Support Vector Machines

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Non-Linearly Separable Data

Data not separate in $\mathbb{R}$

Can we still use SVM?

Yes!
How? Project data to a higher dimensional space.

Projection/Transformation Function

$$\phi : \mathbb{R}^d \rightarrow \mathbb{R}^D$$

where, d = original dimension

D = new dimension

In our example:

$d = 1; D = 2$

Linear SVM:

Maximize

$$L(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j \overline{x_i}.\overline{x_j}$$

such that constriants are satisfied.

$$\downarrow$$

Transformation (ϕ)

$$\downarrow$$

$$L(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j \phi(\overline{x_i}).\phi(\overline{x_j})$$

Steps

1. Compute $\phi(x)$ for each point

$$\phi : \mathbb{R}^d \to \mathbb{R}^D$$

Q. If $D >> d$

Both steps are expensive!

Steps

1. Compute $\phi(x)$ for each point

$$\phi : \mathbb{R}^d \to \mathbb{R}^D$$

2. Computer dot products over $\mathbb{R}^d$ space

Q. If $D >> d$

Both steps are expensive!

Kernel Trick