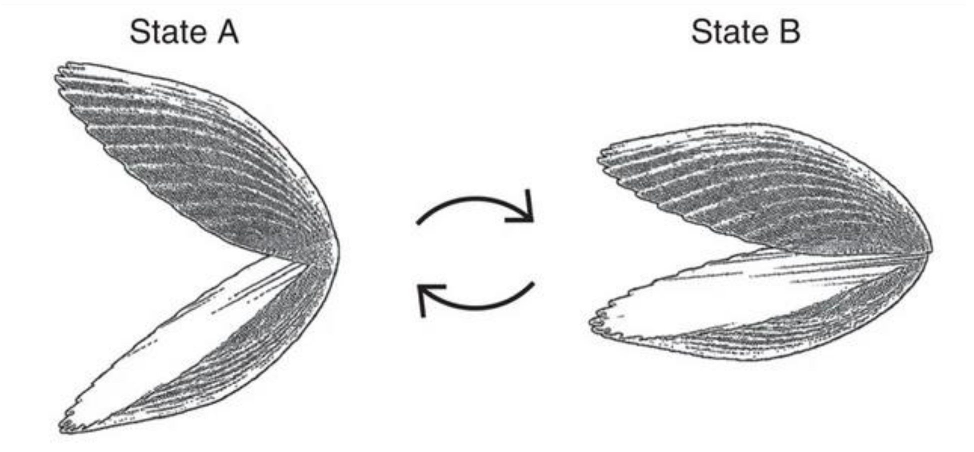
Study of Mechanisms Leading to the Locomotion of Linked Micro-swimmers



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Introduction

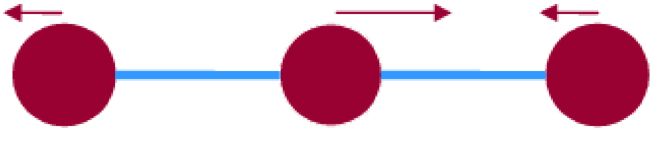
In the usual case of swimming a human being in the water, the displacement gained in the first half period of the cyclic motion is not canceled out by that of the second half period predominantly because of the inertial effects.

Such a mechanism, however, does not work in the microscopic world of biological objects (such as bacteria), where the effects of inertia are not important and the viscous effects dominate. So in the case of a low Reynolds number, we need to break this time-reversal symmetry. This is the basis of Purcell's Scallop Theorem of reciprocal motions. Hence we need to design micro-swimmers with more than one degree of freedom to achieve locomotion.

To study the cause and effects of locomotion of micro-swimmers, we simplify our swimmer to be made up of Spheres and Links. Each link offers us a degree of freedom.

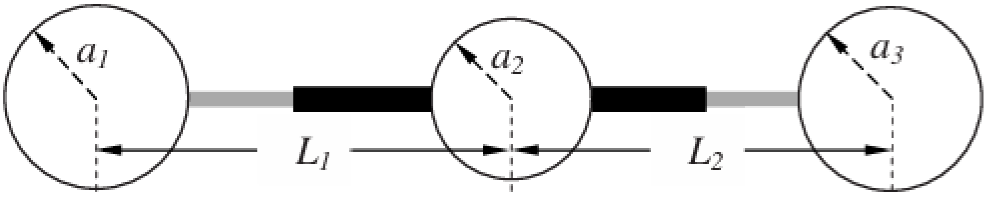
We study the following cases:

- 2 Spheres, 1 Active Link
- 3 Spheres, 2 Active Links (without Oseen interactions)
- 3 Spheres, 2 Active Links (with Oseen interactions)
- 3 Spheres, 1 Active and 1 Elastic Link (detailed analysis is pending, will be added to report soon)



Important Nomenclature

R_i or a_i	Radius of i-th Sphere
v_i	Velocity of i-th Sphere
f_i	Force acting upon i-th Sphere
L_i	Inter-Central Length of i-th Link
l_i	Resting Length of i-th Link
d_i	Deformation Length of i-th Link
L'_i	Rate of change of L_i
l	Sum of l_i, Total Swimmer Size



$$v_1 = \frac{f_1}{6\pi\eta a_1} + \frac{f_2}{4\pi\eta L_1} + \frac{f_3}{4\pi\eta(L_1+L_2)},$$

$$v_2 = \frac{f_1}{4\pi\eta L_1} + \frac{f_2}{6\pi\eta a_2} + \frac{f_3}{4\pi\eta L_2},$$

$$v_3 = \frac{f_1}{4\pi\eta(L_1+L_2)} + \frac{f_2}{4\pi\eta L_2} + \frac{f_3}{6\pi\eta a_3}.$$

Non - Dimensionalization

We will mainly deal with lengths, time, velocities and force.

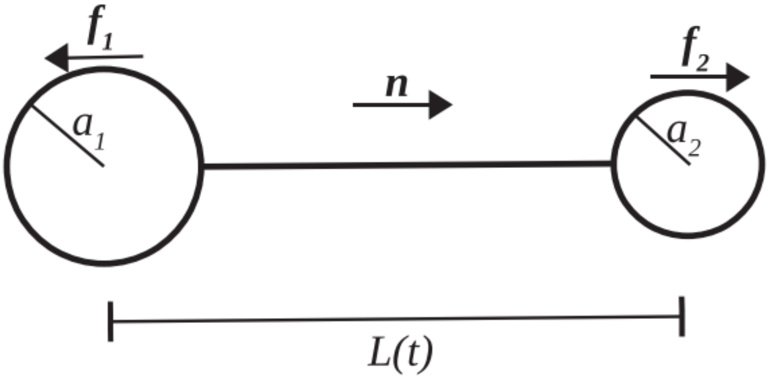
- For length scale we use the total swimmer length.
- For time scale we use the timespan of one deformation cycle.
- Characteristic velocity can be derived from length and time scales
- For force scale, we use the ratio of characteristic velocity and Stoke's Law mobility of the middle sphere (sphere 2)

$$v = f \times mob$$

$$mob = \frac{1}{6\,\pi\mu\,R}$$

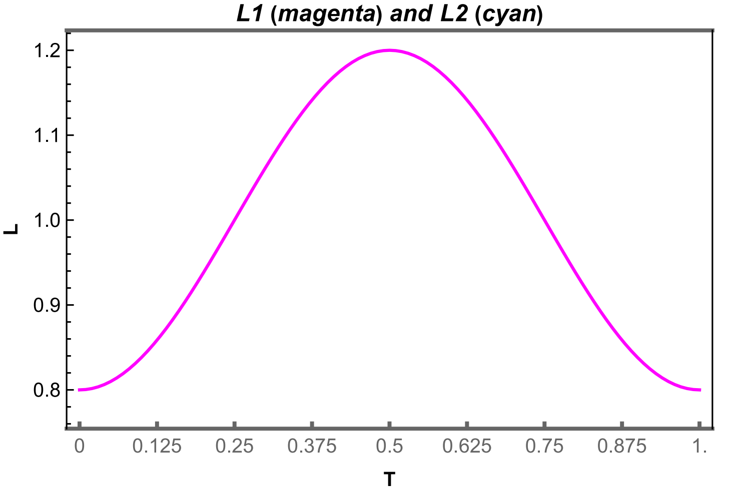
```
R1 = Lchar r1;
R2 = Lchar r2;
R3 = Lchar r3;
L1 = Lchar L1star;
L2 = Lchar L2star;
L1DOT = Vchar L1DOTstar;
L2DOT = Vchar L2DOTstar;
t = Tchar T;
l1 = Lchar x1;
l2 = Lchar x2;
d1 = Lchar e1;
d2 = Lchar e2;
```

