Next Token Generation

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IIT Gandhinagar

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Introduction and Motivation

Acknowledgment and Inspiration

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  - ChatGPT generates text by predicting the next token
  - Same underlying principle scaled to billions of parameters
  - Understanding next token prediction is key to understanding LLMs

Given the sequence “app”, what is the next character?

Problem Formulation

Next Character Prediction as Classification

Classification Task: Predict probability distribution over all

Next Character Prediction as Classification

Input: “app”

Classification Task: Predict probability distribution over all

Next Character Prediction as Classification

Input: "app"

a

p

p

Output: Character Probabilities

Character	Probability
a	0.05
b	0.02
c	0.03
...	...
l	0.35
m	0.01
...	...
y	0.08
z	0.01
- (end)	0.15

Classification Task: Predict probability distribution over all

Case Study: Indian Names Generation

Dataset: Indian Names

Training Dataset

Goal: Learn to generate new, realistic Indian names

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Sample Names:

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Dataset Properties:

- 1000+ unique names
- Diverse regional origins
- Various lengths (4-10 chars)
- Both male & female names
- Rich phonetic patterns

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Assumptions and Constraints

- **Character Set:** Only 26 lowercase letters (a-z)

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Vocabulary Size:

26 letters + 1 hyphen = **27 characters**

Training Data Generation

Generate Training Dataset

Example: "abid" → Training Examples

Context Length: 3 characters

Input (X)				Target (Y)
Char 1	**Char 2**	**Char 3**	**Context**	**Next Char**
-	-	-	"—"	a
-	-	a	"–a"	b
-	a	b	"-ab"	i
a	b	i	"abi"	d
b	i	d	"bid"	-

