Naive Bayes

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July 21, 2025

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Bayesian Networks

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- Edges denote direct impact

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Example

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Example

- Grass can be wet due to multiple reasons:
    - Rain
    - Sprinkler
- Also, if it rains, then sprinkler need not be used.

$P(X_1, X_2, X_3, \ldots, X_N)$ denotes the joint probability, where X_i are random variables.

$$P(X_1, X_2, X_3, \ldots, X_N) = \Pi_{k=1}^{N} P(X_k | parents(X_k))$$

$$P(S, G, R) = P(G|S, R)P(S|R)P(R)$$

Bayesian Networks

Example

Known Random variables

- $P(T)$

Example

Known Random variables

- $P(T)$
- $P(E)$

Known Random variables

- $P(T)$
- $P(E)$
- $P(A|T, E)$

Known Random variables

- $P(T)$
- $P(E)$
- $P(A|T, E)$
- $P(R|E)$

Question

Given, the above, calculate

$$P(A|T)$$

Solution

$$P(A|T) = \frac{P(A, T)}{P(A)}$$

$$= \frac{P(A, T, E) + P(A, T, \bar{E})}{P(A, T, E) + P(A, T, \bar{E}) + P(A, \bar{T}, E) + P(A, \bar{T}, \bar{E})}$$

Medical Diagnosis

You tested positive for a disease.

Well, the test is only 99% accurate.

- $P(Test = +ve|Disease = True) = 0.99$

Also, the disease is a rare one. Only one in 10,000 has it.

Medical Diagnosis

You tested positive for a disease.
Well, the test is only 99% accurate.

- $P(Test = +ve | Disease = True) = 0.99$
- $P(Test = -ve | Disease = False) = 0.99$

Also, the disease is a rare one. Only one in 10,000 has it.

Problem

- $P(T|D) = 0.99$

Given the above, calculate $P(D|T)$. Given the result is positive, what is the probability that someone has the disease

Problem

- $P(T|D) = 0.99$
- $P(\bar{T}|\bar{D}) = 0.99$

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Problem

- $P(T|D) = 0.99$
- $P(\bar{T}|\bar{D}) = 0.99$
- $P(T|\bar{D}) = 0.01$

Given the above, calculate $P(D|T)$. Given the result is positive, what is the probability that someone has the disease

Problem

- $P(T|D) = 0.99$
- $P(\bar{T}|\bar{D}) = 0.99$
- $P(T|\bar{D}) = 0.01$
- $P(\bar{T}|D) = 0.01$

Given the above, calculate $P(D|T)$. Given the result is positive, what is the probability that someone has the disease

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- $P(T|D) = 0.99$
- $P(\bar{T}|\bar{D}) = 0.99$
- $P(T|\bar{D}) = 0.01$
- $P(\bar{T}|D) = 0.01$
- $P(D) = 10^{-4}$

Given the above, calculate $P(D|T)$. Given the result is positive, what is the probability that someone has the disease

Problem

- $P(T|D) = 0.99$
- $P(\bar{T}|\bar{D}) = 0.99$
- $P(T|\bar{D}) = 0.01$
- $P(\bar{T}|D) = 0.01$
- $P(D) = 10^{-4}$
- $P(\bar{D}) = 1 - 10^{-4}$

Given the above, calculate $P(D|T)$. Given the result is positive, what is the probability that someone has the disease

Problem

$$\begin{aligned}
P(D|T) &= \frac{P(T|D)P(D)}{P(T)} \\
&= \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|\bar{D})P(\bar{D})}
\end{aligned} \tag{1}$$

SPAM EMAIL CLASSIFICATION

From the emails construct a vector X. The vector has ones if the word is present, and zeros is the word is absent

Naive Bayes

- Classification model

$$P(x_1, x_2, x_3, \ldots, x_n|y) = P(x_1|y)P(x_2|y)\ldots P(x_N|y)$$

Naive Bayes

- Classification model
- Scalable

$$P(x_1, x_2, x_3, \ldots, x_n | y) = P(x_1|y)P(x_2|y)\ldots P(x_N|y)$$

Naive Bayes

- Classification model
- Scalable
- Generative

$$P(x_1, x_2, x_3, \ldots, x_n | y) = P(x_1|y)P(x_2|y)\ldots P(x_N|y)$$

Quick Question

Why is Naive Bayes model called Naive?

