PHAS1240 Reading Week task 2016: Using a weighted fit to calculate e/m

Revision date: November 4, 2016

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1. Weighted least-squares fit

The fit we did in session 5 assumed that each of the data points were equally valid.

Frequently though, we find that experimental data do not have equal uncertainties, and some data points may have significantly larger uncertainties than others.

We can take the relative sizes of the error bars into account by weighting them proportionally. The weight we will give each data point is

$$w_i = \frac{1}{(\Delta y_i)^2}. \tag{1}$$

If these weights are included in the least-squares derivation, we end up with new expressions for the slope m:

$$m = \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\sum_i w_i \sum_i w_i x_i^2 - \left(\sum_i w_i x_i\right)^2} \tag{2}$$

$$= \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\delta}, \tag{3}$$

and the intercept c:

$$c = \frac{\sum_i w_i x_i^2 \sum_i w_i y_i - \sum_i w_i x_i \sum_i w_i x_i y_i}{\delta}; \tag{4}$$

where we have used an abbreviation for the denominator

$$\delta = \sum_i w_i \sum_i w_i x_i^2 - \left(\sum_i w_i x_i\right)^2, \tag{5}$$

The uncertainties in these quantities are given by

$$\Delta m = \sqrt{\frac{\sum_i w_i}{\delta}} \tag{6}$$

and

$$\Delta c = \sqrt{\frac{\sum_i x_i^2 w_i}{\delta}}. \tag{7}$$

2. Task: calculate e/m

Download the data file "eovermdata2016.csv" from the assignment page on Moodle.

This data is from a student experiment to calculate e/m, the charge-to-mass ratio of the electron. This is a version of an experiment first performed in 1897 by J.J. Thomson (see for example http://en.wikipedia.org/wiki/J._J._Thomson).

In this particular student experiment, a cathode ray tube was used to accelerate electrons through a voltage V. The electron beam was then bent into a circular path using a magnetic field $B = 1.28 \pm 0.01 \times 10^{-3}$ Tesla. The students used a ruler to measure the radius r of the circular orbit of the electron beam. This was quite difficult to measure accurately, and so the uncertainties in the readings are quite large. However, for two of the readings, the student taking the measurement was distracted by a seagull flying past the window and misread the ruler. The student's lab partner noticed this and recorded a much larger uncertainty for these data points.

Your task is to use this data to calculate the experimental value of e/m using two methods:

1. an unweighted least-squares fit (as in session 5)
2. a weighted least-squares fit that takes the uncertainties in the readings into account.

2.1 What you need to do

For this task, you need to prepare a **Jupyter Notebook**, clearly explaining what you are doing at each step, both in terms of code and your analysis of the problem. Your final notebook should read as a clear, self-contained document, similar in style to a Laboratory Formal Report, although you can reference equations both from this script and the Laboratory Data Analysis booklet.

- The force experienced by the electrons is given by:

$$F_r = evB = \frac{mv^2}{r}. \tag{8}$$

  The speed v of the electrons is given by:

$$\frac{1}{2}mv^2 = eV. \tag{9}$$

  Rearrange these equations to give an expression involving e/m in the form of a straight line (hint: use $\sqrt{V}$ as the x-axis quantity and r as the y-axis quantity).

- Write code (you can use your corrected code from session 5 as a starting point) to:
  - Load the data from the file

  - Fit an unweighted straight line, outputting the slope, intercept and uncertainties at full precision.
  - Fit a weighted straight line, outputting the slope, intercept and uncertainties at full precision.
  - Plot the data, with errorbars, and both straight line fits on a single suitably labelled and formatted graph.
    * *Hint:* We covered error bars in the script for session 4.
  - You should include the equation for the fit, with the fitted parameters quoted at the appropriate precision, either on the plot itself, in a code cell output, or quoted in a text cell.
  - Use your results to calculate values of e/m for both the weighted and unweighted fits.

- The accepted value of e/m is $(1.75882002 \pm 0.00000001) \times 10^{11}$ C/kg (CODATA Fundamental Constants 2014). Discuss how the results of both your weighted and unweighted calculations compare to this. In particular, which of your two results would you trust more? Which one should you consider more reliable?

2.2 Important things to note

- Your code should import the data from the same directory as the notebook. Do not rename the data file or make any changes to the data file itself.
- The grading of this assignment is anonymised. ***Do not include your name in the filename or anywhere in the assignment itself.*** Instead, please use your student ID (the 8-digit number on your ID card, for most of you this will start with 16) as the filename. Make sure you type the number correctly. If you have an e-sticker from Student Disability Services, please copy and paste the wording (but not your name!) into a clearly labelled text cell at the top of your assignment.
- ***This is an individual assignment.*** You must work on this on your own, with no help from staff or other students. You may copy sections of code from your own submitted session 5 assignment, and you may copy and paste equations from the provided notebook of equations, but everything else in your submission must be *completely* your own work, or have the source explicitly acknowledged and suitably referenced. We will be using several methods to check for any plagiarised or copied assignments.

2.3 Assessment

Your work will be graded using a rubric markscheme based on the following assessment criteria:

- Whether the values you have calculated are correct (50% of available marks);
- The quality of your plot, code comments, and coding style (35% of available marks).

- The quality and coherence of the text commentary, discussion, and the conclusions you have drawn from your results. (15% of available marks).

The grading will be taking into account that you have significantly more time for this task than the in-session assignments, and you should bear this in mind, particularly when writing your text commentary. Remember that this is a formal assignment, and should be similar in style to a Laboratory Formal Report.

References and links

For a more in-depth discussion of the weighted least-squares fit, see Chapter 6 (and in particular section 6.3) of Hughes and Hase "Measurements and their uncertainties", OUP 2010 (e-book available via the UCL library catalogue), or the "Experimental methods and data analysis" booklet from Lab 1.

You will need to include equations in your Jupyter notebook text cells. There is a separate notebook on Moodle that contains the equations from the script already in LaTeX format that you may freely copy and paste from. For any additional equations, you may find one of the following online LaTeX equation editors useful (you can pick the maths symbols you want from a palette and then copy and paste the LaTeX code into your notebook. See the notebook for Session 3 if you need a refresher).

http://www.sciweavers.org/free-online-latex-equation-editor

http://www.codecogs.com/latex/eqneditor.php