Training Examples

From one name "abid", we create **5 training examples** using a

Representation Learning

The Idea: Character Embeddings

- **Goal:** Learn vector representations for each character

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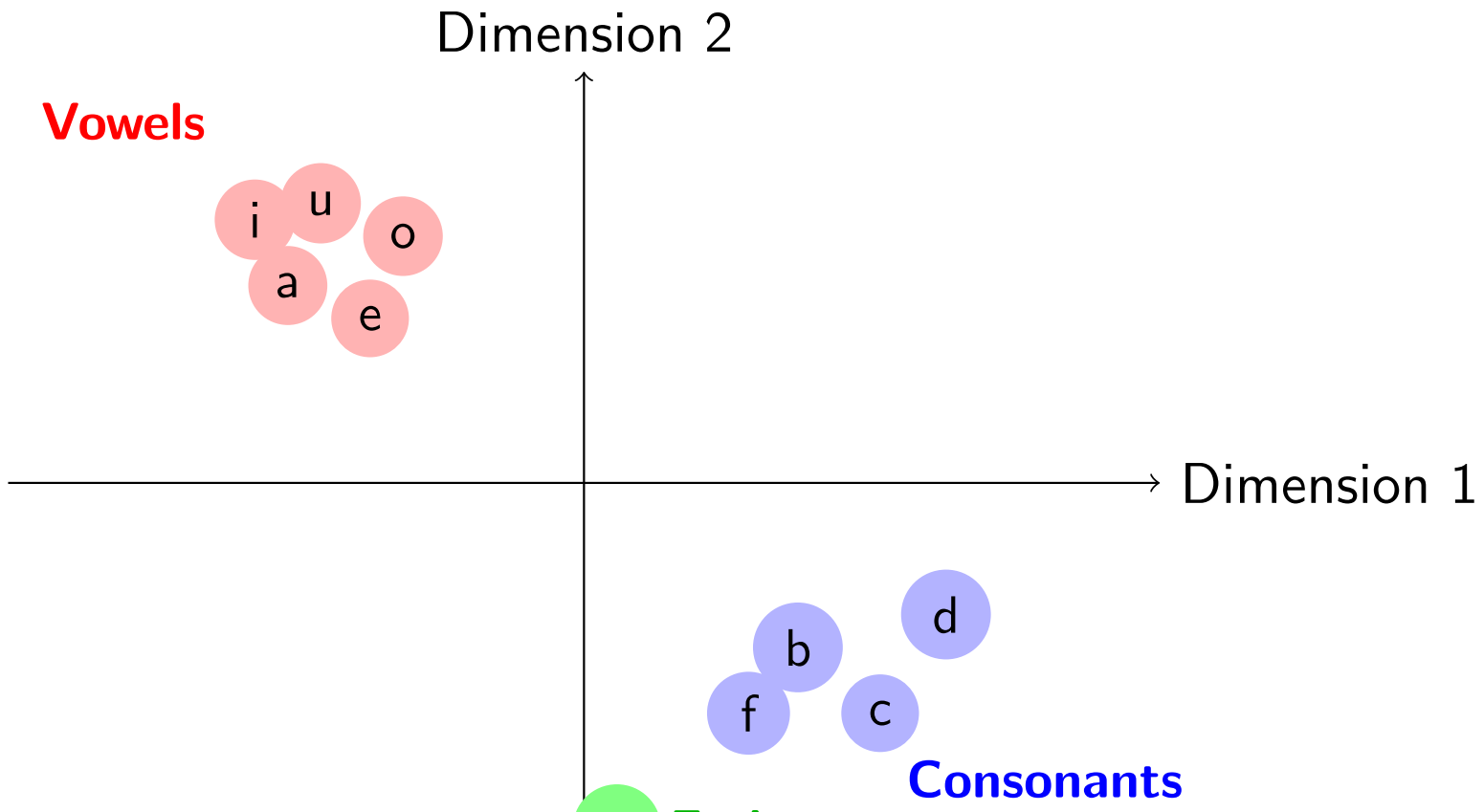
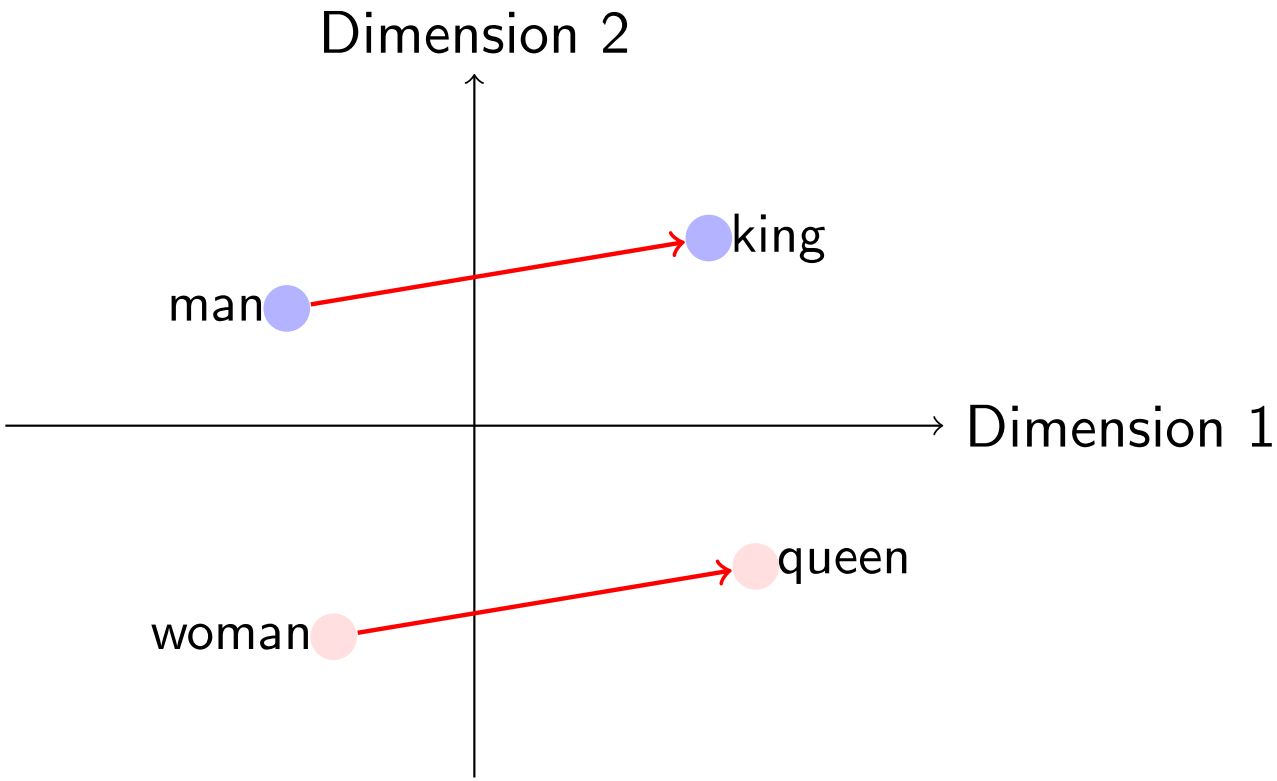


