Decision Trees

Nipun Batra and teaching staff

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IIT Gandhinagar

Discrete Input Discrete Output

How to maintain trust in AI

Beyond developing initial trust, however, creators of AI also must work to maintain that trust. Siau and Wang suggest seven ways of "developing continuous trust" beyond the initial phases of product development:

- Usability and reliability. AI "should be designed to operate easily and intuitively," Siau and Wang write. "There should be no unexpected downtime or crashes."

- Collaboration and communication. AI developers want to create systems that perform autonomously, without human involvement. Developers must focus on creating AI applications that smoothly and easily collaborate and communicate with humans.

- Sociability and bonding. Building social activities into AI applications is one way to strengthen trust. A robotic dog that can recognize its owner and show affection is one example, Siau and Wang write.

- Security and privacy protection. AI applications rely on large data sets, so ensuring privacy and security will be crucial to establishing trust in the applications.

- Interpretability. Just as transparency is instrumental in building initial trust, interpretability – or the ability for a machine to explain its conclusions or actions – will help sustain trust.

Training Data

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Learnt Decision Tree

Medical Diagnosis using Decision Trees
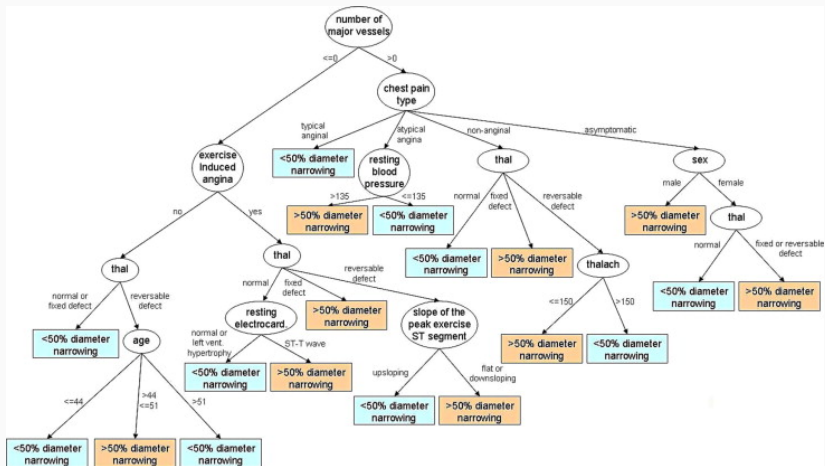
Source: Improving medical decision trees by combining relevant health-care criteria

Leo Brieman

Leo Breiman 1928-2005

Professor of Statistics, UC Berkeley
Verified email at stat.berkeley.edu - Homepage
Data Analysis   Statistics   Machine Learning

TITLE	CITED BY	YEAR
Random forests L Breiman Machine learning 45 (1), 5-32	53816	2001
Classification and Regression Trees L Breiman, JH Friedman, RA Olshen, CJ Stone CRC Press, New York	43992 *	1999
Classification and regression trees L Breiman Chapman & Hall/CRC	43992 *	1984
Bagging predictors L Breiman Machine learning 24 (2), 123-140	22742	1996
Statistical Modeling: The Two Cutures L Breiman	2788 *	2003
Statistical modeling: The two cultures (with comments and a rejoinder by the author) L Breiman Statistical Science 16 (3), 199-231	2772	2001
Estimating optimal transformations for multiple regression and correlation	2096	1985

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Optimal Decision Tree

CONSTRUCTING OPTIMAL BINARY DECISION TREES IS NP-COMPLETE[*]

Laurent HYAFIL

IRIA – Laboria, 78150 Rocquencourt, France

and

Ronald L. RIVEST
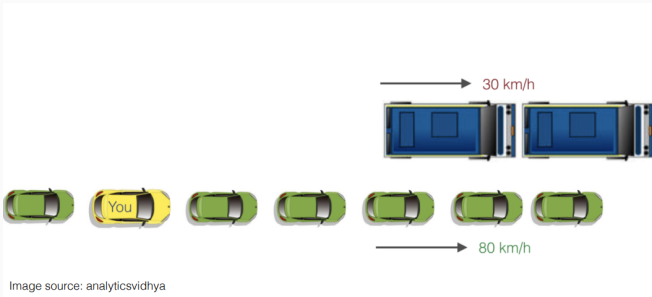
Dept. of Electrical Engineering and Computer Science, M.I.T., Cambridge, Massachusetts 02139, USA

7

Greedy Algorithm

Core idea: At each level, choose an attribute that gives **biggest estimated** performance gain!

Greedy!=Optimal

Towards biggest estimated performance gain

Towards biggest estimated performance gain

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- Would it be trivial if we had 14 Yes or 14 No?

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- Key insight: Problem is "easier" when there is less disagreement

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- For examples, we have 9 Yes, 5 No
- Would it be trivial if we had 14 Yes or 14 No?
- Yes!
- Key insight: Problem is "easier" when there is less disagreement
- Need some statistical measure of "disagreement"

Entropy

Statistical measure to characterize the (im)purity of examples

Entropy

Statistical measure to characterize the (im)purity of examples

$$H(X) = - \sum_{i=1}^{k} p(x_i) \log_2 p(x_i)$$

Notebook: entropy.html

Entropy vs. $P(+)$

Towards biggest estimated performance gain

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- Can we use Outlook as the root node?

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- Can we use Outlook as the root node?

- When Outlook is overcast, we always Play and thus no "disagreement"

Information Gain

Reduction in entropy by partitioning examples (S) on attribute A

$$\text{Gain}(S, A) \equiv \text{Entropy}(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

ID3 (Examples, Target Attribute, Attributes)

- Create a root node for tree

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    - $A \leftarrow$ attribute from Attributes which best classifies Examples

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    - $A \leftarrow$ attribute from Attributes which best classifies Examples
    - Root $\leftarrow A$
    - For each value (v) of A

ID3 (Examples, Target Attribute, Attributes)

- Create a root node for tree
- If all examples are +/-, return root with label = +/-
- If attributes = empty, return root with most common value of Target Attribute in Examples
- Begin
  - A ← attribute from Attributes which best classifies Examples
  - Root ← A
  - For each value (v) of A
    - Add new tree branch : A = v

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  - For each value (v) of A
    - Add new tree branch : $A = v$
    - Examples$_v$: subset of examples that $A = v$

