Matrix Factorization for Movie Recommendation Systems

Nipun Batra

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IIT Gandhinagar

Today's Learning Journey

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- **Step-by-Step**: Building intuition with examples
- **Algorithms**: ALS vs Gradient Descent
- **Practice**: Hands-on understanding

Problem Setup

The Movie Recommendation Challenge

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- You've rated 100 movies out of 15,000

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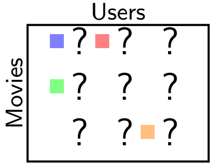

Sparse Rating Matrix

Pop Quiz 1: Understanding the Scale

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But typical users rate only 20-100 movies. What percentage of the matrix is filled?
Answer: $\frac{100}{15000} = 0.67\%$ - extremely sparse!

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- **Notation**: $\Omega = \{(i, j) : a_{ij} \text{ is observed}\}$

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Key Insight: Latent Features

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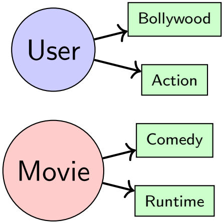

Latent Features

Step 1: Define Movie Features Explicitly

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Sholay	0.95	0.10	0.85
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Batman	0.05	0.80	0.30
Interstellar	0.05	0.95	0.70
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Movie Feature Matrix H $\in \mathbb{R}^{3\times5}$:

$$\mathbf{H} = \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix}$$

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Key Question: How do we learn these w_{ij} values from observed ratings?

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The Magic Formula:

$$\text{Alice's rating} = \text{Alice's preferences} \cdot \text{Sholay's features}$$

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$$\hat{a}_{11} = \mathbf{w}_1^T \mathbf{h}_1 \tag{1}$$

$$= w_{11} \cdot 0.95 + w_{12} \cdot 0.10 + w_{13} \cdot 0.85 \tag{2}$$

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Goal: Find w_{11}, w_{12}, w_{13} such that $\hat{a}_{11} \approx 5$ (Alice's actual rating)

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Pop Quiz 2: Matrix Dimensions

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Key Insight: If $r \ll \min(N, M)$, we have huge parameter reduction!

Learning the Factorization

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Why This is Challenging

Problem Characteristics:

- **Non-convex:** Multiple local minima exist

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Key Insight: While non-convex jointly, it's convex in each matrix individually!

Algorithm 1: Alternating Least Squares (ALS)

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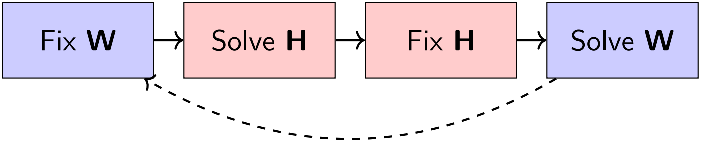

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Update Alice's preferences ($\mathbf{w}_1$):

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$$\mathbf{X}_1 = \begin{bmatrix} 0.95 & 0.10 & 0.85 \\ 1.00 & 0.20 & 0.90 \\ 0.05 & 0.80 & 0.30 \\ 0.05 & 0.95 & 0.70 \\ 0.05 & 0.15 & 0.95 \end{bmatrix} \tag{6}$$

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ALS: Complete Algorithm

Algorithm 1: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

1. **Initialize:** $\mathbf{W}^{(0)} \in \mathbb{R}^{N \times r}$, $\mathbf{H}^{(0)} \in \mathbb{R}^{r \times M}$ randomly

Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

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3. **Check Convergence:** Stop if $\|\mathbf{W}^{(t)}\mathbf{H}^{(t)} - \mathbf{W}^{(t-1)}\mathbf{H}^{(t-1)}\|_F < \epsilon$

Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

Algorithm 2: Gradient Descent

Simultaneous Updates: Update both $\mathbf{W}$ and $\mathbf{H}$ together

Gradient Descent Approach

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Gradients:

$$\frac{\partial L}{\partial \mathbf{w}_i} = -2 \sum_{j:(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)\mathbf{h}_j \tag{9}$$

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- Learning rate α controls step size

SGD: Step-by-Step Example

Example: Alice rates Sholay as 5, but we predict 3.2

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$$\text{Current: } \mathbf{w}_1 = [0.4, 0.2, 0.3], \quad \mathbf{h}_1 = [0.95, 0.10, 0.85] \tag{13}$$

$$\text{Prediction: } \hat{a}_{11} = 0.4 \times 0.95 + 0.2 \times 0.10 + 0.3 \times 0.85 = 0.655 \tag{14}$$

$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

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Updates with $\alpha = 0.01$:

$$\mathbf{w}_1 \leftarrow [0.4, 0.2, 0.3] + 0.01 \times 4.345 \times [0.95, 0.10, 0.85] \tag{16}$$

$$= [0.4413, 0.2043, 0.3369] \tag{17}$$

$$\mathbf{h}_1 \leftarrow [0.95, 0.10, 0.85] + 0.01 \times 4.345 \times [0.4, 0.2, 0.3] \tag{18}$$

$$= [0.9674, 0.1087, 0.8631] \tag{19}$$

Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

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Answers:

1. $e_{ij} = 2 - 4.5 = -2.5$

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Answers:

1. $e_{ij} = 2 - 4.5 = -2.5$
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Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

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Algorithm Comparison and Practical Considerations

ALS vs SGD: Head-to-Head Comparison

Aspect	ALS	SGD
Updates	Alternating	Simultaneous
Convergence	Faster, more stable	Slower, can oscillate
Parallelization	Excellent	Limited
Memory	Higher	Lower
Implementation	Complex	Simple
Hyperparameters	Few (rank r)	Many (α, schedule)
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Regularization: Prevent overfitting

$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

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Bias Terms: Account for global, user, and item biases

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Hands-On Understanding

Let's Build Intuition: Small Example

Our 3×3 rating matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & ? & 2 \\ 4 & 4 & ? \\ ? & 5 & 1 \end{bmatrix}$$

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Constraint: Only minimize error on observed entries!

Step-by-Step ALS Solution

Iteration 1: Initialize randomly

$$\mathbf{W}^{(0)} = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}, \quad \mathbf{H}^{(0)} = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 0.3 & 1.2 & 0.8 \end{bmatrix}$$

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Solve: $\mathbf{w}_1^{(1)} = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$

Continue for all users and movies...

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Master Check

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- 200M users, 15K movies

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Summary and Key Takeaways

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$$\boxed{\text{Collaborative Filtering} = \text{Matrix Factorization} = \text{Dimensionality Reductio}}$$

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Questions?

Thank you for your attention!

Next: Deep learning approaches to recommendation systems