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Modified Model of RLS Adaptive Filter for Noise Cancellation

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Abstract

Recursive Least Square (RLS) is a popular algorithm for noise cancellation in non-stationary signals; however, it demands more computational resources and more difficult mathematical operations. Also, RLS has less performance stability. The present research explores a novel idea of an RLS adaptive noise cancellation through a modified method that uses an RLS adaptive filter by introducing an additional constant multiplier, along with their in-depth analysis. The proposed algorithm is analyzed using three primary performance metrics: mean square error (MSE), signal-to-noise ratio (SNR), and convergence rate. The obtained results demonstrate that the proposed algorithm has reduced the MSE by 79.65%, which leads to an improvement in SNR by 86.16% compared to the traditional RLS algorithm. The additional constant multiplier is optimized for SNR, and the optimized value is found to be equal to 0.65, which gives the best possible SNR value of 19.38 dB. Also, the proposed algorithm has been successfully applied to a real-world scenario in acoustic echo cancellation (AEC). The experimental setup for the echo canceller is simulated on MATLAB to measure the echo canceller efficiency in terms of MSE and echo return loss enhancement. Based on performance evaluation, the proposed algorithm has been found to better echo cancellation in AEC as compared to traditional RLS.

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1 Introduction

There are numerous examples in real-time digital signal processing systems where desired signals are corrupted by unwanted signals referred to as noisy signals. The term "noise" refers to any disturbances caused by the environment, stimuli, or parts of sensors and circuits and does not just refer to thermal or flicker noise [18]. Noisy data exists from a wide variety of internal and external sources. The information could, for instance, have been obtained using unreliable sensors or represent a useful signal element corrupted by transmission over a communication channel [21]. Linear and nonlinear filtering are the two main categories of noise removal techniques. The linear filtering problem is dealt with using a statistical method. Prior knowledge of the message signal's statistical properties and an unknown signal (noise) is necessary for linear filtering. To reduce the cause of additive noise at the output, we apply statistical parameter criteria (Correlation functions and mean) when developing a linear filter with noise as input data. This filter optimization tool's mean-square error (MSE) will be as low as possible. Also, the filter coefficients cannot be predicted in advance if the signal statistics are unknown beforehand or if they change over time, making the usage of linear filters inadequate. Due to the real-time changes in signal and noise characteristics, it is necessary to apply filter algorithms that rapidly converge to the new environment [13]. The resulting stationary input solution is frequently called a Wiener Filter [48]. The Wiener filter, however, cannot handle non-stationary circumstances when dealing with non-stationary input signals. For the optimum filter, non-stationary signals require a time-varying form to be assumed. The Kalman filters [25] offer a very effective strategy. Nevertheless, the Kalman filters required incredibly sophisticated and expensive technology to operate in real time. The topic of adaptive filter noise cancellation currently has a great application impact and research interest. Because their adaptations are based on statistical averages obtained with limited sample numbers, it is known that all adaptive filters capable of adjusting at real-time rates undergo performance losses [44]. The adaptive filter [9, 19, 22, 41, 45, 47] is one of the most effective techniques for reducing predictable and unpredictable noise from the original signal corruption. To extract the correlated signal from the non-stationary input, adaptive filters' impulse responses are modified [29, 38]. Applications of adaptive filters include adaptive noise cancellation [12, 17, 24], adaptive equalization [37], inverse modeling [20], adaptive system identification [6, 26, 43], and linear prediction [30, 36]. Adaptive filters do not have constant filter coefficients [7]. A broad class of adaptive signal processing techniques' tracking capabilities is studied in [15]. In a hydro-acoustic channel with significant reflections, the quality of data transmission is enhanced via adaptive filtering based on recursive least squares (RLS). The obtained data can also be used to evaluate how effectively RLS adaptive filtering improves data transmission quality [4]. A higher convergence blind equalization approach using the Least Squares-based RLS method and a noise reduction unit is also useful [28]. The

RLS algorithm does not require any parameter estimation and can efficiently suppress cardiac interference merely using a single-channel EEG and ECG [8].

The design of a classical RLS adaptive filter is depicted in Fig. 1 [35]. The RLS algorithm minimizes the cost function using weighted linear least squares relating to the input signals by iteratively finding the adaptive filter coefficients. Even without the statistical measurements of the input signals, the RLS algorithm has the ability to update a filter's coefficients automatically. This technique minimizes the weighted sum of the squares of the required signal and estimates errors at every instant [5]. To minimize the weighted sum of the squares error in RLS, a parameter known as the forgetting factor ($0 << \lambda < 1$) is used to forget data samples in the distant past.

The behavior of the RLS algorithm in terms of convergence, misalignment, and stability is significantly influenced by the forgetting factor λ. In the conventional RLS technique, this parameter has a constant value that is positive and less than 1 [34]. It is well known that the algorithm has less tracking ability and better stability and fast convergence if λ is closer to one. Lowering the value of λ improves tracking capabilities but also degrades algorithmic stability and speed convergence. To address these contradictory needs, the variable forgetting factor is applied. Numerous algorithms for variable forgetting factor RLS (VFF-RLS) have been developed (see [2, 3, 31, 40] and references therein).

The RLS algorithm [11, 16, 27, 32, 33, 39, 42] is given by as below:

$$y(\kappa) = X^T(\kappa)h(\kappa) \tag{1}$$

$$e(\kappa) = d(\kappa) - y(\kappa) \tag{2}$$

$$h(\kappa) = h(\kappa - 1) + I(\kappa)e(\kappa) \tag{3}$$

where $I(\kappa)$ is the intermediate gain, $h(\kappa) = [h_0(\kappa), h_1(\kappa), h_2(\kappa)......., h_{M-1}(\kappa)]^T$ is the coefficient vector,$X(\kappa) = [x(\kappa), x(\kappa - 1), x(\kappa - 2)........., x(\kappa - M + 1)]^T$ is

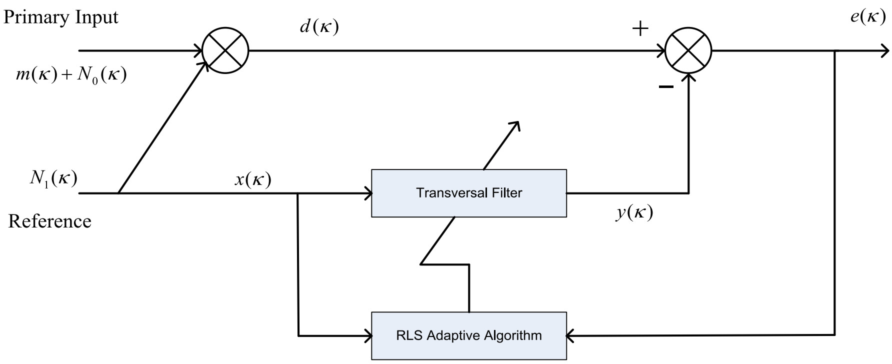


Fig. 1 Traditional RLS adaptive filter

the input vector, e(κ) is the error signal, d(κ) is the desired signal, y(κ) is the output signal, and M is the length of the filter.

