Next Token Generation

Nipun Batra

July 26, 2025

IIT Gandhinagar

Table of Contents

Introduction and Motivation

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**
- Search for *"Neural Networks: Zero to Hero"* to find his comprehensive tutorial series

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**
- Search for *"Neural Networks: Zero to Hero"* to find his comprehensive tutorial series
- These concepts are fundamental to understanding modern language models

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**
- Search for *"Neural Networks: Zero to Hero"* to find his comprehensive tutorial series
- These concepts are fundamental to understanding modern language models
- Direct relevance to **ChatGPT** and other large language models:

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**
- Search for *"Neural Networks: Zero to Hero"* to find his comprehensive tutorial series
- These concepts are fundamental to understanding modern language models
- Direct relevance to **ChatGPT** and other large language models:
    - ChatGPT generates text by predicting the next token

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**
- Search for *"Neural Networks: Zero to Hero"* to find his comprehensive tutorial series
- These concepts are fundamental to understanding modern language models
- Direct relevance to **ChatGPT** and other large language models:
    - ChatGPT generates text by predicting the next token
    - Same underlying principle scaled to billions of parameters

Acknowledgment and Inspiration

- This lecture is inspired by the excellent work of **Andrej Karpathy**
- Search for *"Neural Networks: Zero to Hero"* to find his comprehensive tutorial series
- These concepts are fundamental to understanding modern language models
- Direct relevance to **ChatGPT** and other large language models:
    - ChatGPT generates text by predicting the next token
    - Same underlying principle scaled to billions of parameters
    - Understanding next token prediction is key to understanding LLMs

Given the sequence “app”, what is the next character?

Problem Formulation

Next Character Prediction as Classification

Classification Task: Predict probability distribution over all

Next Character Prediction as Classification

a

p

p

Input: "app"

Classification Task: Predict probability distribution over all

Next Character Prediction as Classification

Output: Character Probabilities

a

p

p

Input: “app”

Character	Probability
a	0.05
b	0.02
c	0.03
...	...
l	0.35
m	0.01
...	...
y	0.08
z	0.01
- (end)	0.15

Classification Task: Predict probability distribution over all

Case Study: Indian Names Generation

Dataset: Indian Names

Training Dataset

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Goal: Learn to generate new, realistic Indian names

Dataset: Indian Names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Dataset Properties:

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Dataset Properties:

- 1000+ unique names

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Dataset Properties:

- 1000+ unique names
- Diverse regional origins

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Dataset Properties:

- 1000+ unique names
- Diverse regional origins
- Various lengths (4-10 chars)

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Dataset Properties:

- 1000+ unique names
- Diverse regional origins
- Various lengths (4-10 chars)
- Both male & female names

Goal: Learn to generate new, realistic Indian names

Training Dataset

Sample Names:

- Abid, Abhidha, Adesh
- Aditya, Arjun, Kiran
- Krishna, Lakshmi, Meera
- Nisha, Priya, Rajesh
- Sunita, Vikash, Zara

Dataset Properties:

- 1000+ unique names
- Diverse regional origins
- Various lengths (4-10 chars)
- Both male & female names
- Rich phonetic patterns

Goal: Learn to generate new, realistic Indian names

Assumptions and Constraints

- **Character Set:** Only 26 lowercase letters (a-z)

Assumptions and Constraints

- **Character Set:** Only 26 lowercase letters (a-z)
- **End Marker:** Hyphen (-) indicates end of name

Assumptions and Constraints

- **Character Set:** Only 26 lowercase letters (a-z)
- **End Marker:** Hyphen (-) indicates end of name
- **Length Constraint:** Names are between 4 and 10 characters

Assumptions and Constraints

- **Character Set:** Only 26 lowercase letters (a-z)
- **End Marker:** Hyphen (-) indicates end of name
- **Length Constraint:** Names are between 4 and 10 characters
- **Simplification:** No spaces, special characters, or uppercase letters

