Ridge Regression

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$\ln f(x) = c_0 + c_1 x + c_2 x^2 + \ldots$ it is $\max |c_i|$

Objective:

$$\text{Minimise } (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})$$
$$\text{s.t. } \boldsymbol{\theta}^T \boldsymbol{\theta} \leq S$$

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This is equivalent to

$$\text{Minimise } (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\theta}) + \delta^2\boldsymbol{\theta}^T\boldsymbol{\theta}$$

$$
\begin{aligned}
\text{Minimise } & (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\theta}) \\
\text{s.t. } & \boldsymbol{\theta}^T \boldsymbol{\theta} \leq S
\end{aligned}
$$

$$
L(\boldsymbol{\theta}, \mu) = (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\theta}) + \mu \left(\boldsymbol{\theta}^T \boldsymbol{\theta} - S \right)
$$

where, $\mu \geq 0$ (and $\mu = \delta^2$)

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There is no
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If $\mu \neq 0$
$\implies \boldsymbol{\theta}^T\boldsymbol{\theta} - S = 0$

Are θ_i all zero for high μ?

$$\mathbf{X}^T\mathbf{X} = \begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix}$$

$$(\mathbf{X}^T\mathbf{X})^{-1} = \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}$$

$$\mathbf{X}^T\mathbf{y} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$

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$$\mathbf{X}^T\mathbf{X} + \mu\mathbf{I} = \begin{bmatrix} 6 & 10 \\ 10 & 32 \end{bmatrix}$$

$$(\mathbf{X}^T\mathbf{X} + \mu\mathbf{I})^{-1} = \frac{1}{92}\begin{bmatrix} 32 & -10 \\ -10 & 6 \end{bmatrix}$$

$$\mathbf{X}^T\mathbf{y} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$

Another interpretation of “regularisation”

- Contrast with update equation for unregularised regression:

- Contrast with update equation for unregularised regression:
- $\boldsymbol{\theta} = \underbrace{\boldsymbol{\theta}}_{\text{No Shrinking } \boldsymbol{\theta}} - \alpha(-2\mathbf{X}^\top \mathbf{y} + 2\mathbf{X}^\top \mathbf{X}\boldsymbol{\theta})$