# Quantum Mechanics Notes

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## 1 Stern Gerlach Experiment

Note that stern gerlach experiment is best way to get the quantum mechanical Intuition of things. The apparatus includes a silver beam , a homogeneous magnetic field source, and a plate to see that. note that silver has one unpaired electron hence it would have some , magnetic moment in out classical land we might expect it to be randomly distributed in all direction but when we do the following experiment we observer that silver beam spilts in row halfs with equal probability instead of spliting in continuous beam which we would have expected if all the atoms were arranged in random direction, this shows that atom were oriented in either$+\hat{z}$ or $-\hat{z}$ direction with equal probability.

but we if we do sequential stern gerlach experiment with first apparatus in $\hat{z}$ direction and 2nd in $\hat{x}$ note that we observe that after passing silver thorugh first apparatus, the $-\hat{z}$ component got removed from ray (we know if cuz if we pass that ray through another similer apparatus in same direction we would get all atoms in $\hat{z}$ direction) so well what do you think after passing it thorugh $\hat{z}$ direction the info bout $-\hat{z}$ is lost?? you cannot be more than wrong!! if we pass resulting ray thorugh a new magnetic field but now in $\hat{x}$ direction then we would expect equal spilts and that what we would get, so far so good. but what if we again pass it through apparatus in $\hat{z}$ direction??, we get ray in bith direction!!, so the info about orientation in $\hat{z}$ was destroyed by doing this experiment in x direction!, Hence at the same time you cant say that if a ray is polarised in x direction or z direction, even if you could do it for oen at a time!.

now suppose that we represent that state of spin up particle in z direction as $|S_z;+\rangle$ defining other analougously.but note that in such a way we conjecture that we can write the x states as

$$|S_x;+\rangle = \frac{1}{\sqrt{2}}|S_z;+\rangle - \frac{1}{\sqrt{2}}|S_z;-\rangle$$

note that now it looks like we can write the states of spin as some basis of a vector space, but what about $S_y$?

if we were to write $|S_y,+\rangle$ purely in terms of $S_z$ states then we won't be able to do it since these $S_y$ states should diffrent from $S_x$, and also in equal probability hence we need something more than real numbers, for out rescue we have complex numbers!!

we can do the following

$$|S_y;+\rangle = \frac{1}{\sqrt{2}}|S_z;+\rangle + \frac{i}{\sqrt{2}}|S_z;-\rangle$$

$$|S_y;-\rangle = \frac{1}{\sqrt{2}}|S_z;+\rangle - \frac{i}{\sqrt{2}}|S_z;-\rangle$$

now we can say that both the states describe our required state equally while being diffrent from the $|S_x;\pm\rangle$

# 2 Bra, Kets , and Operators

## 2.1 Bra and Kets

After discussing why We need Complex vector spaces for our quantum mechanical models we move onto bra-ket notation. Note that set of all posible states of out system is known as ket space, suppose we had out spin system, then all the states form a ket space a single state is called ket, and we can write it as $|\psi\rangle$, for any state $\psi$, but we can also write it as $\langle\psi|$, which is also called bra. now bra is same as ket but just conjugated. we use this lingo because its elegant and would shine when we introduce Operators. for now they are same as inner products hence suppose $\langle psi|\psi\rangle isanumber$ now in stict mathematical sense bra space is just dual space of ket spaces, so theres nothing to worry.

## 2.2 Operators

Note that Operators act on ket space, if ket vector are state that has all the information about the system, then Operators are the operations that we perfom on that ket to get some certain desired values. but as we know that Operators when acted upon kets give another ket, so wheres out observable quantity?? note that operator defines what are called its eigenvalue, hence those are our observables, more formally If A is an Operator and $|\psi\rangle$ is a ket, then $A|\psi\rangle = |\psi'\rangle$ is a ket. but we can decompose $|\psi'\rangle$ into what are called the eigenstates of A. those are kets such that $A|\alpha\rangle = \alpha'|\alpha\rangle$ for some number $\alpha' \in R$. hence we can define eigenvalues of an operator as the numbers that we get when we act upon it.

now since we are so well versed with linear algebra language we wont spend much time in doing low-ly work of understanding operators (/s). Note that Operator will always come before a ket vector i.e $A|\alpha\rangle$ and will always come after bra ,i.e $\langle\alpha|A$ because fundamentally ket is a vector, and bra is a linear functional that takes a ket as input and gives a number as output.

also

$$X|\alpha\rangle = \langle\alpha|X^\dagger$$

where $X^\dagger$ is the adjoint of $X$. and if $X^\dagger = X$ then X is called Hermitian operator.

Now note that we have something called outer product suppose that $|\alpha\rangle$ is ket and $\langle\beta|$ is bra, then we can write outer product as $|\alpha\rangle\langle\beta|$ this is just fancy way of saying the following suppose we have 2 kets and 1 bra then

$$|\alpha\rangle\langle\beta|\gamma\rangle = (|\alpha\rangle\langle\beta|)|\gamma\rangle$$

we denote $\langle\alpha|X|\beta\rangle$ as $(\langle\alpha|X)(\langle\beta|)$ hence the following outer product acts as a Operators. Note that

$$\langle\beta|X|\alpha\rangle = \langle\alpha|X^\dagger|\beta\rangle^*$$

# 3 Eigenstates

We Introduce the following Theorem

**Theorem 1.** *The eigenvalues of a Hermitian operator A are real. and eigenkets Corresponding to diffrent eigenvalues are orthogonal.*

Note that

$$|\alpha\rangle = \sum_\beta c_\beta|\beta\rangle$$

note that $|\alpha\rangle\langle\alpha|$ is just the operator that thats takes the component of an vector along $\alpha$ and giving that.

hence following theorem makes sense

$$\sum_{\alpha'} |\alpha'\rangle\langle\alpha'| = I$$

this is called Completeness Theorem. and the specefic operator is called the projection Operator.