

# Decision Trees

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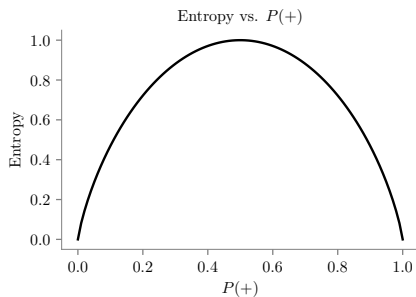
# Information Gain Intuition

- ▶ For examples, we have 9 Yes, 5 No
- ▶ Would it be trivial if we had 14 Yes or 14 No?
- ▶ Yes!
- ▶ Key insight: Problem is “easier” when there is less disagreement
- ▶ Need some statistical measure of “disagreement”

# Entropy Formula

$$H(X) = - \sum_{i=1}^k p(x_i) \log_2 p(x_i)$$

**Notebook: entropy.html**



# Root Node Selection

- ▶ Can we use Outlook as the root node?

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- ▶ Can we use Outlook as the root node?
- ▶ When Outlook is overcast, we always Play and thus no “disagreement”

# What Does Entropy Measure?

**Answer: B) The impurity or “disagreement” in a set of examples** - Higher entropy means more mixed classes, lower entropy means more pure subsets.



# Entropy Calculation Examples

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Overcast	Yes
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Outlook	Play
Rain	Yes
Rain	Yes
Rain	No
Rain	Yes
Rain	No

We have 3 Yes, 2 No

$$\begin{aligned}\text{Entropy} = & -\frac{3}{5} \log_2 \left( \frac{3}{5} \right) - \\ & \frac{2}{5} \log_2 \left( \frac{2}{5} \right) = 0.971\end{aligned}$$

# Information Gain Calculations

- ▶  $\text{Gain}(S_{\text{Outlook}=\text{Sunny}}, \text{Temp}) = \text{Entropy}(2 \text{ Yes}, 3 \text{ No}) -$   
 $(2/5) * \text{Entropy}(0 \text{ Yes}, 2 \text{ No}) - (2/5) * \text{Entropy}(1 \text{ Yes}, 1 \text{ No}) -$   
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# Prediction Example

Prediction for <High Humidity, Strong Wind, Sunny Outlook, Hot Temp> is ?

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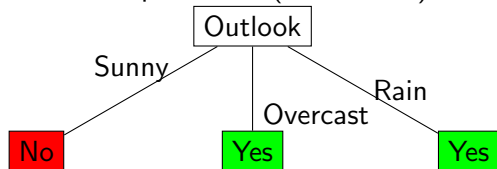
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# Why Outlook is Good Root?

**Answer: B) When Outlook=Overcast, all examples have Play=Yes** - This creates a pure subset with  $\text{entropy}=0$ , maximizing information gain.

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- ▶  $\text{MSE}(S) = 311.34$
- ▶ What about splitting criterion for regression?
- ▶ **MSE Reduction** (not Information Gain!)
- ▶  $\text{MSE Reduction} = \text{MSE}(S) - \sum_v \frac{|S_v|}{|S|} \text{MSE}(S_v)$

# Regression Splitting Criterion

**Answer: C) Mean Squared Error (MSE) Reduction** - For regression, we minimize MSE instead of maximizing information gain.

# Continuous Features

**Answer: B) Use midpoints between consecutive sorted feature values** - This ensures we test all meaningful boundaries between different class regions.

# Leaf Node Predictions

**Answer: C) The mean of target values in that region** - Each leaf predicts the average target value of training samples that reach that leaf.