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- If all examples are $+$/-, return root with label $= +$/-
- If attributes $=$ empty, return root with most common value of Target Attribute in Examples
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  - A $\leftarrow$ attribute from Attributes which best classifies Examples
  - Root $\leftarrow$ A
  - For each value (v) of A
    - Add new tree branch : A $=$ v
    - Examples$_v$: subset of examples that A $=$ v
    - If Examples$_v$ is empty: add leaf with label $=$ most common value of Target Attribute

ID3 (Examples, Target Attribute, Attributes)

- Create a root node for tree
- If all examples are +/-, return root with label = +/-
- If attributes = empty, return root with most common value of Target Attribute in Examples
- Begin
    - A ← attribute from Attributes which best classifies Examples
    - Root ← A
    - For each value (v) of A
        - Add new tree branch : A = v
        - $Examples_v$: subset of examples that A = v
        - If $Examples_v$ is empty: add leaf with label = most common value of Target Attribute
        - Else: ID3 ($Examples_v$, Target attribute, Attributes - A)

Root Node (empty)

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Entropy calculated

We have 14 examples in S: 5 No, 9 Yes

$$\text{Entropy}(S) = -p_{\text{No}} \log_2 p_{\text{No}} - p_{\text{Yes}} \log_2 p_{\text{Yes}}$$
$$= -\frac{5}{14} \log_2 \left(\frac{5}{14}\right) - \frac{9}{14} \log_2 \left(\frac{9}{14}\right) = 0.940$$

Information Gain for Outlook

Outlook	Play
Sunny	No
Sunny	No
Overcast	Yes
Rain	Yes
Rain	Yes
Rain	No
Overcast	Yes
Sunny	No
Sunny	Yes
Rain	Yes
Sunny	Yes
Overcast	Yes
Overcast	Yes
Rain	No

Information Gain for Outlook

Outlook	Play
Sunny	No
Sunny	No
Sunny	No
Sunny	Yes
Sunny	Yes

We have 2 Yes, 3 No

$$\text{Entropy} =$$
$$-\frac{3}{5} \log_2 \left(\frac{3}{5}\right) -$$
$$\frac{2}{5} \log_2 \left(\frac{2}{5}\right) = 0.971$$

Outlook	Play
Sunny	No
Sunny	No
Sunny	No
Sunny	Yes
Sunny	Yes

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Outlook	Play
Overcast	Yes
Overcast	Yes
Overcast	Yes
Overcast	Yes

We have 4 Yes, 0 No

Entropy $= 0$ (pure subset)

Information Gain for Outlook

Outlook	Play
Sunny	No
Sunny	No
Sunny	No
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Outlook	Play
Overcast	Yes
Overcast	Yes
Overcast	Yes
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We have 4 Yes, 0 No

Entropy = 0 (pure subset)

Outlook	Play
Rain	Yes
Rain	Yes
Rain	No
Rain	Yes
Rain	No

We have 3 Yes, 2 No

Entropy =
$-\frac{3}{5}\log_2\left(\frac{3}{5}\right) -$
$\frac{2}{5}\log_2\left(\frac{2}{5}\right) = 0.971$

Information Gain

$$\text{Gain}(S, \text{Outlook}) = \text{Entropy}(S) - \sum_{v \in \{\text{Rain, Sunny, Overcast}\}} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

$$\text{Gain}(S, \text{Outlook}) = \text{Entropy}(S) - \frac{5}{14} \text{Entropy}(S_{\text{Sunny}}) - \frac{4}{14} \text{Entropy}(S_{\text{Overca}}$$

$$= 0.940 - \frac{5}{14} \times 0.971 - \frac{4}{14} \times 0 - \frac{5}{14} \times 0.971 = 0.940 - 0.347 - 0 - 0.347 = 0.24$$
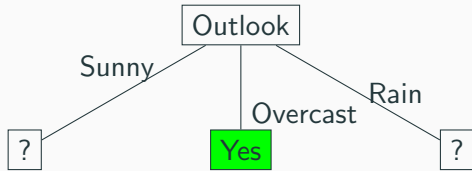
Information Gain

0.25	Outlook
0.15	Humidity
$4.8 \cdot 10^{-2}$	Wind
$2.9 \cdot 10^{-2}$	Temperature

Calling ID3 on Outlook=Sunny

Day	Temp	Humidity	Windy	Play
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

Calling ID3 on Outlook=Sunny

Day	Temp	Humidity	Windy	Play
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

- Gain($S_{\text{Outlook=Sunny}}$, Temp) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(0 Yes, 2 No) -(2/5)*Entropy(1 Yes, 1 No) - (1/5)*Entropy(1 Yes, 0 No)

Calling ID3 on Outlook=Sunny

Day	Temp	Humidity	Windy	Play
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

- $\text{Gain}(S_{\text{Outlook}=\text{Sunny}}, \text{Temp}) = \text{Entropy}(2 \text{ Yes}, 3 \text{ No}) - (2/5)*\text{Entropy}(0 \text{ Yes}, 2 \text{ No}) - (2/5)*\text{Entropy}(1 \text{ Yes}, 1 \text{ No}) - (1/5)*\text{Entropy}(1 \text{ Yes}, 0 \text{ No})$

Calling ID3 on Outlook=Sunny

Day	Temp	Humidity	Windy ‖ Play
D1	Hot	High	Weak ‖ No
D2	Hot	High	Strong ‖ No
D8	Mild	High	Weak ‖ No
D9	Cool	Normal	Weak ‖ Yes
D11	Mild	Normal	Strong ‖ Yes

- Gain($S_{\text{Outlook=Sunny}}$, Temp) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(0 Yes, 2 No) -(2/5)*Entropy(1 Yes, 1 No) - (1/5)*Entropy(1 Yes, 0 No)
- Gain($S_{\text{Outlook=Sunny}}$, Humidity) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(2 Yes, 0 No) -(3/5)*Entropy(0 Yes, 3 No)
  $\implies$ **maximum possible for the set**

Calling ID3 on Outlook=Sunny

Day	Temp	Humidity	Windy	Play
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
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- Gain($S_{\text{Outlook=Sunny}}$, Temp) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(0 Yes, 2 No) -(2/5)*Entropy(1 Yes, 1 No) - (1/5)*Entropy(1 Yes, 0 No)
- Gain($S_{\text{Outlook=Sunny}}$, Humidity) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(2 Yes, 0 No) -(3/5)*Entropy(0 Yes, 3 No)
  $\implies$ **maximum possible for the set**

Calling ID3 on Outlook=Sunny

Day	Temp	Humidity	Windy	Play
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

