Matrix Factorization for Movie Recommendation Systems

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Today's Learning Journey

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- **Step-by-Step**: Building intuition with examples
- **Algorithms**: ALS vs Gradient Descent
- **Practice**: Hands-on understanding

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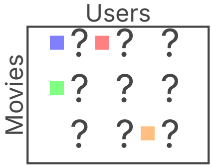

Sparse Rating Matrix

Pop Quiz 1: Understanding the Scale

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But typical users rate only 20-100 movies. What percentage of the matrix is filled?
Answer: $\frac{100}{15000} = 0.67\%$ - extremely sparse!

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- **Notation**: $\Omega = \{(i, j) : a_{ij} \text{ is observed}\}$

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- But we don’t know these explicitly!

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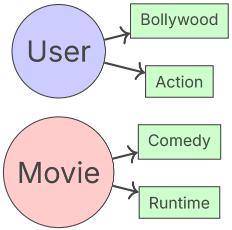

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Batman	0.05	0.80	0.30
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Movie Feature Matrix $\mathbf{H} \in \mathbb{R}^{3\times 5}$:

$$\mathbf{H} = \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix}$$

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Key Question: How do we learn these w_{ij} values from observed ratings?

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$$\mathbf{A}_{3\times 5} = \begin{bmatrix} 5 & 4 & 2 & 3 & 2 \\ ? & 5 & 1 & 4 & ? \\ 4 & ? & 1 & 5 & ? \end{bmatrix} \approx$$

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix} =$$

$$\mathbf{W}_{3\times 3}\mathbf{H}_{3\times 5}$$

Step 4: Understanding the Calculation

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The Magic Formula:

$$\text{Alice's rating} = \text{Alice's preferences} \cdot \text{Sholay's features}$$

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Goal: Find w_{11}, w_{12}, w_{13} such that $\hat{a}_{11} \approx 5$ (Alice's actual rating)

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1. What are the dimensions of $\mathbf{A}$?
2. What are the dimensions of $\mathbf{W}$?
3. What are the dimensions of $\mathbf{H}$?
4. How many parameters do we need to learn?

Answers:

1. $\mathbf{A} \in \mathbb{R}^{N \times M}$

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Key Insight: If $r \ll \min(N, M)$, we have huge param-

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- Ω: set of observed (i, j) pairs

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Problem Characteristics:

- **Non-convex:** Multiple local minima exist

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Key Insight: While non-convex jointly, it's convex in each matrix individually!

ALS: The Big Picture

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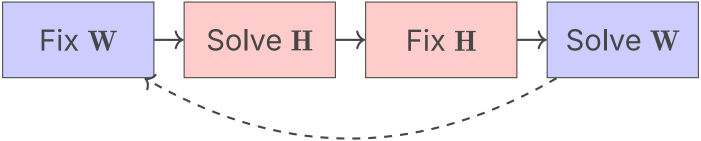

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Least Squares Solution:

$$\boxed{\mathbf{w}_i^* = (\mathbf{X}_i^T \mathbf{X}_i)^{-1} \mathbf{X}_i^T \mathbf{y}_i}$$

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Update Alice's preferences (w_1):
Alice rated: Sholay(5), Swades(4), Batman(2), Interstellar(3), Shawshank(2)

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$$\mathbf{X}_1 = \begin{bmatrix} 0.95 & 0.10 & 0.85 \\ 1.00 & 0.20 & 0.90 \\ 0.05 & 0.80 & 0.30 \\ 0.05 & 0.95 & 0.70 \\ 0.05 & 0.15 & 0.95 \end{bmatrix} \tag{6}$$

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Solution: $\mathbf{w}_1^* = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$
This gives us Alice's feature preferences!

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ALS: Complete Algorithm

Algorithm 1: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

1. **Initialize:** $\mathbf{W}^{(0)} \in \mathbb{R}^{N \times r}$, $\mathbf{H}^{(0)} \in \mathbb{R}^{r \times M}$ randomly

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2. **For** $t = 1, 2, \ldots, T$:

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Algorithm 4: [

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   2) **Update Movies:** For each movie $j = 1, \ldots, M$:

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3. **Check Convergence:** Stop if $\|\mathbf{W}^{(t)}\mathbf{H}^{(t)} - \mathbf{W}^{(t-1)}\mathbf{H}^{(t-1)}\|_F < \epsilon$

Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

Gradient Descent Approach

Simultaneous Updates: Update both $\mathbf{W}$ and $\mathbf{H}$ together

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Gradients:

$$\frac{\partial L}{\partial \mathbf{w}_i} = -2 \sum_{j:(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)\mathbf{h}_j \tag{9}$$

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SGD: Learning Like a Human

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$$\mathbf{w}_i \leftarrow \mathbf{w}_i + \alpha \cdot e_{ij} \cdot \mathbf{h}_j \tag{11}$$

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- Learning rate α controls step size

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Example: Alice rates Sholay as 5, but we predict 3.2

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$$\text{Current: } \mathbf{w}_1 = [0.4, 0.2, 0.3], \quad \mathbf{h}_1 = [0.95, 0.10, 0.85] \tag{13}$$

$$\text{Prediction: } \hat{a}_{11} = 0.4 \times 0.95 + 0.2 \times 0.10 + 0.3 \times 0.85 = 0.655 \tag{14}$$

$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

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$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

Updates with $\alpha = 0.01$:

$$\mathbf{w}_1 \leftarrow [0.4, 0.2, 0.3] + 0.01 \times 4.345 \times [0.95, 0.10, 0.85] \tag{16}$$

$$= [0.4413, 0.2043, 0.3369] \tag{17}$$

$$\mathbf{h}_1 \leftarrow [0.95, 0.10, 0.85] + 0.01 \times 4.345 \times [0.4, 0.2, 0.3] \tag{18}$$

$$= [0.9674, 0.1087, 0.8631] \tag{19}$$

Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

Pop Quiz 3: SGD Understanding

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A user gives a rating of 2 to a movie, but our model predicts 4.5.

1. What is the error e_{ij}?

Pop Quiz 3: SGD Understanding

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ALS vs SGD: Head-to-Head Comparison

Aspect	**ALS**	**SGD**
Updates	Alternating	Simultaneous
Convergence	Faster, more stable	Slower, can oscillate
Parallelization	Excellent	Limited
Memory	Higher	Lower
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When to Use Which?

- **ALS:** Large-scale, production systems (Spark, distributed)
- **SGD:** Online learning, real-time updates, research

Advanced Practical Considerations

Regularization: Prevent overfitting

$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

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Our 3×3 rating matrix:

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Step-by-Step ALS Solution

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Solve: $\mathbf{w}_1^{(1)} = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$
Continue for all users and movies...

Pop Quiz 4: Final Challenge

Master Check

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- 200M users, 15K movies

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The Mathematical Beauty:

$$\boxed{\text{Collaborative Filtering} = \text{Matrix Factorization} = \text{Dimensionality R}}$$

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Questions?

Thank you for your attention!

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