Bias-Variance Tradeoff & Model Complexity

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Outline

Decision Tree Depth: A Story

Observation from Decision Trees:

- As depth increases, training accuracy **improves**

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The Central Question

Why does this happen? How do we choose the right model complexity?

Pop Quiz: Overfitting Intuition

Quick Quiz 1

A decision tree with 1000 levels perfectly classifies all training data but performs poorly on test data. This is likely due to:

a) Underfitting - the model is too simple

Answer: b) The model memorized training specifics instead of learning general patterns!

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c) Perfect generalization

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The Fundamental Decomposition

For any learning algorithm, the expected test error can be decomposed as:

Bias-Variance Decomposition

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- **Variance**: Error due to sensitivity to small changes in training set
- **Noise**: Irreducible error in the data itself

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The Tradeoff

Key insight: Reducing bias typically increases variance, and vice versa!

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Quick Quiz 2

Which scenario represents high variance?

a) A linear model consistently predicts poorly on both training and test sets

Answer: b) High variance means the model is too sensitive to training data changes!

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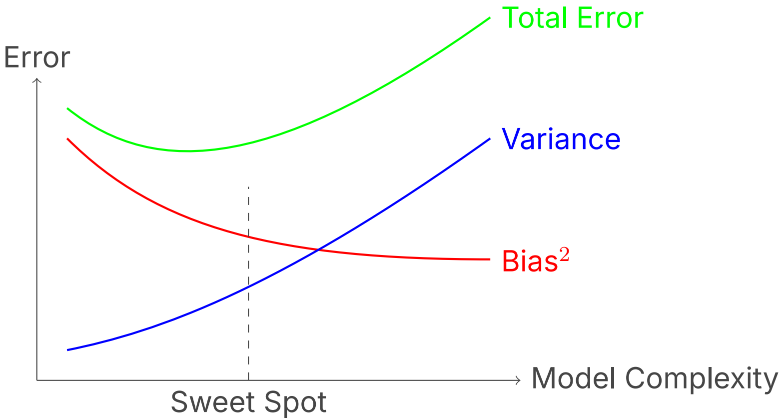
Which scenario represents high variance?

a) A linear model consistently predicts poorly on both training and test sets
b) A model gives very different predictions when trained on slightly different datasets
c) A model that always predicts the average target value

Answer: b) High variance means the model is too sensitive to training data changes!

The Model Complexity Spectrum



Goal: Find the complexity that minimizes total error!

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Pop Quiz: Cross-Validation

Quick Quiz 3

In 5-fold cross-validation with 1000 data points, how many points are used for training in each fold?

a) 200 points

Answer: b) 800 points (4 out of 5 folds are used for training each time)

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  - Training and validation errors are close and reasonably low

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Learning curves help diagnose bias vs variance:

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  - Apply appropriate remedies based on bias vs variance

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Coming Up

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