- Gain($S_{\text{Outlook=Sunny}}$, Temp) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(0 Yes, 2 No) -(2/5)*Entropy(1 Yes, 1 No) - (1/5)*Entropy(1 Yes, 0 No)
- Gain($S_{\text{Outlook=Sunny}}$, Humidity) = Entropy(2 Yes, 3 No) - (2/5)*Entropy(2 Yes, 0 No) -(3/5)*Entropy(0 Yes, 3 No)
  $\implies$ **maximum possible for the set**
- Gain($S_{\text{Outlook=Sunny}}$, Windy) = Entropy(2 Yes, 3 No) - (3/5)*Entropy(1 Yes, 2 No) -(2/5)*Entropy(1 Yes, 1 No)
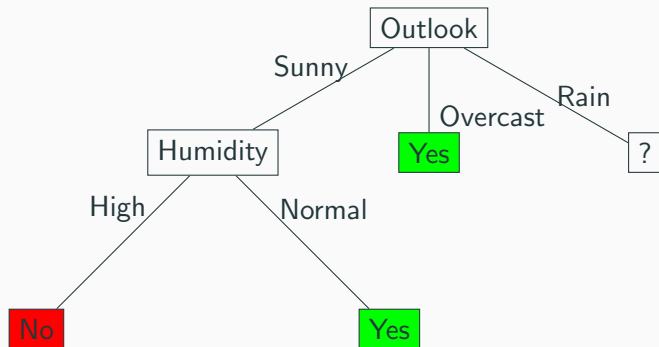
Calling ID3 on (Outlook=Rain)

Day	Temp	Humidity	Windy	Play
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

- The attribute Windy gives the highest information gain

Prediction for Decision Tree

Just walk down the tree!

Prediction for Decision Tree

Just walk down the tree!

Prediction for <High Humidity, Strong Wind, Sunny Outlook, Hot Temp> is ?

Prediction for Decision Tree

Just walk down the tree!

Prediction for <High Humidity, Strong Wind, Sunny Outlook, Hot Temp> is ?
No

Limiting Depth of Tree

Assuming if you were only allowed depth-1 trees, how would it look for the current dataset?

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Apply the same rules, except when depth limit is reached, the leaf node is assigned the most common occurring value in that path.

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node is assigned the most common occurring value in that path.

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Always predicting Yes

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What is depth-0 tree (no decision) for the examples?
Always predicting Yes

What is depth-1 tree (no decision) for the examples?

Limiting Depth of Tree

Assuming if you were only allowed depth-1 trees, how would it look for the current dataset?

Apply the same rules, except when depth limit is reached, the leaf node is assigned the most common occurring value in that path.

What is depth-0 tree (no decision) for the examples?
Always predicting Yes

What is depth-1 tree (no decision) for the examples?

Discrete Input, Real Output

Modified Dataset

Day	Outlook	Temp	Humidity	Wind	Minutes Played
D1	Sunny	Hot	High	Weak	20
D2	Sunny	Hot	High	Strong	24
D3	Overcast	Hot	High	Weak	40
D4	Rain	Mild	High	Weak	50
D5	Rain	Cool	Normal	Weak	60
D6	Rain	Cool	Normal	Strong	10
D7	Overcast	Cool	Normal	Strong	4
D8	Sunny	Mild	High	Weak	10
D9	Sunny	Cool	Normal	Weak	60
D10	Rain	Mild	Normal	Weak	40
D11	Sunny	Mild	High	Strong	45
D12	Overcast	Mild	High	Strong	40
D13	Overcast	Hot	Normal	Weak	35
D14	Rain	Mild	High	Strong	20

Measure of Impurity for Regression?

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- Any guesses?

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- $\text{MSE}(S) = 311.34$

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- **MSE Reduction** (not Information Gain!)

Measure of Impurity for Regression?

- Any guesses?
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- $MSE(S) = 311.34$
- What about splitting criterion for regression?
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Measure of Impurity for Regression?

- Any guesses?
- Mean Squared Error
- $\text{MSE}(S) = 311.34$
- What about splitting criterion for regression?
- **MSE Reduction** (not Information Gain!)
- MSE Reduction $= \text{MSE}(S) - \sum_v \frac{|S_v|}{|S|} \text{MSE}(S_v)$

Gain by splitting on Wind

Wind	Minutes Played
Weak	20
Strong	24
Weak	40
Weak	50
Weak	60
Strong	10
Strong	4
Weak	10
Weak	60
Weak	40
Strong	45
Strong	40
Weak	35
Strong	20

MSE(S)=311.34

Gain by splitting on Wind

Wind	Minutes Played
Weak	20
Strong	24
Weak	40
Weak	50
Weak	60
Strong	10
Strong	4
Weak	10
Weak	60
Weak	40
Strong	45
Strong	40
Weak	35
Strong	20

MSE(S)=311.34

Wind	Minutes Played
Weak	20
Weak	40
Weak	50
Weak	60
Weak	10
Weak	60
Weak	40
Weak	35

$MSE(S_{Wind=Weak}) = 277$, Weight $= \frac{8}{14}$

Wind	Minutes Played
Strong	24
Strong	10
Strong	4
Strong	45
Strong	40
Strong	20

$MSE(S_{Wind=Strong}) = 218$, Weight $= \frac{6}{14}$

MSE Reduction Calculation

Correct calculation for Wind split:

$$\text{MSE Reduction} = \text{MSE}(S) - \text{Weighted Average MSE}$$

$$= 311.34 - \left[\frac{8}{14} \times 277 + \frac{6}{14} \times 218\right] = 311.34 - [158.857 + 93.429] = 311.3$$

Key insight: MSE Reduction > 0 means the split improves our model!

For regression: Use MSE Reduction, NOT Information Gain!

