Decision Trees: From Intuition to Implementation

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Outline

What Are Decision Trees?

- **Intuitive approach:** Make decisions by asking yes/no questions

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Should I play tennis today?

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- If sunny $\rightarrow$ check humidity

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Should I play tennis today?

- If sunny $\rightarrow$ check humidity
- If rainy $\rightarrow$ check wind
- If overcast $\rightarrow$ always play!

Pop Quiz: Decision Tree Basics

Quick Quiz 1

What makes decision trees particularly useful in practice?

a) They always give the highest accuracy

Answer: b) Interpretability is a key advantage of decision trees!

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What makes decision trees particularly useful in practice?

a) They always give the highest accuracy
b) They're easy to interpret and explain to humans
c) They work only for numerical data

Answer: b) Interpretability is a key advantage of decision trees!

Information Gain Intuition

The Core Question: How do we choose the best question to ask?

- Dataset: 9 Yes, 5 No (mixed outcomes)

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Goal

Find the question that **reduces impurity the most** after splitting

Entropy: Measuring Impurity

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Interpretation:

Entropy: Measuring Impurity

Entropy Formula

$$H(X) = -\sum_{i=1}^{k} p(x_i) \log_2 p(x_i)$$

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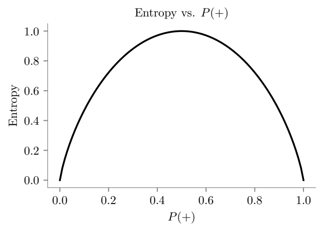

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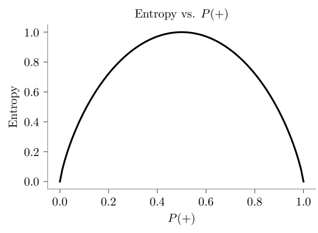

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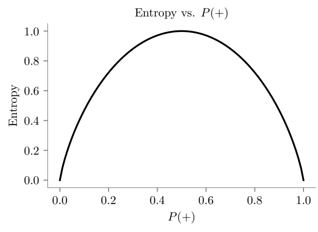

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Quick Quiz 2

For a dataset with 8 positive and 8 negative examples, what is the entropy?

a) $H = 0$ (pure)

Answer: b) $H = -0.5\log_2(0.5) - 0.5\log_2(0.5) = 1$

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a) $H = 0$ (pure)
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c) $H = 0.5$ (moderate impurity)

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Choosing the Root Node

Question: Which feature should we use as the root node?

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$$\text{Gain}(S, A) = H(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} H(S_v)$$

Entropy Calculation Examples

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Outlook	Play
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$H = 0$ (perfect purity!)

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Mixed Subset

Outlook	**Play**
Rain	Yes
Rain	Yes
Rain	No
Rain	Yes
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3 Yes, 2 No

$H = -\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right)$

$= 0.971$ (high impurity)

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Key Insight: We want to create splits that result in low entropy subsets!

Information Gain: Step-by-Step

For Outlook=Sunny subset (2 Yes, 3 No):

- Temp split:

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Winner

Humidity gives the highest information gain for this subset!

Pop Quiz: Information Gain

Quick Quiz 3

Which split would give the highest information gain?

a) Split that creates subsets: (5 Yes, 5 No) and (3 Yes, 2 No)

Answer: b) Pure subsets (entropy = 0) give maximum information gain!

Pop Quiz: Information Gain

Quick Quiz 3

Which split would give the highest information gain?

a) Split that creates subsets: (5 Yes, 5 No) and (3 Yes, 2 No)
b) Split that creates subsets: (8 Yes, 0 No) and (0 Yes, 7 No)

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a) Split that creates subsets: (5 Yes, 5 No) and (3 Yes, 2 No)

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c) Split that creates subsets: (4 Yes, 3 No) and (4 Yes, 4 No)

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Prediction Example

Prediction for $<$High Humidity, Strong Wind, Sunny Outlook, Hot Temp$>$ is ?

Prediction Example

Prediction for $<$High Humidity, Strong Wind, Sunny Outlook, Hot Temp$>$ is ?
No

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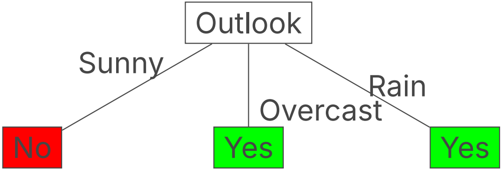

Why Outlook is Good Root?

