A Survey on the Hardware Realization of 2-D State Space Filtering

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Abstract— **Hardware realization of 2-D state space filtering is a big problem of present scenario. In the decade of seventy, the theoretical realizations of 2-D linear state-space models were reported by various researchers. Among these models, the general model is suitable for state space filtering and hardware realization. The beauty of general model SSF is that, it may be transform either in Fornasini-Marchesini (FM) first or second model after slightly modification in hardware. In this article, we present a brief overview of the key developments in hardware realization of the state apace filtering algorithms and architectures.**

Index Terms— **2-D system, FM model, General model, state-space filter realization.**

I. Introduction

DUE to rapid development of technology in information science and computing, the importance of 2-D digital filter are increases. Lot of application of 2-D digital filter for both theoretical and practical aspects such as weather signal, seismographic data processing signal, mechanical vibration signal, telemetry signals, radio astronomy signal, biomedical system, navigation signals, gas absorption and mobile signals etc[1-4]. In the study of 2-D state space filtering (SSF) process, many authors discussed about stability of a system, error estimation, round-off noise minimization, optimization of error feedback, finite word length effects. Due to these uncertainties, the instability has been developed and hence, we have used robust controller that stabilizes the system.

We may design or simulate state space filters with help of either software or hardware based. Software is what makes the hardware function properly and to an optimum level. The Software based architecture design and verification are very easy as compared to hardware based realization of same architecture. The real time performance of software based simulation is very poor while hardware based is very reliable and suitable. A real-time system is described as the one which receives data, process them and returns the results in sufficiently small time to control the environment at that time.

In the decade of 70's, many authors have described the 2-D system dynamics in other way such as linear state-space models for 2-D discrete systems [5-8]. The popular 2-D state space models are Roesser model [5], Fornasini-Marchesini (FM) first model [6], FM second model [7] and General model [8]. Even though the Roesser Model has been proposed to analyse and control linear iterative circuits, but it can also be used in encoding, decoding and image processing where as the FM models and the General model have been extensively used in signal processing and control systems. A brief review on stability analysis of FM-I and FM-II are discussed in [9-10]. As opposed to 1-D systems, the 2-D Lyapunov theorems reported so far gives only sufficient conditions for the stability. The present trend of research in state space filtering is only LMI based analysis. In these papers, the authors have discussed about asymptotic stability of state space system which was verified and simulated by linear matrix inequality (LMI) tool. To solving various problems of robust control, LMI is a powerful tool. Hardware development is costly and requires significant amount of dedicated R&D investment. Due to this most of the authors have focused only on theoretical point of views.

In this paper, we focus on basic hardware realization technique of 2-D state space filtering such as general model, FM-I, FM-II and Roesser model. The general model which is the most efficient structural because they maybe transform either in FM-I or FM-II model after slightly modification in hardware. This paper, therefore, presents a brief survey on the hardware realization of 2-D discrete systems. The organization of paper is as follows. In next section, we have discussed about basic concept of 2-D state space filters with corresponding block diagrams. In section-III, presents a brief survey on hardware realization of 2-D state-space filtering. Finally, some concluding remarks are given in Section IV.

Throughout this paper, the following notation will be taken as: R^n and $R^{n \times n}$ denotes real vector space of dimension $n \times 1$ and $n \times n$ respectively. The local input vector is defined as $\boldsymbol{u}(i,j) \in R^{m \times 1}$, $\boldsymbol{x}(i,j)$ is $n \times 1$ state vector, $\boldsymbol{A}_1 \in R^{n \times n}$, $\boldsymbol{A}_2 \in R^{n \times n}$, $\boldsymbol{A}_3 \in R^{n \times n}$, $\boldsymbol{B}_1 \in R^{n \times l}$, $\boldsymbol{B}_2 \in R^{n \times l}$, $\boldsymbol{B}_3 \in R^{n \times l}$, $\boldsymbol{C} \in R^{n \times n}$, and $\boldsymbol{D} \in R^{m \times l}$ and $y(i,j)$ is a scalar output.