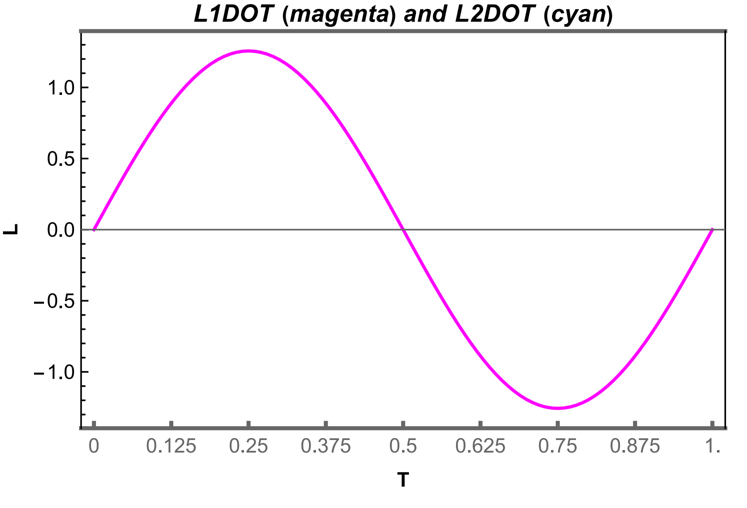
2 Spheres, 1 Active Link



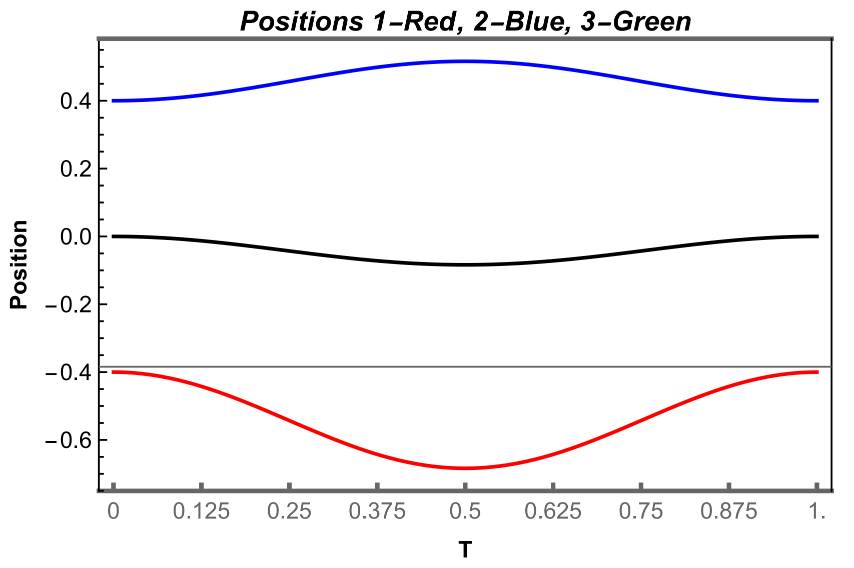
The active link is deformed harmonically as a function of time. We expect no net displacement over one complete cycle due to the scallop theorem. Only one degree of freedom is present, hence the cycle is necessarily time-reversal symmetric.

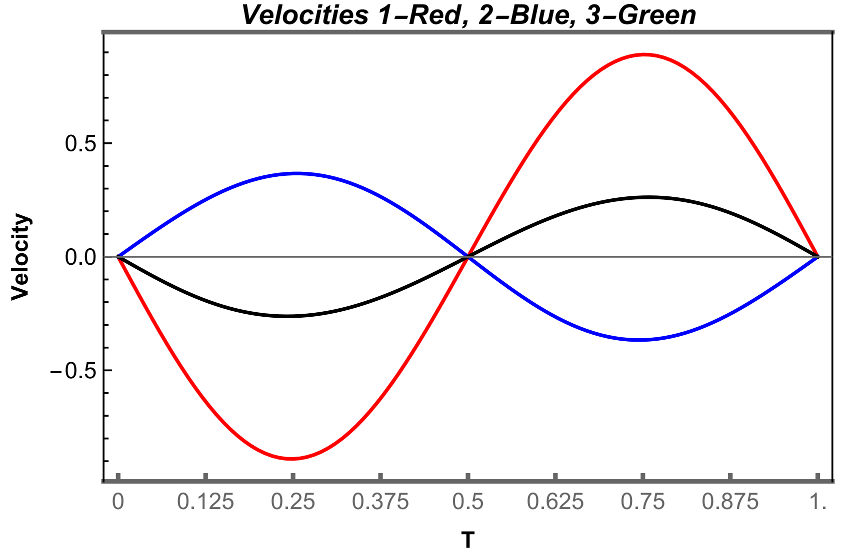
```
Clear[u1]
L1star = x1 + e1 u1[t];
L1DOTstar = D[L1star, T];

Clear[ω]
u1[t_] := -Cos[ω t];
Tcycle = 2 π / ω;

l = 1;
ω = 1;
```

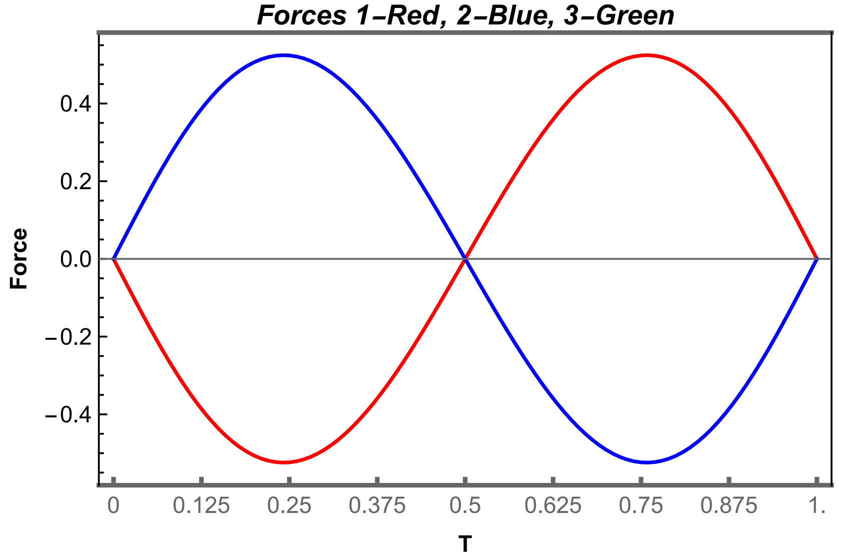




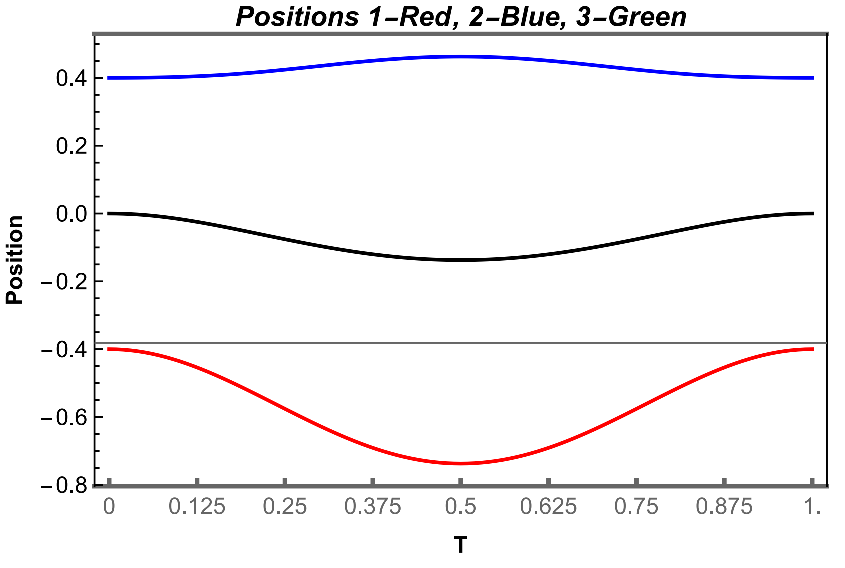
```
r1 = 0.1;
r2 = 0.2;
x1 = 1;
e1 = 0.2;
```

