

Support Vector Machines

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Non-Linearly Separable Data

Data not separable in \mathbb{R}

Can we still use SVM?

Yes!

How? Project data to a higher dimensional space.

Projection/Transformation Function

$$\phi : \mathbb{R}^d \rightarrow \mathbb{R}^D$$

where, d = original dimension

D = new dimension

In our example:

$$d = 1; D = 2$$

Linear SVM:

Maximize

$$L(\alpha) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j y_i y_j \bar{x}_i \cdot \bar{x}_j$$

such that constraints are satisfied.



Transformation (ϕ)



$$L(\alpha) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j y_i y_j \phi(\bar{x}_i) \cdot \phi(\bar{x}_j)$$

Steps

1. Compute $\phi(x)$ for each point

$$\phi : \mathbb{R}^d \rightarrow \mathbb{R}^D$$

Q. If $D \gg d$

Both steps are expensive!

Steps

1. Compute $\phi(x)$ for each point

$$\phi : \mathbb{R}^d \rightarrow \mathbb{R}^D$$

2. Computer dot products over \mathbb{R}^D space

Q. If $D \gg d$

Both steps are expensive!

Kernel Trick