II. Basic State Space Filtering

In this section, we have discussed the basics of 2-d state space filters. Each model has specific advantage and application. Even though the Roesser Model has been proposed to analyse and control linear iterative circuits, but it can also be used in data encoding-decoding and image processing where as the FM models and the General model have been extensively used in control system and signal processing.

A. General Model State Space Filtering

The state space representation of 2-D discrete linear shift invariant General model [8] can be defined by

$$\begin{aligned}\boldsymbol{x}(i+1,j+1) &= \boldsymbol{A}_1 x(i,j+1)+\boldsymbol{A}_2 x(i+1,j)+\boldsymbol{A}_3\boldsymbol{x}(i,j) \\ &\quad +\boldsymbol{B}_1\boldsymbol{u}(i,j+1)+\boldsymbol{B}_2\boldsymbol{u}(i+1,j)+\boldsymbol{B}_3\boldsymbol{u}(i,j)\end{aligned} \tag{1.1a}$$

$$y(i,j)=\boldsymbol{C}\,\boldsymbol{x}(i,j)+\boldsymbol{D}\,\boldsymbol{u}(i,j) \tag{1.1b}$$

$$i\ge 0,\; j\ge 0, \tag{1.1c}$$

The initial conditions of the system are that there exist two positive integers r_1 and r_2 such that,

$$\boldsymbol{x}(i,0)=0,\;\; i\ge r_1,\quad \boldsymbol{x}(0,j)=0,\quad j\ge r_2 \tag{1.2}$$

The equilibrium $\boldsymbol{x}(i,j)=0$ of above systems is said to be globally asymptotically stable if

$$\lim_{i\to\infty \text{ and / or } j\to\infty}\boldsymbol{x}(i,j)=\lim_{i+j\to\infty}\boldsymbol{x}(i,j)=0 \tag{1.3}$$

Transfer function for the General model is given as:

$$\begin{aligned}\boldsymbol{H}(z_1,z_2)&=\boldsymbol{C}(z_1z_2\boldsymbol{I}-z_2\boldsymbol{A}_1-z_1\boldsymbol{A}_2-\boldsymbol{A}_3)^{-1}\\ &\quad\times(z_2\boldsymbol{B}_1-z_1\boldsymbol{B}_2-\boldsymbol{B}_3)+\boldsymbol{D}\end{aligned} \tag{1.4}$$

The block diagram of 2-D general model state space filtering is shown in fig. 1. The block contains simple multiplier, adder and shifter circuits.

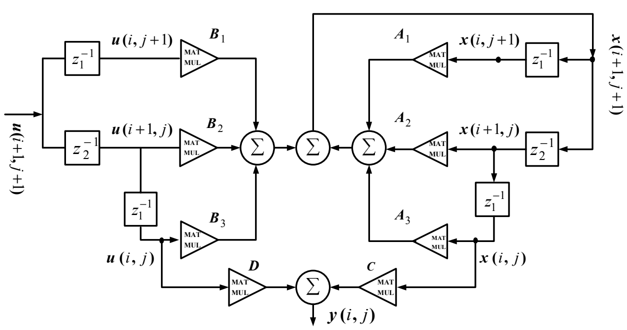

Fig: 1 Block diagram 2-D general model state-space filter.

B. Fornasini-Marchesini First Model (FM-I)

The FM first model [9] may be used to describe a linear shift-invariant 2-D discrete system. The block diagram of FM-I model is depicted in figure 2.

$$\begin{aligned}\boldsymbol{x}(i+1,j+1)&=\boldsymbol{A}_1x(i,j+1)+\boldsymbol{A}_2x(i+1,j)+\boldsymbol{A}_3\boldsymbol{x}(i,j)\\ &\quad+\boldsymbol{B}\boldsymbol{u}(i,j)\end{aligned} \tag{2.1a}$$