Answer: B) When Outlook=Overcast, all examples have Play=Yes - This creates a pure subset with entropy=0, maximizing information gain.

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- MSE Reduction = $\mathrm{MSE}(S) - \sum_v \frac{|S_v|}{|S|} \mathrm{MSE}(S_v)$

Regression Splitting Criterion

Answer: C) Mean Squared Error (MSE) Reduction - For regression, we minimize MSE instead of maximizing information gain.

Continuous Features

Answer: B) Use midpoints between consecutive sorted feature values - This ensures we test all meaningful boundaries between different class regions.

Leaf Node Predictions

Answer: C) The mean of target values in that region - Each leaf predicts the average target value of training samples that reach that leaf.

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Stop growing tree before it becomes too complex:

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- **Minimum impurity decrease**: Only split if improvement > threshold

Advantages: Simple, computationally efficient
Disadvantages: May stop too early, miss good splits later

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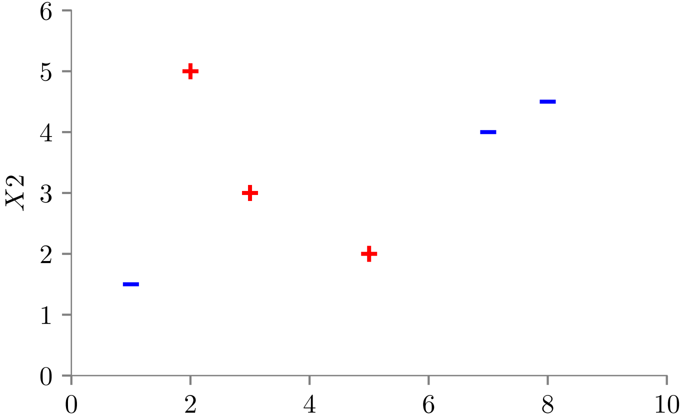
X2
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1
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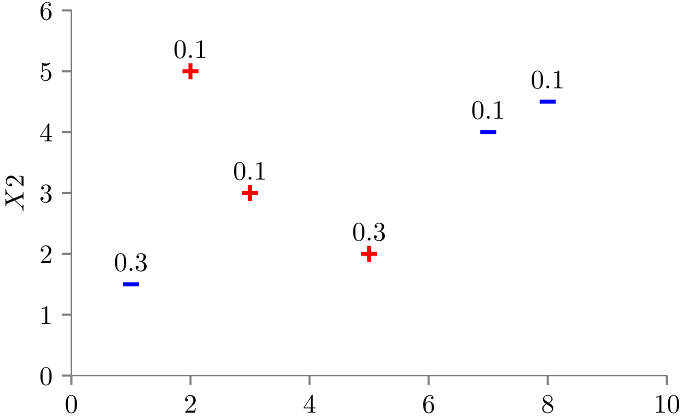
8
10



