Conventions, Accuracy Metrics, Classification, Regression

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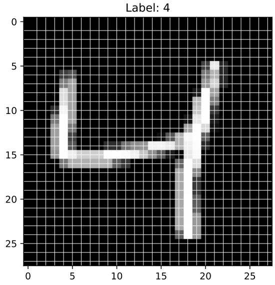
Digit Recognition Problem

Let us work on the digit recognition problem.

Notebook: rule-based-vs-ml.html



Rule-based Approach for Digit Recognition

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- There can be some cases of 4 where the width of each stroke is different

Apple Quality Features

- Size

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- Colour

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Let us modify our data table for now.

Colour	**Size**	**Texture**	**Condition**
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad

Training Set Components

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2. Output or Response Variable

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2. Output vector ($\mathbf{y} \in \mathbb{R}^{n}$) containing output variable for n samples.

Dataset Example

Example (after encoding): $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ (Orange=1, Small=0, Smooth=1)

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- Complete dataset: $\mathcal{D} = \{(\mathbf{x}_i^\top, y_i)\}_{i=1}^n$

Machine Learning Goal

Learn f: Condition $= f(\text{colour, size, texture})$

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1. From Training Dataset
2. To Predict the condition for the Testing set

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- No! Since, the test set is only a sample from all possible inputs.

Training vs Test Sets

Both the training set and the test set are samples drawn from the hidden true distribution (also sometimes called population)

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More discussion later once we study bias and variance

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# People	**Temp (C)**	**Energy (kWh)**
4000	30	30
4200	30	32
4200	35	40
3000	20	?
1000	45	?

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    - How much rainfall will fall?

Accuracy Calculation

$$\text{Accuracy} = \frac{|\{i : y_i = \hat{y}_i\}|}{n} = \frac{3}{5} = 0.6$$

Accuracy Notation

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- **Alternative: Indicator function notation**

$$\text{Accuracy} = \frac{\sum_{i=1}^{n} \mathbf{1}[y_i = \hat{y}_i]}{n}$$

$$\text{where } \mathbf{1}[\text{condition}] = \begin{cases} 1 & \text{if condition is true} \\ 0 & \text{if condition is false} \end{cases}$$

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- Both notations are mathematically equivalent and commonly used in ML literature

When Precision/Recall Matter

Cases for this:

- Cancer Screening

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Precision Metric

$$\text{Precision} = \frac{|\{i : y_i = \hat{y}_i = \text{Good}\}|}{|\{i : \hat{y}_i = \text{Good}\}|} = \frac{2}{4} = 0.5$$

"the fraction of relevant instances among the retrieved instances", i.e. "out of the number of times we predict Good, how many times is the condition actually Good"

Accuracy vs Precision/Recall

$$\begin{aligned}
\text{Accuracy} &= \frac{98}{100} = 0.98 \\
\text{Recall} &= \frac{0}{1} = 0 \\
\text{Precision} &= \frac{0}{1} = 0
\end{aligned}$$

Confusion Matrix

		Ground Truth	
		Yes	No
Predicted	Yes	True Positive	False Positive
	No	False Negative	True Negative

Example Metrics

$$
\begin{array}{c} \\ \text{Pred Positive} \\ \text{Pred Negative} \end{array}
\begin{array}{c}
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$$\text{Recall} = \frac{\text{T.P}}{\text{T.P} + \text{F.P}}$$

$$\text{Precision} = \frac{\text{T.P}}{\text{T.P} + \text{F.N}}$$

Mean Error Issues

Is there any downside with using mean error?

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Pop Quiz

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Which metrics should you use for imbalanced datasets?

1. Accuracy only

Answer: c) Precision, recall, and F1-score give a more complete picture!

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- **Visualization is crucial:** Always plot your data (Anscombe's Quartet lesson)
- **Use baselines:** Simple baseline models help validate your approach

Summary: Evaluation Metrics

Task	Common Metrics	When to Use
Classification	Accuracy, Precision, Recall, F1 Confusion Matrix	Balanced/Imbalanced Multi-class problems
Regression	MSE, RMSE, MAE Mean Error	Continuous prediction Check for bias

Remember: Choose metrics based on your problem's characteristics and business requirements!