Matrix Factorization for Movie Recommendation Systems

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August 1, 2025

Today’s Learning Journey

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- **Step-by-Step**: Building intuition with examples
- **Algorithms**: ALS vs Gradient Descent
- **Practice**: Hands-on understanding

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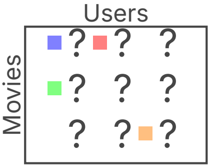

Sparse Rating Matrix

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Answer: $\frac{100}{15000} = 0.67\%$ - extremely sparse!

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- **Notation**: $\Omega = \{(i, j) : a_{ij} \text{ is observed}\}$

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Before We Dive In: A Simple Question

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- But we don't know these explicitly!

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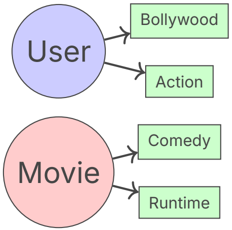

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Movie Feature Matrix $\mathbf{H} \in \mathbb{R}^{3\times 5}$:

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Key Question: How do we learn these w_{ij} values from observed ratings?

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$$\boxed{\mathbf{A} \approx \mathbf{W}\mathbf{H}}$$

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$$\mathbf{A}_{3\times 5} = \begin{bmatrix} 5 & 4 & 2 & 3 & 2 \\ ? & 5 & 1 & 4 & ? \\ 4 & ? & 1 & 5 & ? \end{bmatrix} \approx$$

$$\underbrace{\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix}}_{\mathbf{W}_{3\times 3}\mathbf{H}_{3\times 5}} =$$

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- Action-ness: 0.10 (low)

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The Magic Formula:

$$\text{Alice's rating} = \text{Alice's preferences} \cdot \text{Sholay's features}$$

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$$\begin{aligned} \hat{a}_{11} &= \mathbf{w}_1^T \mathbf{h}_1 && (1) \\ &= w_{11} \cdot 0.95 + w_{12} \cdot 0.10 + w_{13} \cdot 0.85 && (2) \end{aligned}$$

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$$\hat{a}_{11} = \mathbf{w}_1^T \mathbf{h}_1 \tag{1}$$
$$= w_{11} \cdot 0.95 + w_{12} \cdot 0.10 + w_{13} \cdot 0.85 \tag{2}$$

Goal: Find w_{11}, w_{12}, w_{13} such that $\hat{a}_{11} \approx 5$ (Alice's actual rating)

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4. How many parameters do we need to learn?

Answers:

1. $\mathbf{A} \in \mathbb{R}^{N \times M}$

Pop Quiz 2: Matrix Dimensions

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Key Insight: If $r \ll \min(N, M)$, we have huge param-

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- Ω: set of observed (i, j) pairs

Why This is Challenging

Problem Characteristics:

- **Non-convex:** Multiple local minima exist

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Key Insight: While non-convex jointly, it's convex in each matrix individually!

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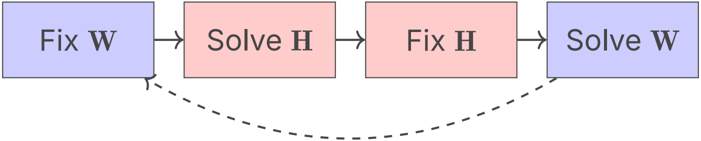

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Least Squares Solution:

$$\boxed{\mathbf{w}_i^* = (\mathbf{X}_i^T \mathbf{X}_i)^{-1} \mathbf{X}_i^T \mathbf{y}_i}$$

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Update Alice's preferences ($\mathbf{w}_1$):
Alice rated: Sholay(5), Swades(4), Batman(2), Interstellar(3), Shawshank(2)

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$$\mathbf{y}_1 = \begin{bmatrix} 5 \\ 4 \\ 2 \\ 3 \\ 2 \end{bmatrix} \tag{5}$$

$$\mathbf{X}_1 = \begin{bmatrix} 0.95 & 0.10 & 0.85 \\ 1.00 & 0.20 & 0.90 \\ 0.05 & 0.80 & 0.30 \\ 0.05 & 0.95 & 0.70 \\ 0.05 & 0.15 & 0.95 \end{bmatrix} \tag{6}$$

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Solution: $\mathbf{w}_1^* = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$
This gives us Alice's feature preferences!

