

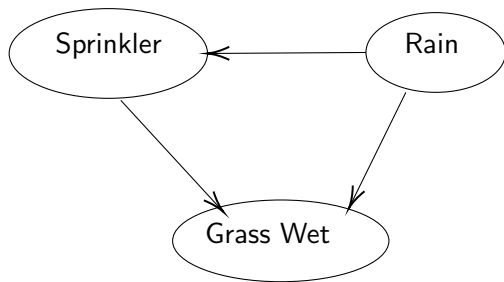
Naive Bayes

Nipun Batra

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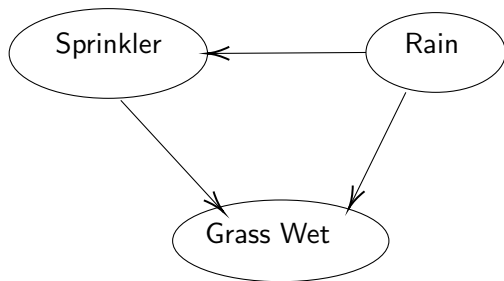
IIT Gandhinagar

Bayesian Networks



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Bayesian Networks



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- Edges denote direct impact

Example

- Grass can be wet due to multiple reasons:

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 - Sprinkler

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- Grass can be wet due to multiple reasons:
 - Rain
 - Sprinkler
- Also, if it rains, then sprinkler need not be used.

$P(X_1, X_2, X_3, \dots, X_N)$ denotes the joint probability, where X_i are random variables.

$$P(X_1, X_2, X_3, \dots, X_N) = \prod_{k=1}^N P(X_k | \text{parents}(X_k))$$

$$P(S, G, R) = P(G|S, R)P(S|R)P(R)$$

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- Each email corresponds to vector/feature of length N containing zeros or ones.

- Classification model

Naive Bayes

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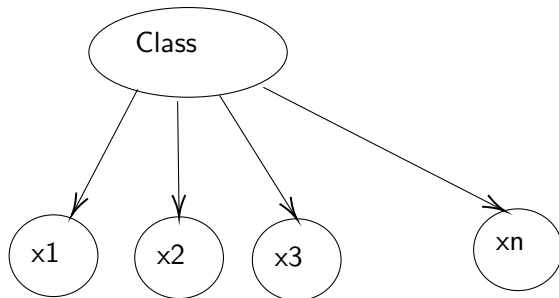
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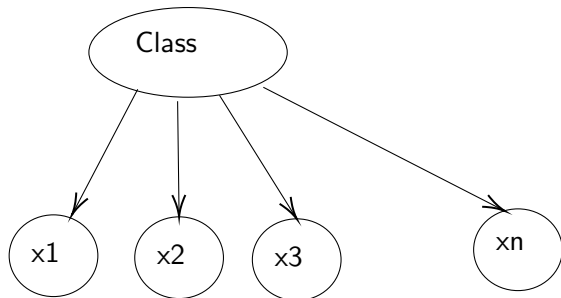
- Classification model
- Scalable
- Generative and Bayesian
- Usually a simple/good baseline
- We want to model $P(\text{class}(y) \mid \text{features}(x))$
- We can use Bayes rule as follows:

$$P(\text{class}(y) \mid \text{features}(x)) = \frac{P(\text{features}(x) \mid \text{class}(y))P(\text{class}(y))}{P(\text{features}(x))}$$

Quick Question

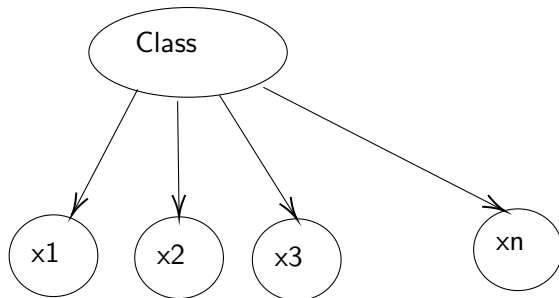


Quick Question



$$P(x_1, x_2, x_3, \dots, x_N | y) = P(x_1 | y) P(x_2 | y) \dots P(x_N | y)$$

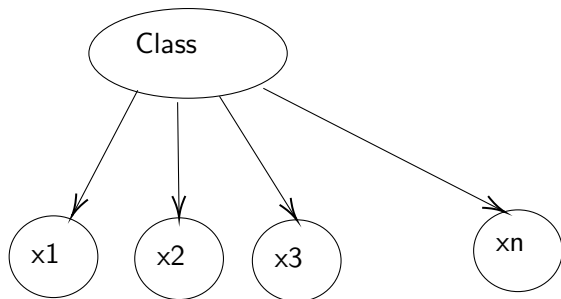
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Why is Naive Bayes model called Naive?

