Next Token Generation & Word Embeddings

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Outline

From Discrete Symbols to Neural Networks

Challenge: How do we feed text into neural networks?

- Neural networks work with numbers, not words

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Our Task: Character-Level Next Token Prediction

Given a sequence like "hello", predict the next character

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Our Task: Character-Level Next Token Prediction

Given a sequence like "hello", predict the next character

- Input: "h", "e", "l", "l" $\rightarrow$ Output: "o"

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Given a sequence like "hello", predict the next character

- Input: "h", "e", "l", "l" $\rightarrow$ Output: "o"
- Learn patterns in character sequences

Pop Quiz: Text Representation

Quick Quiz 1

Why can't we directly feed text into neural networks?

a) Text is too long for neural networks

Answer: b) Neural networks perform mathematical operations requiring numerical inputs!

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a) Text is too long for neural networks
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c) Text doesn't contain useful information

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Building Our Character Vocabulary

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One-Hot Encoding: Each character becomes a 27-dimensional vector

- $'a'$: [1, 0, 0, ..., 0]

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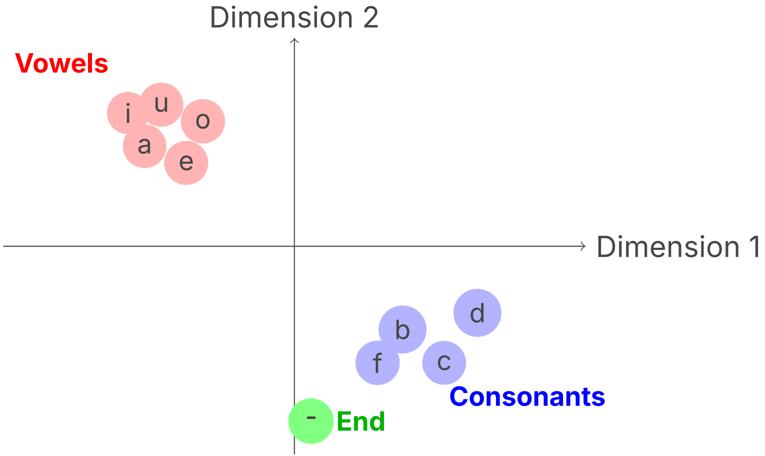
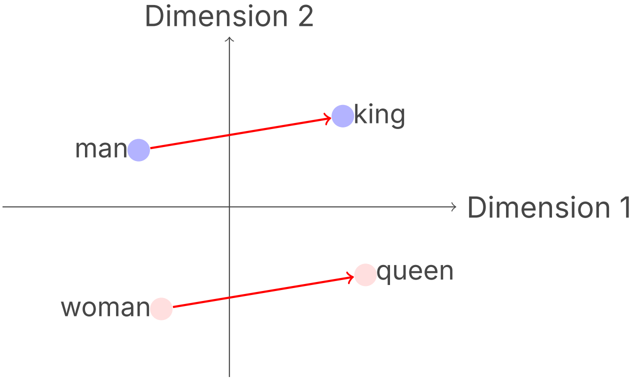
One-Hot Encoding: Each character becomes a 27-dimensional vector

- 'a': [1, 0, 0, ..., 0]
- 'b': [0, 1, 0, ..., 0]
- Very sparse representation!


Dimension 2
Vowels
i
u
o
a
e
Dimension 1
d
b
f
c
Consonants
-
End

Word2Vec Analogy Example

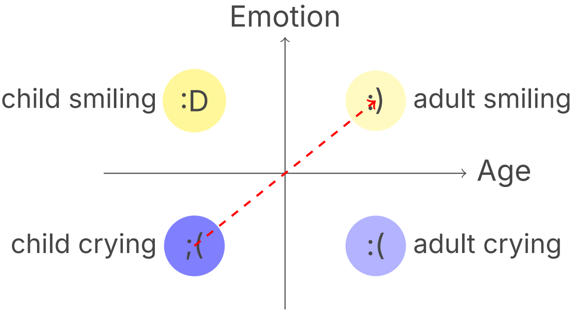
Classic Word2Vec Relationship



Relationship: queen $\approx$ king - man + woman

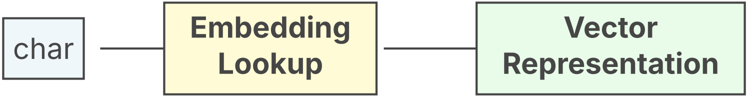
Analogy with Emotions

Emotional Expression Analogy



Relationship: child crying = child smiling + adult crying - adult smiling

Embedding Matrix/Table Concept



Process: Character $\rightarrow$ Lookup in Embedding Table $\rightarrow$ Dense Vector

