K-Nearest Neighbors

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Parametric vs Non-Parametric Models

	Parametric	Non-Parametric
Parameter	Number of parameters is fixed w.r.t dataset size	Number of parameters grows w.r.t. to an increase in dataset size
Speed	Quicker (as the number of parameters are less)	Longer (as number of parameters are less)
Assumptions	Strong Assumptions (like linearity in Linear Regression)	Very few (sometimes no) assumptions
Examples	Linear Regression	KNN, Decision Tree

Lazy vs Eager Strategies

	Lazy	Eager
Train Time	0	$\neq 0$
Test	Long (due to comparison with train data)	Quick (as only "parameters" are involved)
Memory	Store/Memorise entire data	Store only learnt parameters
Utility	Useful for online settings	
Examples	KNN	Linear Regression, Decision Tree

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- What is the **aggregation function** that is going to be used?
- What are the **number of neighbors** that you are going to take into consideration?
- What is the **computational complexity** of the algorithm that you are implementing?

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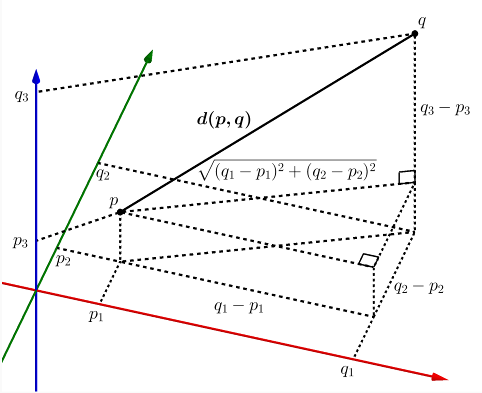

Euclidean Distance

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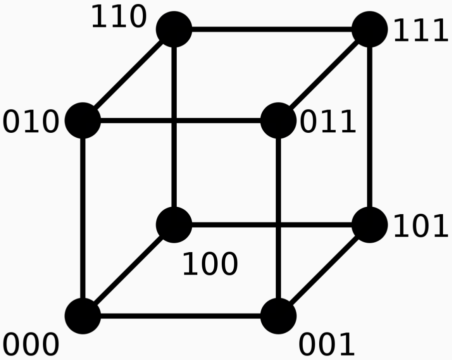

Hamming Distance

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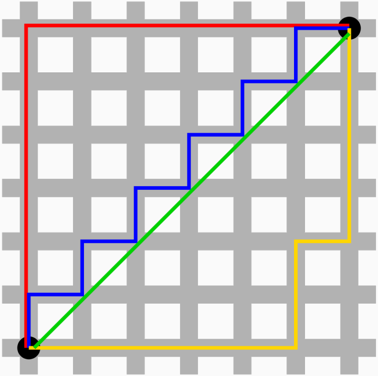

Manhattan Distance

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High values of K will result in smoother decision boundaries $\implies$ lower variance but also higher bias

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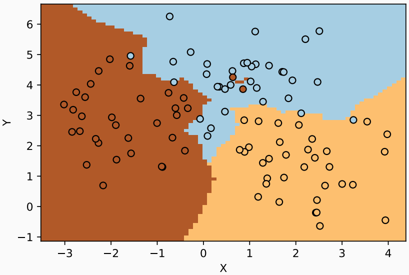

K = 3

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Aggregating data

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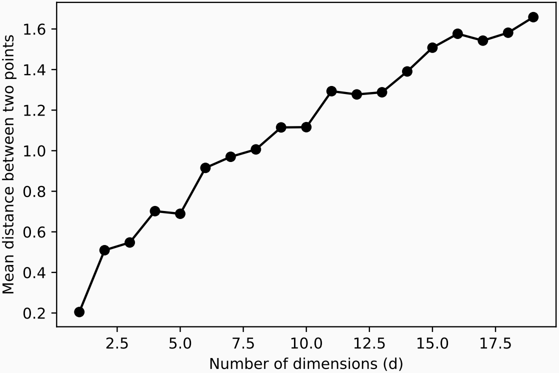

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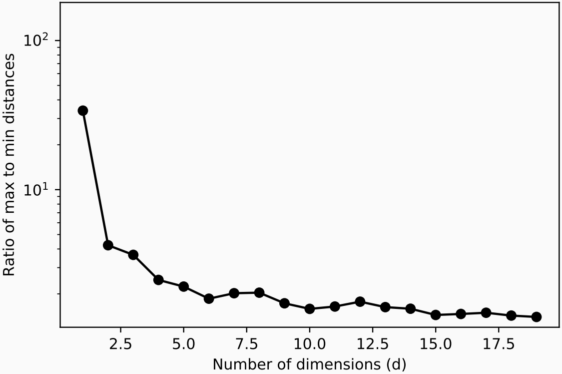

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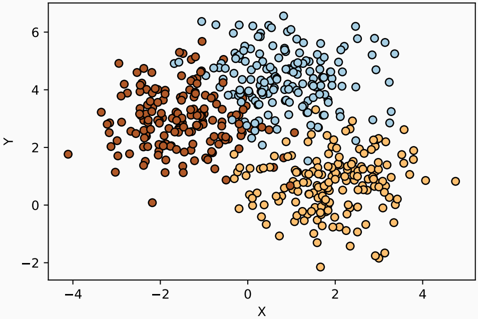
Doing an exhaustive search over all the points is time consuming, especially if you have a large number of data points.



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- Vector approximation files
- Greedy search in proximity neighborhood graphs

Locality sensitive hashing

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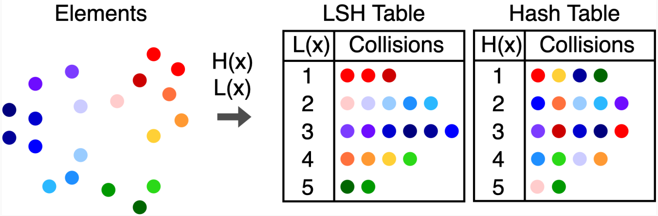

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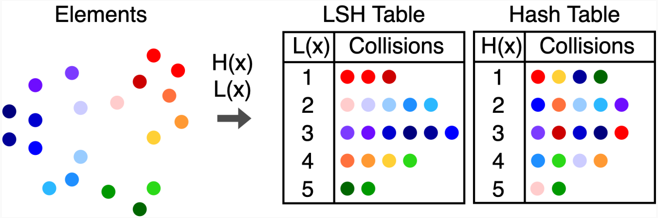

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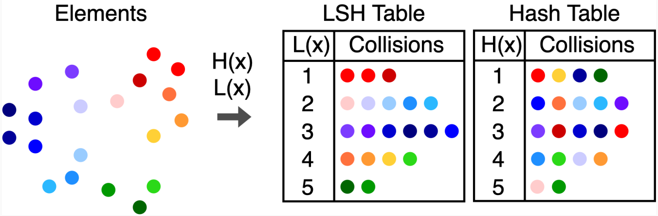
For such cases, all elements in a bin would be given the same label, which again can be decided on the basis of different aggregation methods



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