Next Token Generation

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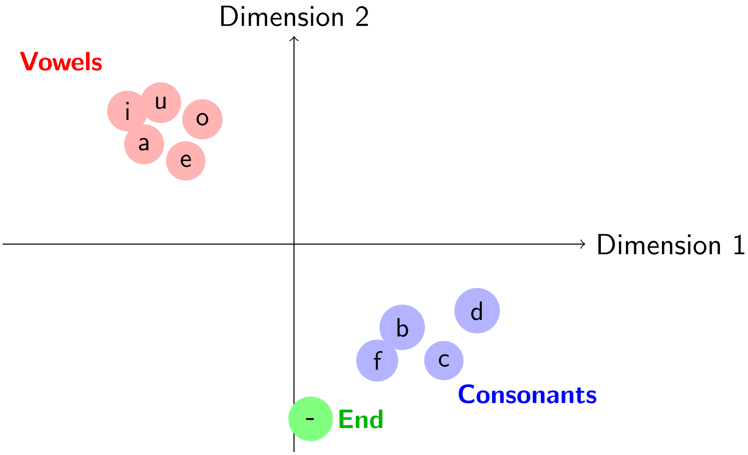
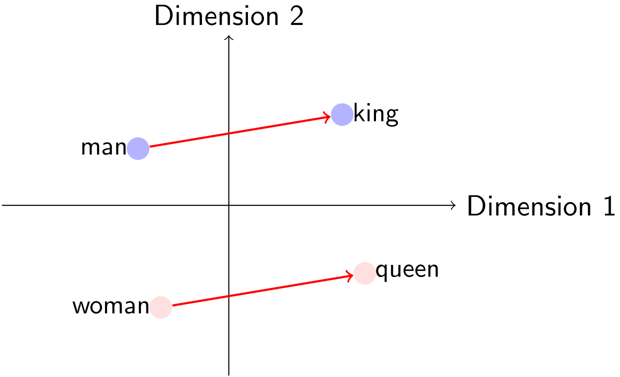
Vocabulary Size:

26 letters $+$ 1 hyphen $=$ **27 characters**


Dimension 2
Vowels
i
u
o
a
e
Dimension 1
d
b
f
c
Consonants
-
End

Word2Vec Analogy Example

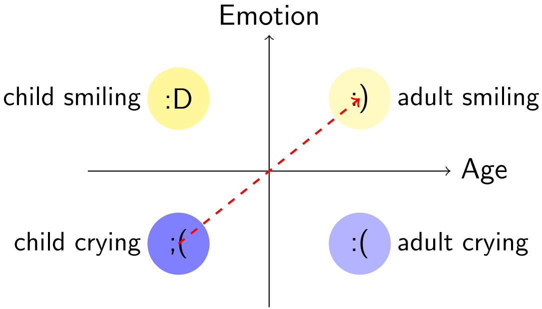
Classic Word2Vec Relationship



Relationship: queen ≈ king - man + woman

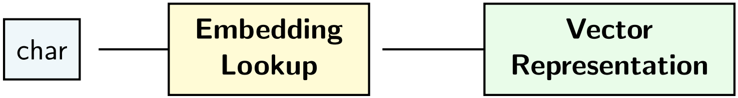
Analogy with Emotions

Emotional Expression Analogy



Relationship: child crying = child smiling + adult crying - adult smiling

Embedding Matrix/Table Concept



Process: Character $\rightarrow$ Lookup in Embedding Table $\rightarrow$ Dense Vector

Embedding Table Structure

27 × K Embedding Matrix

Char	D1	D2	...	DK
a	0.2	-0.1	...	0.8
b	-0.3	0.5	...	-0.2
c	0.1	0.3	...	0.4
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
z	0.7	-0.4	...	0.1
-	0.0	0.9	...	-0.5

Key Point

Each character maps to a K-dimensional vector.

Learnable Parameters

- **Embedding Matrix:** 27 $\times$ K parameters

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  - MLP: (context_size $\times$ K) $\rightarrow$ hidden $\rightarrow \ldots \rightarrow$ 27

Example: 2D Embeddings for “abi”

Embedding Matrix (27 × 2)

Input: X = [“a”, “b”, “i”]

	D1	D2
a	0.2	-0.1
b	-0.3	0.5
...	...	...
i	0.1	0.3
...	...	...
z	0.7	-0.4
-	0.0	0.9

[0.2, -0.1]

[-0.3, 0.5]

[0.1, 0.3]

Concatenate the Embeddings

Feature Vector Construction

a: [0.2, -0.1]

b: [-0.3, 0.5]

i: [0.1, 0.3]

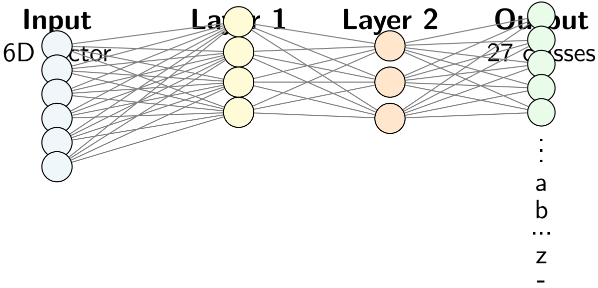
concat → [0.2, -0.1, -0.3, 0.5, 0.1, 0.3]

6D feature vector

Result

3 chars $\times$ 2D embeddings $=$ 6D input to neural network

Multi-Layer Perceptron Architecture



Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

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  4. Repeat for all training examples

Sampling from the Learned Model

Test Input: "abi"

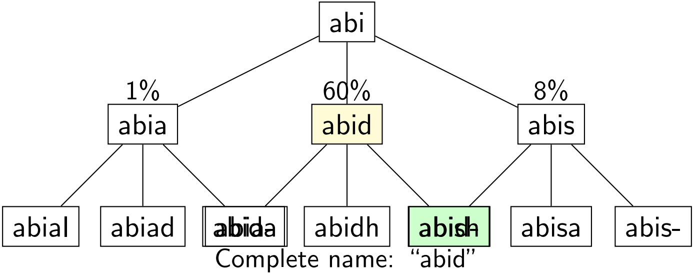
Predicted Probability Distribution

Next Char	Probability	Next Char	Probability
a	0.01	n	0.05
b	0.01	o	0.02
c	0.03	p	0.01
d	**0.60**	q	0.00
e	0.02	r	0.03
f	0.01	s	0.08
...	...	...	...
-	0.05	z	0.01

Most Likely Continuation

"abi" $\rightarrow$ "abid" (60

Generation Tree Structure



Recursive Process: Sample next character, append, repeat until end token

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- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

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  - $T \to \infty$: More uniform (random)

Temperature Variations

Context: "abi" $\rightarrow$ Next character probabilities

Char	**T=0.5 (Low)**	**T=1.0 (Default)**	**T=2.0 (High)**
a	0.001	0.01	0.08
d	**0.95**	**0.60**	**0.25**
s	0.01	0.08	0.12
h	0.005	0.03	0.09
-	0.02	0.05	0.11
others	0.015	0.23	0.35

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  - Billions of parameters instead of thousands

From Character-Level to ChatGPT

Our Model
Character-level
MLP
27 vocab
∼1K params

Scale up →

ChatGPT
Token-level
Transformer
50K+ vocab
∼175B params

Same fundamental principle: Predict the next token!