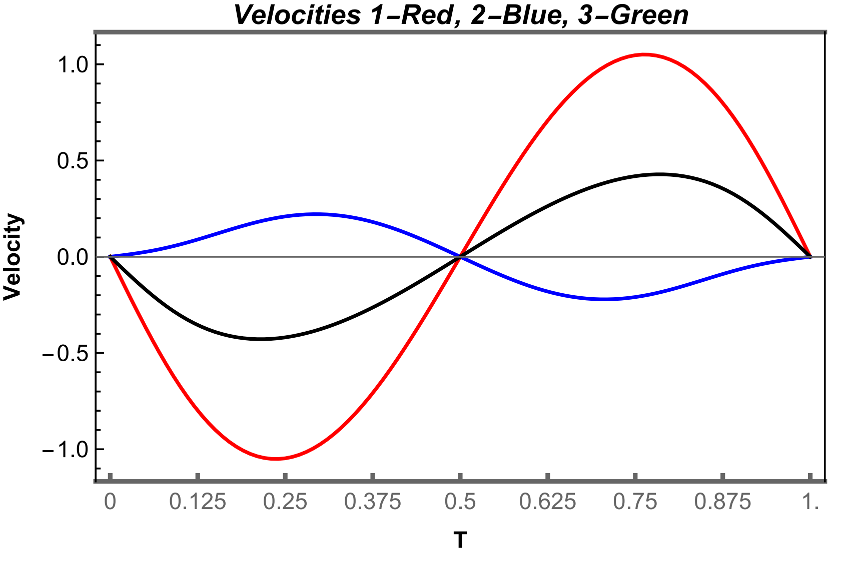


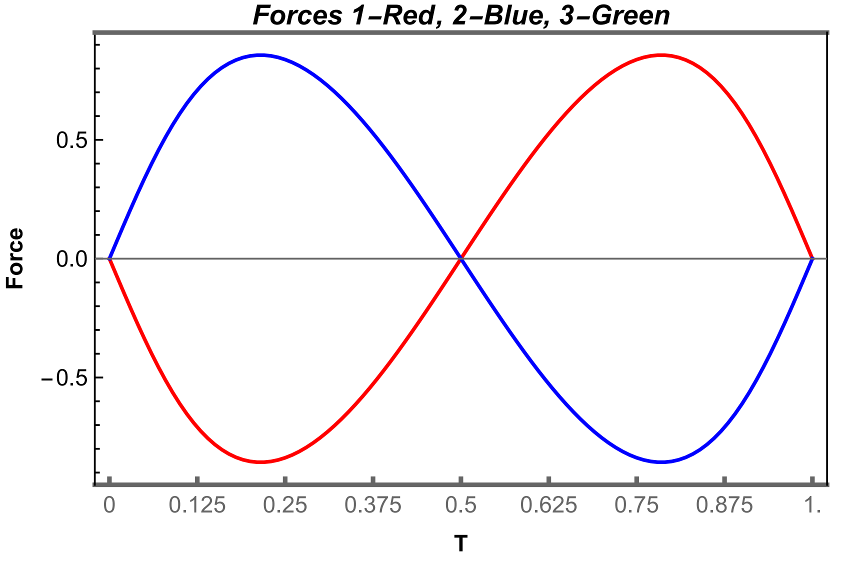




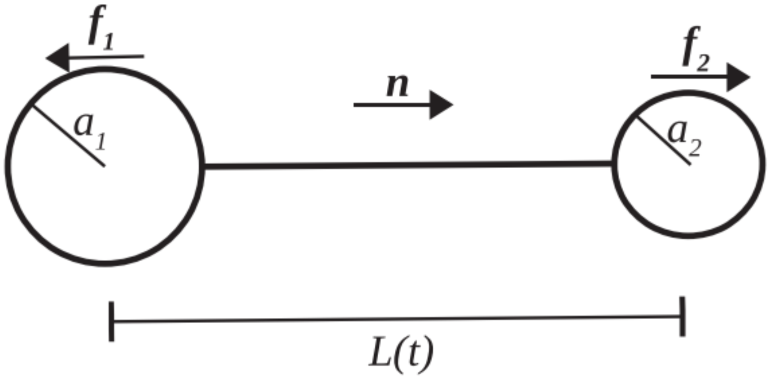
```
r1 = 0.25;
r2 = 0.5;
x1 = 1;
e1 = 0.2;
```







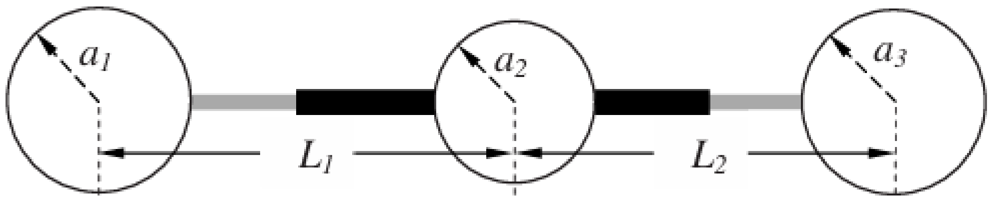
2 Spheres, 1 Active Link



Inferences:

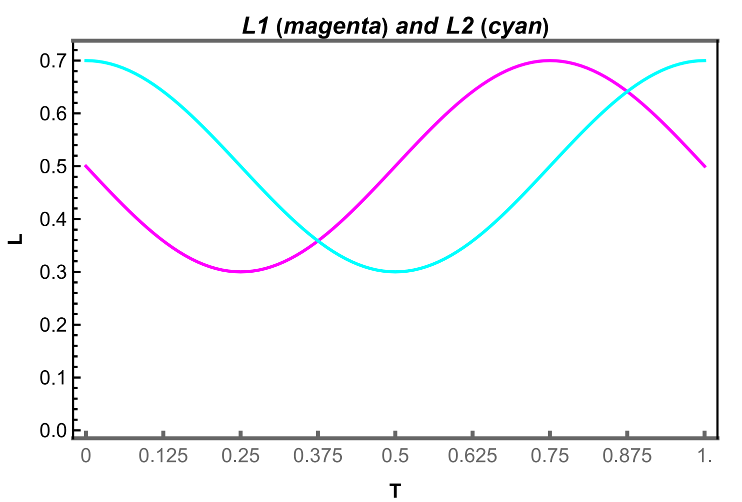
1. Net displacement is indeed zero, due to the scallop theorem.
2. Instantaneous displacement occurs towards the smaller sphere. This may be due to a higher mobility of a small sphere.
3. When the spheres are brought close together (increasing the value of r1 and r2 while keeping x1 constant) there appears to be an added skew in the behaviour. This shows that in contracted states, the spheres exert some significant interaction force on each other.