Assumptions and Constraints

- **Character Set:** Only 26 lowercase letters (a-z)
- **End Marker:** Hyphen (-) indicates end of name
- **Length Constraint:** Names are between 4 and 10 characters
- **Simplification:** No spaces, special characters, or uppercase letters

- **Character Set:** Only 26 lowercase letters (a-z)
- **End Marker:** Hyphen (-) indicates end of name
- **Length Constraint:** Names are between 4 and 10 characters
- **Simplification:** No spaces, special characters, or uppercase letters

Vocabulary Size:

26 letters + 1 hyphen = **27 characters**

Training Data Generation

Example: "abid" → Training Examples

Context Length: 3 characters

Input (X)				**Target (Y)**
Char 1	**Char 2**	**Char 3**	**Context**	**Next Char**
-	-	-	"—"	a
-	-	a	"–a"	b
-	a	b	"-ab"	i
a	b	i	"abi"	d
b	i	d	"bid"	-

Training Examples

From one name "abid", we create **5 training examples** using a

Representation Learning

- **Goal:** Learn vector representations for each character

The Idea: Character Embeddings

- **Goal:** Learn vector representations for each character
- **Hypothesis:** Similar characters should have similar embeddings

The Idea: Character Embeddings

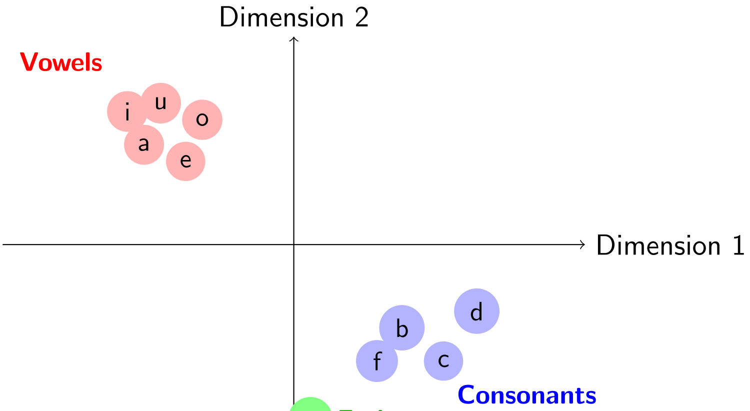
- **Goal:** Learn vector representations for each character
- **Hypothesis:** Similar characters should have similar embeddings
- **Benefit:** Capture semantic relationships between characters

The Idea: Character Embeddings

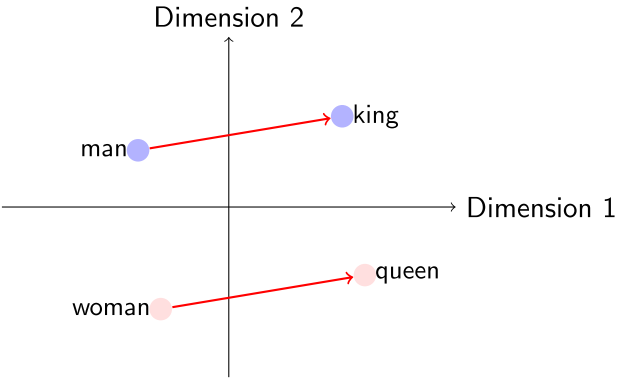
- **Goal:** Learn vector representations for each character
- **Hypothesis:** Similar characters should have similar embeddings
- **Benefit:** Capture semantic relationships between characters

The Idea: Character Embeddings

- **Goal:** Learn vector representations for each character
- **Hypothesis:** Similar characters should have similar embeddings
- **Benefit:** Capture semantic relationships between characters