Compared to NLMS and LMS, the RLS algorithm converges faster. It has a greater capability for noise cancellation, but it also uses more processing resources and includes more challenging mathematical calculations. RLS filter output suffers from low-performance stability.

The main contributions in this paper are three-fold:

- A modified RLS adaptive filter is proposed, which is another promising approach toward further reduction in noise cancellation by introducing an additional constant multiplier to the traditional RLS adaptive filter.
- Based on the proposed model, the more accurate expression for the "estimation-noise" has been derived in terms of constant multiplication factor.
- The dependency of the performance characteristics such as MSE, SNR, and filter output on the additionally introduced constant multiplier of the modified filter is significantly analyzed. Further, the constant multiplier is optimized with the help of simulation.

The remainder of this paper is organized as follows. In Sect. 2, the methodology of the modified model and algorithm has been proposed. Section 3 presents the experimental results of computer simulations that verify the developed theory and its application in AEC. Section 4 is a brief conclusion.

2 The Methodology of the Proposed Model and Algorithm

The ANC methodology of the proposed modified RLS algorithm is depicted in Fig. 2. Modified RLS adaptive filtering consists of an additional multiplier, 'β.'

The objective of adaptive filtering is to minimize the error which is the difference between desired signal and filter output ($e(\kappa) = d(\kappa) - y(\kappa)$). So here our target is to reduce that difference to minimize the error. A pre multiplier 'β' helps in reducing the

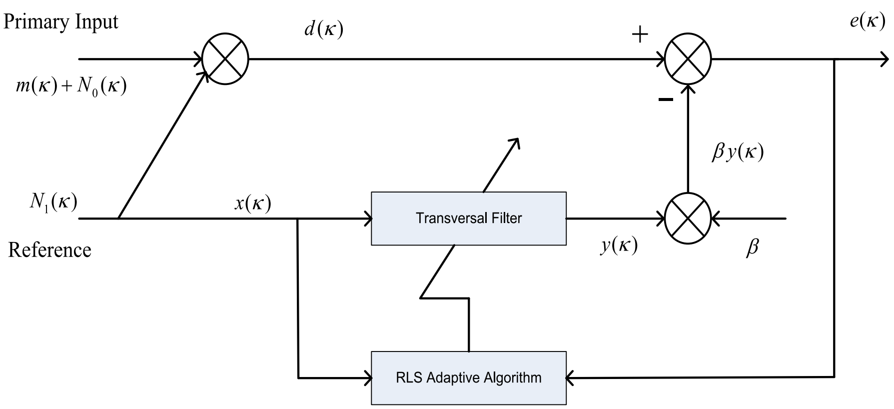


Fig. 2 ANC methodology of modified RLS adaptive filtering concept

difference between the desired signal and filter output $(d(\kappa) - y(\kappa))$, which minimizes the error $(e(\kappa) = d(\kappa) - \beta y(\kappa))$ before applying it to the adaptive algorithm. Thus, β aids in reducing the noise with better accuracy in noise cancellation and also less computational time for updating the filter coefficient in comparison to traditional RLS.

An information signal 'm(κ)' is propagated across a channel to a sensor in an adaptive noise canceller. Noise '$N_0(\kappa)$' is added to the channel uncorrelated to the information signals. The primary input is a noise-filled information signal.

The second sensor now provides the reference input to the canceller, which receives a noise signal '$N_1(\kappa)$' correlated with the noise '$N_0(\kappa)$' and uncorrelated with the information signal. Filtering error is given by

$$e(\kappa) = d(\kappa) - \beta y(\kappa) = d(\kappa) - \beta X^T(\kappa)h(\kappa) \tag{4}$$

weighted sum-square error is calculated by:

$$\varepsilon(\kappa) = \sum_{m=0}^{\kappa} \lambda^{\kappa-m}|e(m)|^2 = \sum_{m=0}^{\kappa} \lambda^{\kappa-m}\Big(d(m) - \beta X^T(m)h(m)\Big)^2 \tag{5}$$

where 'λ' is the forgetting factor (0 << λ < 1) used to forget data samples in the distant past.

For minima, differentiating Eq. (5) with respect to h(κ) and equating to zero, we obtain

$$2\sum_{m=0}^{\kappa} \lambda^{\kappa-m}(d(m) - \beta X^T(m)h(m))\beta X(m) = 0$$

or

$$\left[\sum_{m=0}^{\kappa} \lambda^{\kappa-m} X^T(m)X(m)\right]\beta h(\kappa) = \sum_{m=0}^{\kappa} \lambda^{\kappa-m} d(m)X(m)$$

or

$$\beta \hat{R}_X(\kappa)h(\kappa) = \hat{r}_{dX}(\kappa) \tag{6}$$

The above equation is called a normal equation for LS estimation, where, $\hat{R}_X(\kappa) = \sum_{m=0}^{\kappa} \lambda^{\kappa-m} X^T(m)X(m)$ i.e., estimation for autocorrelation matrix and $\hat{r}_{dX}(\kappa) = \sum_{m=0}^{\kappa} \lambda^{\kappa-m} d(m)X(m)$ i.e., estimation for cross-correlation. Now, the solution to the normal equation is given by:

$$h(\kappa) = \frac{1}{\beta}\hat{R}_X^{-1}(\kappa)\hat{r}_{dX}(\kappa), \beta \neq 0 \tag{7}$$

But in the above equation, matrix inversion is involved, which is computationally complex. So, we will find the above inverse recursively.