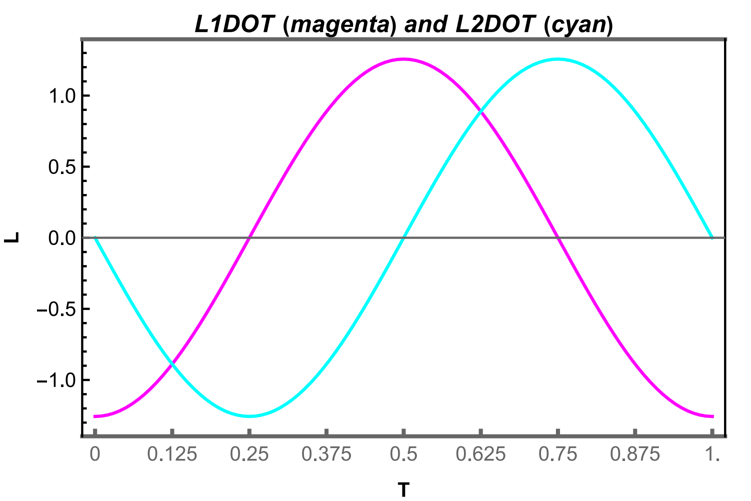
3 Spheres, 2 Active Link without Oseen Interactions



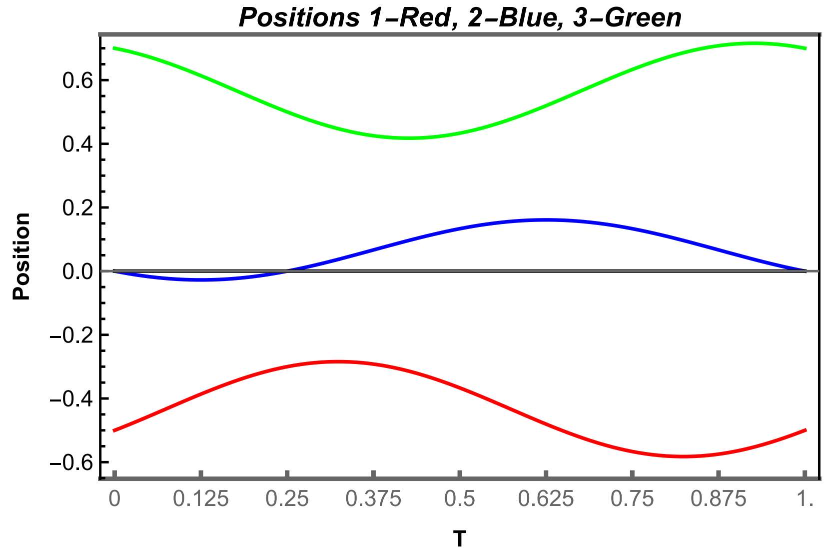
The active links are deformed harmonically as a function of time. Since we added another degree of freedom, we expect that there should be some net displacement over one cycle.

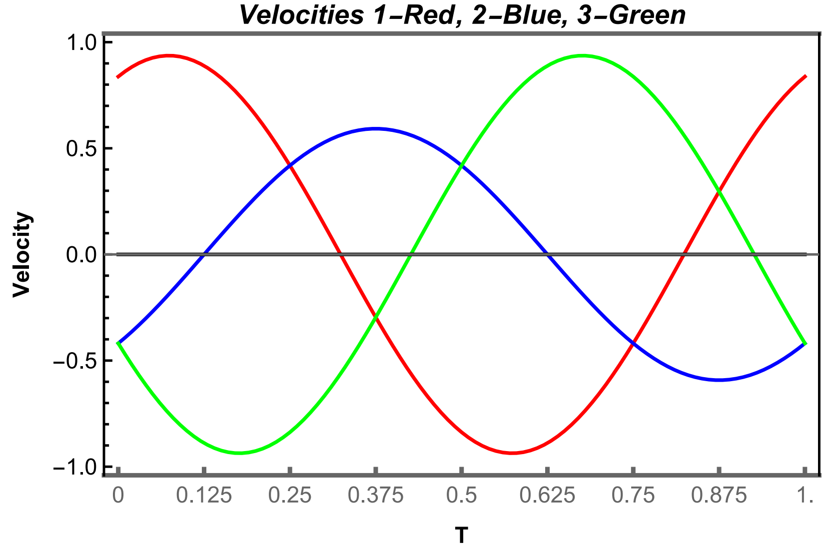
```
Clear[u1, u2]
L1star = x1 + e1 u1[t];
L2star = x2 + e2 u2[t];
L1DOTstar = D[L1star, T];
L2DOTstar = D[L2star, T];

Clear[ω, ϕ1, ϕ2]
u1[t_] := Cos[ω t + ϕ1];
u2[t_] := Cos[ω t + ϕ2];
Tcycle = 2 π / ω;

l = 1;
ω = 1;
ϕ1 = π / 2;
ϕ2 = 0;
```

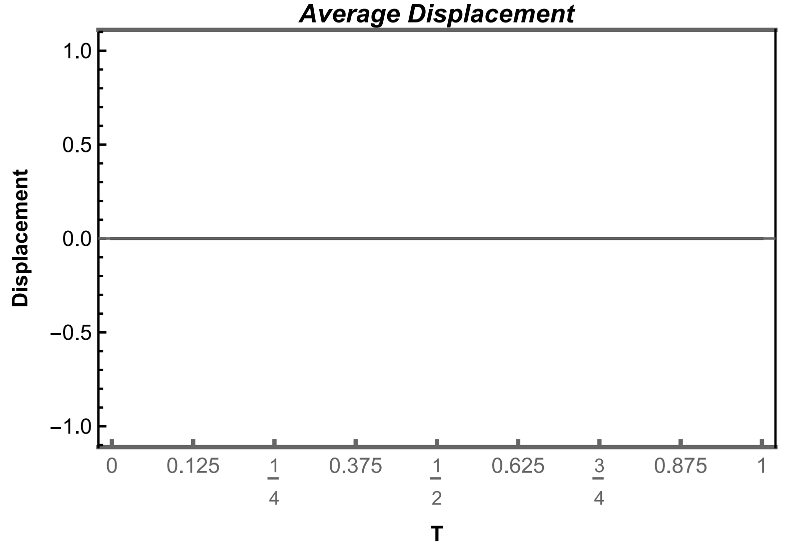




```
r1 = 0.1;
r2 = 0.1;
r3 = 0.1;
x1 = 0.5;
x2 = 0.5;
e1 = 0.2;
e2 = 0.2;
```



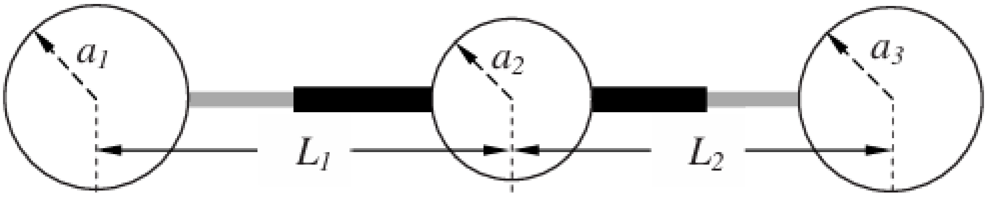




Inferences:

