

# Subgradient

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- Generalises gradient to convex but non-differentiable problems

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- Examples:

# Subgradient

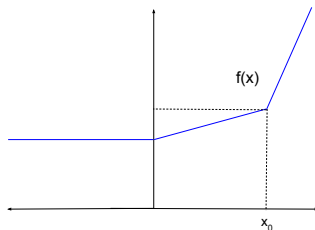
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- Examples:

# Subgradient

- Generalises gradient to convex but non-differentiable problems
- Examples:
  - $f(x) = |x|$

# Task at hand

- TASK: find derivative of  $f(x)$  at  $x = x_0$



# Solution

- Construct a differentiable  $g(x)$

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  - Intersecting  $f(x)$  at  $x = x_0$

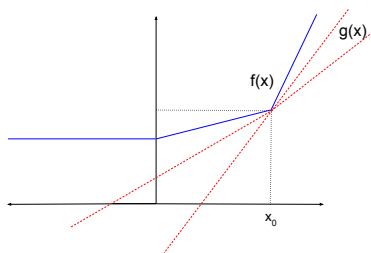


# Solution

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  - Intersecting  $f(x)$  at  $x = x_0$

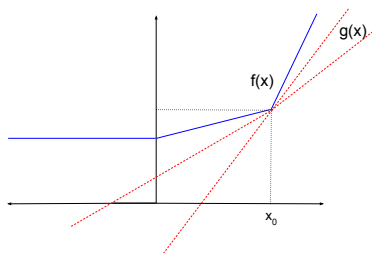
# Solution

- Construct a differentiable  $g(x)$ 
  - Intersecting  $f(x)$  at  $x = x_0$
  - Below or on  $f(x)$  for all  $x$



# Solution

- Compute slope of  $g(x)$  at  $x = x_0$



## Another Example: $f(x) = |x|$

- Subgradient of  $f(x)$  belongs to  $[-1, 1]$

