

K-Nearest Neighbors

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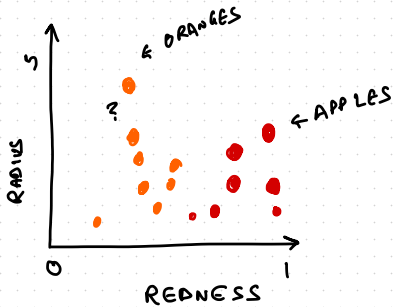
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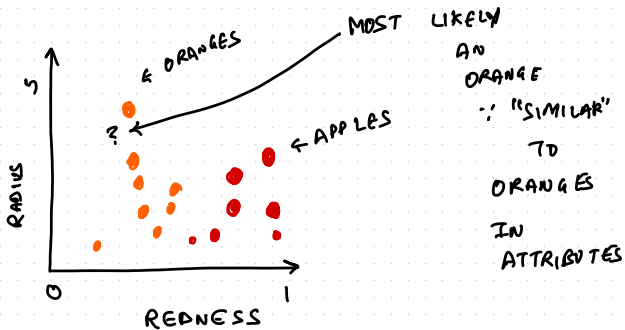
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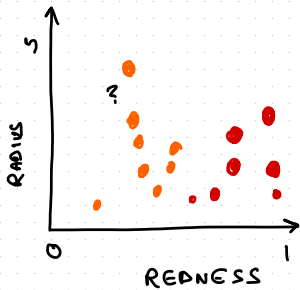
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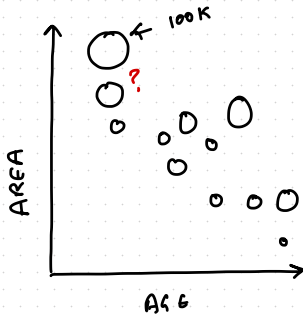
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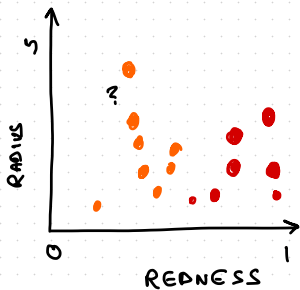
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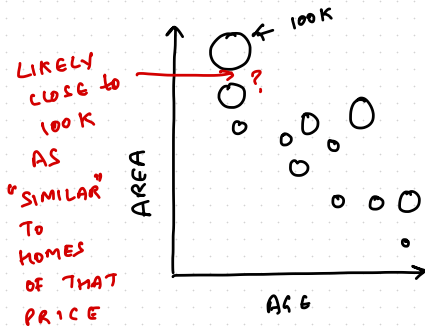
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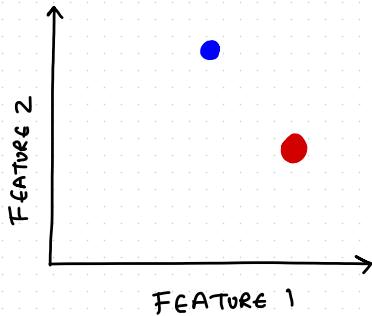
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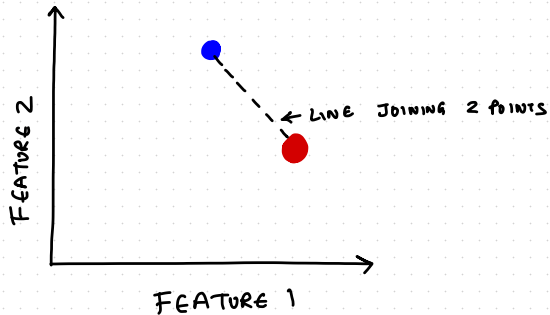
REGRESSION