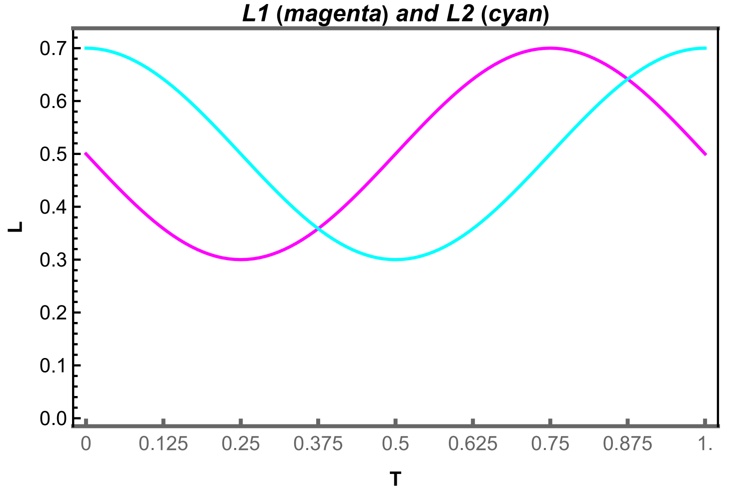
1. Net displacement is zero since there are no Oseen interactions at play. They are necessary for a micro-swimmer to experience propulsion

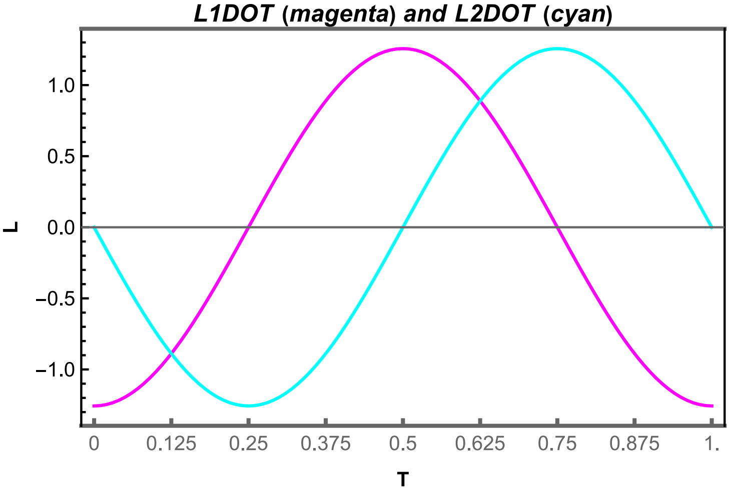
3 Spheres, 2 Active Link with Oseen Interactions



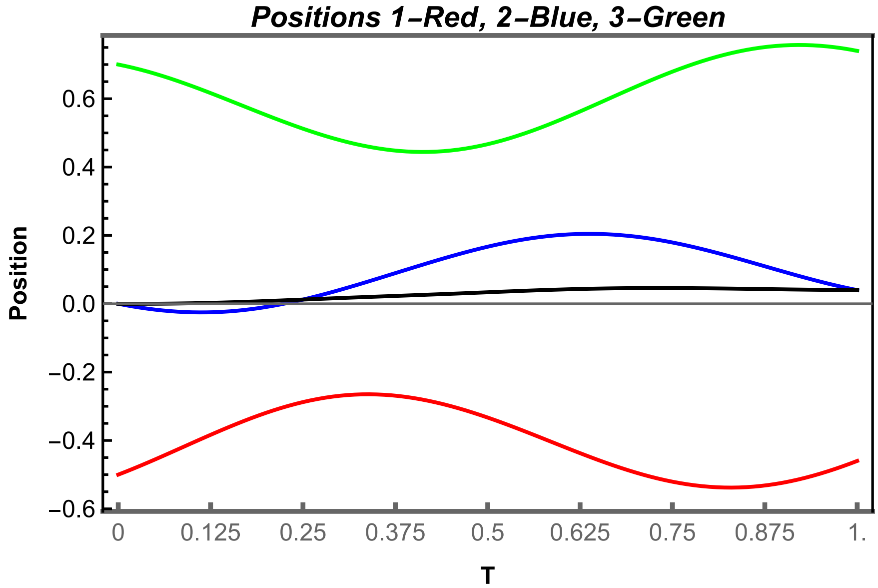
The active links are deformed harmonically as a function of time. Since we added another degree of freedom and also activated Oseen interactions, we expect that there should be some net displacement over one cycle.

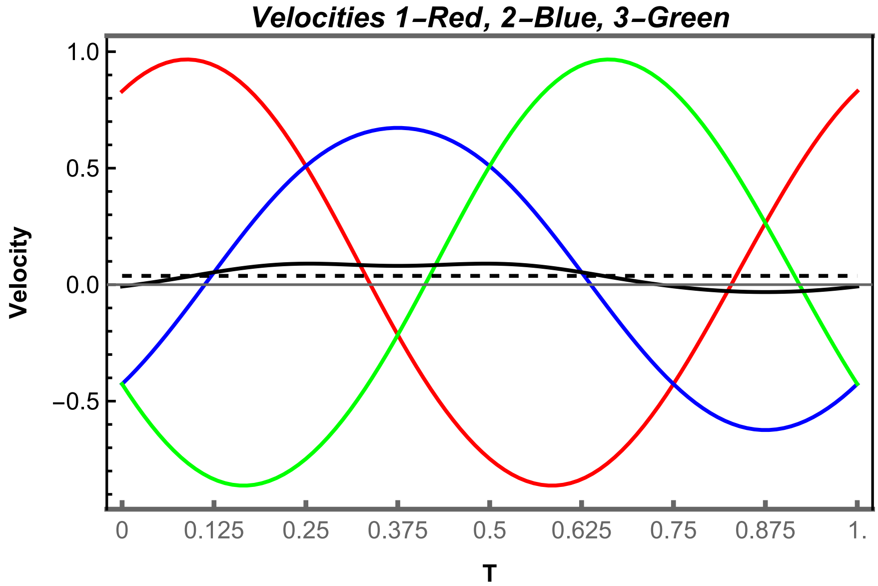
```
Clear[u1, u2]
L1star = x1 + e1 u1[t];
L2star = x2 + e2 u2[t];
L1DOTstar = D[L1star, T];
L2DOTstar = D[L2star, T];

Clear[ω, ϕ1, ϕ2]
u1[t_] := Cos[ω t + ϕ1];
u2[t_] := Cos[ω t + ϕ2];
Tcycle = 2 π / ω;

l = 1;
ω = 1;
ϕ1 = π / 2;
ϕ2 = 0;
```

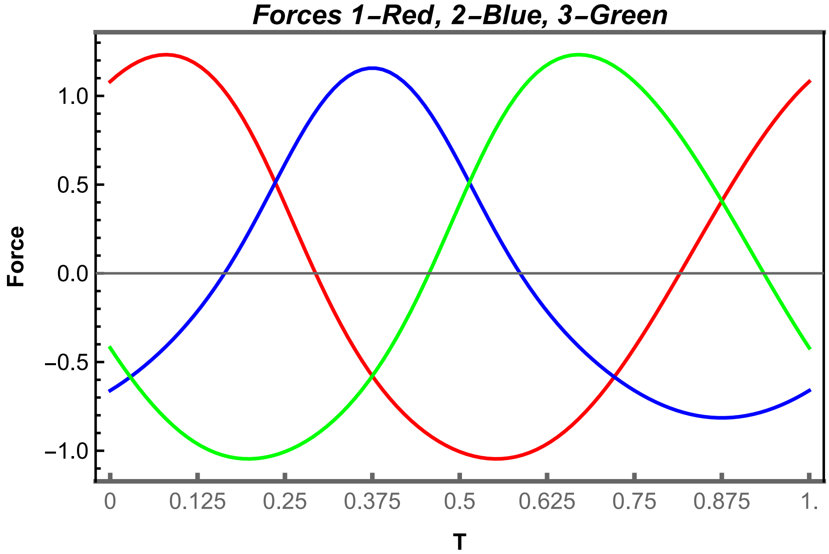




```
r1 = 0.1;
r2 = 0.1;
r3 = 0.1;
x1 = 0.5;
x2 = 0.5;
e1 = 0.2;
e2 = 0.2;
```