MSE Reduction for Regression Trees

Notebook: decision-tree-real-output.html
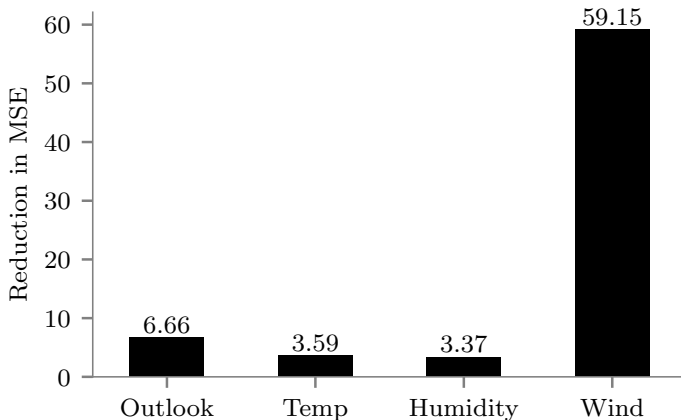
Assume a tree like this is learnt …
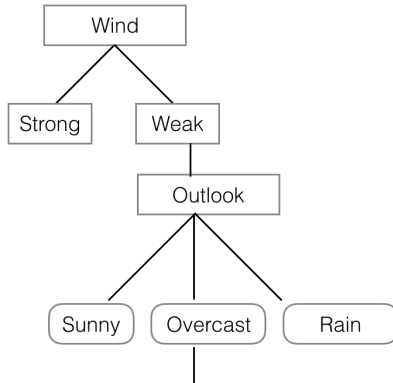
	Day	Outlook	Temp	Humidity	Wind	Minutes Played
2	D3	Overcast	Hot	High	Weak	40
12	D13	Overcast	Hot	Normal	Weak	35

Method 1
Mins
Played=(40+35)
/2

Wind

Strong   Weak

Outlook

Sunny   Overcast   Rain

37.5
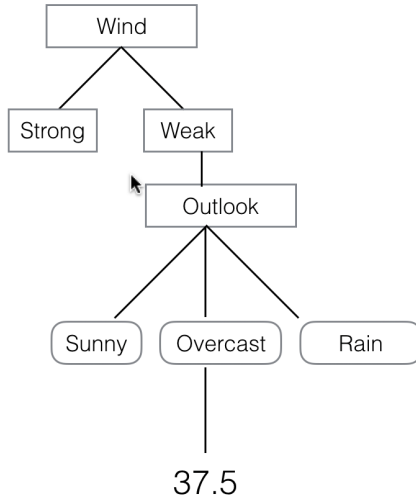
Real Input Discrete Output

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- How do you find splits?

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- How do you find splits?
- Sort by attribute

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- How do you find splits?
- Sort by attribute
- Find potential split points (midpoints).

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- How do you find splits?
- Sort by attribute
- Find potential split points (midpoints).
- For the above example, we have 5 potential splits: 44, 54, 66, 76, 85

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- How do you find splits?
- Sort by attribute
- Find potential split points (midpoints).
- For the above example, we have 5 potential splits: 44, 54, 66, 76, 85
- Calculate the weighted impurity for each split

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- How do you find splits?
- Sort by attribute
- Find potential split points (midpoints).
- For the above example, we have 5 potential splits: 44, 54, 66, 76, 85
- Calculate the weighted impurity for each split
- Choose the split with the lowest impurity

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 44

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 44
- LHS has 1 No and 0 Yes; RHS has 3 Yes and 2 No

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 44
- LHS has 1 No and 0 Yes; RHS has 3 Yes and 2 No
- Entropy for LHS $= 0$, Entropy for RHS $= 0.971$

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 44
- LHS has 1 No and 0 Yes; RHS has 3 Yes and 2 No
- Entropy for LHS = 0, Entropy for RHS = 0.971
- Weighted Entropy = 0.971*5/6 = 0.808

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 54

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 54
- LHS has 2 No and 0 Yes; RHS has 3 Yes and 1 No

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 54
- LHS has 2 No and 0 Yes; RHS has 3 Yes and 1 No
- Entropy for LHS $= 0$, Entropy for RHS $= 0.811$

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 54
- LHS has 2 No and 0 Yes; RHS has 3 Yes and 1 No
- Entropy for LHS $= 0$, Entropy for RHS $= 0.811$
- Weighted Entropy $= 0.811*4/6 = 0.541$

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 66

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 66
- LHS has 2 No and 1 Yes; RHS has 2 Yes and 1 No

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 66
- LHS has 2 No and 1 Yes; RHS has 2 Yes and 1 No
- Entropy for LHS $= 0.918$, Entropy for RHS $= 0.918$

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 66
- LHS has 2 No and 1 Yes; RHS has 2 Yes and 1 No
- Entropy for LHS = 0.918, Entropy for RHS = 0.918
- Weighted Entropy = 0.918*3/6 + 0.918*3/6 = 0.918

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 76

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 76
- LHS has 2 No and 2 Yes; RHS has 1 Yes and 1 No

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 76
- LHS has 2 No and 2 Yes; RHS has 1 Yes and 1 No
- Entropy for LHS $= 1$, Entropy for RHS $= 1$

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

- Consider split at 76
- LHS has 2 No and 2 Yes; RHS has 1 Yes and 1 No
- Entropy for LHS = 1, Entropy for RHS = 1
- Weighted Entropy = 1*4/6 + 1*2/6 = 1

Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

Notebook: decision-tree-real-input-discrete-output.html
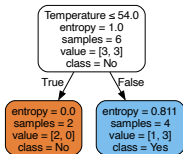
Finding splits

Day	Temperature	PlayTennis
D1	40	No
D2	48	No
D3	60	Yes
D4	72	Yes
D5	80	Yes
D6	90	No

Notebook: decision-tree-real-input-discrete-output.html
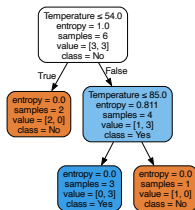
Example (DT of depth 1)

Notebook: decision-tree-real-input-discrete-output.html
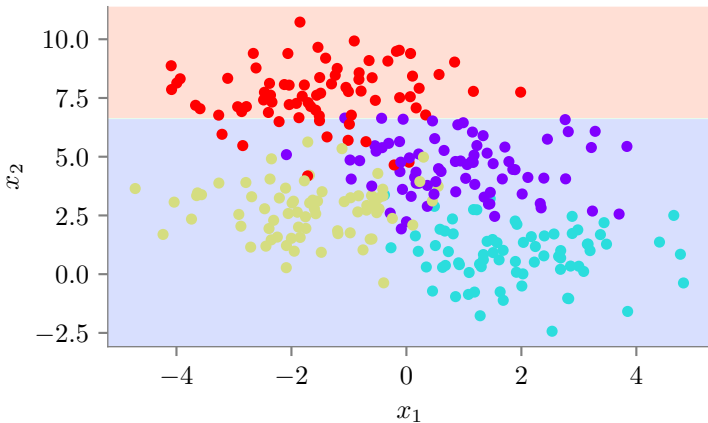
Example (DT of depth 2)

