Next Token Generation

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IIT Gandhinagar

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Introduction and Motivation

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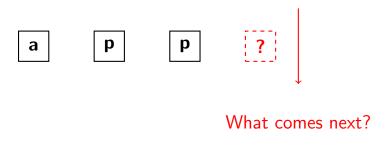
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- Direct relevance to ChatGPT and other large language models:
 - ChatGPT generates text by predicting the next token
 - Same underlying principle scaled to billions of parameters
 - Understanding next token prediction is key to understanding LLMs

The Fundamental Question



Given the sequence "app", what is the next character?

Problem Formulation



Next Character Prediction as Classification

a

g

р

Input: "app"

Next Character Prediction as Classification

Output: Character Probabilities

a

р

р

Input: "app"

Character	Probability
а	0.05
b	0.02
С	0.03
I	0.35
m	0.01
•••	
у	0.08
z	0.01
- (end)	0.15

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Case Study: Indian Names Generation

Training Dataset

Training Dataset

Training Dataset

Abid

Training Dataset

- Abid
- Abhidha

Training Dataset

- Abid
- Abhidha
- Adesh

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

Training Dataset

Abid

Kiran

- Abhidha
- Adesh
- Aditya
- Arjun
- •

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

- Kiran
- Krishna

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

- Kiran
- Krishna
- Lakshmi

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
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- Kiran
- Krishna
- Lakshmi
- Meera

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

- Kiran
- Krishna
- Lakshmi
- Meera
- Nisha

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

- Kiran
- Krishna
- Lakshmi
- Meera
- Nisha
- :

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- •

- Kiran
- Krishna
- Lakshmi
- Meera
- Nisha
- :

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- :

- Kiran
- Krishna
- Lakshmi
- Meera
- Nisha
- :

Priya

Training Dataset

- Abid
- Abhidha
- Adesh
- Aditya
- Arjun
- :

- Kiran
- Krishna
- Lakshmi
- Meera
- Nisha
- :

- Priya
- Rajesh

Training Dataset

- Abid
- Abhidha
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- Kiran
- Krishna
- Lakshmi
- Meera
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- Priya
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Goal: Learn to generate new, realistic Indian names

Dataset: Indian Names

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Vocabulary Size:

26 letters + 1 hyphen = 27 characters

Training Data Generation

Generate Training Dataset

Example: "abid" → **Training Examples**

Context Length: 3 characters

	Target (Y)			
Char 1	Char 2	Char 3	Context	Next Char
-	-	-	"—"	а
-	-	a	"-a"	b
-	a	b	"-ab"	i
a	b	i	"abi"	d
b	i	d	"bid"	-

Training Examples

From one name "abid", we create **5 training examples** using a 7/100

Representation Learning

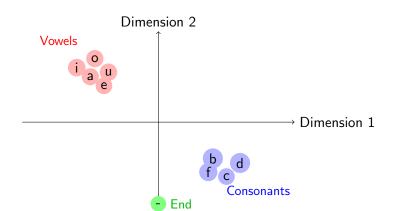
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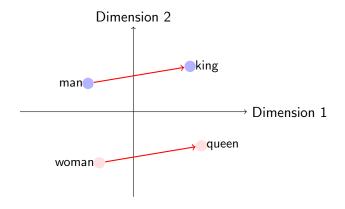
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Word2Vec Analogy Example

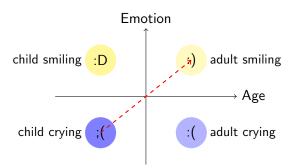
Classic Word2Vec Relationship



 $\textbf{Relationship:} \ \, \mathsf{queen} \approx \mathsf{king} \, \text{-} \, \mathsf{man} \, + \, \mathsf{woman}$

Analogy with Emotions

Emotional Expression Analogy



Relationship: child crying = child smiling + adult crying - adult smiling

Embedding Architecture

Embedding Matrix/Table Concept



Process: Character \rightarrow Lookup in Embedding Table \rightarrow Dense Vector

Embedding Table Structure

27 × K Embedding Matrix

Character	Dim 1	Dim 2		Dim K
а	0.2	-0.1		0.8
b	-0.3	0.5		-0.2
С	0.1	0.3		0.4
:	:	:	٠	i i
Z	0.7	-0.4		0.1
-	0.0	0.9		-0.5

Key Point

Each character maps to a K-dimensional dense vector representation.