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  - ▶ Rules that are too specific to training data
- ▶ **Solution:** Pruning to control model complexity

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- ▶ **Maximum features:** Consider only subset of features at each split
- ▶ **Minimum impurity decrease:** Only split if improvement  $>$  threshold

**Advantages:** Simple, computationally efficient

**Disadvantages:** May stop too early, miss good splits later

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  4. Select  $\alpha$  with best cross-validation performance

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- ▶ **Optimal pruning:** Balances bias and variance
- ▶ **Cross-validation:** Essential for finding this balance

# Practical Pruning Guidelines

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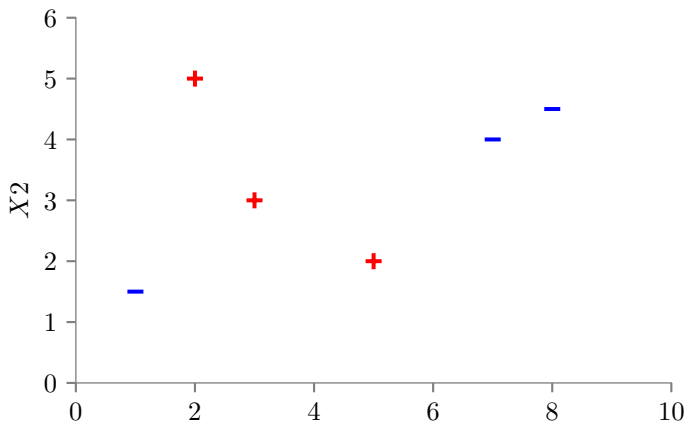
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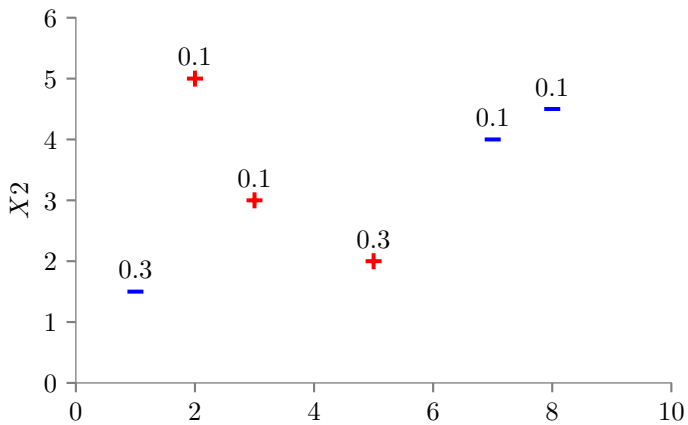
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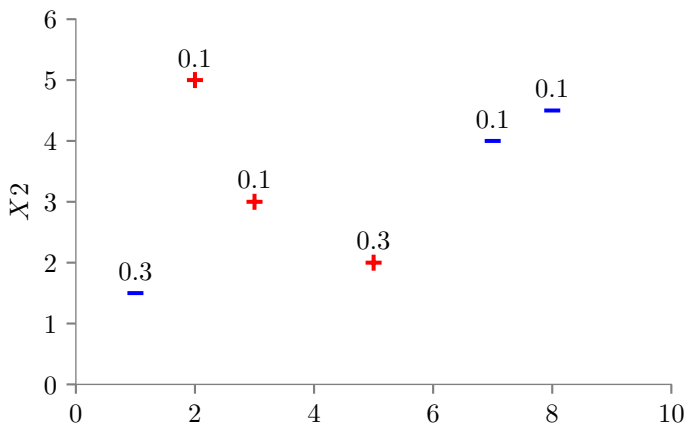
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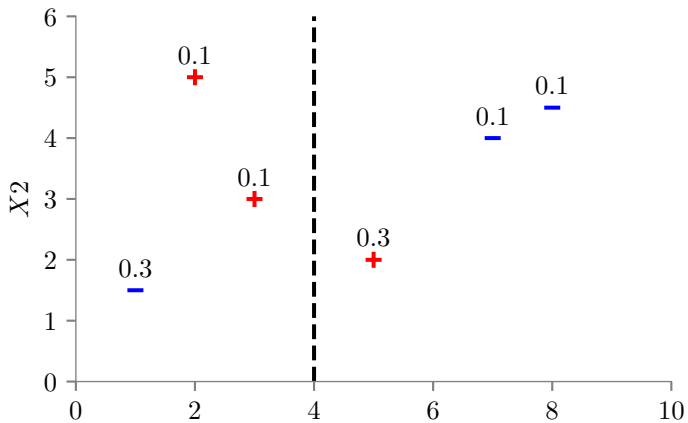