Notebook: decision-tree-real-input-discrete-output.html
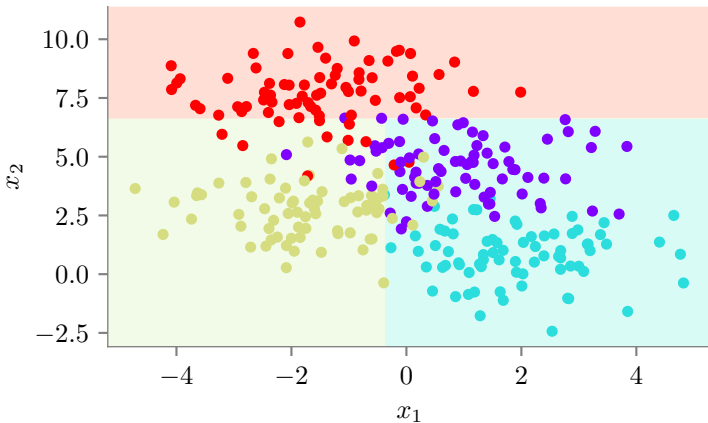
Example (DT of depth 3)

Notebook: decision-tree-real-input-discrete-output.html
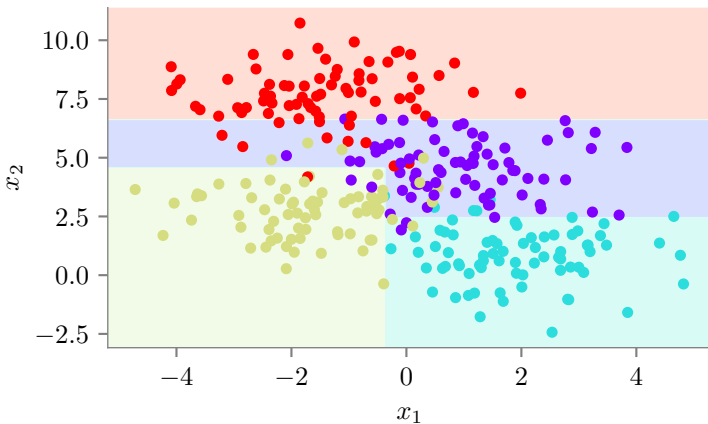
Example (DT of depth 4)

Notebook: decision-tree-real-input-discrete-output.html
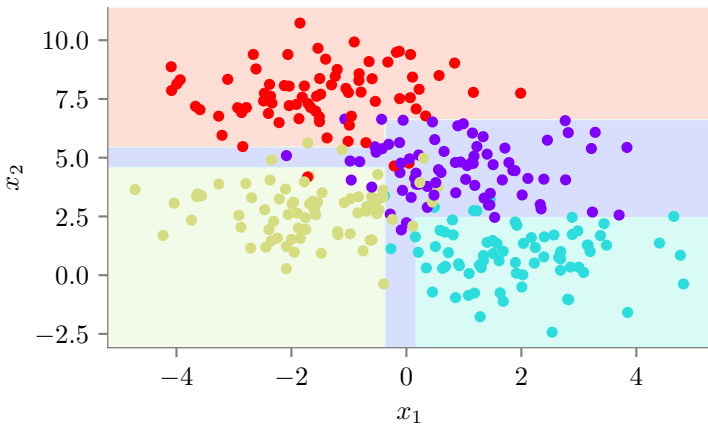
Example (DT of depth 5)

Notebook: decision-tree-real-input-discrete-output.html
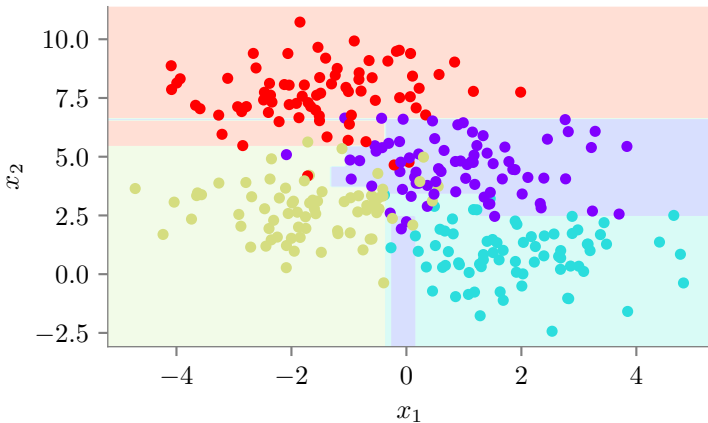
Example (DT of depth 6)

Notebook: decision-tree-real-input-discrete-output.html
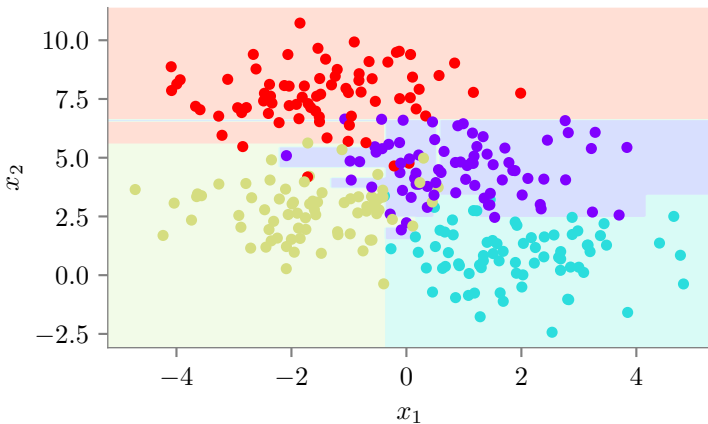
Example (DT of depth 7)

Notebook: decision-tree-real-input-discrete-output.html
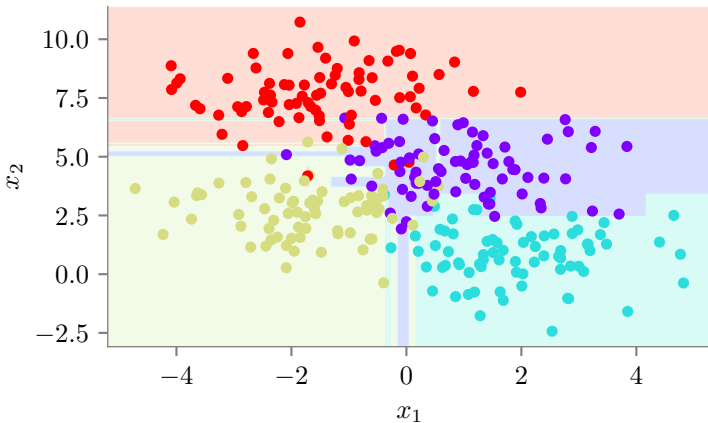
Example (DT of depth 8)