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ALS: Complete Algorithm

Algorithm 1: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

1. **Initialize:** $\mathbf{W}^{(0)} \in \mathbb{R}^{N \times r}$, $\mathbf{H}^{(0)} \in \mathbb{R}^{r \times M}$ randomly

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   1) **Update Users:** For each user $i = 1, \ldots, N$:

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Algorithm 4: [

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2. **For** $t = 1, 2, \ldots, T$:
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   2) **Update Movies:** For each movie $j = 1, \ldots, M$:

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Algorithm 5: [

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3. **Check Convergence:** Stop if $\|\mathbf{W}^{(t)}\mathbf{H}^{(t)} - \mathbf{W}^{(t-1)}\mathbf{H}^{(t-1)}\|_F < \epsilon$

Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

Gradient Descent Approach

Simultaneous Updates: Update both $\mathbf{W}$ and $\mathbf{H}$ together

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Gradients:

$$\frac{\partial L}{\partial \mathbf{w}_i} = -2 \sum_{j:(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)\mathbf{h}_j \tag{9}$$

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SGD: Learning Like a Human

Imagine you’re learning someone’s taste in movies…

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$$\mathbf{w}_i \leftarrow \mathbf{w}_i + \alpha \cdot e_{ij} \cdot \mathbf{h}_j \tag{11}$$

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- Learning rate α controls step size

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Example: Alice rates Sholay as 5, but we predict 3.2

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$$\text{Current: } \mathbf{w}_1 = [0.4, 0.2, 0.3], \quad \mathbf{h}_1 = [0.95, 0.10, 0.85] \tag{13}$$

$$\text{Prediction: } \hat{a}_{11} = 0.4 \times 0.95 + 0.2 \times 0.10 + 0.3 \times 0.85 = 0.655 \tag{14}$$

$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

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$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

Updates with $\alpha = 0.01$:

$$\mathbf{w}_1 \leftarrow [0.4, 0.2, 0.3] + 0.01 \times 4.345 \times [0.95, 0.10, 0.85] \tag{16}$$

$$= [0.4413, 0.2043, 0.3369] \tag{17}$$

$$\mathbf{h}_1 \leftarrow [0.95, 0.10, 0.85] + 0.01 \times 4.345 \times [0.4, 0.2, 0.3] \tag{18}$$

$$= [0.9674, 0.1087, 0.8631] \tag{19}$$

Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

1. What is the error e_{ij}?

Pop Quiz 3: SGD Understanding

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Answers:

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ALS vs SGD: Head-to-Head Comparison

Aspect	ALS	SGD
Updates	Alternating	Simultaneous
Convergence	Faster, more stable	Slower, can oscillate
Parallelization	Excellent	Limited
Memory	Higher	Lower
Implementation	Complex	Simple
Hyperparameters	Few (rank r)	Many (α, schedule)
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When to Use Which?

- **ALS:** Large-scale, production systems (Spark, distributed)
- **SGD:** Online learning, real-time updates, research

Advanced Practical Considerations

Regularization: Prevent overfitting

$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

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Let's Build Intuition: Small Example

Our 3×3 rating matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & ? & 2 \\ 4 & 4 & ? \\ ? & 5 & 1 \end{bmatrix}$$

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Constraint: Only minimize error on observed entries!

Step-by-Step ALS Solution

Iteration 1: Initialize randomly

$$\mathbf{W}^{(0)} = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}, \quad \mathbf{H}^{(0)} = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 0.3 & 1.2 & 0.8 \end{bmatrix}$$

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Solve: $\mathrm{w}_1^{(1)} = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$
Continue for all users and movies...

Pop Quiz 4: Final Challenge

Master Check

You're Netflix's lead ML engineer. You have:

- 200M users, 15K movies

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The Mathematical Beauty:

$$\boxed{\text{Collaborative Filtering} = \text{Matrix Factorization} = \text{Dimensionality R}}$$

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Questions?

Thank you for your attention!

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