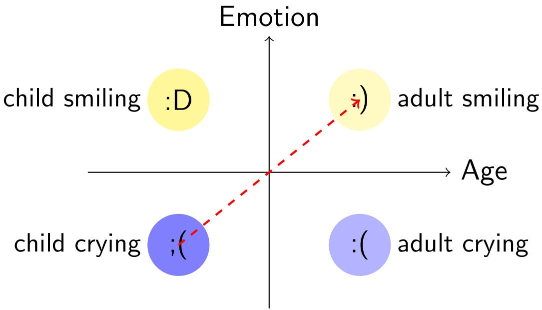


Classic Word2Vec Relationship



Relationship: queen ≈ king - man + woman

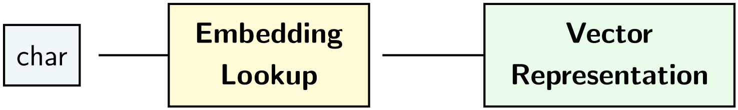
Emotional Expression Analogy



Relationship: child crying = child smiling + adult crying - adult smiling

Embedding Architecture

Embedding Matrix/Table Concept



Process: Character → Lookup in Embedding Table → Dense Vector

Embedding Table Structure

27 × K Embedding Matrix

Char	D1	D2	...	DK
a	0.2	-0.1	...	0.8
b	-0.3	0.5	...	-0.2
c	0.1	0.3	...	0.4
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
z	0.7	-0.4	...	0.1
-	0.0	0.9	...	-0.5

Key Point

Each character maps to a K-dimensional vector.

Learnable Parameters

- **Embedding Matrix:** $27 \times \mathrm{K}$ parameters

Learnable Parameters

- **Embedding Matrix:** $27 \times$ K parameters
    - Initially random

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters
  - Initially random
  - Updated during training via backpropagation

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters
  - Initially random
  - Updated during training via backpropagation
  - Learns meaningful character representations

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters
    - Initially random
    - Updated during training via backpropagation
    - Learns meaningful character representations
- **Neural Network Weights:** MLP parameters

Learnable Parameters

- **Embedding Matrix:** $27 \times$ K parameters
  - Initially random
  - Updated during training via backpropagation
  - Learns meaningful character representations
- **Neural Network Weights:** MLP parameters
  - Transform concatenated embeddings to output

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters
  - Initially random
  - Updated during training via backpropagation
  - Learns meaningful character representations
- **Neural Network Weights:** MLP parameters
  - Transform concatenated embeddings to output
  - Learn classification patterns

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters
  - Initially random
  - Updated during training via backpropagation
  - Learns meaningful character representations
- **Neural Network Weights:** MLP parameters
  - Transform concatenated embeddings to output
  - Learn classification patterns
- **Total Learnable Parameters:**

Learnable Parameters

- **Embedding Matrix:** $27 \times K$ parameters
  - Initially random
  - Updated during training via backpropagation
  - Learns meaningful character representations
- **Neural Network Weights:** MLP parameters
  - Transform concatenated embeddings to output
  - Learn classification patterns
- **Total Learnable Parameters:**
  - Embedding: $27 \times K$

Learnable Parameters

- **Embedding Matrix:** 27 $\times$ K parameters
  - Initially random
  - Updated during training via backpropagation
  - Learns meaningful character representations
- **Neural Network Weights:** MLP parameters
  - Transform concatenated embeddings to output
  - Learn classification patterns
- **Total Learnable Parameters:**
  - Embedding: 27 $\times$ K
  - MLP: (context_size $\times$ K) $\rightarrow$ hidden $\rightarrow$ ... $\rightarrow$ 27

Neural Network Architecture

Embedding Matrix (27 × 2)

Input: X = [“a”, “b”, “i”]

	D1	D2
a	0.2	-0.1
b	-0.3	0.5
...	...	...
i	0.1	0.3
...	...	...
z	0.7	-0.4
-	0.0	0.9

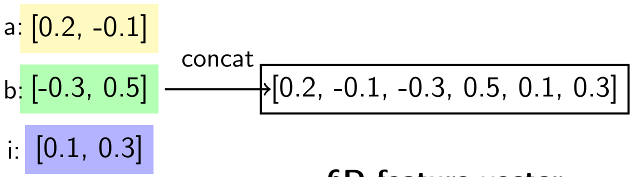
[0.2, -0.1]

