# LAB 3
# NON LINEAR EQUATIONS

## BISECTION METHOD

### Objective :

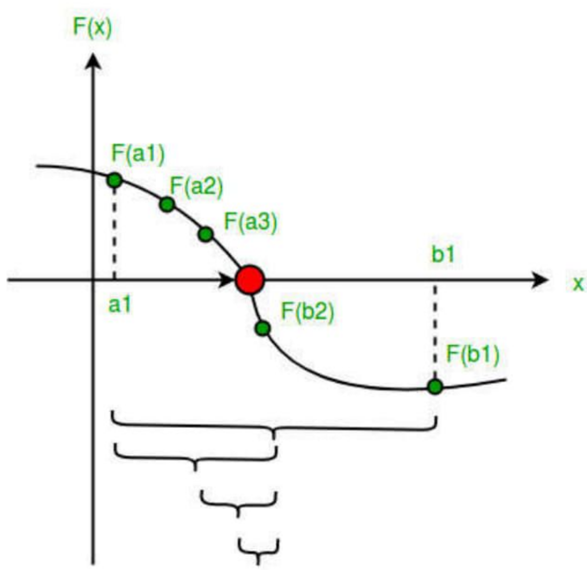
Write an algorithm for solution of nonlinear equations by bisection method. Implement this algorithm by PYTHON coding and fill the following observation table.

### Introduction :

Suppose f is a continuous function defined on the interval [a, b], with f(a) and f(b) of opposite sign. By the intermediate value theorem, there exists p in (a, b) with *f(p) = 0*. Although the procedure will work for the case when f(a)and f(b) have opposite signs and there is more than one root in the interval (a, b), we assume for simplicity that the root in this interval is unique .The method calls for a repeated halving of subintervals of (a, b) and at each step, locating the half containing p.

To begin, set a1 = a and b1 = b, and let p1 be the midpoint of [a, b]; that is,

p1 = 1/2 (a1+b1)



## Theorem :

An equation $f(x)=0$, where $f(x)$ is a real continuous function, has at least one root between $x_\ell$ and $x_u$ if $f(x_\ell)f(x_u)<0$ (See Figure 1).

Note that if $f(x_\ell)f(x_u)>0$, there may or may not be any root between $x_\ell$ and $x_u$ (Figures 2 and 3). If $f(x_\ell)f(x_u)<0$, then there may be more than one root between $x_\ell$ and $x_u$ (Figure 4). So the theorem only guarantees one root between $x_\ell$ and $x_u$.

## Algorithm for the bisection method

The steps to apply the bisection method to find the root of the equation $f(x)=0$ are

1. Choose $x_\ell$ and $x_u$ as two guesses for the root such that $f(x_\ell)f(x_u)<0$, or in other words, $f(x)$ changes sign between $x_\ell$ and $x_u$.
2. Estimate the root, $x_m$, of the equation $f(x)=0$ as the mid-point between $x_\ell$ and $x_u$ as

   $$x_m = \frac{x_\ell + x_u}{2}$$

3. Now check the following
   a) If $f(x_\ell)f(x_m)<0$, then the root lies between $x_\ell$ and $x_m$; then $x_\ell = x_\ell$ and $x_u = x_m$.
   b) If $f(x_\ell)f(x_m)>0$, then the root lies between $x_m$ and $x_u$; then $x_\ell = x_m$ and $x_u = x_u$.
   c) If $f(x_\ell)f(x_m)=0$; then the root is $x_m$. Stop the algorithm if this is true.
4. Find the new estimate of the root

   $$x_m = \frac{x_\ell + x_u}{2}$$

   Find the absolute relative approximate error as

   $$\left|\in_a\right| = \left|\frac{x_m^{\text{new}} - x_m^{\text{old}}}{x_m^{\text{new}}}\right| \times 100$$

   where

   $x_m^{\text{new}}$ = estimated root from present iteration

   $x_m^{\text{old}}$ = estimated root from previous iteration

5. Compare the absolute relative approximate error $|\in_a|$ with the pre-specified relative error tolerance $\in_s$. If $|\in_a| > \in_s$, then go to Step 3, else stop the algorithm. Note one should also check whether the number of iterations is more than the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user about it.

| S# | Functions | Starting Interval | Tolerance | No. of Iterations | Root |
|---|---|---|---|---|---|
| 1 | Cos( $x$ ) – 1.3 $x$ - 0 | - | 0.01 | | |
| | | - | 0.001 | | |
| | | - | 0.0001 | | |
| 2 | $x$ Cos( $x$ ) – 2x$^2$ + 3 $x$ – 1 = 0 | - | 0.01 | | |
| | | - | 0.001 | | |
| | | - | 0.0001 | | |
| 3 | 2 $x$ cos(2 $x$ ) – ( $x$ +1)2 = 0 | - | 0.01 | | |
| | | - | 0.001 | | |
| | | - | 0.0001 | | |

**Problem :** You are working for 'DOWN THE TOILET COMPANY' that makes floats for ABC commodes. The floating ball has a specific gravity of 0.6 and has a radius of 5.5 cm. You are asked to find the depth to which the ball is submerged when floating in water.

