PRACTICE 2 SOL

MUM

① $x(t) = \cos(t) + \sin(2t)$

$\omega_0 = 1 \quad X_1 = X_{-1} = \frac{1}{2} \quad X_2 = \frac{1}{2i} \quad X_{-2} = -\frac{1}{2i}$

• $H(\frac{\pi}{4}) = 1+i \quad H(-\frac{\pi}{4}) = 1-i$

$H(\frac{7\pi}{4}) = H(-\frac{\pi}{4} + 2\pi) = 1-i$

② $H(\omega) = \cos(\omega)$

$x(n) = 2 + \cos(\frac{\pi}{2}n + \frac{\pi}{4}) + \sin(\pi n + \frac{3\pi}{2})$

ω	$H(\omega)$	$\|H(\omega)\|$	$\measuredangle H(\omega)$
0	1	1	0
$\frac{\pi}{2}$	0	0	$-$
π	-1	1	$-\pi$

$y(n) = 2 + \sin(\pi n + \frac{3\pi}{2} - \pi)$

③ $H(0) = 0$

$H(\frac{\pi}{2}) = 0 \Rightarrow H(-\frac{\pi}{2}) = 0$

$H(\pi) = 1$

$\tilde{H}(\omega) = (e^{-i\omega} - \underset{-1}{e^{0}})(e^{-i\omega} - \underset{+i}{e^{-i\frac{\pi}{2}}})(e^{-i\omega} - \underset{-i}{e^{i\frac{\pi}{2}}})$

$= (e^{-i\omega} - 1)(e^{-i2\omega} + 1)$

$= e^{-i3\omega} - e^{-i2\omega} + e^{-i\omega} - 1$

$\tilde{H}(\pi) = -4$

$H(\omega) = \frac{1}{4}(1 - e^{-i\omega} + e^{-i2\omega} - e^{-i3\omega})$

$y(n) = \frac{1}{4}(x(n) - x(n-1) + x(n-2) - x(n-3))$

④ $y(n) + y(n-1) = 2x(n) - 5x(n-2)$

$H(\omega) = \frac{2 - 5e^{-i2\omega}}{1 + e^{-i\omega}}$

• $H(\omega) = \frac{1 + e^{-i\omega}}{e^{-i\omega}}$

$x(n) = \cos(\frac{\pi}{2}n + \frac{\pi}{4}) + \sin(\pi n + \frac{3\pi}{2})$

ω	$H(\omega)$		$\|H(\omega)\|$	$\measuredangle H(\omega)$
$\frac{\pi}{2}$	$\frac{1-i}{-i}$	$i+1$	$\sqrt{2}$	$-\frac{7\pi}{4}$
π	0	0	0	

$x(n) = \sqrt{2}\cos(\frac{\pi}{2}n + \frac{\pi}{4} - \frac{7\pi}{4})$

⑤ $x(t) = \begin{cases} \frac{\pi}{a} & -a \le t \le a \\ 0 & \end{cases} \Longleftrightarrow X(\omega) = 2\pi \frac{\sin(a\omega)}{a\omega}$

$x(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 0 & \end{cases}$

$\Rightarrow a = 1$

SCALE BY $\frac{1}{\pi}$

SHIFT BY $e^{-i\omega}$

$X(\omega) = 2e^{-i\omega}\frac{\sin(\omega)}{\omega}$

⑥ $x(n) = e^{i\omega n}$

$y(n) = \sum_{k=-\infty}^{\infty} h(k) e^{i\omega(n-k)}$

$= \underbrace{\sum_{k=-\infty}^{\infty} h(k) e^{-ik\omega}}_{H(\omega)} \underbrace{e^{i\omega n}}_{\text{DTFT}}$

(7) $X_1(\omega) = \delta(\omega-\pi) + \delta(\omega+\pi)$

$\rightarrow$ CTFT^{-1}

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\delta(\omega-\pi)+\delta(\omega+\pi)\right) e^{i\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left(e^{-i\pi t} + e^{i\pi t}\right)$$

$$= \frac{1}{\pi}\cos(\pi t)$$

- $X_2(\omega) = \cos(\omega)$

$\rightarrow$ DTFT^{-1}

$$x(n) = \frac{1}{2\pi}\int_{0}^{2\pi} \cos(\omega) e^{i\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\frac{1}{2}\int_0^{\pi} e^{i\omega} e^{i\omega n}\, d\omega + \frac{1}{2}\int_0^{\pi} e^{-i\omega} e^{i\omega n}\, d\omega \right.$$

E.G. $\Rightarrow e^{i\omega(n-1)} \rightarrow 2\pi\delta(n-1)$

$$= \frac{1}{2}\delta(n+1) + \frac{1}{2}\delta(n-1)$$

(8) $h(n) = \delta(n) + \delta(n-2)$

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-i\omega n}$$

$$H(\omega) = 1 + e^{-i2\omega}$$

(9) IS LIKE (7)

(10) $x(n) = \cos(\frac{\pi}{2} n)$ $\quad \omega_0 = \frac{\pi}{2}$

$N=4$ $\quad x = 1\ 0\ {-1}\ 0$

$X_0 = 1 - 1 = 0$

$X_1 = 1 - e^{-i\frac{2\pi}{2}} = 2$

$X_2 = 1 - e^{-i2\cdot\frac{2\pi}{2}} = 0$

$X_3 = 1 - e^{-i3\cdot\frac{2\pi}{2}} = 2$

(11) PRACTICE THE BLOK DIAGRAMS ON SEPARATE SHEET.