We have an autocorrelation matrix

$$\hat{R}_X(\kappa) = \sum_{m=0}^{\kappa} \lambda^{\kappa-m} X^T(m)X(m) = \lambda \sum_{m=0}^{\kappa-1} \lambda^{\kappa-m-1} X^T(m)X(m) + X^T(\kappa)X(\kappa)$$

or

$$\hat{R}_X(\kappa) = \lambda \hat{R}_X(\kappa-1) + X^T(\kappa)X(\kappa) \tag{8}$$

Similarly, cross-correlation is given by recursive relation as,

$$\hat{r}_{dX}(\kappa) = \lambda \hat{r}_{dX}(\kappa-1) + d(\kappa)X(\kappa) \tag{9}$$

Now we will find an inverse autocorrelation matrix to solve Eq. (7)

$$\hat{R}_X^{-1}(\kappa) = [\lambda \hat{R}_X(\kappa-1) + X^T(\kappa)X(\kappa)]^{-1} \tag{10}$$

Applying the matrix inversion lemma [23] to Eq. (10), we have

$$\hat{R}_X^{-1}(\kappa) = \frac{1}{\lambda}\hat{R}_X^{-1}(\kappa-1) - \frac{\frac{1}{\lambda}\hat{R}_X^{-1}(\kappa-1)X(\kappa)X^T(\kappa)\frac{1}{\lambda}\hat{R}_X^{-1}(\kappa-1)}{1 + X^T(\kappa)\frac{1}{\lambda}\hat{R}_X^{-1}(\kappa-1)X(\kappa)} \tag{11}$$

Let $P(\kappa) = \hat{R}_X^{-1}(\kappa)$, then, Eq. (11) can be written as

$$P(\kappa) = \frac{1}{\lambda}\left[P(\kappa-1) - \frac{P(\kappa-1)X(\kappa)X^T(\kappa)P(\kappa-1)}{\lambda + X^T(\kappa)P(\kappa-1)X(\kappa)}\right] \tag{12}$$

Let us define intermediate gain

$$I(\kappa) = \frac{P(\kappa-1)X(\kappa)}{\lambda + X^T(\kappa)P(\kappa-1)X(\kappa)} \tag{13}$$

Equation (12) is now rewriteable as,

$$P(\kappa) = \frac{1}{\lambda}[P(\kappa-1) - I(\kappa)X^T(\kappa)P(\kappa-1)] \tag{14}$$

By rearranging and solving Eq. (13) and Eq. (14), we get

$$I(\kappa) = P(\kappa)X(\kappa) \tag{15}$$

Using Eq. (7) and Eq. (9), we get.
$h(\kappa) = \frac{1}{\beta}P(\kappa)[\lambda \hat{r}_{dX}(\kappa-1) + d(\kappa)X(\kappa)]$, where $\hat{R}_X^{-1}(\kappa) = P(\kappa)$.

or

$$h(\kappa) = \frac{1}{\beta} P(\kappa)\lambda \hat{r}_{dX}(\kappa - 1) + \frac{1}{\beta} P(\kappa) d(\kappa) X(\kappa)$$

put the value $P(\kappa)$ and $I(\kappa)$ from Eqs. (14) and (15),

$$h(\kappa) = \frac{1}{\beta\lambda}\Big[P(\kappa-1) - I(\kappa)X^T(\kappa)P(\kappa-1)\Big]\lambda\hat{r}_{dX}(\kappa-1) + \frac{1}{\beta} I(\kappa)d(\kappa)$$

or

$$h(\kappa) = \frac{1}{\beta} P(\kappa-1)\hat{r}_{dX}(\kappa-1) - \frac{1}{\beta} I(\kappa)X^T(\kappa)P(\kappa-1)\hat{r}_{dX}(\kappa-1) + \frac{1}{\beta} I(\kappa)d(\kappa)$$

or

$$h(\kappa) = h(\kappa-1) - I(\kappa)X^T(\kappa)h(\kappa-1) + \frac{1}{\beta} I(\kappa)d(\kappa)$$

or

$$h(\kappa) = h(\kappa-1) + \frac{1}{\beta} I(\kappa)[d(\kappa) - \beta X^T(\kappa)h(\kappa-1)]$$

The final weight update equation is given by:

$$h(\kappa) = h(\kappa-1) + \frac{1}{\beta} I(\kappa)e(\kappa) \tag{16}$$

2.1 Minimum Mean Square Error

The error signal is mathematically given by Eq. (17) and is equal to the difference between the primary input and the filter output multiplied by factor β

$$e(\kappa) = m(\kappa) + N_0(\kappa) - \beta y(\kappa) \tag{17}$$

Squaring both sides of Eq. (17), we get,

$$e^2(\kappa) = m^2(\kappa) + [N_0(\kappa) - \beta y(\kappa)]^2 + 2m(\kappa)[N_0(\kappa) - \beta y(\kappa)]$$

Taking expected value

$$E[e^2(\kappa)] = E[m^2(\kappa)] + E[\{N_0(\kappa) - \beta y(\kappa)\}^2] + 2E[m(\kappa)\{N_0(\kappa) - \beta y(\kappa)\}]$$

As y(κ) and $N_0(\kappa)$ is uncorrelated with information signal m(n), therefore

$$E[m(\kappa)\{N_0(\kappa) - \beta y(\kappa)\}] = 0 \tag{18}$$

And minimum mean square error is given by:

$$E_{\min}[e^2(\kappa)] = E[m^2(\kappa)] + E_{\min}[\{N_0(\kappa) - \beta y(\kappa)\}^2] \tag{19}$$

In the weight update equation for the modified RLS algorithm as given in Eq. (16), filter weight is updated in such a way that $E[e^2(\kappa)]$ is minimized and therefore $E[(N_0(\kappa)\text{-}\beta y(\kappa))^2]$ is also minimized. For an ideal value,

$$E[\{N_0(\kappa) - \beta y(\kappa)\}^2] = 0 \tag{20}$$

From Eq. (19),

$$E[e^2(\kappa)] = E[m^2(\kappa)] \;\; \Rightarrow e(\kappa) = m(\kappa) \tag{21}$$

The error signals should be equal to the message signals for the ideal adaptive noise cancellation, as can be seen from Eq. (21) (see Sect. 3 of reference [46]), but as an ideal system does not exist, that means error signals have some part of the reduced noise, i.e., $e(\kappa) = m(\kappa) + \Delta N\prime$.