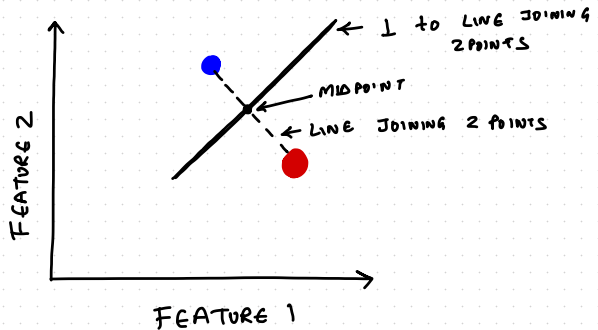
VORONOI DIAGRAM FOR 1-NN



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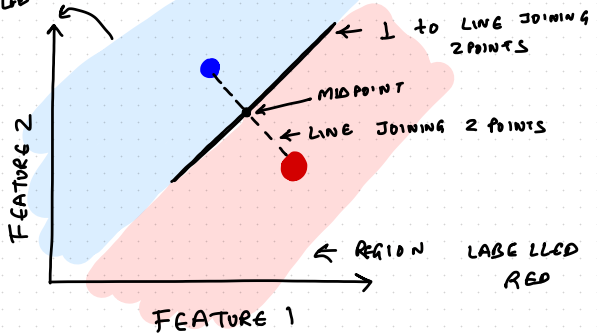


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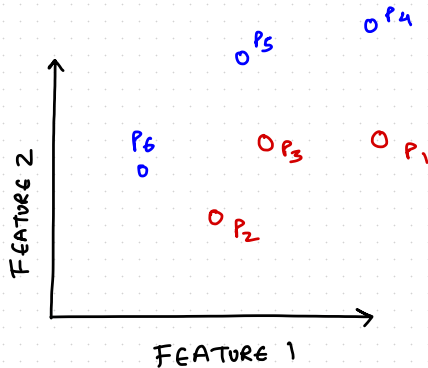


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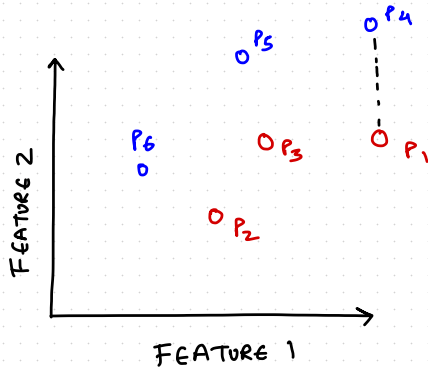
REGION LABELLED BLUE



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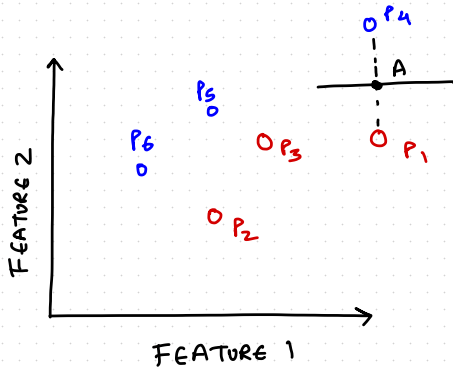


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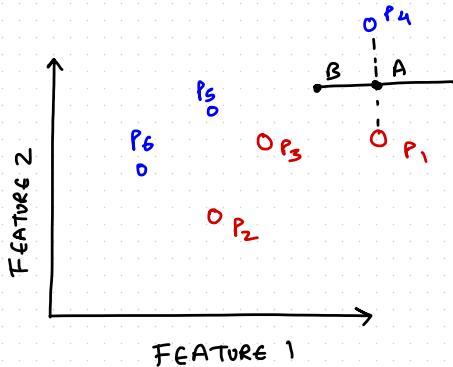
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A: MID PT B/W P_1 & P_4



