Matrix Factorization for Movie Recommendation Systems

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- **Algorithms**: ALS vs Gradient Descent
- **Practice**: Hands-on understanding

Problem Setup

The Movie Recommendation Challenge

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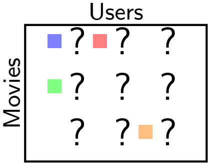

Sparse Rating Matrix

Pop Quiz 1: Understanding the Scale

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Answer: $200 \times 10^6 \times 15 \times 10^3 = 3 \times 10^{12}$ possible ratings!
But typical users rate only 20-100 movies. What percentage of the matrix is filled?
Answer: $\frac{100}{15000} = 0.67\%$ - extremely sparse!

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- **Notation**: $\Omega = \{(i, j) : a_{ij} \text{ is observed}\}$

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- Can we predict Carol's rating for Swades?

Key Insight: Latent Features

Before We Dive In: A Simple Question

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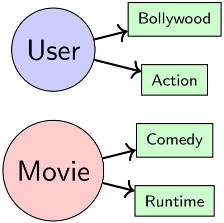

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Batman	0.05	0.80	0.30
Interstellar	0.05	0.95	0.70
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Movie Feature Matrix H $\in \mathbb{R}^{3\times5}$:

$$\mathbf{H} = \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix}$$

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Key Question: How do we learn these w_{ij} values from observed ratings?

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$$\mathbf{A}_{3\times 5} = \begin{bmatrix} 5 & 4 & 2 & 3 & 2 \\ ? & 5 & 1 & 4 & ? \\ 4 & ? & 1 & 5 & ? \end{bmatrix} \approx$$

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix} = \mathbf{W}_{3\times 3}\mathbf{H}_{3\times 5}$$

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The Magic Formula:

$$\text{Alice's rating} = \text{Alice's preferences} \cdot \text{Sholay's features}$$

Step 4: Detailed Calculation Example

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$$\hat{a}_{11} = \mathbf{w}_1^T \mathbf{h}_1 \tag{1}$$

$$= w_{11} \cdot 0.95 + w_{12} \cdot 0.10 + w_{13} \cdot 0.85 \tag{2}$$

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Goal: Find w_{11}, w_{12}, w_{13} such that $\hat{a}_{11} \approx 5$ (Alice's actual rating)

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1. What are the dimensions of $\mathbf{A}$?
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Answers:

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Key Insight: If $r \ll \min(N, M)$, we have huge parameter reduction!

Learning the Factorization

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Objective: Minimize prediction error on observed ratings only

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- Ω: set of observed (i, j) pairs

Problem Characteristics:

- **Non-convex:** Multiple local minima exist

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Key Insight: While non-convex jointly, it's convex in each matrix individually!

Algorithm 1: Alternating Least Squares (ALS)

Alternating Least Squares Strategy:

ALS: The Big Picture

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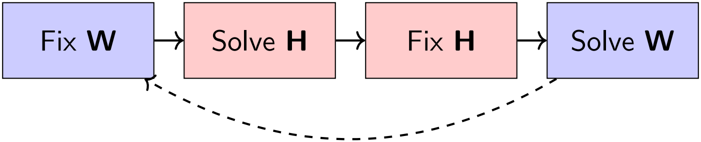

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Update Alice's preferences ($\mathbf{w}_1$):

Alice rated: Sholay(5), Swades(4), Batman(2), Interstellar(3), Shawshank(2)

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ALS: Complete Algorithm

Algorithm 1: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

1. **Initialize:** $\mathbf{W}^{(0)} \in \mathbb{R}^{N \times r}$, $\mathbf{H}^{(0)} \in \mathbb{R}^{r \times M}$ randomly

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3. **Check Convergence:** Stop if $\|\mathbf{W}^{(t)}\mathbf{H}^{(t)} - \mathbf{W}^{(t-1)}\mathbf{H}^{(t-1)}\|_F < \epsilon$

Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

Algorithm 2: Gradient Descent

Gradient Descent Approach

Simultaneous Updates: Update both $\mathbf{W}$ and $\mathbf{H}$ together

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Gradients:

$$\frac{\partial L}{\partial \mathbf{w}_i} = -2 \sum_{j:(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)\mathbf{h}_j \tag{9}$$

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SGD: Learning Like a Human

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- Learning rate α controls step size

SGD: Step-by-Step Example

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$$\text{Current: } \mathbf{w}_1 = [0.4, 0.2, 0.3], \quad \mathbf{h}_1 = [0.95, 0.10, 0.85] \tag{13}$$

$$\text{Prediction: } \hat{a}_{11} = 0.4 \times 0.95 + 0.2 \times 0.10 + 0.3 \times 0.85 = 0.655 \tag{14}$$

$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

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Updates with $\alpha = 0.01$:

$$\mathbf{w}_1 \leftarrow [0.4, 0.2, 0.3] + 0.01 \times 4.345 \times [0.95, 0.10, 0.85] \tag{16}$$

$$= [0.4413, 0.2043, 0.3369] \tag{17}$$

$$\mathbf{h}_1 \leftarrow [0.95, 0.10, 0.85] + 0.01 \times 4.345 \times [0.4, 0.2, 0.3] \tag{18}$$

$$= [0.9674, 0.1087, 0.8631] \tag{19}$$

Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

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Answers:

1. $e_{ij} = 2 - 4.5 = -2.5$

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Answers:

1. $e_{ij} = 2 - 4.5 = -2.5$
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Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

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Algorithm Comparison and Practical Considerations

ALS vs SGD: Head-to-Head Comparison

Aspect	ALS	SGD
Updates	Alternating	Simultaneous
Convergence	Faster, more stable	Slower, can oscillate
Parallelization	Excellent	Limited
Memory	Higher	Lower
Implementation	Complex	Simple
Hyperparameters	Few (rank r)	Many (α, schedule)
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Regularization: Prevent overfitting

$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

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Hands-On Understanding

Let's Build Intuition: Small Example

Our 3×3 rating matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & ? & 2 \\ 4 & 4 & ? \\ ? & 5 & 1 \end{bmatrix}$$

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Constraint: Only minimize error on observed entries!

Step-by-Step ALS Solution

Iteration 1: Initialize randomly

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Solve: $\mathbf{w}_1^{(1)} = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$

Continue for all users and movies...

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Master Check

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- 200M users, 15K movies

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$$\boxed{\text{Collaborative Filtering} = \text{Matrix Factorization} = \text{Dimensionality Reductio}}$$

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Questions?

Thank you for your attention!

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