Next Token Generation

Nipun Batra

IIT Gandhinagar

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Inspiration and Relevance

Inspired by the great lecture from Andrej Karpathy

- Search for ”Neural Networks: Zero to Hero” to find the original lecture

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- This approach is fundamental to modern language models
- **Direct connection to ChatGPT:**
  - Same core principle: predict the next token
  - Scaled up from characters to words/subwords
  - Uses transformer architecture instead of MLP

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What is the Next Character?

$\boxed{\textbf{app}}$ **?**

What is the next character?

Classification Task

app **?**

We can pose this as a classification task

Classification Task

app ?

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Input:
app

Classification Task

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We can pose this as a classification task

Input:
app

Output: Probability Distribution

Char	Prob	Char	Prob
a	0.01	n	0.01
b	0.01	o	0.01
c	0.01	p	0.01
...	...	...	...
l	**0.45**	z	0.01
m	0.01	-	0.05

Generate Indian Names

Specific Problem: Generate Indian Names

Dataset:

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Dataset:

Abid
Abhidha
Adesh
Aditya
Agam
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- Collection of Indian names
- Each name represents a sequence

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- Collection of Indian names
- Each name represents a sequence
- Goal: Learn to generate similar names

Assumptions

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1. **Character set:** Only use 26 lowercase characters (a-z)

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2. **End marker:** A hyphen (-) indicates the end character
3. **Length constraint:** Names are between 4 and 10 characters

Total vocabulary size: 26 + 1 = 27 characters

Generate Training Dataset

Creating Training Data from "abid"

Using history/context of 3 characters:

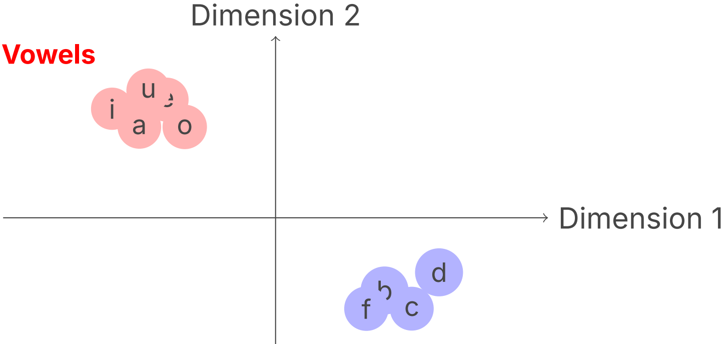
X (Input)		Y (Target)
[-, -, -]	$\rightarrow$	a a>-, a
	$\rightarrow$	b a, b>a, b
	$\rightarrow$	i a, b, i>a, b, i
	$\rightarrow$	d b, i, d>b, i, d
	$\rightarrow$	-

Result: 5 training examples from one name "abid"

Representation Learning

Important Idea: Representation Learning

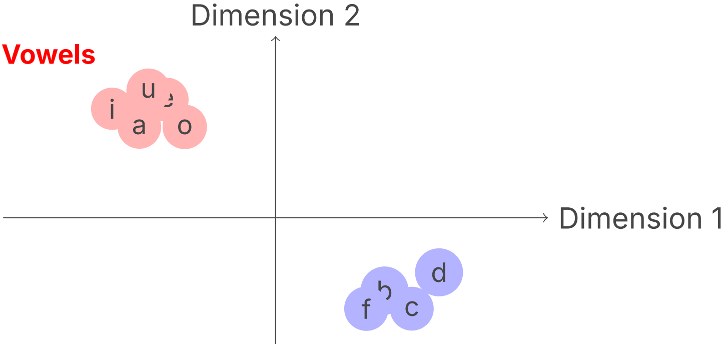
- Learn a vector representation for each character



Representation Learning

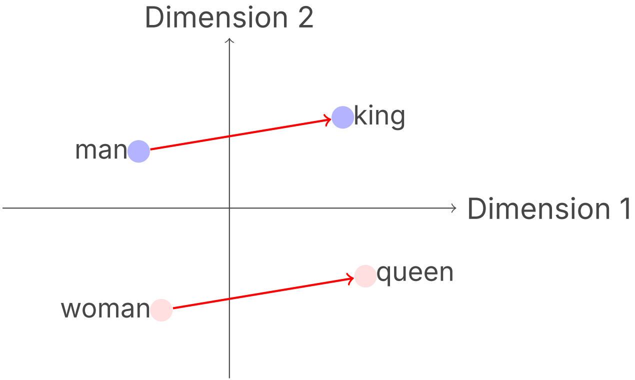
Important Idea: Representation Learning

- Learn a vector representation for each character
- Hope that similar characters will be closer in vector space