It assumes that the features are independent during modelling, which is generally not the case.

$$P(y|x_1, x_2, \ldots, x_N) = \frac{P(x_1, x_2, \ldots, x_N|y)P(y)}{P(x_1, x_2, \ldots, x_N)}$$

Spam Mail Classification

Probability of x_i being a spam email

$$P(x_i = 1 | y = 1) = \frac{\text{Count}(x_i = 1 \text{ and } y = 1)}{\text{Count } (y = 1)}$$

Similarly,

$$P(x_i = 0 | y = 1) = \frac{\text{Count}(x_i = 0 \text{ and } y = 1)}{\text{Count } (y = 1)}$$

Spam Mail classification

$$P(y = 1) = \frac{\text{Count } (y = 1)}{\text{Count } (y = 1) + \text{Count } (y = 0)}$$

Similarly,

$$P(y = 0) = \frac{\text{Count } (y = 0)}{\text{Count } (y = 1) + \text{Count } (y = 0)}$$

Example

lets assume that dictionary is $[w_1, w_2, w_3]$

Index	w_1	w_2	w_3	y
1	0	0	0	1
2	0	0	0	0
3	0	0	0	1
4	1	0	0	0
5	1	0	1	1
6	1	1	1	0
7	1	1	1	1
8	1	1	0	0
9	0	1	1	0
10	0	1	1	1

Spam Classification

if y=0

- $P(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$

$P(y=0) = 0.5$
Similarly, if y=1

$P(y=1) = 0.5$

Spam Classification

if y=0

- $P(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$
- $P(w_2 = 0|y = 0) = \frac{2}{5} = 0.4$

$P(y=0) = 0.5$
Similarly, if y=1

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Spam Classification

if y=0

- $P(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$
- $P(w_2 = 0|y = 0) = \frac{2}{5} = 0.4$
- $P(w_3 = 0|y = 0) = \frac{3}{5} = 0.6$

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Similarly, if y=1

- $P(w_1 = 1 | y = 1) = \frac{2}{5} = 0.4$

$P(y=1) = 0.5$

Spam Classification

if y=0

- $P(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$
- $P(w_2 = 0|y = 0) = \frac{2}{5} = 0.4$
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Similarly, if y=1

- $P(w_1 = 1|y = 1) = \frac{2}{5} = 0.4$
- $P(w_2 = 1|y = 1) = \frac{1}{5} = 0.2$

$P(y=1) = 0.5$

Spam Classification

if y=0

- $P(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$
- $P(w_2 = 0|y = 0) = \frac{2}{5} = 0.4$
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Similarly, if y=1

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- $P(w_2 = 1|y = 1) = \frac{1}{5} = 0.2$
- $P(w_3 = 1|y = 1) = \frac{3}{5} = 0.6$

$P(y=1) = 0.5$

Spam Classification

Given, test email 0,0,1, classify it. Using naive bayes rule, we can do the following,

$$P(y = 1|w_1 = 0, w_2 = 0, w_3 = 1) \frac{P(w_1 = 0|y = 1)P(w_2 = 0|y = 1)P(w_3}{P(w_1 = 0, w_2 = 0, w_3}$$

Gaussian naive Bayes

We have classes $C_1, C_2, C_3, \ldots, C_k$
There is a continuous attribute x
For Class k

- $\mu_k = Mean(x|y(x) = C_k)$

Gaussian naive Bayes

We have classes $C_1, C_2, C_3, \ldots, C_k$
There is a continuous attribute x
For Class k

- $\mu_k = Mean(x|y(x) = C_k)$
- $\sigma_k^2 = Variance(x|y(x) = C_k)$

Guassian Naive Bayes

Now for $x =$ some observation 'v'

$$P(x = v | C_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp^{\frac{-(v-\mu_k)^2}{2\sigma_k^2}}$$

Wikipedia Example

Height	Weight	Footsize	Gender
6	180	12	M
5.92	190	11	M
5.58	170	12	M
5.92	165	10	M
5	100	6	F
5.5	100	6	F
5.42	130	7	F
5.75	150	7	F

Example

	Male	Female
Mean (height)	5.855	5.41
Variance (height)	3.5×10^{-2}	9.7×10^{-2}
Mean (weight)	176.25	132.5
Variance (weight)	1.22×10^2	5.5×10^2
Mean (Foot)	11.25	7.5
Variance (Foot)	9.7×10^{-1}	1.67

Classify the Person

Given height = 6ft, weight = 13.5 lbs, feet = 8 inches, classify if it's male or female.

Classify the Person

Given height = 6ft, weight = 13.5 lbs, feet = 8 inches, classify if
it's male or female.
It is female!