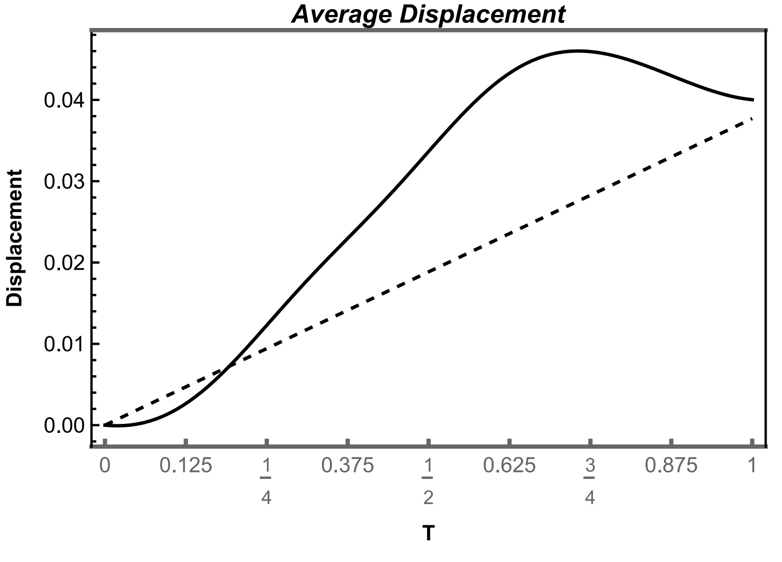


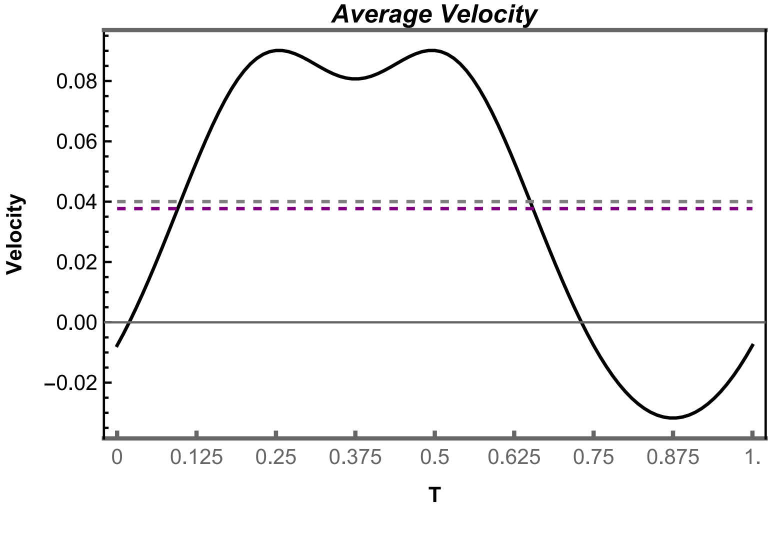


Such configuration of the three spheres swimmer has been explored in literature and we have a theoretical expression for the time-averaged velocity

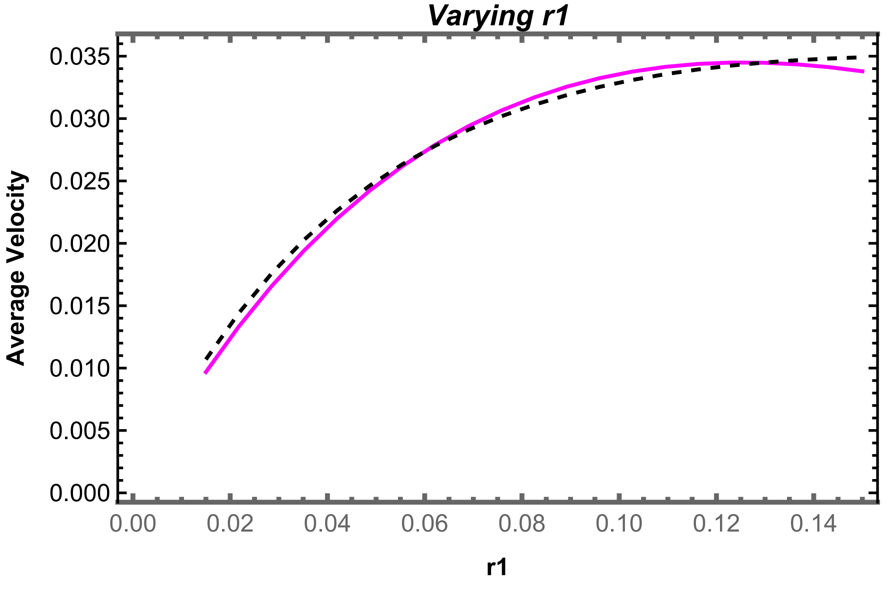
$$K = \frac{3a_1a_2a_3}{(a_1+a_2+a_3)^2}\left[\frac{1}{l_1^2}+\frac{1}{l_2^2}-\frac{1}{(l_1+l_2)^2}\right].$$

$$\overline{V_0} = \frac{K}{2} d_1 d_2 \omega \sin(\varphi_1 - \varphi_2).$$



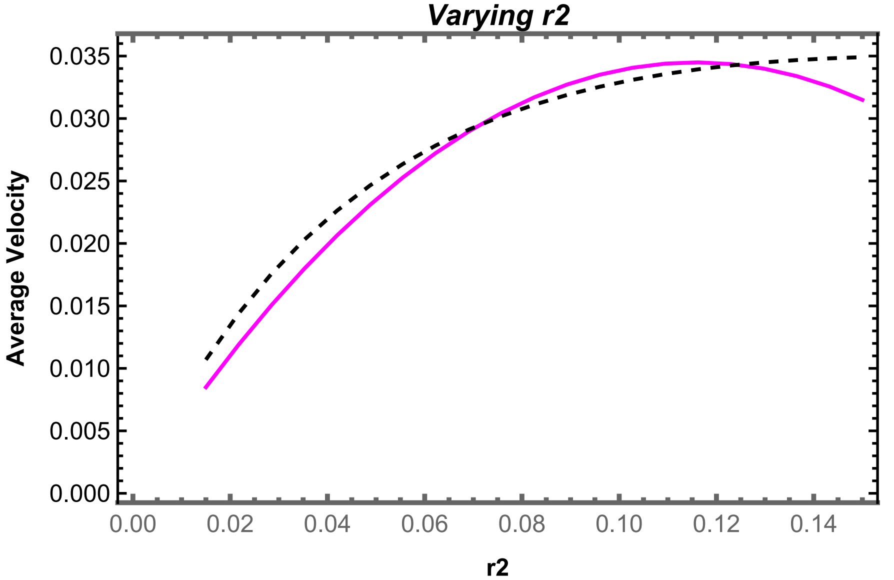


Since we are experiencing net displacement, we can now analyse over variations of the geometric values at play.