The value of pre-multiplier β has been calculated from the minimum mean square condition given in Eq. (20) as follows,

$$\beta(k) = \frac{No(k)}{y(k)} \text{ and } \beta(Z) = \frac{No(Z)}{y(Z)} \text{ where } \beta(Z) \text{ is } \text{ z} - \text{transform} \tag{22}$$

where Y(Z) and N_0(Z) are z-transforms of y(κ) and $N_0(\kappa)$, respectively.

The choice of β can have a significant impact on the stability and tracking performance of the algorithm. To rate the stability of the algorithm with respect to β, a pole-zero analysis of the modified RLS system can be performed to assess its stability. The pole-zero plot can help to understand the effects of β on the system's poles and zeros. If poles move outside the unit circle in the Z-plane, it can indicate instability. So, it should be ensured that the poles remain within the unit circle. Also, simulation and testing help in finding stability. Simulating the algorithm with different values of β and observing its behavior in response to various parameters, convergence, tracking performance, MSE, and SNR can rate the stability.

2.2 Convergence Rate of Error Signal

If filter length M = 1 and the desired signal d(κ) = d, input vector x(κ) = x, and gain I(κ) = k, then the filter output and the error signal are written as follows

$$y(\kappa) = h_0(\kappa)x \tag{23}$$

$$e(\kappa) = d - \beta y(\kappa) \tag{24}$$

The modified RLS algorithm's weight update equation is given by:

$$h_0(\kappa) = h_0(\kappa - 1) + \frac{1}{\beta} ke(\kappa) \tag{25}$$

Now from Eq. (24),

$$e(\kappa - 1) = d - \beta y(\kappa - 1)$$

or

$$e(\kappa - 1) = d - \beta x h_0(\kappa - 1)$$

or

$$e(\kappa - 1) = d - \beta x[h_0(\kappa) - \frac{1}{\beta} ke(\kappa)] = d - \beta x h_0(\kappa) + kxe(\kappa)$$

or

$$e(\kappa - 1) = e(\kappa) + kxe(\kappa) = e(\kappa)[1 + kx]$$

or

$$e(\kappa) = \left(\frac{1}{1 + kx}\right) e(\kappa - 1) \tag{26}$$

Similarly, if filter length M = 2 and take input vector x(κ) = x(κ-1) = x, desired signal d(κ) = d, and gain I(κ) = I(κ-1) = k, then filter output and the error signal is written as follows

$$y(\kappa) = [h_0(\kappa) + h_1(\kappa)]x \tag{27}$$

$$e(\kappa) = d - \beta y(\kappa) = d - \beta[h_0(\kappa) + h_1(\kappa)]x \tag{28}$$

Now, weight update equations for the modified RLS algorithm are given as follow:

$$h_0(\kappa) = h_0(\kappa - 1) + \frac{1}{\beta} ke(\kappa) \;\; and \; h_1(\kappa) = h_1(\kappa - 1) + \frac{1}{\beta} ke(\kappa)$$

Now from Eq. (28),

$$e(\kappa - 1) = d - \beta y(\kappa - 1)$$

or

$$e(\kappa - 1) = d - \beta[h_0(\kappa - 1) + h_1(\kappa - 1)]x$$

or

$$e(\kappa-1)=d-\beta x[h_0(\kappa)+h_1(\kappa)-\frac{2}{\beta}ke(\kappa)]=d-\beta x[h_0(\kappa)+h_1(\kappa)]+2kxe(\kappa)$$

or

$$e(\kappa-1)=e(\kappa)+2kxe(\kappa)=e(\kappa)[1+2kx]$$

or

$$e(\kappa)=\left(\frac{1}{1+2kx}\right)e(\kappa-1) \tag{29}$$

In general convergence rate is given by the following equation:

$$e(\kappa)=\left(\frac{1}{1+Mkx}\right)e(\kappa-1) \tag{30}$$

or

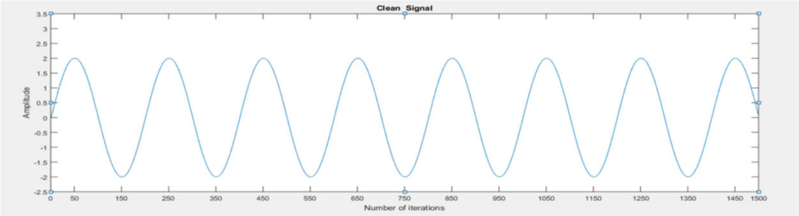
$$e(\kappa)=\left(\frac{1}{1+I(\kappa)X(\kappa)}\right)e(\kappa-1) \tag{31}$$

3 Simulation and Analysis

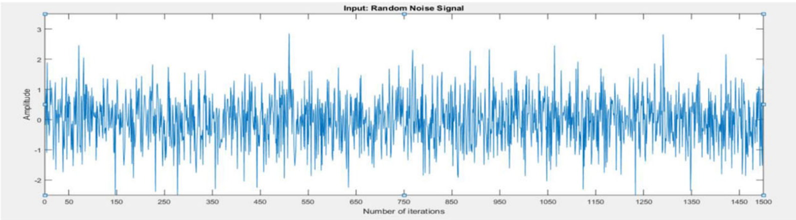
In this section, the performance of the proposed algorithm is simulated in MATLAB; and then, the statistical parameters are analyzed. The three primary performance metrics: mean square error (MSE), signal-to-noise ratio (SNR), and convergence rate are used to analyze the performance of the algorithm. Also, the proposed algorithm has been successfully applied to a real-world scenario in acoustic echo cancellation (AEC). The parameters, MSE and ERLE are used to measure the echo canceller efficiency. The details are in the following sub-section.

3.1 Comparative Analysis of Proposed Algorithm

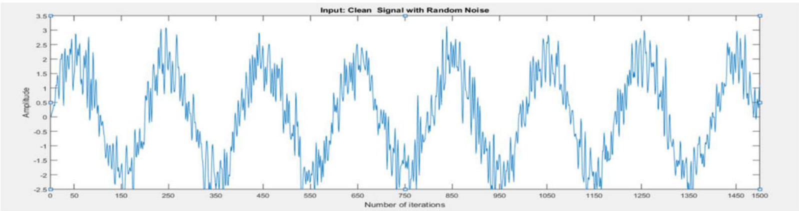
Figure 3 illustrates the simulation results for noise cancellation using a modified RLS algorithm with filter length M = 19 and forgetting factor λ = 1. Figure 3a shows the sinusoidal signal taken as clean signal (information signal) at the following parameters: amplitude = 2, phase = 0, and frequency = 0.005 cycles/sample. Figure 3b shows the AWGN signal with zero mean and noise variance = 0.8. Figure 3c shows the mixed signal (desired signal), i.e., information signal with noisy data. Figure 3d shows mean square error, which confirms that MSE for the modified RLS algorithm is much less than in comparison to the traditional RLS and hence improved the accuracy in adaptive noise cancellation.