The equation that gives the depth $x$ to which the ball is submerged under water is given by

$$x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Use the bisection method of finding roots of equations to find the depth $x$ to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration, and the number of significant digits at least correct at the end of each iteration.

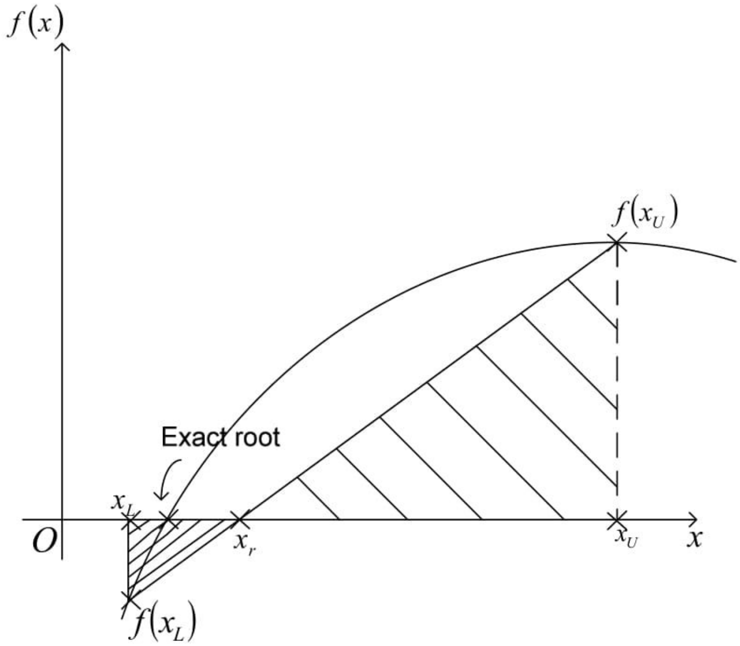
| Iteration | $x_\ell$ | $x_u$ | $x_m$ | $\lvert\in_a\rvert$ % | $f(x_m)$ |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |

# REGULA FALSI

## Objective

Follow the algorithm of the false-position method of solving a nonlinear equation, apply the false-position method to find roots of a nonlinear equation.

## Diagram




Every next Root finding formula in False Position Method

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

Find the root of $f(x) = (x-4)^2(x+2) = 0$, using the initial guesses of $x_L = -2.5$ and $x_U = -1.0$, and a pre-specified tolerance of $\in_s = 0.1\%$.

2. Root of $f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$ for false-position method. using the initial guesses of *xL* –=2.5 and *xU* –=1.0, and a pre-specified tolerance of ∈*s* =0.1% .

## Theorem :

The steps to apply the this to find the root of the equation $f(x)=0$ are as follows.

1. Choose $x_L$ and $x_U$ as two guesses for the root such that $f(x_L)f(x_U)<0$, or in other words, $f(x)$ changes sign between $x_L$ and $x_U$.

2. Estimate the root, $x_r$ of the equation $f(x)=0$ as

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

3. Now check the following

If $f(x_L)f(x_r)<0$, then the root lies between $x_L$ and $x_r$; then $x_L = x_L$ and $x_U = x_r$.

If $f(x_L)f(x_r)>0$, then the root lies between $x_r$ and $x_U$; then $x_L = x_r$ and $x_U = x_U$.

If $f(x_L)f(x_r)=0$, then the root is $x_r$. Stop the algorithm.

4. Find the new estimate of the root

$$x_r = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

Find the absolute relative approximate error as

$$\left|\in_a\right| = \left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| \times 100 \quad \text{where}$$

$x_r^{new}$ = estimated root from present iteration

$x_r^{old}$ = estimated root from previous iteration

5. Compare the absolute relative approximate error $\left|\in_a\right|$ with the pre-specified relative error tolerance $\in_s$. If $\left|\in_a\right| > \in_s$, then go to step 3, else stop the algorithm. Note one should also check whether the number of iterations is more than the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user about it.

Note that the false-position and bisection algorithms are quite similar. The only difference is the formula used to calculate the new estimate of the root $x_r$ as shown in steps #2 and #4!

## Observations

| S# | Functions | Starting Interval | Tolerance | No. of Iterations | Root |
|---|---|---|---|---|---|
| 1 | Cos( $x$ ) – 1.3 $x$ - 0 | - | 0.01 | | |
| | | - | 0.001 | | |
| | | - | 0.0001 | | |
| 2 | $x$ Cos( $x$ ) – $2x^2$ + 3 $x$ – 1 = 0 | - | 0.01 | | |
| | | - | 0.001 | | |
| | | - | 0.0001 | | |
| 3 | 2 $x$ cos(2 $x$ ) – ( $x$ +1)2 = 0 | - | 0.01 | | |
| | | - | 0.001 | | |
| | | - | 0.0001 | | |

## Conclusion