Word2Vec Analogy Example

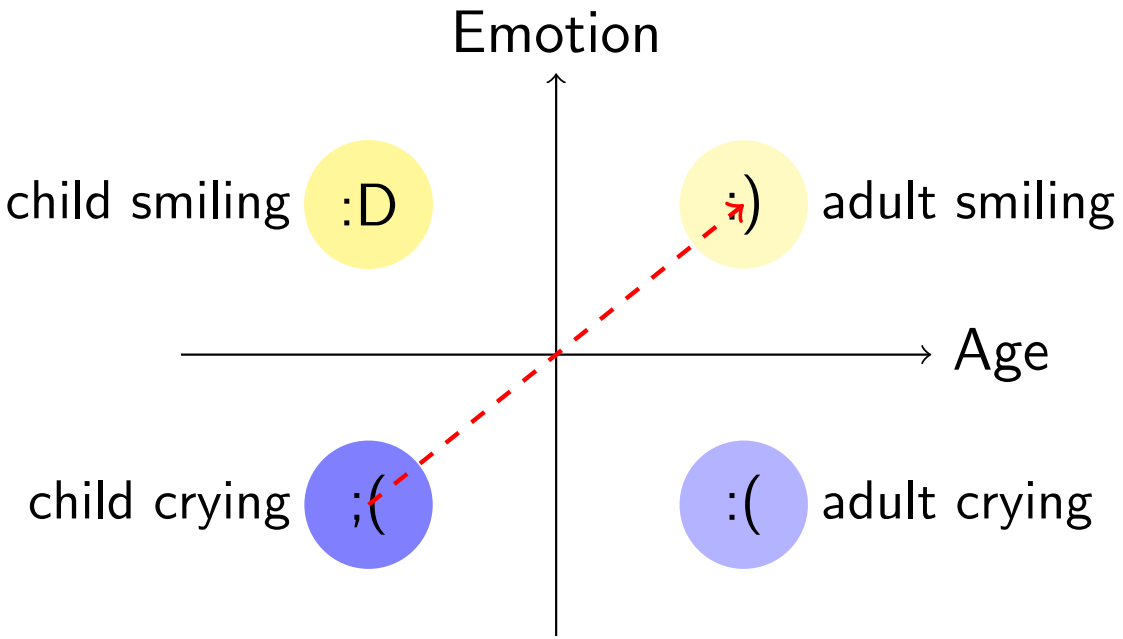
Classic Word2Vec Relationship



Relationship: queen $\approx$ king - man $+$ woman

Analogy with Emotions

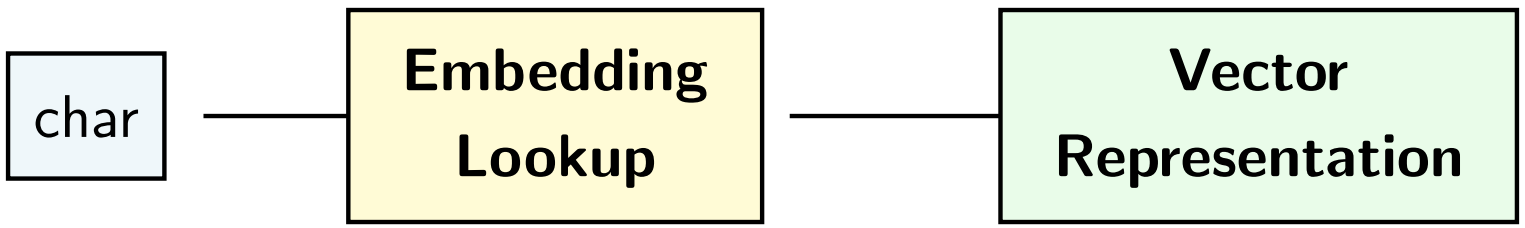
Emotional Expression Analogy



Relationship: child crying = child smiling + adult crying - adult smiling

Embedding Architecture

Embedding Matrix/Table Concept



Process: Character $\rightarrow$ Lookup in Embedding Table $\rightarrow$ Dense Vector

Embedding Table Structure

$27 \times K$ Embedding Matrix

Char	D1	D2	...	DK
a	0.2	-0.1	...	0.8
b	-0.3	0.5	...	-0.2
c	0.1	0.3	...	0.4
⋮	⋮	⋮	⋱	⋮
z	0.7	-0.4	...	0.1
-	0.0	0.9	...	-0.5

Key Point

Each character maps to a K-dimensional vector.