Quick Question



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Why is Naive Bayes model called Naive?

Naive assumption x_i and x_{i+1} are independent given y

$$\text{i.e. } p(x_2 | x_1, y) = p(x_2 | y)$$

It assumes that the features are independent during modelling, which is generally not the case.

What do we need to predict?

$$P(y|x_1, x_2, \dots, x_N) = \frac{P(x_1, x_2, \dots, x_N|y)P(y)}{P(x_1, x_2, \dots, x_N)}$$

Spam Mail Classification

Probability of x_i being a spam email

$$P(x_i = 1|y = 1) = \frac{\text{Count}(x_i = 1 \text{ and } y = 1)}{\text{Count}(y = 1)}$$

Similarly,

$$P(x_i = 0|y = 1) = \frac{\text{Count}(x_i = 0 \text{ and } y = 1)}{\text{Count}(y = 1)}$$

Spam Mail classification

$$P(y = 1) = \frac{\text{Count}(y = 1)}{\text{Count}(y = 1) + \text{Count}(y = 0)}$$

Similarly,

$$P(y = 0) = \frac{\text{Count}(y = 0)}{\text{Count}(y = 1) + \text{Count}(y = 0)}$$

Example

lets assume that dictionary is $[w_1, w_2, w_3]$

Index	w_1	w_2	w_3	y
1	0	0	0	1
2	0	0	0	0
3	0	0	0	1
4	1	0	0	0
5	1	0	1	1
6	1	1	1	0
7	1	1	1	1
8	1	1	0	0
9	0	1	1	0
10	0	1	1	1

Spam Classification

if $y=0$

- $P(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$

$$P(y=0) = 0.5$$

Similarly, if $y=1$

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Similarly, if $y=1$

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$$\begin{aligned} & P(y = 1 | w_1 = 0, w_2 = 0, w_3 = 1) \\ = & \frac{P(w_1 = 0 | y = 1)P(w_2 = 0 | y = 1)P(w_3 = 1 | y = 1)P(y = 1)}{P(w_1 = 0, w_2 = 0, w_3 = 1)} \\ = & \frac{0.6 \times 0.8 \times 0.6 \times 0.5}{Z} \end{aligned}$$

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$\frac{P(y=1|w_1=0,w_2=0,w_3=1)}{P(y=0|w_1=0,w_2=0,w_3=1)} = 2 > 1$. Thus, classified as a spam example.

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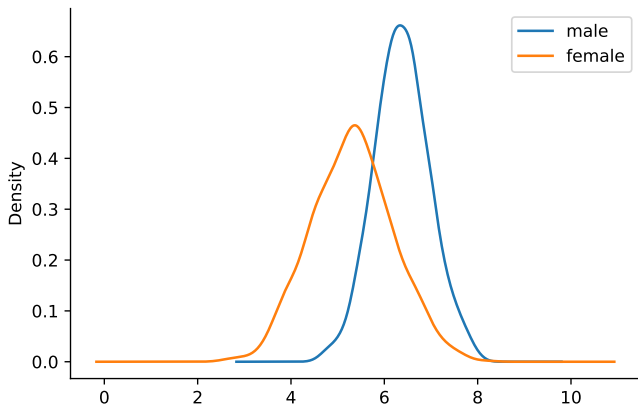
- “This product is pathetic”. We would assume the sentiment of such a sentence to be negative. Why? Presence of “pathetic”
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- Good is positive, Bad is negative

Naive Bayes for email/sentiment analysis

- “This product is pathetic”. We would assume the sentiment of such a sentence to be negative. Why? Presence of “pathetic”
- Naive Bayes would store the probabilities of words belonging to positive or negative sentiment.
- Good is positive, Bad is negative
- What about: This product is not bad. Naive Bayes is very naive and does not account for sequential aspect of data.