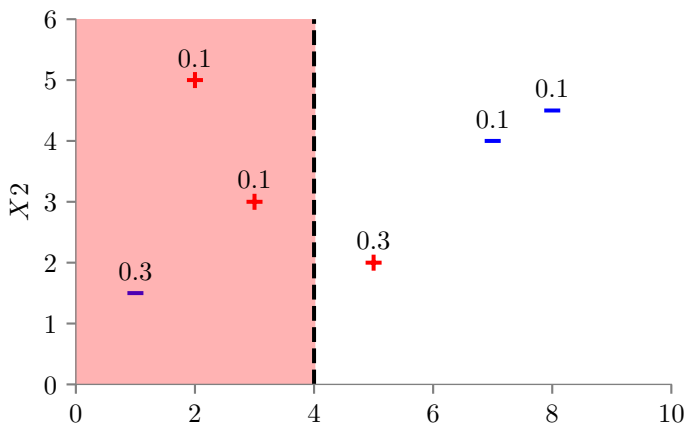


$$\text{Entropy} = -P(+)\log_2 P(+)-P(-)\log_2 P(-)$$

$$P(+)=\frac{0.1+0.1+0.3}{1}=0.5,\quad P(-)=\frac{0.3+0.1+0.1}{1}=0.5$$



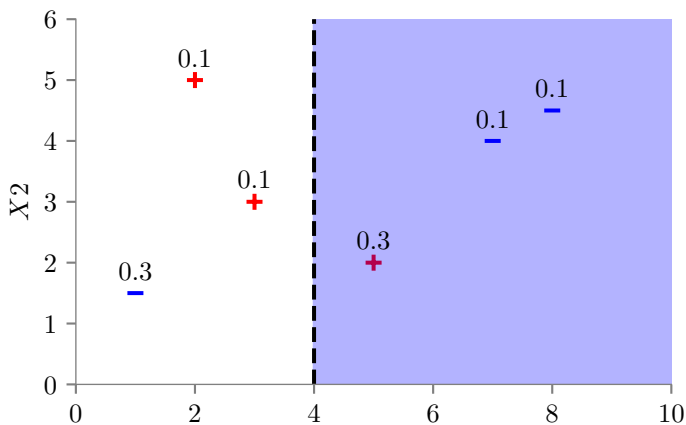
Candidate Line:  $X_1 = 4(X_1^*)$



Entropy of  $X_1 \leq X_1^* = E_{S(X_1 < X_1^*)}$

$$P(+)=\frac{0.1+0.1}{0.1+0.1+0.3}=\frac{2}{5}$$

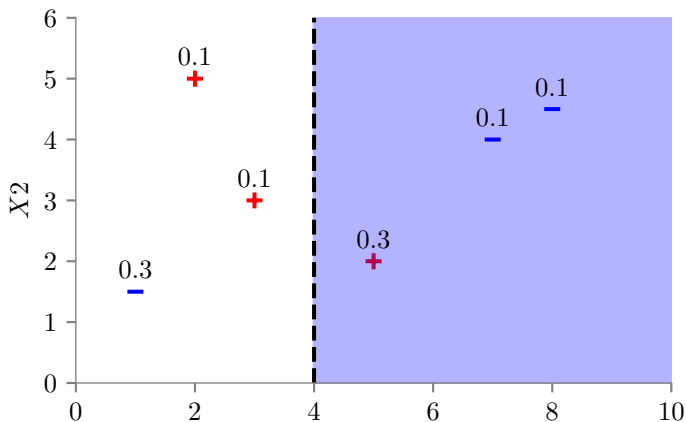
$$P(-)=\frac{3}{5}$$



Entropy of  $X_1 > X_1^* = E_{S(X_1 > X_1^*)}$

$$P(+)=\frac{3}{5}$$

$$P(-)=\frac{2}{5}$$



$$\text{IG}(X_1 = X_1^*) = E_S - \frac{0.5}{1} \cdot E_{S(X_1 < X_1^*)} - \frac{0.5}{1} \cdot E_{S(X_1 > X_1^*)}$$