[-0.3, 0.5]

[0.1, 0.3]

Concatenate the Embeddings

Feature Vector Construction

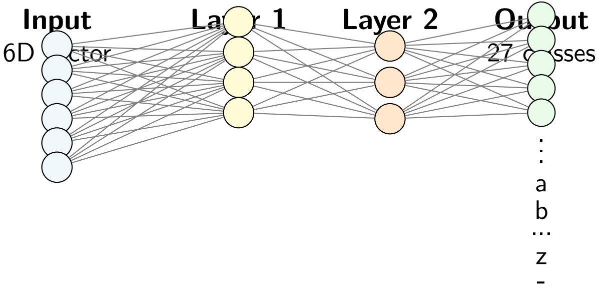


6D feature vector

Result

3 chars $\times$ 2D embeddings $=$ 6D input to neural network

Multi-Layer Perceptron Architecture



Training and Loss Function

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N}\sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \quad (1)$$

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N}\sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \qquad (1)$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations (27 $\times$ K parameters)

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations (27 $\times$ K parameters)
  2. **MLP Weights:** Neural network parameters for classification

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N}\sum_{c=1}^{27} y_{i,c}\log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations ($27 \times$ K parameters)
  2. **MLP Weights:** Neural network parameters for classification
- **Training Process:**

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations ($27 \times$ K parameters)
  2. **MLP Weights:** Neural network parameters for classification
- **Training Process:**
  1. Forward pass: Input $\rightarrow$ Embeddings $\rightarrow$ Concatenate $\rightarrow$ MLP $\rightarrow$ Probabilities

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations ($27 \times$ K parameters)
  2. **MLP Weights:** Neural network parameters for classification
- **Training Process:**
  1. Forward pass: Input $\rightarrow$ Embeddings $\rightarrow$ Concatenate $\rightarrow$ MLP $\rightarrow$ Probabilities
  2. Compute cross-entropy loss

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations ($27 \times$ K parameters)
  2. **MLP Weights:** Neural network parameters for classification
- **Training Process:**
  1. Forward pass: Input $\rightarrow$ Embeddings $\rightarrow$ Concatenate $\rightarrow$ MLP $\rightarrow$ Probabilities
  2. Compute cross-entropy loss
  3. Backward pass: Update both embeddings and MLP weights

Training Objective

- **Loss Function:** Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c}) \tag{1}$$

- **What we learn:**
  1. **Embedding Matrix:** Character representations ($27 \times$ K parameters)
  2. **MLP Weights:** Neural network parameters for classification
- **Training Process:**
  1. Forward pass: Input $\rightarrow$ Embeddings $\rightarrow$ Concatenate $\rightarrow$ MLP $\rightarrow$ Probabilities
  2. Compute cross-entropy loss
  3. Backward pass: Update both embeddings and MLP weights
  4. Repeat for all training examples

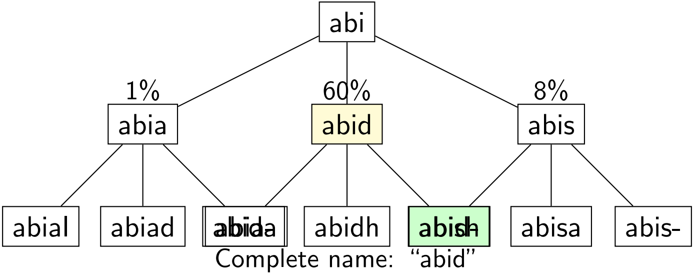
Text Generation

Sampling from the Learned Model

Test Input: "abi"

Predicted Probability Distribution

Next Char	Probability	Next Char	Probability
a	0.01	n	0.05
b	0.01	o	0.02
c	0.03	p	0.01
d	**0.60**	q	0.00
e	0.02	r	0.03
f	0.01	s	0.08
...	...	...	...
-	0.05	z	0.01