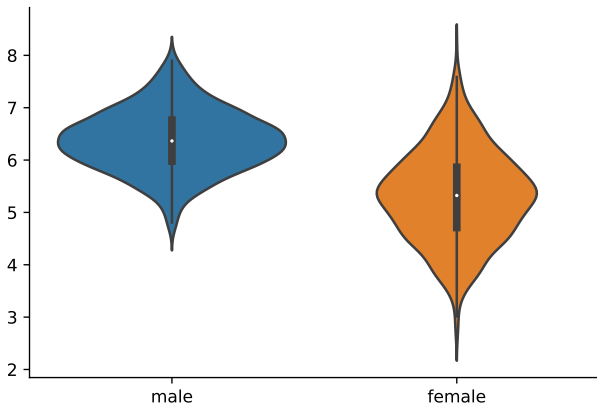
Gaussian Naive Bayes

Let us generate some normally distributed height data assuming
Height (male) $\sim \mathcal{N}(\mu_1 = 6.1, \sigma_1^2 = 0.6)$ and Height (female)
 $\sim \mathcal{N}(\mu_2 = 5.3, \sigma_2^2 = 0.9)$



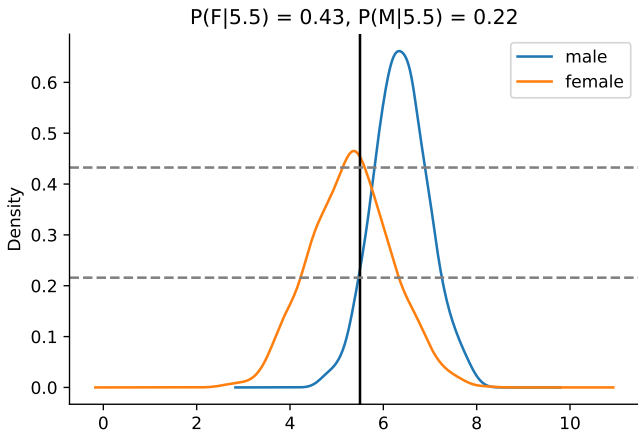
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Gaussian Naive Bayes

Would you expect a person to height 5.5 as a female or male? And why?



Gaussian Naive Bayes

We have classes $C_1, C_2, C_3, \dots, C_k$

There is a continuous attribute x

For Class k

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- $\mu_k = \text{Mean}(x|y(x) = C_k)$
- $\sigma_k^2 = \text{Variance}(x|y(x) = C_k)$

Guassian Naive Bayes

Now for $x =$ some observation ' v '

$$P(x = v | C_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp \frac{-(v-\mu_k)^2}{2\sigma_k^2}$$

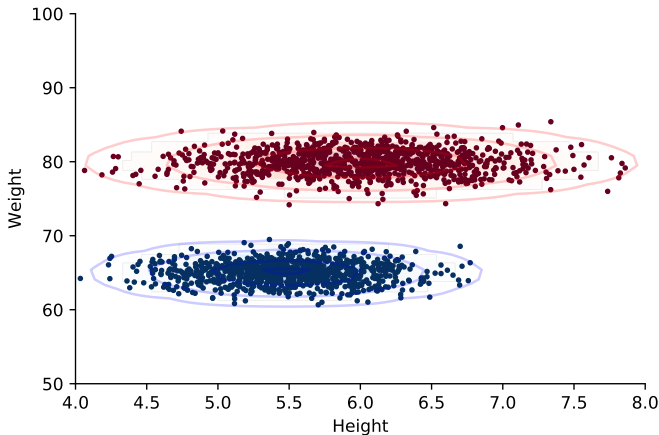
Gaussian Naive Bayes (2d example)

Would you expect a person to height 5.5 and weight 80 as a female or male? And why?

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Note: no cross covariance! Remember all features are independent.



Wikipedia Example

Height	Weight	Footsize	Gender
6	180	12	M
5.92	190	11	M
5.58	170	12	M
5.92	165	10	M
5	100	6	F
5.5	100	6	F
5.42	130	7	F
5.75	150	7	F

Example

	Male	Female
Mean (height)	5.855	5.41
Variance (height)	3.5×10^{-2}	9.7×10^{-2}
Mean (weight)	176.25	132.5
Variance (weight)	1.22×10^2	5.5×10^2
Mean (Foot)	11.25	7.5
Variance (Foot)	9.7×10^{-1}	1.67

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- $$P(130lbs|F) = \frac{1}{\sqrt{2\pi \times 550}} \times \exp \frac{-(132.5-130)^2}{2 \times 550} = .0167$$
- Finally, we get probability of female given data is greater than the probability of class being male given data.