• Embedding Matrix: 27 × K parameters

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 - Initially random

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- Total Learnable Parameters:
 - Embedding: 27 × K
 - MLP: (context_size \times K) \rightarrow hidden \rightarrow ... \rightarrow 27

Neural Network Architecture

Example: 2D Embeddings for "abi"

Embedding Matrix (27
$$\times$$
 2)

Input: X = ["a", "b", "i"]

$$\begin{bmatrix}
D1 & D2 \\
a & 0.2 & -0.1 \\
b & -0.3 & 0.5 \\
... & ... & ... \\
\vdots & 0.1 & 0.3 \\
... & ... & ... \\
z & 0.7 & -0.4
\end{bmatrix}$$
[0.2, -0.1]
[-0.3, 0.5]
[0.1, 0.3]

Concatenate the Embeddings

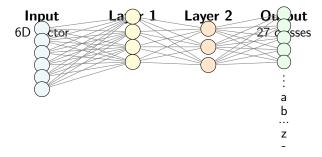
Feature Vector Construction

a:
$$[0.2, -0.1]$$
b: $[-0.3, 0.5]$
concatenate
$$[0.2, -0.1, -0.3, 0.5, 0.1, 0.3]$$
i: $[0.1, 0.3]$
6-dimensional feature vector

Result

Context of 3 characters \times 2D embeddings = 6-dimensional input to neural network

Multi-Layer Perceptron Architecture



Training and Loss Function

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c})$$
 (1)

• Loss Function: Cross-entropy loss for multi-class classification

$$\mathcal{L} = -\sum_{i=1}^{N} \sum_{c=1}^{27} y_{i,c} \log(\hat{y}_{i,c})$$
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• What we learn:

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 - 1. Forward pass: Input \rightarrow Embeddings \rightarrow Concatenate \rightarrow MLP \rightarrow Probabilities

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 - 4. Repeat for all training examples

Text Generation

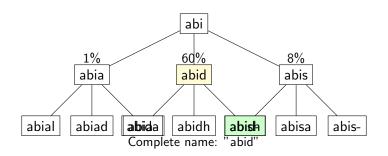
Sampling from the Learned Model

Test Input: "abi"

Predicted Probability Distribution

Next Char	Probability	Next Char	Probability
а	0.01	n	0.05
b	0.01	О	0.02
С	0.03	р	0.01
d	0.60	q	0.00
е	0.02	r	0.03
f	0.01	s	0.08
-	0.05	z	0.01

Generation Tree Structure



Recursive Process: Sample next character, append, repeat until end token

Temperature and Sampling Strategies

• Standard Softmax:

$$P(y_i) = \frac{e^{z_i}}{\sum_{j=1}^{27} e^{z_j}}$$
 (2)

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- Temperature Effects:
 - T = 1: Standard probabilities
 - $T \rightarrow 0$: More peaked (deterministic)
 - $T \to \infty$: More uniform (random)

Temperature Variations

Context: "abi" \rightarrow Next character probabilities

Character	T = 0.5	T = 1.0	T = 2.0
	(Low)	(Default)	(High)
a	0.001	0.01	0.08
d	0.95	0.60	0.25
S	0.01	0.08	0.12
h	0.005	0.03	0.09
-	0.02	0.05	0.11
others	0.015	0.23	0.35

• Low T: More conservative, predictable names

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	(Low)	(Default)	(High)
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others	0.015	0.23	0.35

• Low T: More conservative, predictable names

• **High T:** More creative, diverse names

Summary and Applications

• Core Idea: Next token prediction as classification

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- Representation Learning: Character embeddings capture similarity

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- **Architecture:** Embeddings + MLP for sequence modeling

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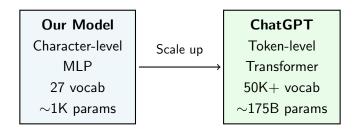
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 - Billions of parameters instead of thousands

From Character-Level to ChatGPT



Same fundamental principle: Predict the next token!