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters

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  - Initially random

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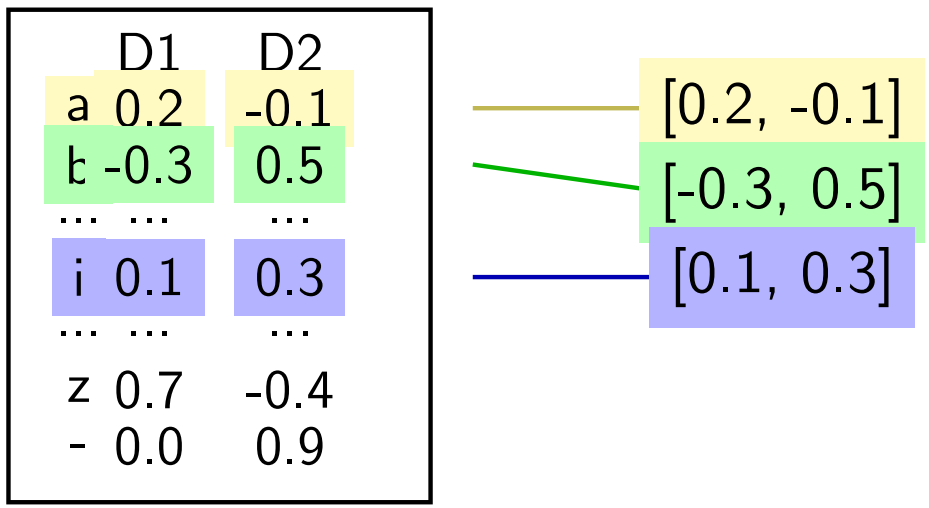
  - Embedding: $27 \times K$
  - MLP: (context_size $\times$ K) $\rightarrow$ hidden $\rightarrow$ ... $\rightarrow$ 27

Neural Network Architecture

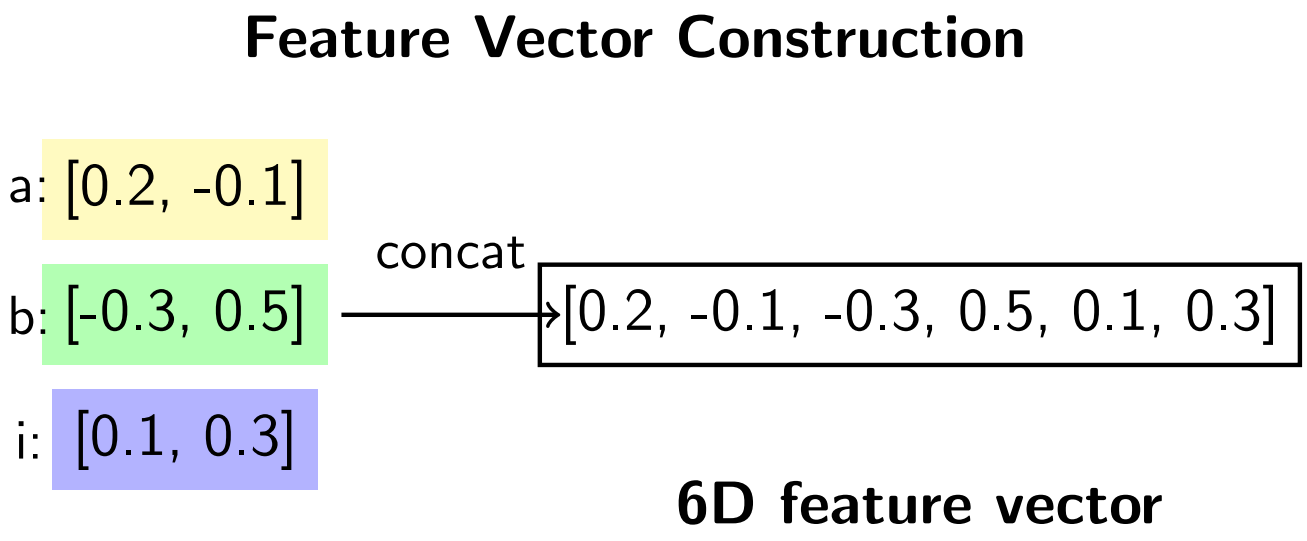
Example: 2D Embeddings for “abi”

Input: X = [“a”, “b”, “i”]

Embedding Matrix (27 × 2)