$$y(i,j)=\boldsymbol{C}\,\boldsymbol{x}(i,j) \tag{2.1b}$$

$$i\ge 0,\quad j\ge 0, \tag{2.1c}$$

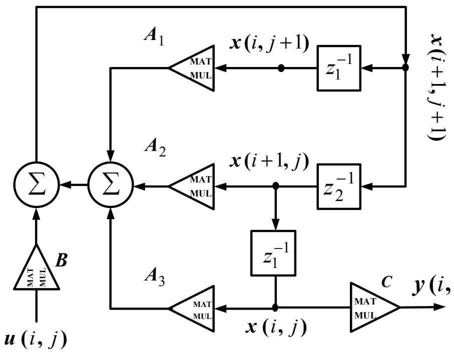

Fig: 2 Block diagram of 2-D FM first model

The initial condition of this model is same as above and transfer function is defined as:

$$\boldsymbol{H}(z_1,z_2)=z_1z_2\boldsymbol{C}(\boldsymbol{I}-z_1\boldsymbol{A}_1-z_1\boldsymbol{A}_1-z_1z_2\boldsymbol{A}_3)^{-1}\boldsymbol{B} \tag{2.2}$$

The state-space model (2.1a) is asymptotically stable [9] if and only if

$$\det(\boldsymbol{I}-z_1\boldsymbol{A}_1-z_2\boldsymbol{A}_2-z_1z_2\boldsymbol{A}_3)^{-1}\ne 0 \tag{2.3a}$$

for all

$$(z_1,z_2)\in\overline{\mathrm{U}^2} \tag{2.3b}$$

where

$$\overline{\mathrm{U}^2}=\{(z_1,z_2):|z_1|\le 1,|z_2|\le 1\}.$$

C. Fornasini-Marchesini Second Model (FM-II)

The FM first model [10] may be used to describe a linear shift-invariant 2-D discrete system. The block diagram of this model is depicted in figure 3.

$$\begin{aligned}\boldsymbol{x}(i+1,j+1)&=\boldsymbol{A}_1\boldsymbol{x}(i,j+1)+\boldsymbol{A}_2\boldsymbol{x}(i+1,j)\\ &\quad+\boldsymbol{B}_1u(i,j+1)+\boldsymbol{B}_2u(i+1,j)\end{aligned} \tag{3.1a}$$

$$y(i,j)=\boldsymbol{C}\,\boldsymbol{x}(i,j)+\boldsymbol{D}\,\boldsymbol{u}(i,j) \tag{3.1b}$$

$$i\ge 0,\quad j\ge 0 \tag{3.1c}$$

The initial and asymptotic stability of this model is same as above and transfer function is defined as:

$$\begin{aligned}\boldsymbol{H}(z_1,z_2)&=\boldsymbol{C}(\boldsymbol{I}-z_1\boldsymbol{A}_1-z_2\boldsymbol{A}_2)^{-1}\\ &\quad\times(z_1\boldsymbol{B}_1+z_2\boldsymbol{B}_2)+\boldsymbol{D}\end{aligned} \tag{3.2}$$

The state-space model (2.1a) is asymptotically stable [10] if and only if

$$\det(\boldsymbol{I}-z_1\boldsymbol{A}_1-z_2\boldsymbol{A}_2)\ne 0 \tag{3.3a}$$

for all

$$(z_1,z_2)\in\overline{\mathrm{U}^2} \tag{3.3b}$$

where

$$\overline{\mathrm{U}^2}=\{(z_1,z_2):|z_1|\le 1,|z_2|\le 1\}.$$

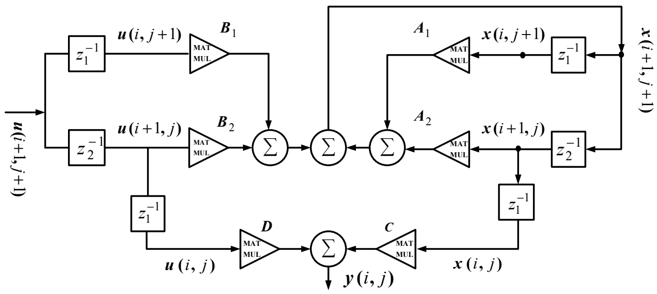

Fig: 3 Block diagram of 2-D FM second model

D. The Local State-Space Model (Roesser Model)

The Roesser model is generally used for linear image processing. An image is a generalization of a temporal signal in that it is defined over two spatial dimensions instead of a single temporal dimension. The mathematically it may be depicted in eqn. (4). The block diagram of Roesser model is shown in figure 4.

