# Coordinate Descent

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- Another optimisation method (akin to gradient descent)
- Objective: $\min_{\boldsymbol{\theta}} f(\boldsymbol{\theta})$
- Key idea: Sometimes difficult to find minimum for all coordinates
- ..., but, easy for each coordinate
- turns into a one-dimensional optimisation problem

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- No step-size to choose!

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- Picking next coordinate: random, round-robin
- No step-size to choose!
- Converges for Lasso objective

# Coordinate Descent : Example

Learn $y = \theta_0 + \theta_1 x$ on following dataset, using coordinate descent where initially $(\theta_0, \theta_1) = (2, 3)$ for 2 iterations.

| **x** | **y** |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

# Coordinate Descent : Example

Our predictor, $\hat{y} = \theta_0 + \theta_1 x$

Error for $i^{th}$ datapoint, $\epsilon_i = y_i - \hat{y}_i$

$\epsilon_1 = 1 - \theta_0 - \theta_1$

$\epsilon_2 = 2 - \theta_0 - 2\theta_1$

$\epsilon_3 = 3 - \theta_0 - 3\theta_1$

$$\text{MSE} = \frac{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2}{3} = \frac{14 + 3\theta_0^2 + 14\theta_1^2 - 12\theta_0 - 28\theta_1 + 12\theta_0\theta_1}{3}$$

## Iteration 0

$$\text{MSE} = \frac{1}{3}(14 + 3\theta_0^2 + 14\theta_1^2 - 12\theta_0 - 28\theta_1 + 12\theta_0\theta_1)$$

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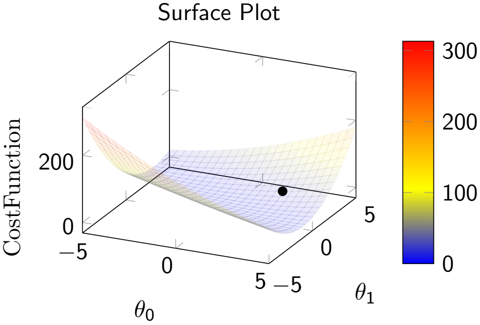

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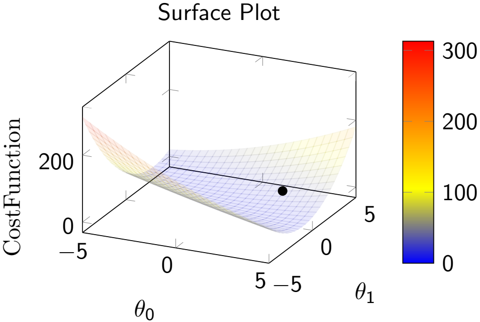


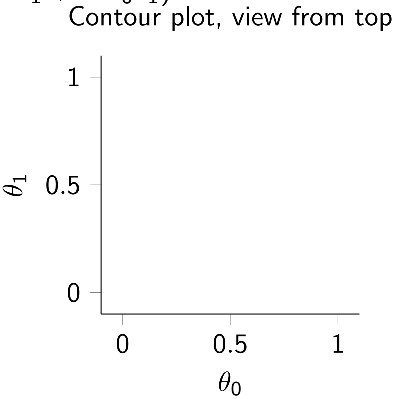

**Iteration 1**

INIT: $\theta_0 = 2$ and $\theta_1 = 3$

$\theta_1 = 3$ optimize for $\theta_0$

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$\theta_1 = 3$ optimize for $\theta_0$

$\frac{\partial \text{MSE}}{\partial \theta_0} = 6\theta_0 + 24 = 0$

$\theta_0 = -4$

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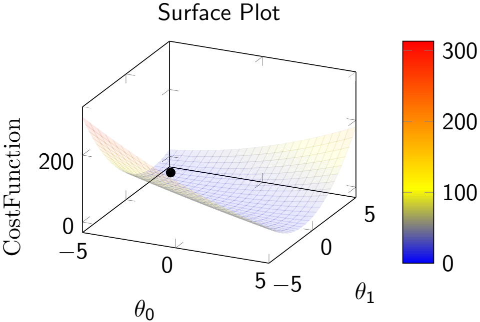