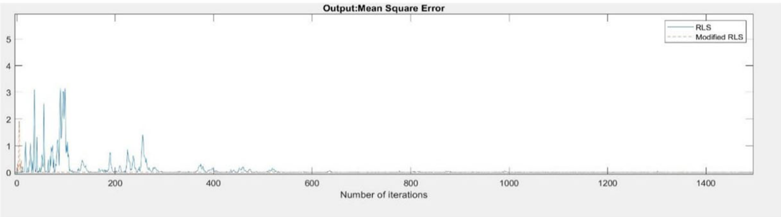
(a) Information Signal



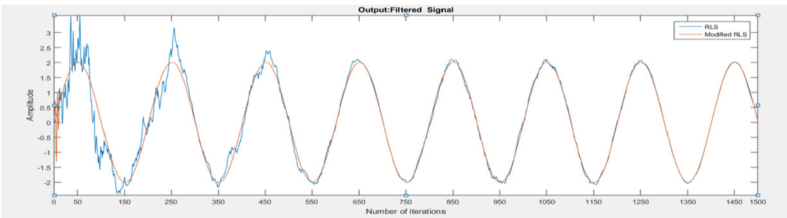
(b) Random Noise Signal



(c) Information Signal with Noisy Signal



(d) Mean Square Error



(e) Filtered Output

Fig. 3 Simulation results for adaptive noise cancellation using Modified RLS algorithm **a** information signal **b** random noise signal **c** information signal with noisy signal **d** mean square error **e** filtered output

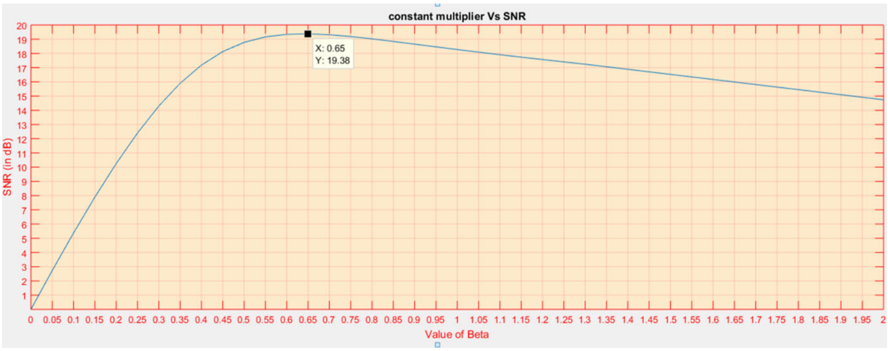


Fig. 4 Optimization of constant multiplier ‘β’

Figure 3e shows adaptive filter output for traditional RLS and modified RLS algorithm. It can be seen from the results that the modified RLS algorithm is more stable and has a fast convergence rate in comparison to the traditional RLS algorithm. It can also be observed that the filtered output by modified RLS takes very less time (approx. 50 iterations) to stable in comparison to the traditional RLS algorithm, which takes 650 iterations approx.

Figure 4 indicates that the optimized value of constant multiplier ‘β’ is 0.65, which gives the SNR value of 19.38 dB. As further the value of ‘β’ increases beyond 0.65, SNR decreases.

The choice of the range of β can significantly impact the performance and stability of the algorithm.

From Eq. 7, it is clear that an inverse autocorrelation matrix can’t be a null matrix that results in $\beta \neq 0$.

Also, β can’t be negative because the target is to minimize the error ($e(\kappa) = d(\kappa) - \beta y(\kappa)$), and if the negative value of β is selected, then error will increase.

Thus, it can be concluded that of $\beta > 0$. Also, it depends on performance requirements for a particular application. For the experiment mentioned, the values of β are selected between 0 and 2 ($0 < \beta < 2$) based on the performance of the algorithm. Finally, we reach the conclusion that the values of β are limited to some extent but there is no fixed, one-size-fits-all stopping criterion, as ‘β’ depends on the specific problem trying to solve, the data characteristics, and performance requirements. It can be tried with different values for the ‘β’ and observe how well the algorithm performs on a specific dataset. Experiment with a range of values and see which one yields the best results in terms of convergence, accuracy, or any other relevant performance metric.

Table. 1 compares the performance parameters for the traditional and modified RLS algorithm in adaptive noise cancellation.

The MSE is one of the quantitative approaches for computing stability. MSE quantifies how closely the adaptive filter’s output tracks the desired signal over time. A decreasing average value of MSE demonstrates that the filter is converging and stable. It can be observed that the traditional RLS having an average value of MSE equal to 0.0172 is less stable in comparison to the modified RLS which has an average value

Table 1 Performance comparison between traditional RLS and modified RLS

S. no	Parameter	Traditional RLS	Modified RLS	Result
1	MSE	0.0172	0.0035	79.65% reduced
2	SNR (in dB)	10.41	19.38	86.16% improved
3	Stability	Less stable	High stable	Modified RLS is highly stable as compared to traditional RLS
4	Convergence Rate	Slow (650 iterations approx.)	Fast (50 iterations approx.)	Compared to conventional RLS, modified RLS has a rapid convergence rate
5	Convergence Time	80.145 m sec	45.817 m sec	Compared to conventional RLS, modified RLS has a lesser value of convergence time

of MSE equal to 0.0035. It can also be seen that traditional RLS has slow convergence rate (650 iterations approx.) as compared to modified RLS (50 iterations approx.). The simulation results for the ANC shows that the traditional RLS and modified RLS have convergence time of 80.145 ms and 45.817 ms respectively.

3.2 Acoustic Echo Cancellation

This section provides an application of the proposed filter design by considering an example of a AEC process. An AEC is a signal processing technology used in various communication systems to eliminate or reduce the echo that occurs during audio or voice calls. Echo in communication systems [10, 14] can be annoying and disruptive, making it difficult for people to have clear conversations. Acoustic echo cancellation is primarily used in telephony, video conferencing, and other real-time communication applications.

