Conventions, Accuracy Metrics, Classification, Regression

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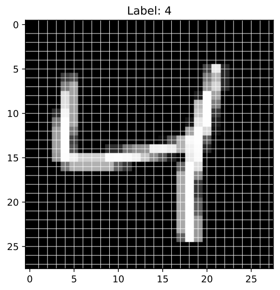
Digit Recognition Problem

Let us work on the digit recognition problem.

Notebook: rule-based-vs-ml.html



Rule-based Approach for Digit Recognition

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- There can be some cases of 4 where the width of each stroke is different

Apple Quality Features

- Size

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Colour	**Size**	**Texture**	**Condition**
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad

Training Set Components

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2. Output or Response Variable

Dataset Notation

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2. Output vector ($\mathbf{y} \in \mathbb{R}^{n}$) containing output variable for n samples.

Dataset Example

Example (after encoding): $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ (Orange=1, Small=0, Smooth=1)

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- Complete dataset: $\mathcal{D} = \{(\mathbf{x}_i^\top, y_i)\}_{i=1}^n$

Machine Learning Goal

Learn f: Condition $= f(\text{colour, size, texture})$

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1. From Training Dataset
2. To Predict the condition for the Testing set

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- No! Since, the test set is only a sample from all possible inputs.

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More discussion later once we study bias and variance

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# People	Temp (C)	Energy (kWh)
4000	30	30
4200	30	32
4200	35	40
3000	20	?
1000	45	?

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  - Examples - Predicting:
    - How much energy will campus consume?
    - How much rainfall will fall?

Accuracy Calculation

$$\text{Accuracy} = \frac{|\{i : y_i = \hat{y}_i\}|}{n} = \frac{3}{5} = 0.6$$

Accuracy Notation

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- **Alternative: Indicator function notation**

$$\text{Accuracy} = \frac{\sum_{i=1}^{n} \mathbf{1}[y_i = \hat{y}_i]}{n}$$

$$\text{where } \mathbf{1}[\text{condition}] = \begin{cases} 1 & \text{if condition is true} \\ 0 & \text{if condition is false} \end{cases}$$

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- Both notations are mathematically equivalent and commonly used in ML literature

When Precision/Recall Matter

Cases for this:

- Cancer Screening

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- Planet Detection

Precision Metric

$$\text{Precision} = \frac{|\{i : y_i = \hat{y}_i = \text{Good}\}|}{|\{i : \hat{y}_i = \text{Good}\}|} = \frac{2}{4} = 0.5$$

"the fraction of relevant instances among the retrieved instances", i.e. "out of the number of times we predict Good, how many times is the condition actually Good"

Accuracy vs Precision/Recall

$$\text{Accuracy} = \frac{98}{100} = 0.98$$
$$\text{Recall} = \frac{0}{1} = 0$$
$$\text{Precision} = \frac{0}{1} = 0$$

Confusion Matrix

		Ground Truth	
		Positive	Negative
Predicted	Positive	True Positive (TP)	False Positive (FP)
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Key Insight: Each cell represents a different type of prediction outcome

Precision: "How accurate are my positive predictions?"

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Focus: Of all items I predicted as positive, how many were actually positive?

Recall: "How many actual positives did I find?"

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Focus: Of all items that are actually positive, how many did I correctly identify?

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Let's say we're testing for a disease:

		Actually Has Disease	
		Yes	No
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Mean Error Issues

Is there any downside with using mean error?

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Is there any downside with using mean error? Errors can get cancelled out

Pop Quiz

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Which metrics should you use for imbalanced datasets?

1. Accuracy only

Answer: c) Precision, recall, and F1-score give a more complete picture!

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- **Use baselines:** Simple baseline models help validate your approach

Summary: Evaluation Metrics

Task	Common Metrics	When to Use
Classification	Accuracy, Precision, Recall, F1 Confusion Matrix	Balanced/Imbalanced Multi-class problems
Regression	MSE, RMSE, MAE Mean Error	Continuous prediction Check for bias

Remember: Choose metrics based on your problem's characteristics and business requirements!