$$\begin{bmatrix} \boldsymbol{x}^h(i+1,j) \\ \boldsymbol{x}^v(i,j+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^h(i,j) \\ \boldsymbol{x}^v(i,j) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B_1} \\ \boldsymbol{B_2} \end{bmatrix} u(i,j) \quad (4.1a)$$

$$y(i,j) = \begin{bmatrix} \boldsymbol{C}_1 & \boldsymbol{C}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^h(i,j) \\ \boldsymbol{x}^v(i,j) \end{bmatrix} + \boldsymbol{D}u(i,j) \quad (4.1b)$$

where $\boldsymbol{x}^h \in R^{n1}$ is the horizontal state vector, $\boldsymbol{x}^v \in R^{n2}$ is the vertical state vector, $\boldsymbol{x} \in R^n$, $n = n_1 + n_2$ is the local state vector, u and y are the input and output respectively. Equation (4.1) has same initial condition as discussed above.

The 2-D rational transfer function of LSSM is represented as:

$$\boldsymbol{H}(z_1, z_2) = \boldsymbol{C}(z_1 z_2 \boldsymbol{I}_1 \boldsymbol{I}_2 - \boldsymbol{A})^{-1} \boldsymbol{B} + \boldsymbol{D} \quad (4.2)$$

where $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ are $n_1 \times n_1$ & $n_2 \times n_2$ identity matrices respectively.

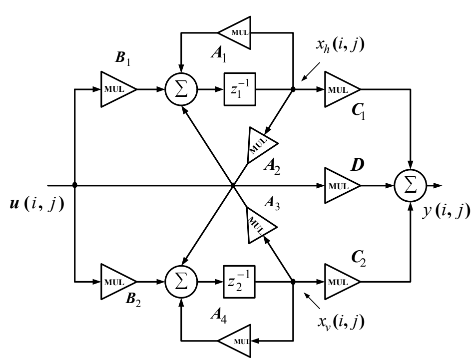

Fig:4 Block diagram of 2-D Roesser model

III. A Brief Survey

In this Section, we will discuss the briefing of exiting literature survey on hardware realization of 2-D state space filtering.

A vast variety of papers may be found in literature, in which realization of 2-D systems has been discussed for various state-space models [11-19].

Realization of 2-D filter based on the Attasi model from pulse response data is discussed in [13]. Koo et *al.* [14] have presented realization techniques for 2-D transfer function. In [15], the authors have realized a 2-D transfer function from measured input-output data. A canonical form realization for the 2-D Roesser model is discussed in [16]. Other hardware realizations of Roesser state-space model given by Koonar et al. [17] have developed an algorithm for realization of a 2-D transfer function. It has been shown in [18], that the algorithm present in [17] is invalid. New hardware realization techniques of 2-D state space digital filters are presented in [19]. In this paper, author has implemented a half-plane local state-space model. The architecture is simplest and realized easily. A basic cell unit has implemented which associates with summer and multipliers for fixed and floating point arithmetic. This small cell is further used to design the state space filters. An application based examples are presented for checking and verification of efficient performance. The one is used to illustrate the realization of SSF is free from overflow oscillation. The others are used to demonstrate the extension of this realization approach to higher dimensions digital filters i.e. 3-D SSF digital filters. S. Sridharan has suggested a block state-space filter which is used for both fixed and floating point arithmetic and compares its signal to noise ratio and overflow problem which was occurred in most of hardware based architecture realization [20]. The real-time two dimensional (LSS) digital filtering is introduced in [21], which has used systolic arrays first time. The processing elements of systolic arrays are used ROM for multiplication instead of multipliers. Since, the multipliers have replaced by ROM's then we may avoid the need to initially load the coefficients. The coefficients of the filter are fixed in the cells of systolic arrays. The advantage of using ROM is that it eliminates the quantized error because the partial product terms generated during multiplication are stored in ROM. This systolic design is valid for image of 512x512 but extend up to larger size. For implementing these architectures, authors had used digital ICs instead of HDL coding which is used in present state of art for simulation and verification of digital hardware design. Keshab and David [22] suggested a new technique for designing of recursive 2-D state space filtering. This technique is valid for both one and two dimensional filters. Another method to realization of state space 2-D linear digital system is given by Ali M. Eydgahi [23]. The object of this paper is to obtain realization of a 2-D transfer function matrix in the Roesser state-space model. A systolic architecture for 2-D state-space filter, which also introduce some VLSI structures for high speed applications has been discussed in [24].