Figure 5 shows the AEC with a double talk detector (DTD) [1]. The core task of a DTD is to identify instances where both the near-end and far-end talkers in a communication session are speaking at the same time. This simultaneous speech can create challenges for echo cancellation and can lead to audio artifacts or reduced call quality.

The signal $x(\kappa)$ is the far-end speech that goes through the acoustic impulse response $h(\kappa)$, resulting in the echo signal, $y(\kappa)$. The echo signal is picked up by the microphone together with the near-end signal $v(\kappa)$, resulting in the microphone signal $d(\kappa)$. The output of the adaptive filter, $\hat{y}(\kappa)$, provides a replica of the

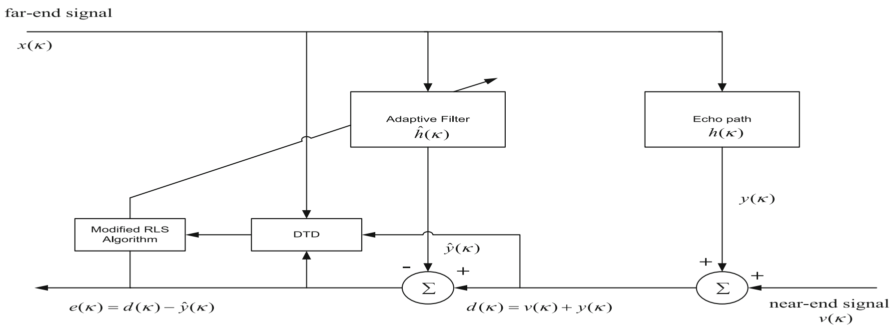


Fig. 5 AEC with DTD

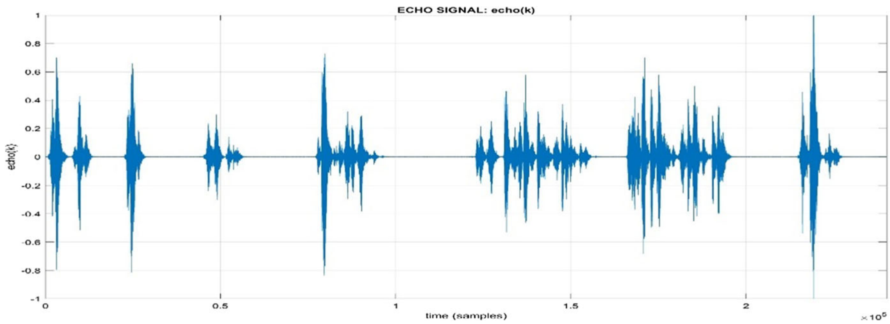


Fig. 6 Echo Signal

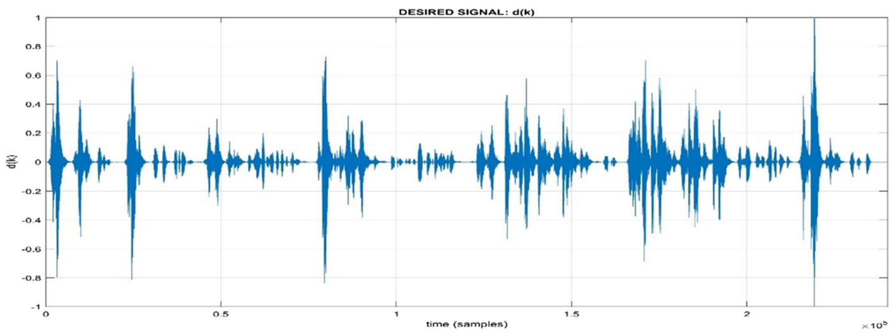


Fig. 7 Desired Signal

echo, which will be subtracted from the microphone signal resulting in an error signal, $e(\kappa) = v(\kappa) + y(\kappa) - \hat{y}(\kappa)$. The adaptive filter works to minimize the echo $(y(\kappa) - \hat{y}(\kappa))$ to zero to obtain only near-end signal $v(\kappa)$ in the ideal case.

The adaptive filter is a central component in AEC systems, playing a crucial role in identifying and canceling the echo present in audio signals during communication. The adaptive algorithm (RLS and Modified RLS) is applied in acoustic echo cancellation to cancel the effect of echo in the system. The measure of the echo canceller efficiency is expressed by the mean square error (MSE) and echo return loss enhancement (ERLE).

The experiments considered an AEC with DTD, as shown in Fig. 5, are also simulated in MATLAB. The length of the adaptive filter is set to 2000 coefficients. The acoustic echo path is measured using an 8000 Hz sampling rate. Figures 6 and 7 show the echo signal and desired signal.

The impulse response of the acoustic echo path and estimated system is plotted in Fig. 8.

The output of the adaptive filter for RLS and Modified RLS is shown in Fig. 9. This output is subtracted from the desired signal to get the error signal which is shown in Fig. 10

Normalized cross-correlation between error and microphone signal is used for double talk detection as shown in Fig. 11. The decision statistic is compared to a threshold value ($V_{th} = 0.0$). If the decision statistic is greater than the threshold, then only the far-end signal is present and if it is less than the threshold, then double-talk or only near-end talks or both sides are silent.

The MSE is shown in Fig. 12. MSE measures how much the algorithm minimizes the echo. The lower the value of MSE results the better echo cancellation efficiency. The average value of the MSE for both algorithms is given in Table 2.

