**Assignment 6, Practice questions**
MECHTRON/SFWTENG 3MX3 2021
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# 1 Short

What are the Fourier series coefficients for the sifnal $x(t) = cos(t) + sin2t)$.

For a disctete signal we know that $H(\frac{\pi}{4}) = 1 + i$. What is the value at $H(-\frac{\pi}{4})$ and $H(\frac{3\pi}{4})$

# 2 Frequency Response

Given the frequency response

$$H(\omega) = \cos(\omega)$$

compute the output to the signal $x(n) = 2 + \cos(\frac{\pi}{2}t + \frac{\pi}{4}) + \sin(\pi n + \frac{3\pi}{2})$

# 3 Filter Design

Give the difference equation of a causal, minimum order, discrete, real valued system that removes any signal with the normalized frequency of 0 and $\frac{\pi}{2}$ and passes the signal with the normalized frequency $\pi$ with a gain of 1.

# 4 Frequency Response

- Give the frequency response of the system

$$y(n) + y(n-1) = 2x(n) - 5x(n-2)$$

- Given the frequency response

$$H(\omega) = \frac{1 + e^{-i\omega}}{e^{-i\omega}}$$

compute the output to the signal $x(n) = \cos(\frac{\pi}{2}n + \frac{\pi}{4}) + \sin(\pi n + \frac{3\pi}{2})$

# 5 Fourier Transform

A CTFT (FT) tables states that

$$x(t) = \begin{cases} \frac{\pi}{a} & -a \leq t \leq a \\ 0 & \text{else} \end{cases} \quad \Longleftrightarrow \quad X(\omega) = 2\pi \frac{\sin(a\omega)}{a\omega}$$

Use the table to compute the Fourier transform of

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 2 \\ 0 & \text{else} \end{cases}$$

# 6 Convolution

We use convolution to compute the output of a system given the impulse response $h(n)$. Given the impulse response $h(n)$ compute the output to the signal $x(n) = e^{i\omega n}$. Explain how your result is related to the DTFT.

# 7 Time and Frequency Domain

Given the following two signals in the frequency domain:

$$X_1(\omega) = \delta(\omega - \pi) + \delta(\omega + \pi) \quad X_2(\omega) = \cos(\omega)$$

1. Which one corresponds to a discrete and which to a continuous signal?
2. Compute the signal that corresponds to $X_1(\omega)$.

# 8 DTFT

The Discrete Time Fourier Transform and its inverse are given by:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-in\omega} \quad x(n) = \frac{1}{2\pi}\int_0^{2\pi} X(\omega)e^{i\omega n}d\omega$$

Given is some system by $h(n) = \delta(n) + \delta(n-2)$, compute $H(\omega)$ using the DTFT (you have to use the DTFT and not just guess the solution to get points).

# 9 Fourier Transforms

Compute the signals in the time domain that correspond to the following two frequency domain representations. Note, one signal has to be continuous, the other discrete!

$$\begin{aligned} X_1(\omega) &= \delta(\omega - \pi) + \delta(\omega + \pi) \\ X_2(\omega) &= \cos(\omega) \end{aligned}$$

You can use that $\cos(a) = \frac{1}{2}(e^{ia} + e^{-ia})$ and $\int_0^{2\pi} e^{ik\omega}d\omega = 2\pi$ if $k = 0$ and 0 otherwise. The CTFT and the DTFT are given on other pages!

# 10 FT

$x(n) = cos(\frac{\pi}{2}n)$ and $N = 4$, compute the FT coefficients $X_k$.

# 11 Block Diagrams

Practice your block Diagrams (separate sheet).