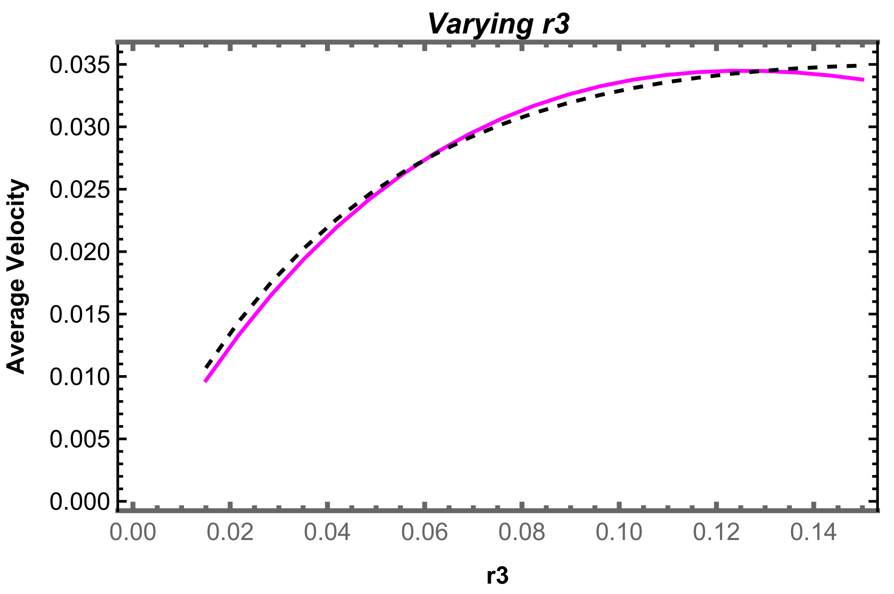


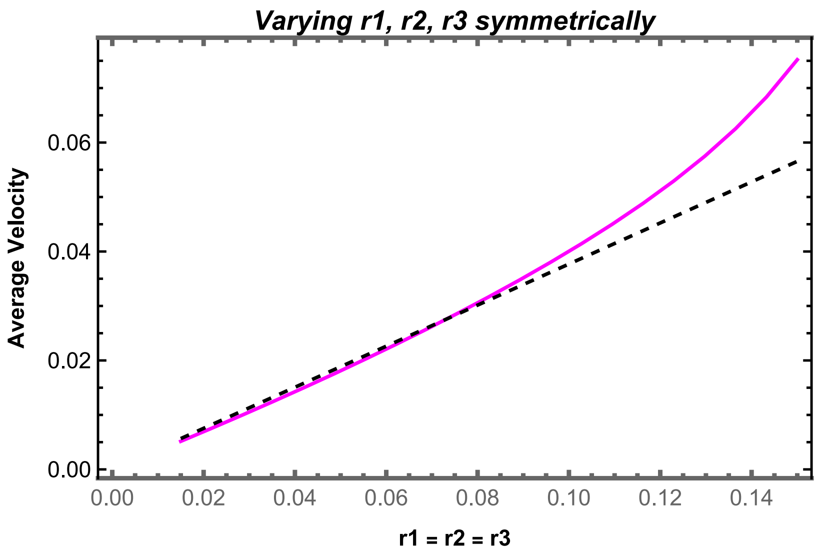
$$x2 = 0.5;$$
$$e1 = 0.2;$$
$$e2 = 0.2;$$

$$rmax = \frac{1}{2} Min[\{x1 - e1, x2 - e2\}];$$
$$rmin = \frac{rmax}{10};$$
$$rdiff = \frac{(rmax - rmin)}{20};$$

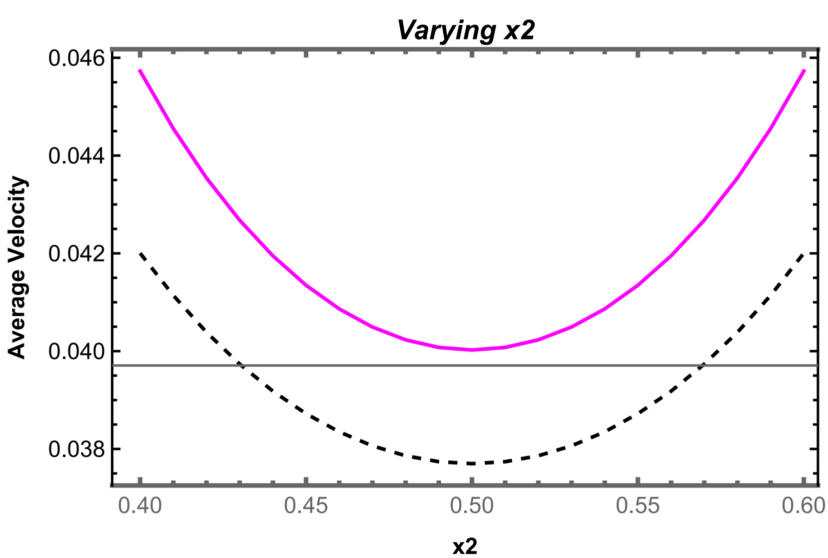


Observation:
The variation for r1 and r3 is the same. This means that the orientation of the swimmer does not matter, only the deformation mechanism matters. (r1, r2, r3) and (r3, r2, r1) give the same locomotion



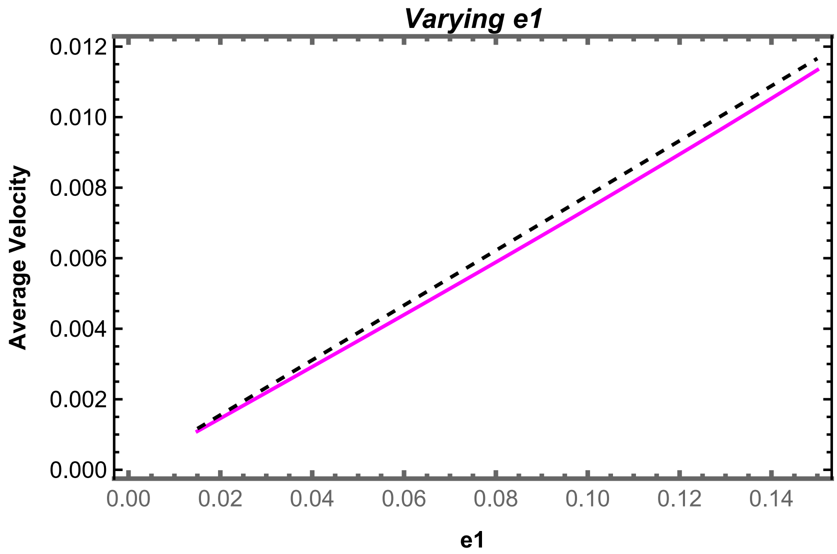



```
x2 = 0.5;
e1 = 0.2;
e2 = 0.2;

rmax = 1/2 Min[{x1 - e1, x2 - e2}];
rmin = rmax/10;
rdiff = (rmax - rmin)/20;
```