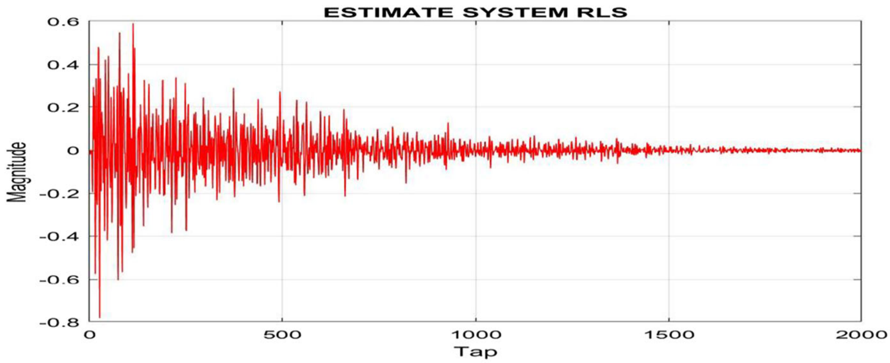
Figure 13 shows the comparative ERLE between RLS and modified RLS. ERLE depends on the algorithm used for the adaptive filter. A higher value of ERLE demonstrates better echo cancellation. The ERLE is defined as

$$\text{ERLE} = 10\log_{10}\left(\frac{desired\ signal\ power}{error\ signal\ power}\right)$$

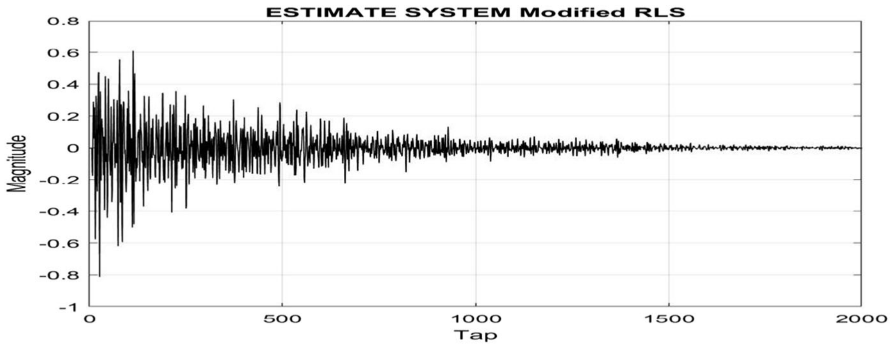
Table 2 shows the comparative average value of MSE and ERLE between the algorithms used in AEC. The results show that the proposed algorithm has better echo cancellation efficiency with an average value of MSE equal to 1.24×10^{-3} as compared to traditional RLS with an average value of MSE equal to 2.38×10^{-3}. Also, it can be observed that the average value of ERLE in AEC has improved to 17.746 dB from 9.568 dB.

Table 3 demonstrates the comparison of the traditional RLS and Modified RLS algorithm used in AEC for different filter orders based on the average value of MSE and ERLE. It can be observed that, for the higher length of the filter, echo cancellation efficiency in AEC gets improved as compared to a lower filter length. However, the filter order should be taken carefully by the application of the filter.

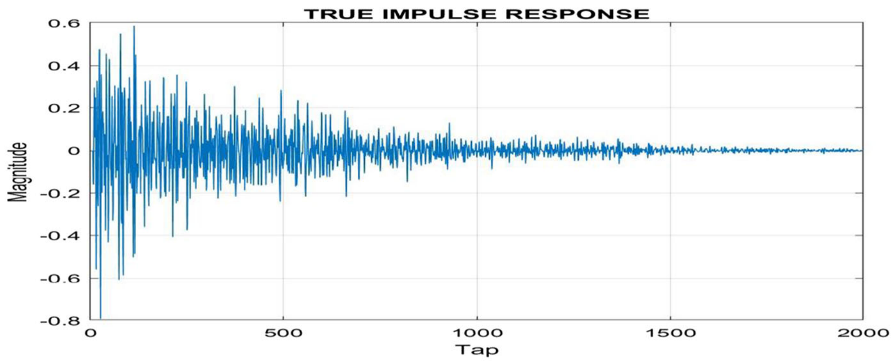
Figure 14 shows the performance evaluation of traditional RLS and the Modified RLS algorithm based on MSE and ERLE.



(a) Impulse response of estimate system RLS



(b) Impulse response of estimate system Modified RLS



(c) Impulse response of acoustic echo path

Fig. 8 Impulse responses **a** estimate system RLS **b** estimate system modified RLS **c** true impulse response