Embedding Table Structure

27 × K Embedding Matrix

Char	D1	D2	...	DK
a	0.2	-0.1	...	0.8
b	-0.3	0.5	...	-0.2
c	0.1	0.3	...	0.4
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
z	0.7	-0.4	...	0.1
-	0.0	0.9	...	-0.5

Key Point

Each character maps to a K-dimensional vector.

Learnable Parameters

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  - MLP: (context_size × K) → hidden → ... → 27

Example: 2D Embeddings for “abi”

Embedding Matrix (27 × 2)

Input: X = [“a”, “b”, “i”]

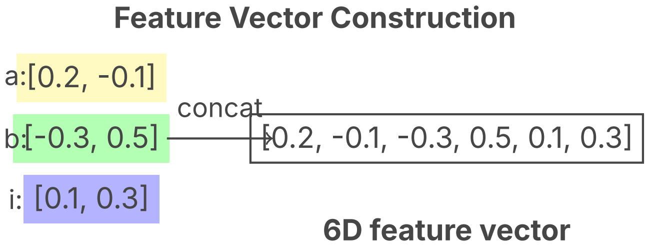
	D1	D2
a	0.2	-0.1
b	-0.3	0.5
…	…	…
i	0.1	0.3
…	…	…
z	0.7	-0.4
-	0.0	0.9

[0.2, -0.1]

[-0.3, 0.5]

[0.1, 0.3]

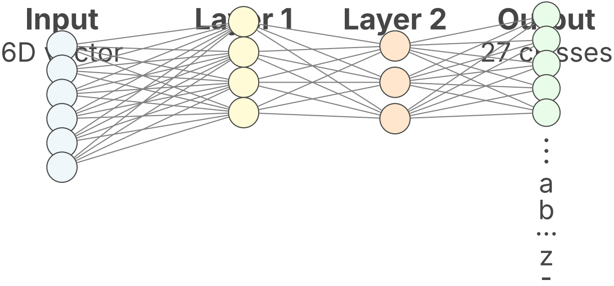
Concatenate the Embeddings



Result

3 chars × 2D embeddings = 6D input to neural network

Multi-Layer Perceptron Architecture



Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

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  4. Repeat for all training examples

Sampling from the Learned Model

Test Input: “abi”

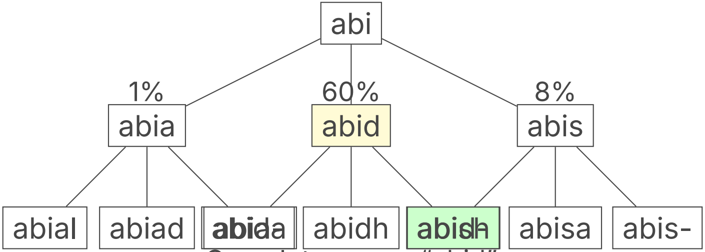
Predicted Probability Distribution

Next Char	Probability	Next Char	Probability
a	0.01	n	0.05
b	0.01	o	0.02
c	0.03	p	0.01
d	**0.60**	q	0.00
e	0.02	r	0.03
f	0.01	s	0.08
...	...	...	...
-	0.05	z	0.01

Most Likely Continuation

“abi” → “abid” (60

Generation Tree Structure



Complete name: “abid”

Recursive Process: Sample next character, append, repeat until end token

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- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

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  - $T \to \infty$: More uniform (random)

Temperature Variations

Context: "abi" → Next character probabilities

Char	T=0.5 (Low)	T=1.0 (Default)	T=2.0 (High)
a	0.001	0.01	0.08
d	**0.95**	**0.60**	**0.25**
s	0.01	0.08	0.12
h	0.005	0.03	0.09
-	0.02	0.05	0.11
others	0.015	0.23	0.35

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  - Transformer architecture instead of MLP
  - Billions of parameters instead of thousands

From Character-Level to ChatGPT

Our Model
Character-level
MLP
27 vocab
∼1K params

Scale up

ChatGPT
Token-level
Transformer
50K+ vocab
∼175B params

Same fundamental principle: Predict the next token!