10 Points]
- Use Master Theorem, to calculate the time complexity of the following [15 Points]
  1. $T(n) = 6T(n/2) + n^3$
  2. $T(n) = 9T(n/2) + n^2$
  3. $T(n) = 4T(n/2) + 1$
- Use Iteration Method, to calculate the time complexity of the following [15 Points]

1. $T(n) = 4T(n/2) + n$, (T(1) = 1)
2. $T(n) = 1 + T(n/2)$, (T(1) = 0)

- Given an array A of n numbers, suggest an $O(n)$ expected time algorithm to determine whether there is a number in A that appears more than $n/2$ times. Show its correctness using Loop Invariant. [10 Points]

- Solve 1.2.1, 1.2.2, 1.2.3, 1.1, 2.1.1, 2.1.2, 2.1.3, 2.1.4, 2.2.1, 2.2.2, 2.2.3, 2.2.4, 2.3.1, 2.3.2, 2.3.4, 2.3.5, 2.3.6, 2.3.7, 2.1, 2.4 from book [For Practise Only. Not necessary to be submitted. But some questions may appear in MidTerm.]

- For each of the following questions, indicate whether it is T (True) or F (False) and justify using some examples e.g. assuming a function? [20 Points]

  1. For all positive f(n), f(n) + o(f(n)) = Θ(f(n)).
  2. For all positive f(n), g(n) and h(n), if f(n) = O(g(n)) and f(n) = Ω(h(n)), then g(n) + h(n) = Ω(f(n)).
  3. If f(n)= O(g(n)) and f(n)= Ω(g(n)), then we have $(f(n))^2 = \Theta((g(n))^2)$
  4. If f(n) = O(g(n)) and f(n) = Ω (g(n)), then we have f(n) = g(n)