Notebook: decision-tree-real-input-discrete-output.html
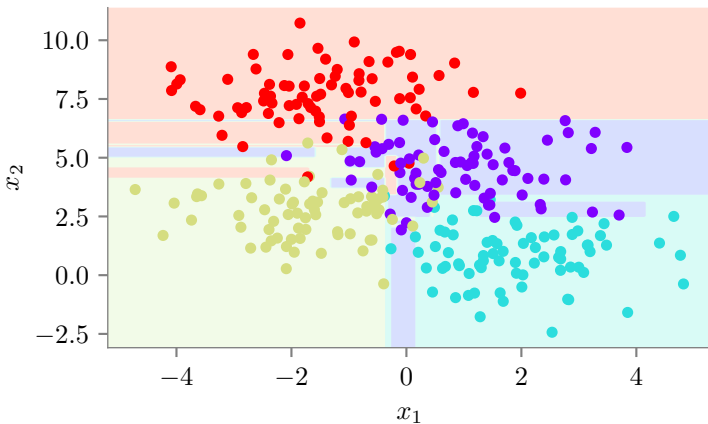
Example (DT of depth 9)

Notebook: decision-tree-real-input-discrete-output.html
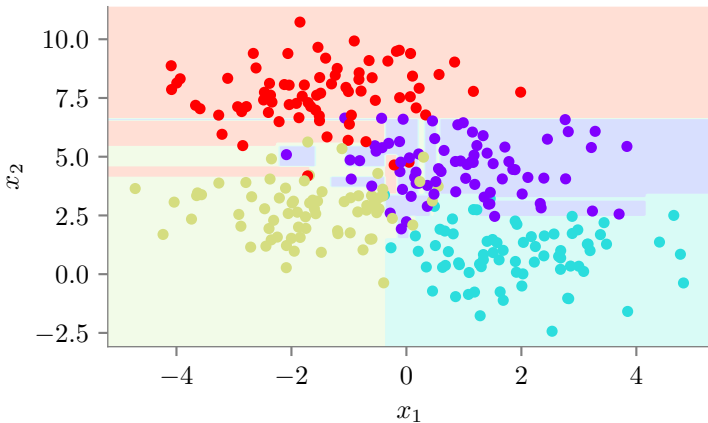
Real Input Real Output

Example 1

Let us consider the dataset given below

Notebook: decision-tree-real-input-real-output.html
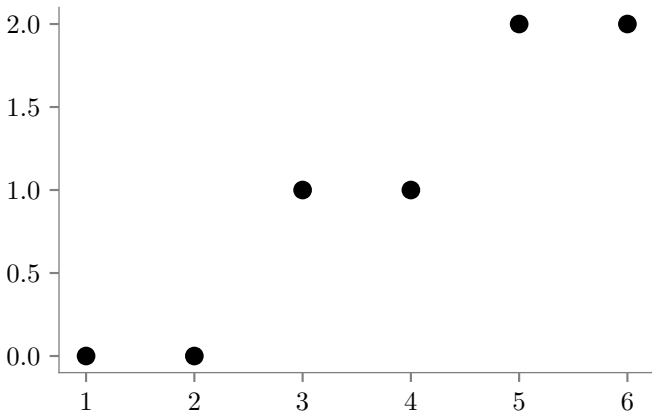
Example 1

What would be the prediction for decision tree with depth 0?

Notebook: decision-tree-real-input-real-output.html
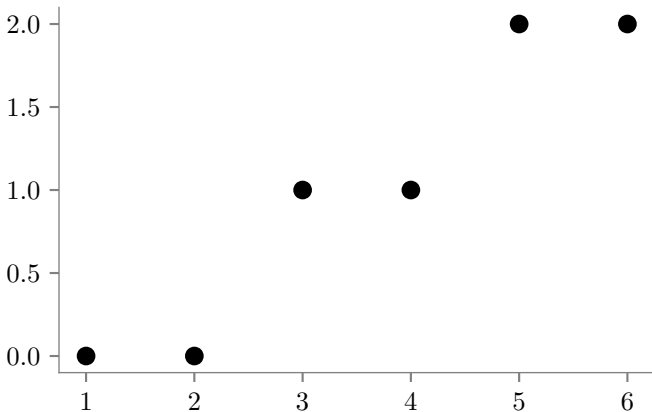
Example 1

Prediction for decision tree with depth 0.
Horizontal dashed line shows the predicted Y value. It is the average of Y values of all datapoints.

Notebook: decision-tree-real-input-real-output.html

Example 1

What would be the decision tree with depth 1?

Notebook: decision-tree-real-input-real-output.html

Example 1

Decision tree with depth 1

Notebook: decision-tree-real-input-real-output.html

Example 1

The Decision Boundary
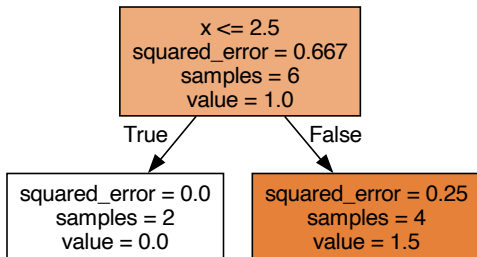
Example 1

What would be the decision tree with depth 2 ?

Notebook: decision-tree-real-input-real-output.html

Example 1

Decision tree with depth 1

Example 1

The Decision Boundary

Objective Function for Regression Trees

Feature is denoted by X and target by Y.

Let the split be at $X = s$.

Define regions: $R_1 = \{x : x \leq s\}$ and $R_2 = \{x : x > s\}$.

Objective Function for Regression Trees

Feature is denoted by X and target by Y.

Let the split be at $X = s$.

Define regions: $R_1 = \{x : x \leq s\}$ and $R_2 = \{x : x > s\}$.

For each region, compute the mean prediction:

$c_1 = \frac{1}{|R_1|} \sum_{x_i \in R_1} y_i$

$c_2 = \frac{1}{|R_2|} \sum_{x_i \in R_2} y_i$

Objective Function for Regression Trees

Feature is denoted by X and target by Y.

Let the split be at $X = s$.

Define regions: $R_1 = \{x : x \leq s\}$ and $R_2 = \{x : x > s\}$.