In this paper, first the state-space realizations of 2-D IIR filters are discussed and then using systolic array realizations, a direct form implementation and a transformed implementation, are obtained to improve local speedup. This architecture is also applicable for multiple column of image together. The 2-D recursive digital filter represented by a 2-D block state-space model is considered in [25] and a pipelined SIMD architecture is developed for the purpose of 2-D block processing. This type of architecture is particularly useful when similar operations on a large set of data are to be performed. Studies in [24] is further carried out in [25] for high speed architectures of 2-D state space recursive filter. An advanced state update architecture based on 2-D systolic array is proposed and it is shown that adjustable throughput rates for different requirements can be obtained due to flexible and modular global speedup architecture. Due to higher data throughput rate and better performance as compared to the corresponding to the simple 2-D filter, block processing is preferred in the past few years. M. *Y.* Dabbagh and W. E. Alexander [26] give a new architecture of block state-space 2-D digital filter achieves linear speedup and involve matrix and vector operation. Jinyun Zhang et al. describe VLSI implementations of real-time 1-D IIR filters based on the state-space technique [27]. This paper is extension of [21] and it also used direct form systolic array for realization. The authors have used an advanced state calculation array and a cascade- form of second-order structures for transforming the state matrix. These advanced technique increases throughput and efficiency by (N + 1) times at the cost of two times increased in hardware. The author has also discussed block processing technique for realization of same SSF. The size and speed is big problem for any hardware based design. To reduce this problem in the designing of 2-D SSF, Pradeep Mishra et al. introduces a new technique for realization of filtering using fewer multipliers [28] and the saving in cost of hardware. The author compares this new technique to that architecture which was discussed in [19]. In [29], author has used distributive arithmetic technique for designing of systolic arrays. In this technique, they used ROM for multiplication. Jin Yun Zhang and Willem Steenaart [30] have proposed very attractive 2-D systolic arrays for high speed applications. The block processing technique has used to increase throughput rate and simple input/output scheme for increased hardware. The state update matrix is changed by recursive multiplications and the advanced state vector can be calculated. The throughput rate may be very high and it is adjustable by changing the number of column processors. Several other methods of hardware realization techniques are given in [31]-[35]. The hardware realization of 2-D adaptive filter based on FM-II model is discussed in [33].

From the above, most of the authors has discussed on Roesser model. The VLSI design of 2-D general model state space digital system is discussed in [36]. The author has design hardware architecture of general model which may be used as either FM-I or FM-II model. It may be converted to FM model after slightly change in equation (1) i.e. when $\boldsymbol{B}_1 = \boldsymbol{B}_2 = 0$ and $\boldsymbol{B}_3 = \boldsymbol{B}$ then equation (1.1) become Fornasini-Marchesini First Model which is defined by equation (2.1) and similarly When $\boldsymbol{A}_3 = \boldsymbol{B}_3 = 0$ then equation (1.1) become Fornasini-Marchesini second Model defined by equation (3.1). The output equation of FM-II model are same as general model while output equation of FM-I model may be achieve by putting $\boldsymbol{D} = 0$ then output equation (1.2) become equation (2.2).

IV. Conclusion

The concluding remarks on this review paper is that the hardware realization of 2-D state space filtering. It is worth noting that all the filtering architectures reported in this review are based on the Roesser model. Roesser Model has been proposed to analyse and control linear iterative circuits, but it can also be used in encoding, decoding and image processing where as the FM models and the General model have been extensively used in signal processing and control. However, designing a state-space filtering architecture based on the General model, which is a super set of the FM first model and FM second model, is much more complex and tedious than the Roesser model.

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