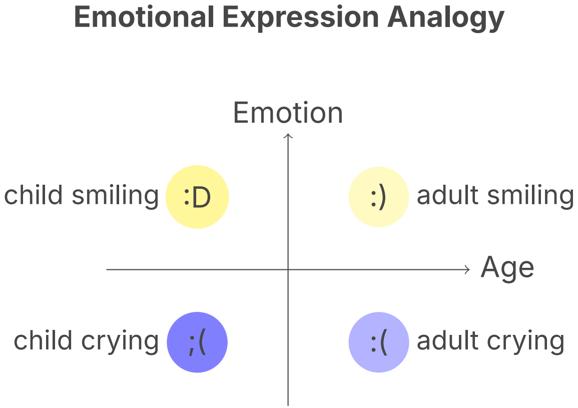
Word2Vec Reference

Classic Word2Vec Relationship



Relationship: queen $\approx$ king - man + woman

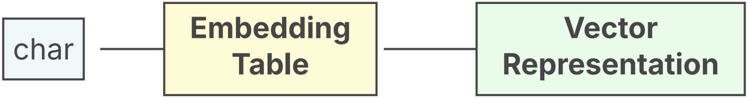
Analogy with Smileys



Relationship: child crying = child smiling + adult crying - adult smiling

Embedding Matrix/Table

Main Idea: Embedding Matrix/Table



Process: Character $\rightarrow$ Lookup in Embedding Table $\rightarrow$ Dense Vector

27 × K Embedding Matrix

Embedding Table Structure

Char	D1	D2	...	DK
a	0.2	-0.1	...	0.8
b	-0.3	0.5	...	-0.2
c	0.1	0.3	...	0.4
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
z	0.7	-0.4	...	0.1
-	0.0	0.9	...	-0.5

This overall becomes a 27 × K dimensional matrix

Learnable Matrix

This matrix is learnable!

- **Initially:** Random values

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- **After training:** Contains meaningful character representations
- **Similar characters:** Will have similar embedding vectors

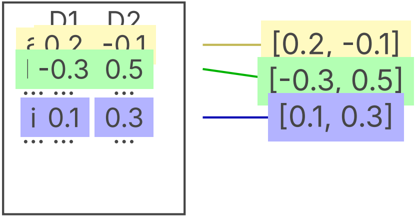
The network learns both the embeddings AND the classification weights!

Overall Architecture (2D Example)

Example with X = "abi" and 2D embeddings

Embedding Matrix (27 × 2)

Input: X = ["a", "b", "i"]



Concatenate the Embeddings

Feature Vector Creation for X = "abi"

a: [0.2, -0.1]

b: [-0.3, 0.5] $\xrightarrow{\text{concat}}$ [0.2, -0.1, -0.3, 0.5, 0.1, 0.3]

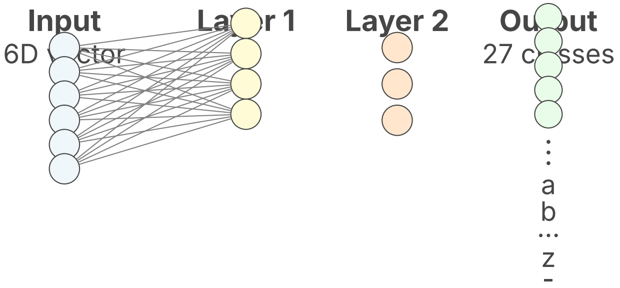
i: [0.1, 0.3]

6D feature vector

The feature vector pulls up embeddings and concatenates them

Multi-Layer Perceptron

Neural Architecture



Eventually shows 27-class output vector

Cross-Entropy Loss

Learning Process

- **Loss Function:** Use cross-entropy loss to learn

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  - Forward pass: Input → Embeddings → Concatenate → MLP → Probabilities

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- **Training Process:**
  - Forward pass: Input → Embeddings → Concatenate → MLP → Probabilities
  - Compute cross-entropy loss against true next character
  - Backward pass: Update both embeddings and MLP weights

Generate/Sample from Learned Model

Test Input: "abi"

Probability vector for next character:

Next Char	Probability	Next Char	Probability
a	0.01	n	0.05
b	0.01	o	0.02
c	0.03	p	0.01
d	**0.60**	...	...
...	...	z	0.01

- ABIA would be 1%

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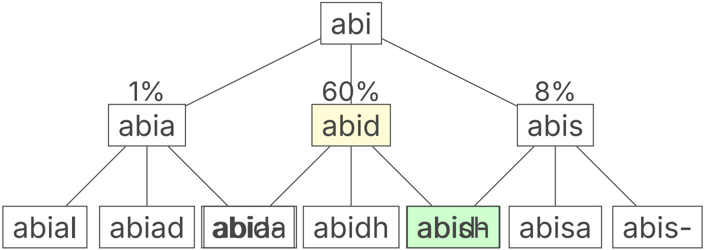
- ABIA would be 1%
- ABIB would be 1%
- **ABID would be 60%**

Tree Structure

Generation as Tree Structure



Had we chosen A, it starts a new branch. Had we chosen D, it starts a new branch, etc.

Temperature Term

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{1}$$

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- **Temperature Effects:**

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  - $T \to 0$: Very low temperature → more peaked (deterministic)

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- **Temperature Effects:**
  - $T = 1$: Default/standard probabilities
  - $T \to 0$: Very low temperature → more peaked (deterministic)
  - $T \to \infty$: Very high temperature → more uniform (random)

Temperature Variations

How sampling differs across temperatures

Next Char	Default T=1.0	Low T T=0.5	High T T=2.0
a	0.01	0.001	0.08
d	**0.60**	**0.95**	**0.25**
s	0.08	0.01	0.12
h	0.03	0.005	0.09
-	0.05	0.02	0.11
others	0.23	0.015	0.35

- **Low Temperature:** Conservative, predictable generation

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- **Low Temperature:** Conservative, predictable generation
- **High Temperature:** Creative, diverse generation