Recursive Process: Sample next character, append, repeat until end token

Temperature and Sampling Strategies

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

- **Temperature-scaled Softmax:**

$$P(y_i) = \frac{e^{z_i/T}}{\sum_{j=1}^{27} e^{z_j/T}} \tag{3}$$

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

- **Temperature-scaled Softmax:**

$$P(y_i) = \frac{e^{z_i/T}}{\sum_{j=1}^{27} e^{z_j/T}} \tag{3}$$

- **Temperature Effects:**

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

- **Temperature-scaled Softmax:**

$$P(y_i) = \frac{e^{z_i/T}}{\sum_{j=1}^{27} e^{z_j/T}} \tag{3}$$

- **Temperature Effects:**
  - $T = 1$: Standard probabilities

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

- **Temperature-scaled Softmax:**

$$P(y_i) = \frac{e^{z_i/T}}{\sum_{j=1}^{27} e^{z_j/T}} \tag{3}$$

- **Temperature Effects:**
  - $T = 1$: Standard probabilities
  - $T \to 0$: More peaked (deterministic)

Temperature in Softmax

- **Standard Softmax:**

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}} \tag{2}$$

- **Temperature-scaled Softmax:**

$$P(y_i) = \frac{e^{z_i/T}}{\sum_{j=1}^{27} e^{z_j/T}} \tag{3}$$

- **Temperature Effects:**
  - $T = 1$: Standard probabilities
  - $T \to 0$: More peaked (deterministic)
  - $T \to \infty$: More uniform (random)

Temperature Variations

Context: "abi" → Next character probabilities

Char	T=0.5 (Low)	T=1.0 (Default)	T=2.0 (High)
a	0.001	0.01	0.08
d	**0.95**	**0.60**	**0.25**
s	0.01	0.08	0.12
h	0.005	0.03	0.09
-	0.02	0.05	0.11
others	0.015	0.23	0.35

- **Low T:** Conservative, predictable

Context: "abi" $\rightarrow$ Next character probabilities

Char	T=0.5 (Low)	T=1.0 (Default)	T=2.0 (High)
a	0.001	0.01	0.08
d	**0.95**	**0.60**	**0.25**
s	0.01	0.08	0.12
h	0.005	0.03	0.09
-	0.02	0.05	0.11
others	0.015	0.23	0.35

- **Low T:** Conservative, predictable
- **High T:** Creative, diverse

Summary and Applications

Key Takeaways

- **Core Idea:** Next token prediction as classification

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights
- **Generation:** Autoregressive sampling with temperature control

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights
- **Generation:** Autoregressive sampling with temperature control
- **Applications:** Foundation for modern language models

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights
- **Generation:** Autoregressive sampling with temperature control
- **Applications:** Foundation for modern language models
    - GPT models use the same principle

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights
- **Generation:** Autoregressive sampling with temperature control
- **Applications:** Foundation for modern language models
  - GPT models use the same principle
  - Scaled to words/subwords instead of characters

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights
- **Generation:** Autoregressive sampling with temperature control
- **Applications:** Foundation for modern language models
  - GPT models use the same principle
  - Scaled to words/subwords instead of characters
  - Transformer architecture instead of MLP

Key Takeaways

- **Core Idea:** Next token prediction as classification
- **Representation Learning:** Character embeddings capture similarity
- **Architecture:** Embeddings + MLP for sequence modeling
- **Training:** Joint learning of embeddings and classifier weights
- **Generation:** Autoregressive sampling with temperature control
- **Applications:** Foundation for modern language models
  - GPT models use the same principle
  - Scaled to words/subwords instead of characters
  - Transformer architecture instead of MLP
  - Billions of parameters instead of thousands

From Character-Level to ChatGPT

Our Model

Character-level

MLP

27 vocab

$\sim$1K params

Scale up $\longrightarrow$

ChatGPT

Token-level

Transformer

50K+ vocab

$\sim$175B params

Same fundamental principle: Predict the next token!