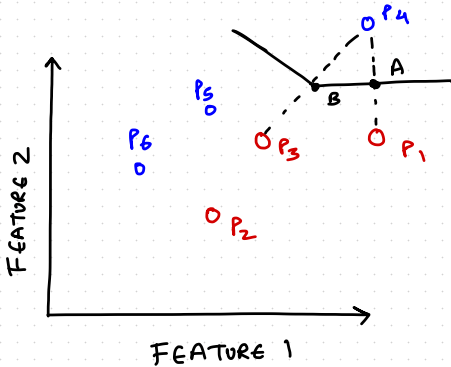
VORONOI DIAGRAM FOR 1-NN

A: MID PT B/W P_1 & P_4
B: CLOSER TO P_3 than P_1



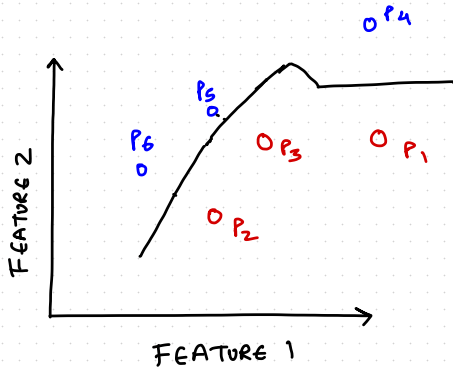
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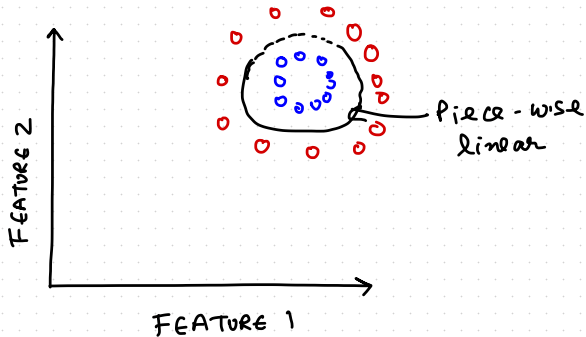
VORONOI DIAGRAM FOR 1-NN

DECISION
BOUNDARY IS
PIECE-WISE
LINEAR

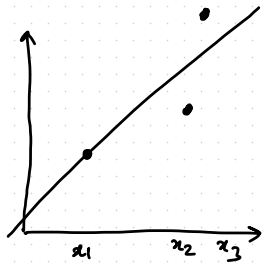


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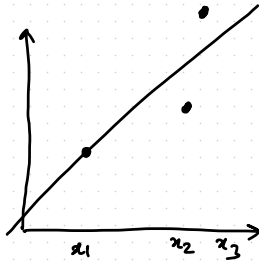


LINEAR REGRESSION

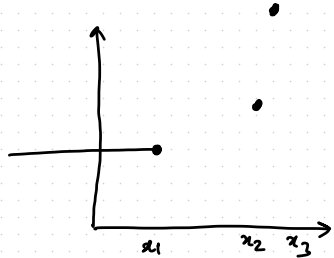


1NN REGRESSION

LINEAR REGRESSION

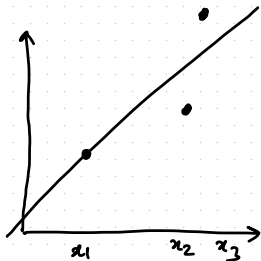


1NN REGRESSION

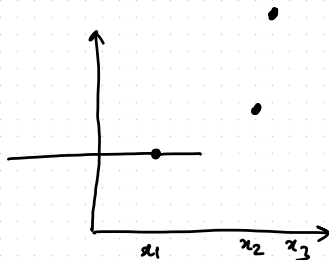


$x < x_1$: NN is (x_1, y_1)

LINEAR REGRESSION



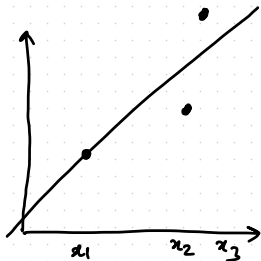
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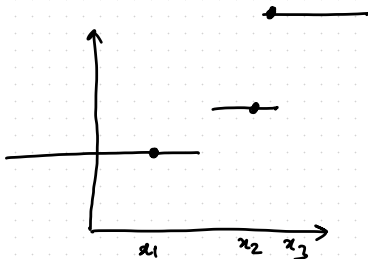
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LINEAR REGRESSION