For each region, compute the mean prediction:

$c_1 = \frac{1}{|R_1|} \sum_{x_i \in R_1} y_i$

$c_2 = \frac{1}{|R_2|} \sum_{x_i \in R_2} y_i$

The loss function is:

$$\text{Loss}(s) = \sum_{x_i \in R_1} (y_i - c_1)^2 + \sum_{x_i \in R_2} (y_i - c_2)^2$$

Objective Function for Regression Trees

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Our objective is to find the optimal split:

Algorithm: Finding the Optimal Split

1. Sort all data points (x_i, y_i) in increasing order of x_i.

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1. Sort all data points (x_i, y_i) in increasing order of x_i.

2. Evaluate the loss function for all candidate splits:

$$s = \frac{x_i + x_{i+1}}{2} \text{ for } i = 1, 2, \ldots, n-1$$

Algorithm: Finding the Optimal Split

Draw a regression tree for $Y = \sin(X)$, $0 \leq X \leq 2\pi$

A Question!

Dataset of $Y = \sin(X)$, $0 \leq X \leq 7$ with 10,000 points

Notebook: decision-tree-real-input-real-output.html
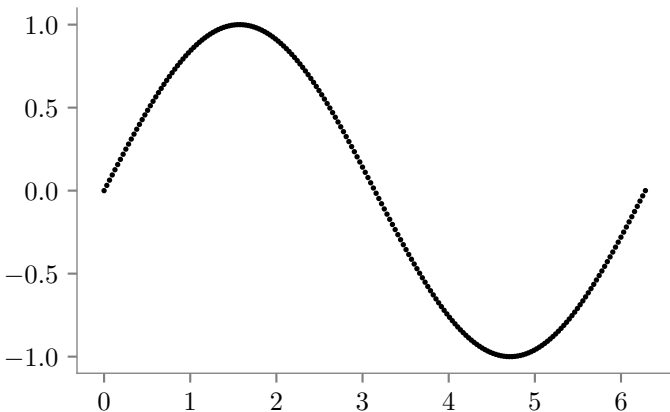
A Question!

Regression tree of depth 1

Notebook: decision-tree-real-input-real-output.html

x <= 3.142
squared_error = 0.498
samples = 200
value = -0.0

True / False

squared_error = 0.096
samples = 100
value = 0.633

squared_error = 0.096
samples = 100
value = -0.633
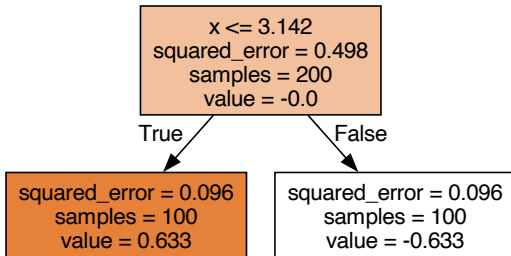
A Question!

Decision Boundary

Notebook: decision-tree-real-input-real-output.html
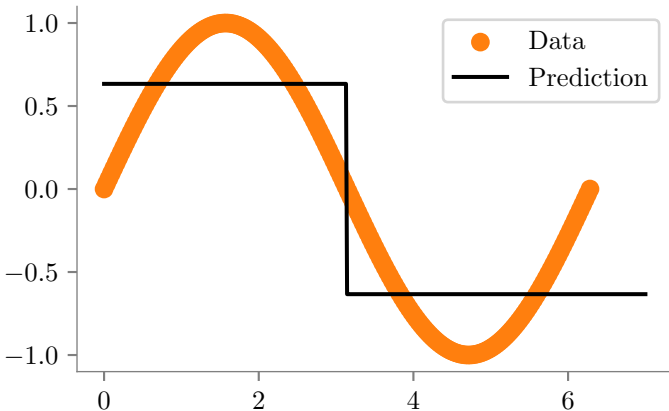
A Question!

Regression tree with no depth limit is too big to fit in a slide.
It has of depth 4. The decision boundaries are in figure below.

Notebook: decision-tree-real-input-real-output.html
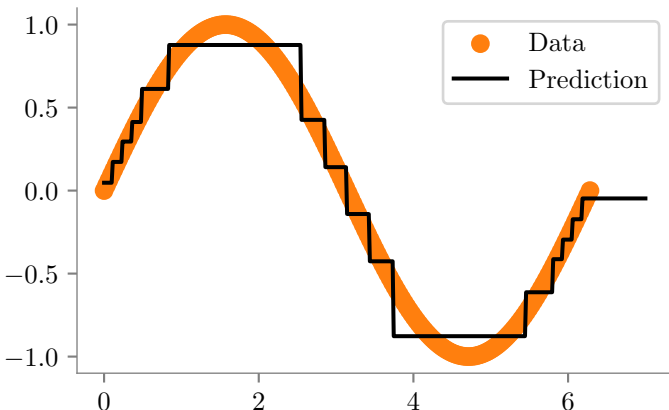
- Interpretability an important goal

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- Decision trees: well known interpretable models

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- Decision trees: well known interpretable models
- Learning optimal tree is hard
- Greedy approach:
- Recursively split to maximize "performance gain"
- Issues:
  - Can overfit easily!
  - Empirically not as powerful as other methods
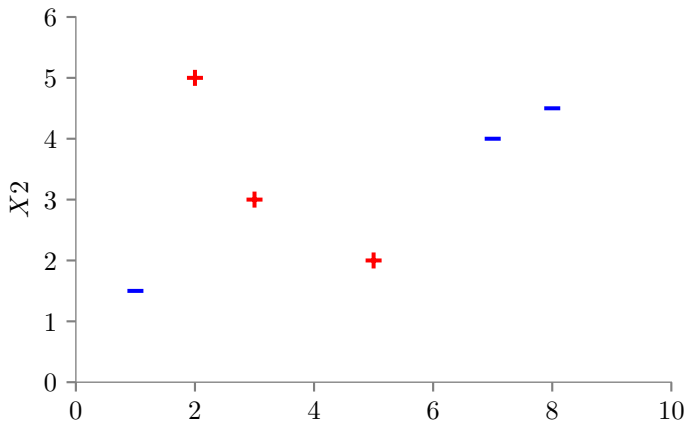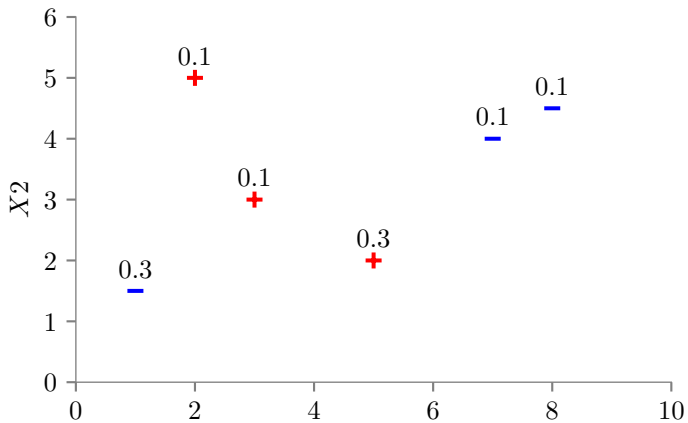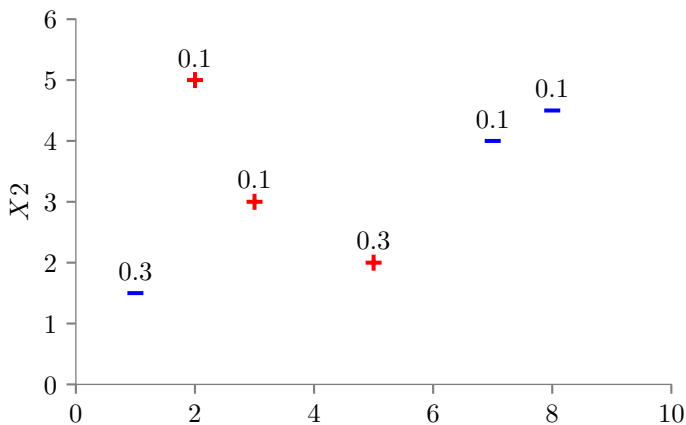
$$\text{Entropy} = -P(+)\log_2 P(+) - P(-)\log_2 P(-)$$