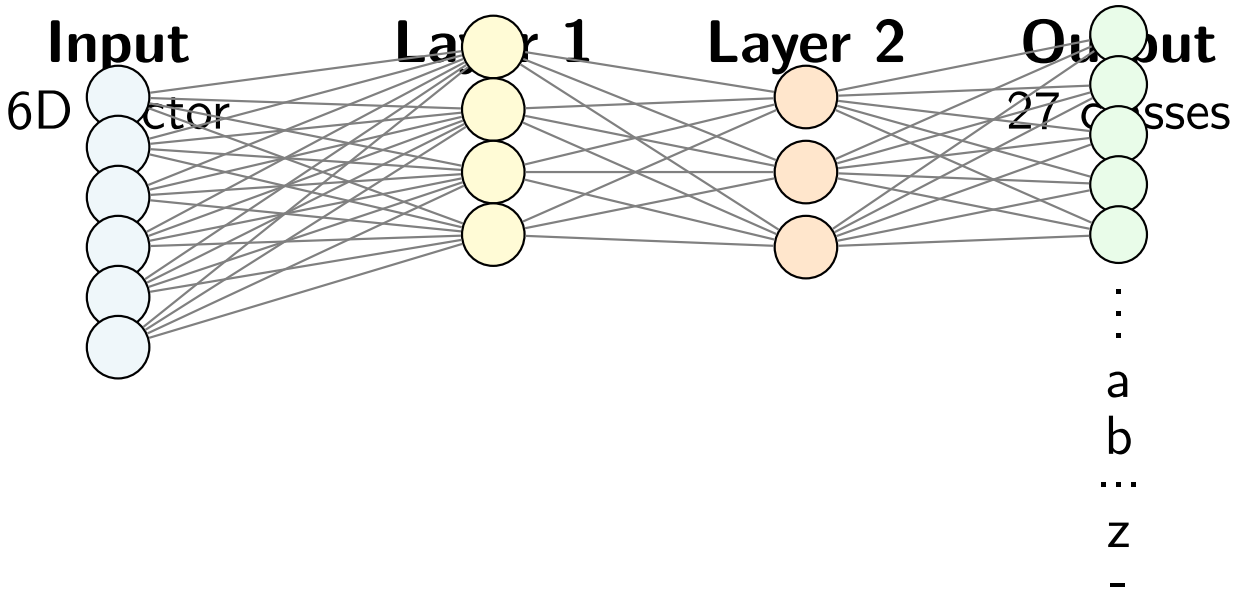
Concatenate the Embeddings



Result

3 chars $\times$ 2D embeddings $=$ 6D input to neural network

Multi-Layer Perceptron Architecture



Training and Loss Function

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N}\sum_{c=1}^{27} y_{i,c}\log(\hat{y}_{i,c}) \tag{1}$$

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  3. Backward pass: Update both embeddings and MLP weights
  4. Repeat for all training examples