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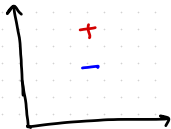


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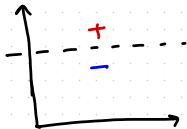
$\frac{x_1 + x_2}{2} < x < \frac{x_2 + x_3}{2}$: NN is (x_2, y_2)

KNN IS NON-PARAMETRIC



LINEAR MODEL

KNN IS NON-PARAMETRIC

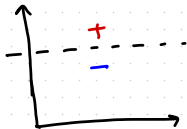


DECS
BOUNDARY

LINEAR MODEL

$$y = mx + c \quad (\# \text{ params} = 2)$$

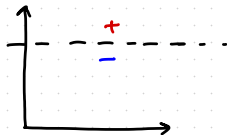
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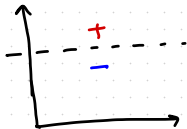
DECISION
BOUNDARY



KNN (K=1)

DECISION
BOUNDARY (LIKE $y = mx + c$)

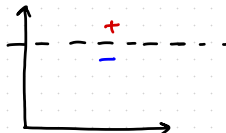
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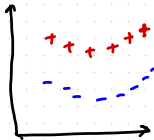


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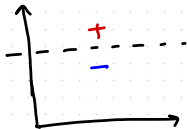
DECISION
BOUNDARY

(LIKE $y = mx + c$)

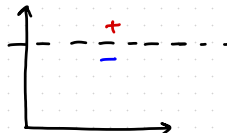
ADD DATA



KNN IS NON-PARAMETRIC

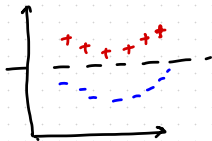


DECISION BOUNDARY
 LINEAR MODEL
 $y = mx + c$ (#params = 2)

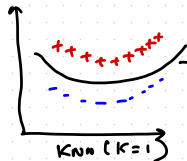


KNN (K=1)
 DECISION BOUNDARY (LIKE $y = mx + c$)

ADD DATA



DECISION BOUNDARY
 LINEAR MODEL
 $y = mx + c$ (2 params)



KNN (K=1)
 #PARAMS $\gg 2$ (AT LEAST 10,311)

PARAMETRIC

#PARAMS FIXED
WRT DATASET SIZE

MAKE ASSUMPTIONS
(LIKE FUNCTIONAL FORM)

USUALLY QUICKER

Eg: LINEAR MODELS,
SVM (LINEAR, POLYNOMIAL)

NON-PARAMETRIC

#PARAMS GROWS
WRT DATASET SIZE

LESSER ASSUMPTIONS

USUALLY SLOWER

Eg: KNN, DT,
SVM (with
RBF)

Parametric vs Non-Parametric Models

	Parametric	Non-Parametric
Parameter	Number of parameters is fixed w.r.t dataset size	Number of parameters grows w.r.t. to an increase in dataset size
Speed	Quicker (as the number of parameters are less)	Longer (as number of parameters are less)
Assumptions	Strong Assumptions (like linearity in Linear Regression)	Very few (sometimes no) assumptions
Examples	Linear Regression	KNN, Decision Tree

Lazy vs Eager Strategies

	Lazy	Eager
Train Time	0	$\neq 0$
Test	Long (due to comparison with train data)	Quick (as only "parameters" are involved)
Memory	Store/Memorise entire data	Store only learnt parameters
Utility	Useful for online settings	
Examples	KNN	Linear Regression, Decision Tree