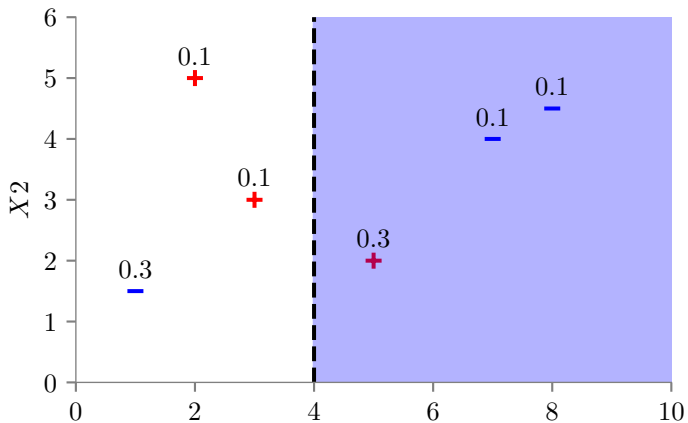$$P(+) = \frac{0.1 + 0.1 + 0.3}{1} = 0.5, \quad P(-) = \frac{0.3 + 0.1 + 0.1}{1} = 0.5$$

Weighted Entropy
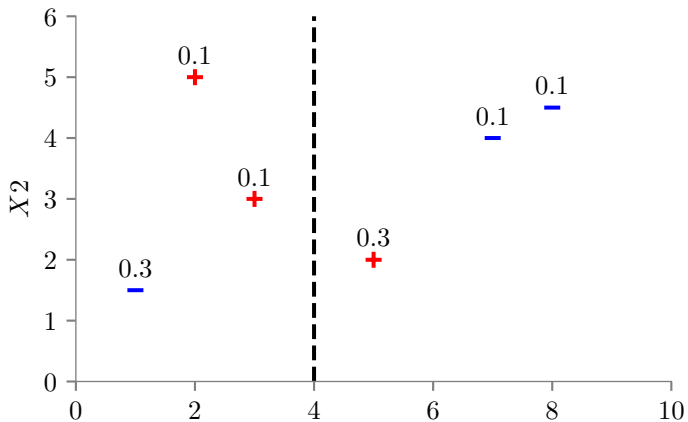
Candidate Line: $X1 = 4(X1^*)$
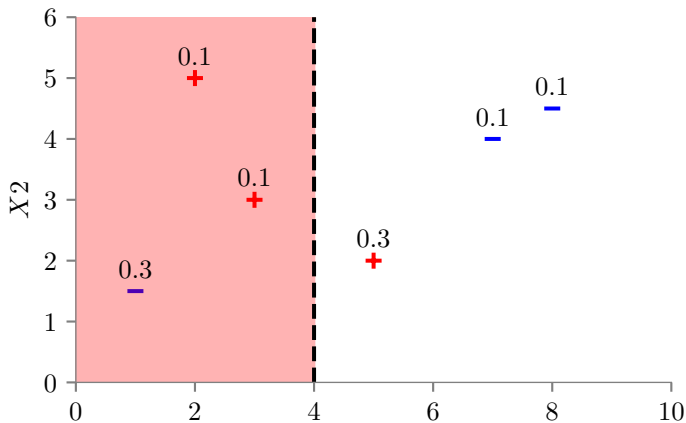
Entropy of $X1 \leq X1^* = E_{S(X1 < X1^*)}$

$$P(+) = \frac{0.1 + 0.1}{0.1 + 0.1 + 0.3} = \frac{2}{5}$$

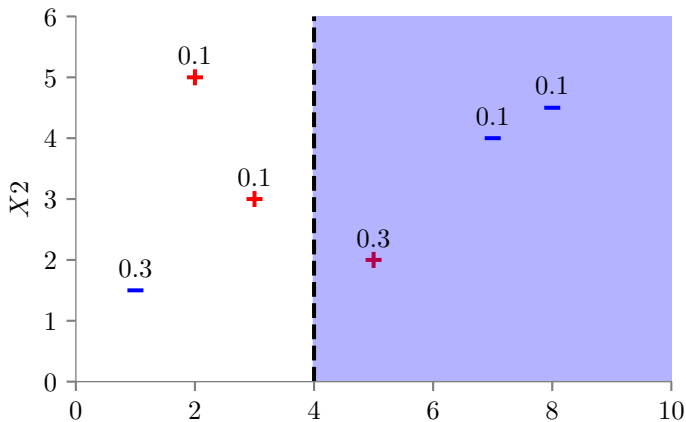Entropy of $X_1 > X_1^* = E_{S(X_1 > X_1^*)}$

$$P(+) = \frac{3}{5}$$

$$\text{IG}(X_1 = X_1^*) = E_S - \frac{0.5}{1} \cdot E_{S(X_1 < X_1^*)} - \frac{0.5}{1} \cdot E_{S(X_1 > X_1^*)}$$