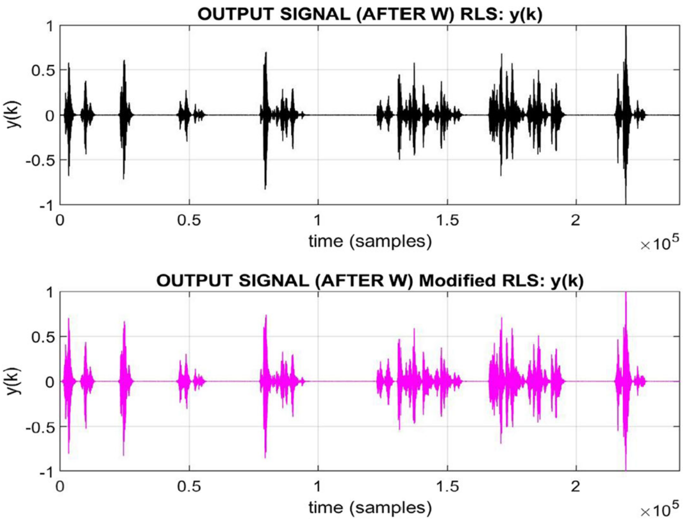


Fig. 9 Output signal for RLS and modified RLS after weight update

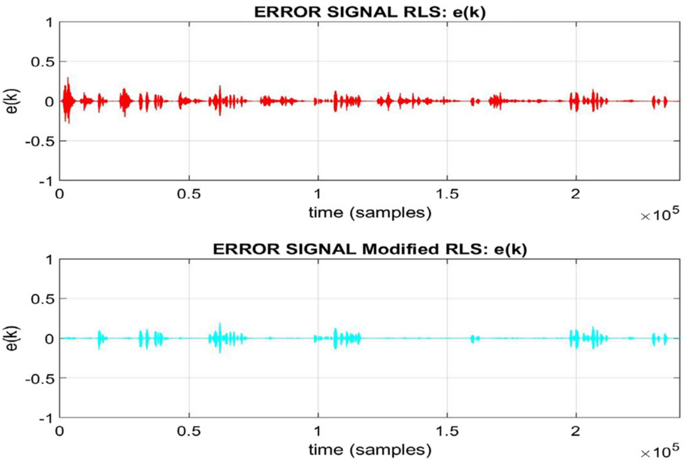


Fig. 10 Error signal for RLS and modified RLS

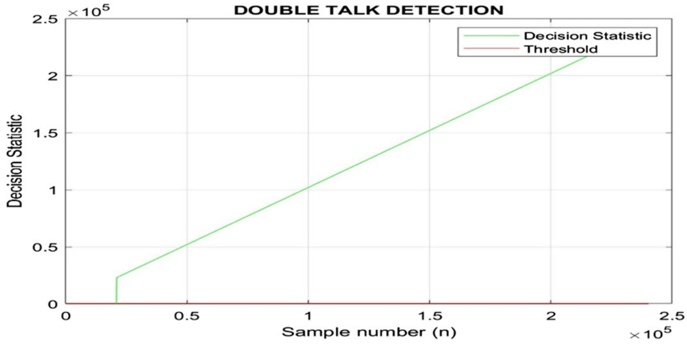


Fig. 11 Double-talk-detection

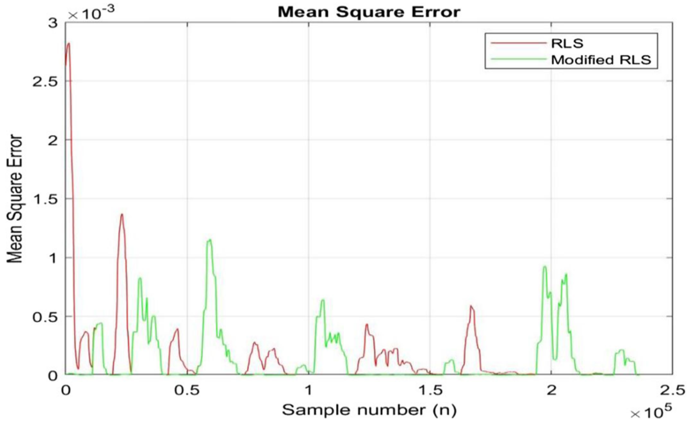


Fig. 12 Mean square error

Table 2 Average value of MSE and ERLE (filter order = 2000)

Algorithm	MSE	ERLE (dB)
RLS	0.000238	9.568
Modified RLS	0.000124	17.746

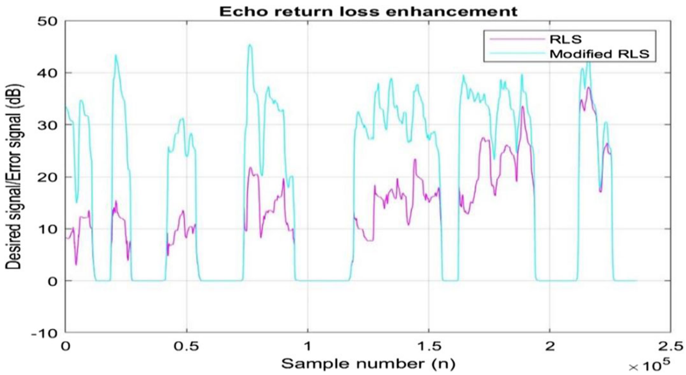
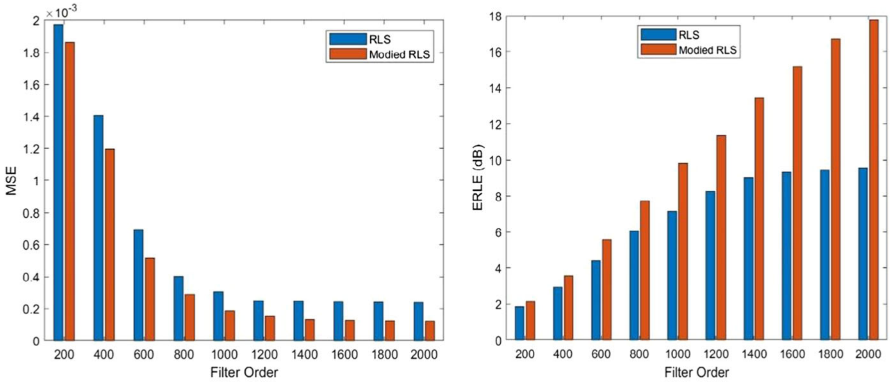


Fig. 13 Comparative ERLE (max = 50 dB) curve

Table 3 Comparison of RLS and modified RLS algorithm used in AEC for different filter orders based on the average value of MSE and ERLE

Filter order	Algorithm			
	RLS		Modified RLS	
	MSE	ERLE	MSE	ERLE
200	1.972×10^{-3}	1.854530	1.860×10^{-3}	2.139807
400	1.403×10^{-3}	2.959801	1.197×10^{-3}	3.559409
600	0.692×10^{-3}	4.438739	0.519×10^{-3}	5.570065
800	0.402×10^{-3}	6.072639	0.290×10^{-3}	7.734348
1000	0.305×10^{-3}	7.134487	0.185×10^{-3}	9.840115
1200	0.251×10^{-3}	8.266248	0.154×10^{-3}	11.380414
1400	0.248×10^{-3}	9.007726	0.135×10^{-3}	13.444999
1600	0.244×10^{-3}	9.325227	0.128×10^{-3}	15.187595
1800	0.240×10^{-3}	9.424519	0.125×10^{-3}	16.727259
2000	0.238×10^{-3}	9.568	0.124×10^{-3}	17.746



(a) Performance Evaluation based on MSE

(b) Performance Evaluation based on ERLE

Fig. 14 Performance evaluation of traditional RLS and Modified RLS algorithm **a** based on MSE **b** based on ERLE

4 Conclusion

In this work, a modified adaptive algorithm has been designed and successfully applied to a real-world scenario in AEC. The performance of the proposed RLS algorithm is compared with traditional RLS. It has been shown that the modified adaptive algorithm has reduced the MSE (79.65%) and hence the improvement in SNR (86.16%) in comparison to the traditional RLS algorithm. Also, the proposed RLS algorithm has a convergence time of 45.817 ms which represents better convergence as compared to traditional RLS which has a convergence time of 80.145 m sec. The optimized value of constant multiplier 'β' is 0.65, which gives the best SNR value of 19.38 dB. Further, echo canceller efficiency in AEC is evaluated in terms of MSE and ERLE. The experimental setup for AEC has been simulated in MATLAB with an adaptive filter length of 2000 and a sampling rate of 8000 Hz and observed that the proposed RLS has better echo canceller efficiency with respect to MSE and ERLE in comparison to traditional RLS.

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Data Availability The datasets generated during and/or analyzed during the current study are available on reasonable request.

Declarations

Conflict of interest All authors have no conflict of interest.

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