Text Generation

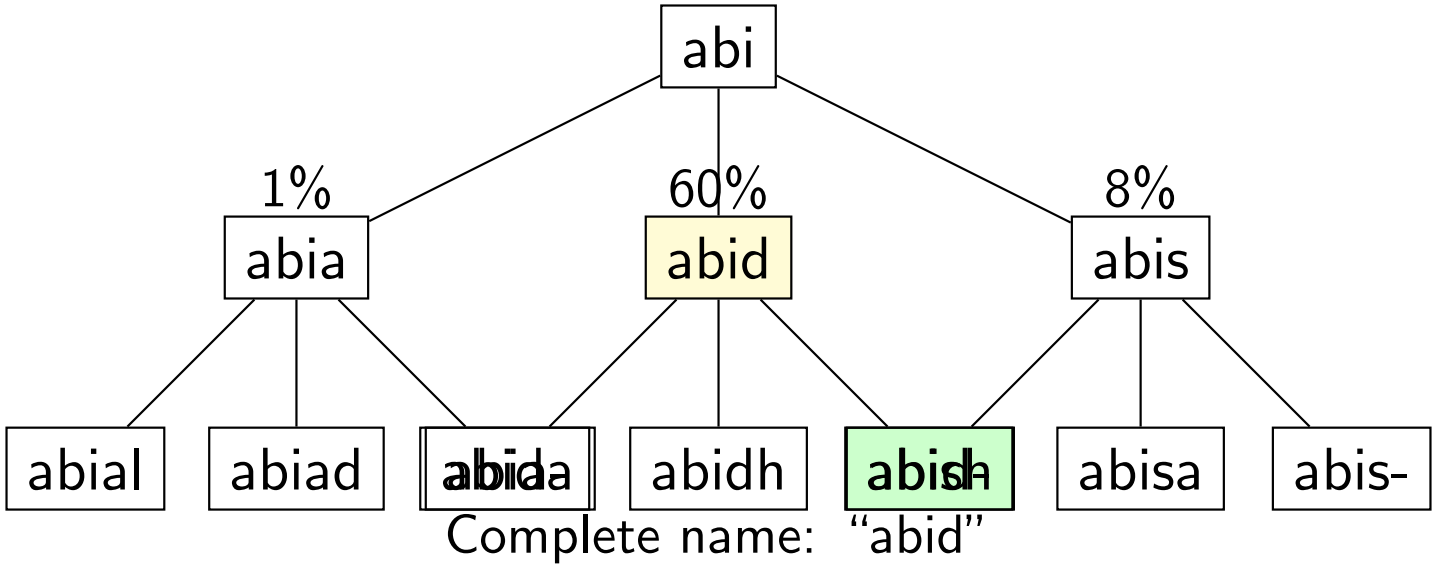
Sampling from the Learned Model

Test Input: "abi"

Predicted Probability Distribution

Next Char	Probability	Next Char	Probability
a	0.01	n	0.05
b	0.01	o	0.02
c	0.03	p	0.01
d	**0.60**	q	0.00
e	0.02	r	0.03
f	0.01	s	0.08
...	...	...	...
-	0.05	z	0.01

Generation Tree Structure



Recursive Process: Sample next character, append, repeat until end token

Temperature and Sampling Strategies

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

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- **Temperature Effects:**
  - $T = 1$: Standard probabilities
  - $T \to 0$: More peaked (deterministic)
  - $T \to \infty$: More uniform (random)

Temperature Variations

Context: "abi" $\rightarrow$ Next character probabilities

Char	T=0.5 (Low)	T=1.0 (Default)	T=2.0 (High)
a	0.001	0.01	0.08
d	**0.95**	**0.60**	**0.25**
s	0.01	0.08	0.12
h	0.005	0.03	0.09
-	0.02	0.05	0.11
others	0.015	0.23	0.35

- **Low T:** Conservative, predictable

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- **High T:** Creative, diverse

Summary and Applications

Key Takeaways

- **Core Idea:** Next token prediction as classification

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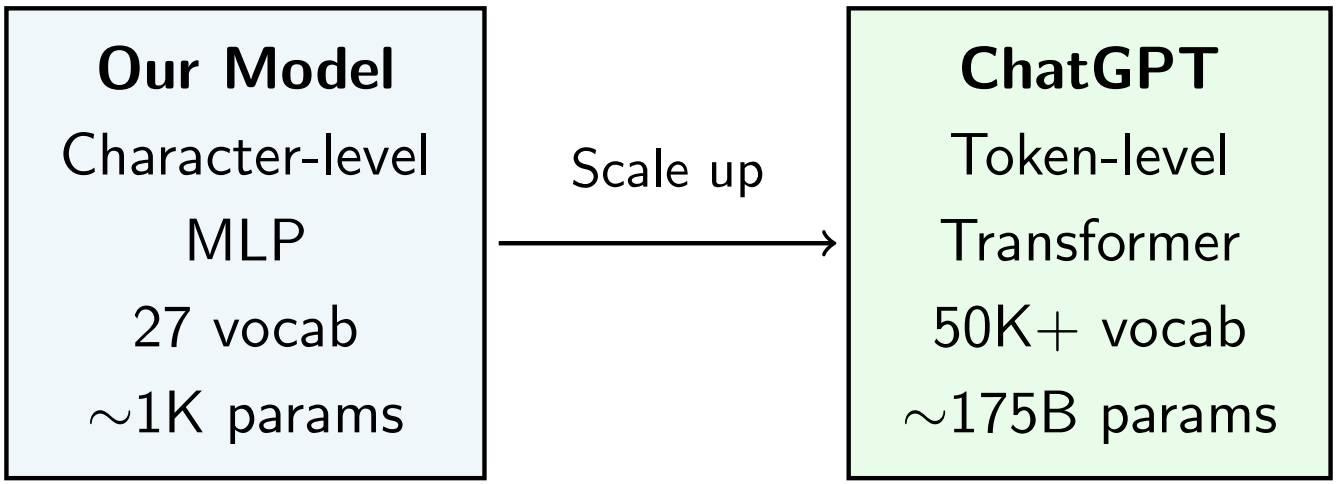
  - Transformer architecture instead of MLP
  - Billions of parameters instead of thousands

From Character-Level to ChatGPT



Same fundamental principle: Predict the next token!