Important Considerations

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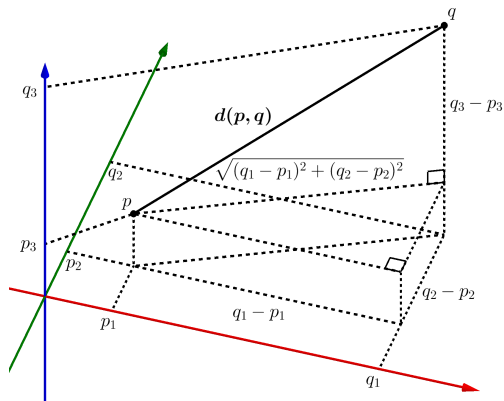
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- What is the **aggregation function** that is going to be used?
- What are the **number of neighbors** that you are going to take into consideration?
- What is the **computational complexity** of the algorithm that you are implementing?

Important Considerations: Distance Metric

The Distance Metric acts as a *measure of similarity* between the points.

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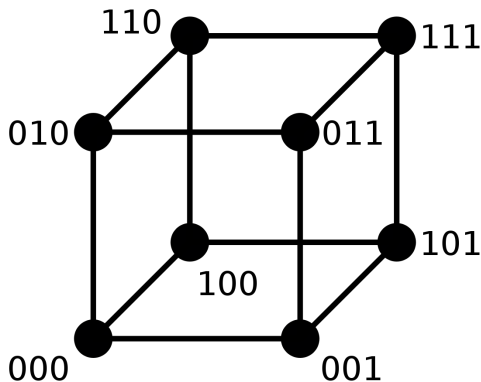
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Euclidean Distance

Important Considerations: Distance Metric

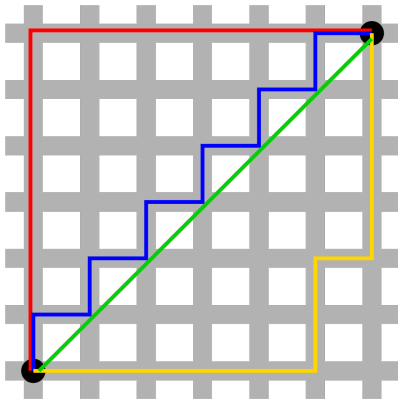
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Hamming Distance

Important Considerations: Distance Metric

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Manhattan Distance

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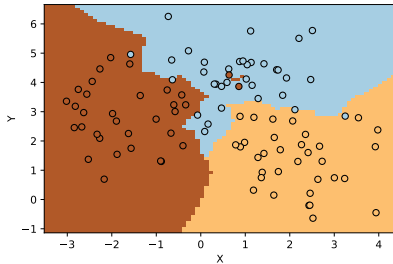
Low values of K will result in each point having a very high influence on the final output \implies noise will influence the result

High values of K will result in smoother decision boundaries

\implies lower variance but also higher bias

Important Considerations: Value of K

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$K = 3$

Important Considerations: Value of K

Aggregating data

There are different ways to go about aggregating the data from the K nearest neighbors.

- Median

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KNN Algorithm

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 2. Predict y^*

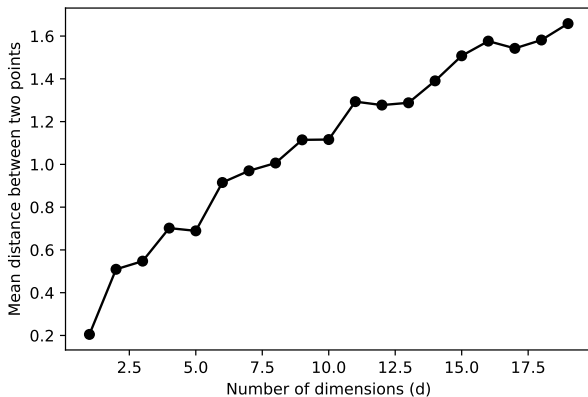
Curse of Dimensionality

With an increase in the number of dimensions:

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1. the distance between points starts to increase



For a uniformly random dataset

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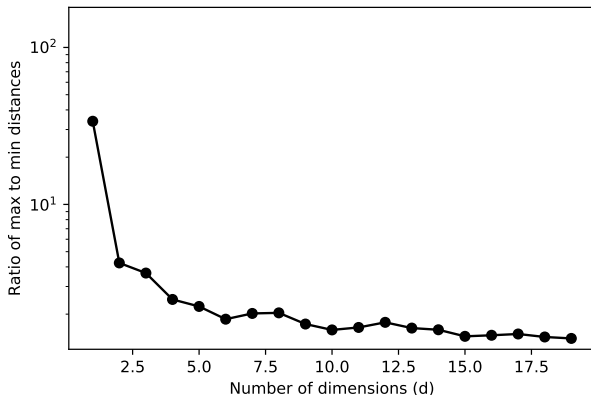
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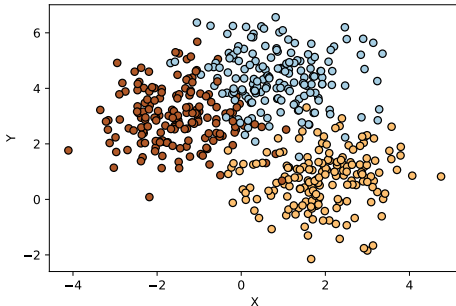
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Approximate Nearest Neighbors

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Example of a big dataset

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Such techniques include:

- Locality sensitive hashing

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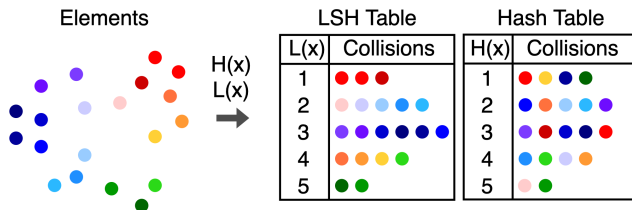
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- Greedy search in proximity neighborhood graphs

Locality sensitive hashing

Normal hash functions $H(x)$ try to keep the collision of points across bins uniform.

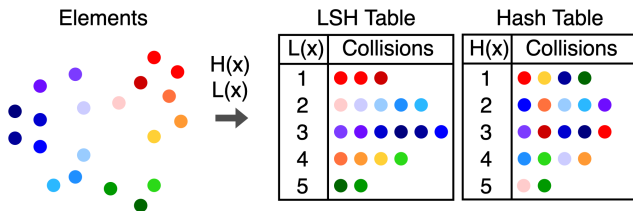


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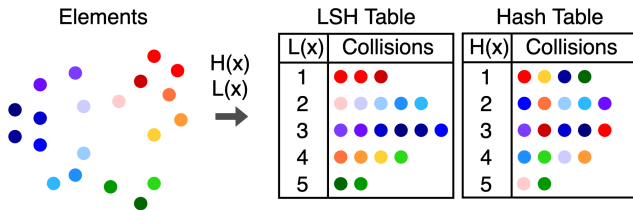


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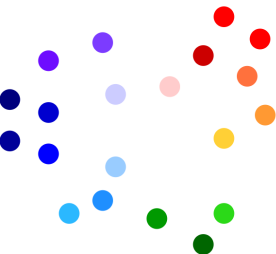
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For such cases, all elements in a bin would be given the same label, which again can be decided on the basis of different aggregation methods



Example of a big dataset






Elements








$H(x)$
 $L(x)$



LSH Table

L(x)	Collisions
1	
2	
3	
4	
5	

Hash Table

H(x)	Collisions
1	
2	
3	
4	
5	

Pop Quiz: KNN Concepts

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4. What is the time complexity of finding k nearest neighbors naively?

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- **Curse of Dimensionality:** Performance degrades in high dimensions
- **Scalability:** Approximate methods needed for large datasets