Naive Bayes

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Naive assumption x_i and x_{i+1} are independent given y

$$\text{i.e. } p\left(x_2 \mid x_1, y\right) = p\left(x_2 \mid y\right)$$

$$P(y = 1 | w_1 = 0, w_2 = 0, w_3 = 1)$$

$$= \frac{P(w_1 = 0|y = 1)P(w_2 = 0|y = 1)P(w_3 = 1|y = 1)P(y = 1)}{P(w_1 = 0, w_2 = 0, w_3 = 1)}$$

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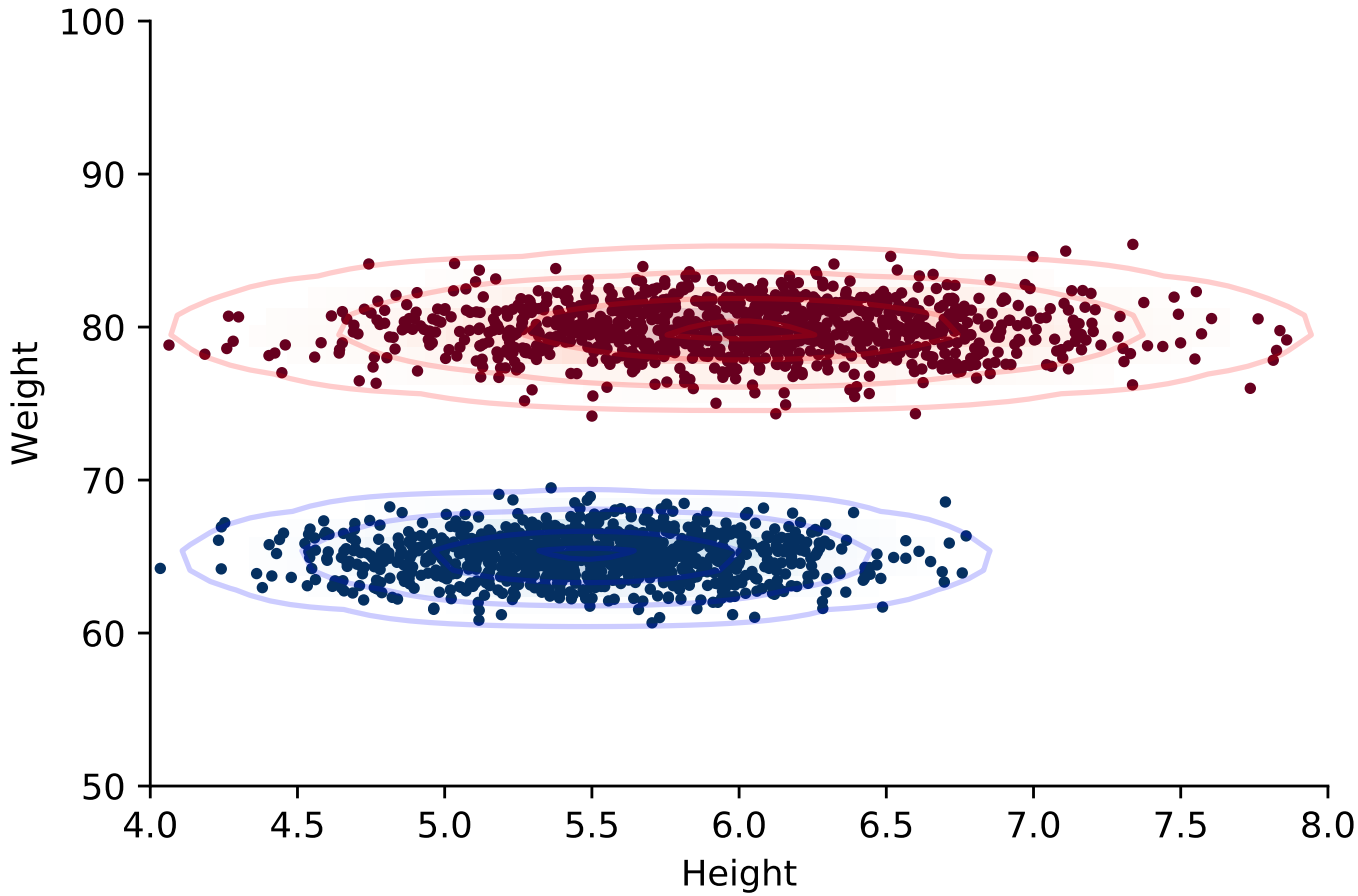
$\frac{P(y=1|w_1=0,w_2=0,w_3=1)}{P(y=0|w_1=0,w_2=0,w_3=1)} = 2 > 1$. Thus, classified as a spam example.

Note: no cross covariance! Remember all features are independent.



Wikipedia Example

Height	Weight	Footsize	Gender
6	180	12	M
5.92	190	11	M
5.58	170	12	M
5.92	165	10	M
5	100	6	F
5.5	100	6	F
5.42	130	7	F
5.75	150	7	F

Example

	Male	Female
Mean (height)	5.855	5.41
Variance (height)	3.5×10^{-2}	9.7×10^{-2}
Mean (weight)	176.25	132.5
Variance (weight)	1.22×10^{2}	5.5×10^{2}
Mean (Foot)	11.25	7.5
Variance (Foot)	9.7×10^{-1}	1.67

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- Finally, we get probability of female given data is greater than the probability of class being male given data.