0.1
0.1
0.1
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X2
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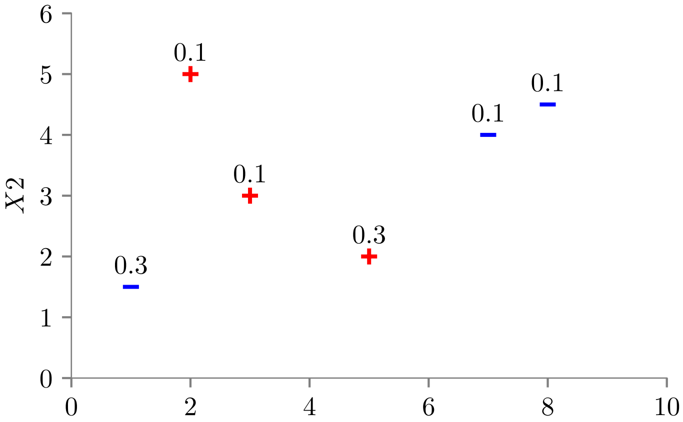
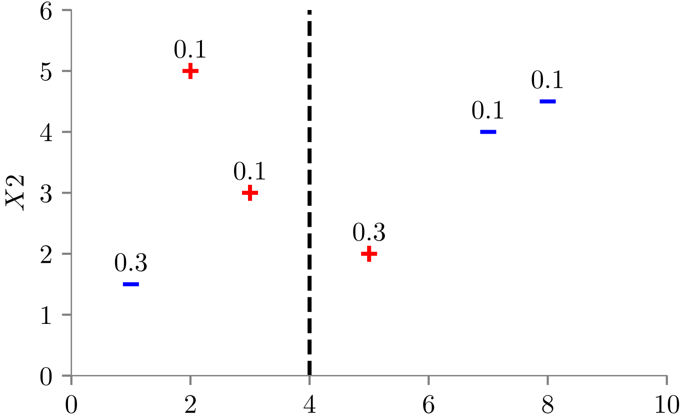
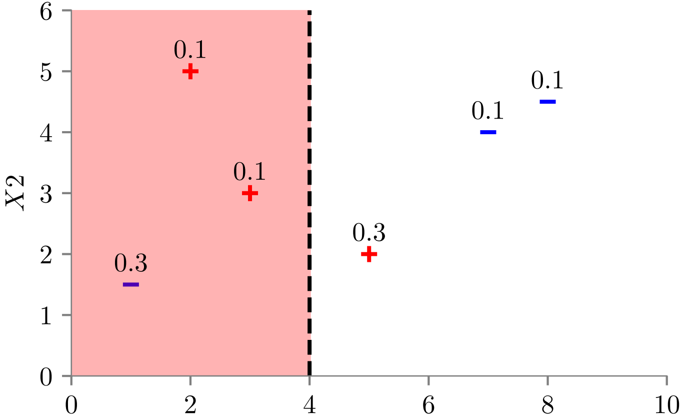


$$\text{Entropy} = -P(+) \log_2 P(+) - P(-) \log_2 P(-)$$

$$P(+) = \frac{0.1 + 0.1 + 0.3}{1} = 0.5, \quad P(-) = \frac{0.3 + 0.1 + 0.1}{1} = 0.5$$



Candidate Line: $X1 = 4(X1^*)$



Entropy of $X1 \leq X1^* = E_{S(X1<X1^*)}$

$$P(+) = \frac{0.1 + 0.1}{0.1 + 0.1 + 0.3} = \frac{2}{5}$$

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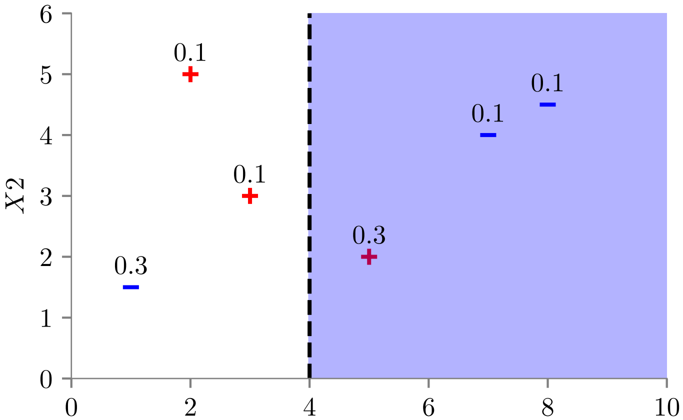

Entropy of $X_1 > X_1^* = E_{S(X_1 > X_1^*)}$

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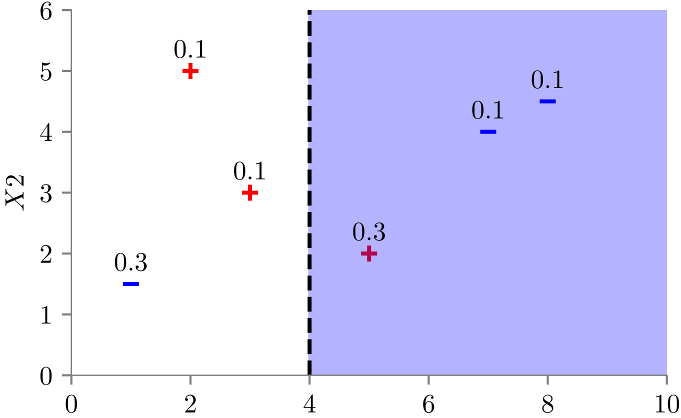

$$\mathsf{IG}(X_1 = X_1^*) = E_S - \frac{0.5}{1} \cdot E_{S(X_1 < X_1^*)} - \frac{0.5}{1} \cdot E_{S(X_1 > X_1^*)}$$