Maths for ML

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3. $$\epsilon^T \epsilon = \sum \epsilon_i^2$$

Scalar Property

4. For a scalar s

$$s = s^T$$

Gradient Vector

5.

$$\nabla_\theta s = \begin{bmatrix} \frac{\partial s}{\partial \theta_1} \\ \frac{\partial s}{\partial \theta_2} \\ \vdots \\ \frac{\partial s}{\partial \theta_N} \end{bmatrix}$$

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Z has a property $Z_{ij} = Z_{ji} \implies Z^T = Z$

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$$\theta^T Z\theta = e\theta_1^2 + 2f\theta_1\theta_2 + g\theta_2^2$$

The term $\theta^T Z\theta$ is a scalar.

Matrix Rank

Since X has fewer rows than columns, its maximum rank is equal to the maximum number of linearly independent rows. And because neither row is linearly dependent on the other row, the matrix has 2 linearly independent rows; so its rank is 2.

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Below, with an example, we illustrate the relationship between a matrix and its inverse.

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Not every square matrix has an inverse; but if a matrix does have an inverse, it is unique.