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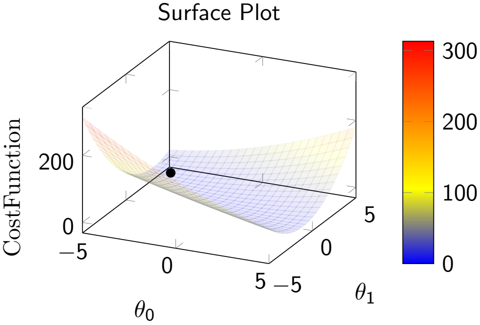


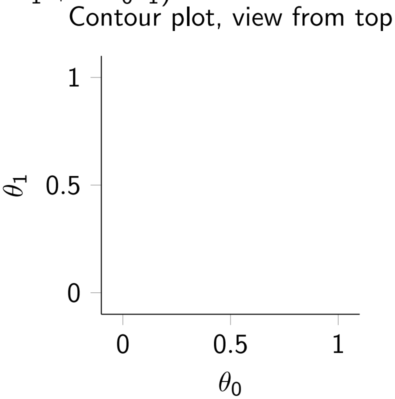

**Iteration 2**

INIT: $\theta_0 = -4$ and $\theta_1 = 3$

$\theta_0 = -4$ optimize for $\theta_1$

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$\theta_0 = -4$ optimize for $\theta_1$

$\theta_1 = 2.7$

# Iteration 2

$$\text{MSE} = \frac{1}{3}(14 + 3\theta_0^2 + 14\theta_1^2 - 12\theta_0 - 28\theta_1 + 12\theta_0\theta_1)$$

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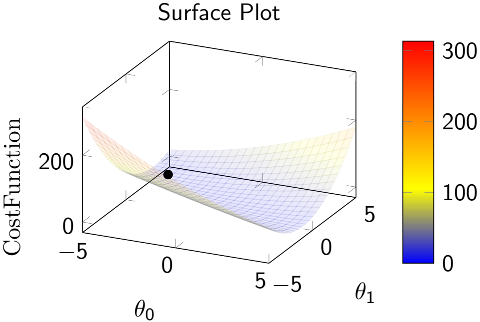

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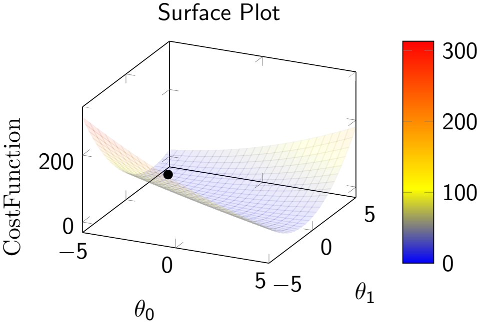


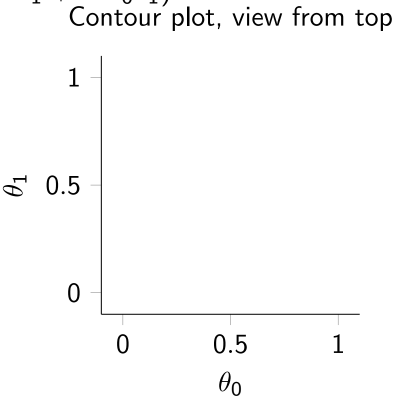

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**Iteration 3**

INIT: $\theta_0 = -4$ and $\theta_1 = 2.7$

$\theta_1 = 2.7$ optimize for $\theta_0$

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$\theta_1 = 2.7$ optimize for $\theta_0$

$\theta_0 = -3.4$

# Iteration 3

$$\text{MSE} = \frac{1}{3}(14 + 3\theta_0^2 + 14\theta_1^2 - 12\theta_0 - 28\theta_1 + 12\theta_0\theta_1)$$

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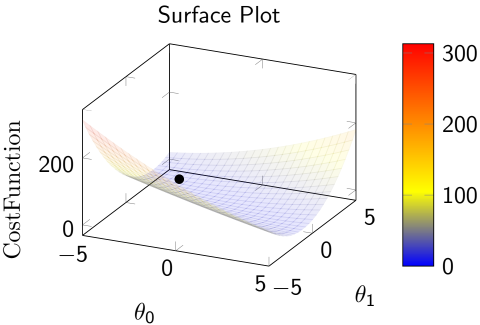

# Iteration 3

$$\text{MSE} = \frac{1}{3}(14 + 3\theta_0^2 + 14\theta_1^2 - 12\theta_0 - 28\theta_1 + 12\theta_0\theta_1)$$