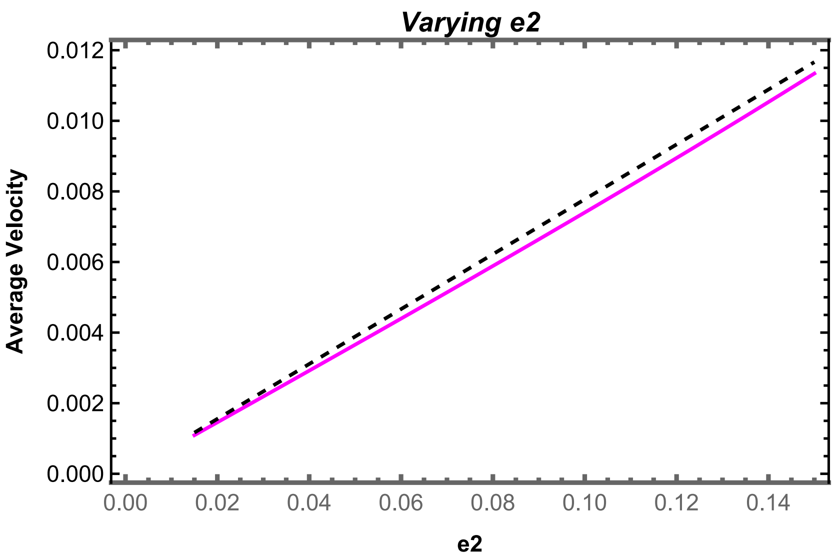


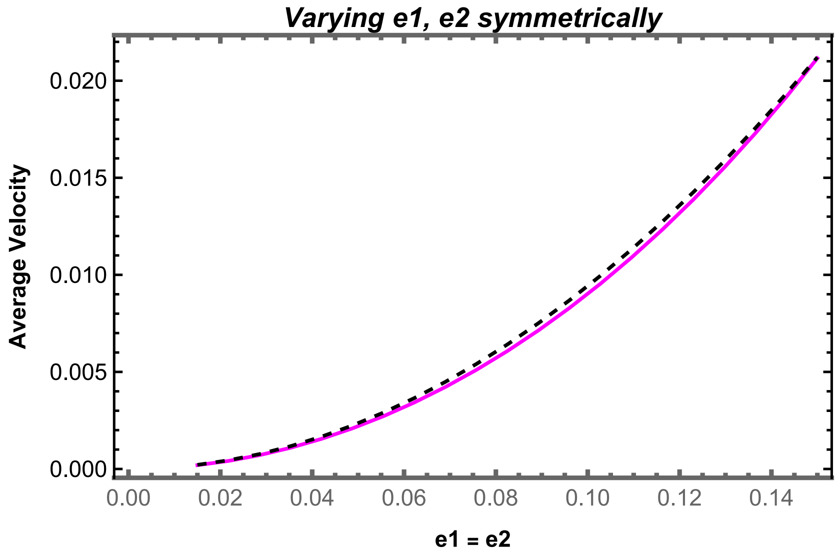

```
r1 = 0.1;
r2 = 0.1;
r3 = 0.1;
rmax = Max[{r1, r2, r3}];
e1 = 0.2;
e2 = 0.2;
emax = Max[{e1, e2}];

xmin = 2 rmax + emax;
xmax = 1 - xmin;
xdiff = (xmax - xmin)/20;
```

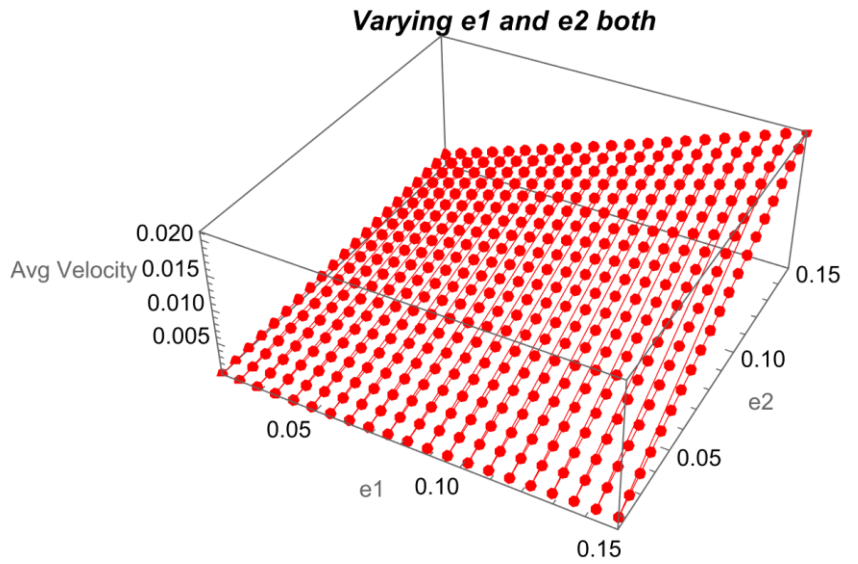
Observation:
More asymmetry in the resting lengths of the links leads to more displacement.

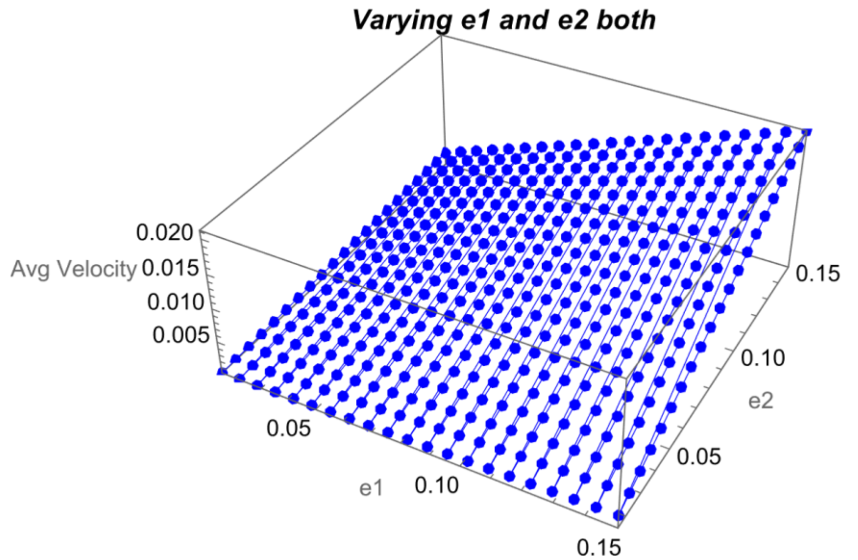


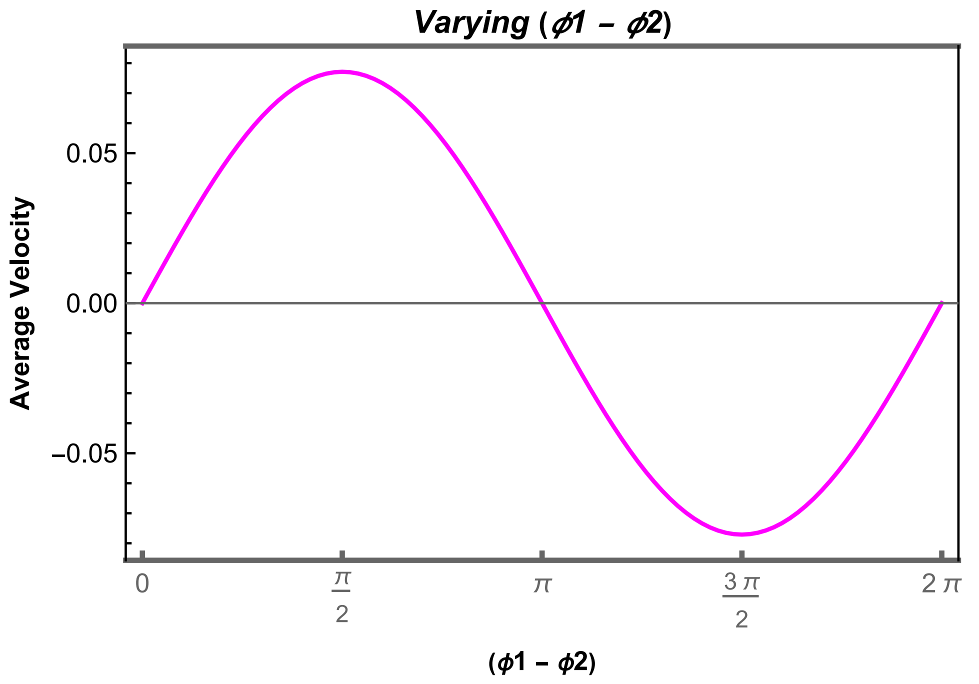

```
r1 = 0.1;
r2 = 0.1;
r3 = 0.1;
rmax = Max[{r1, r2, r3}];
x2 = 0.5;
emax = 1/2 Min[{x1 - 2 rmax, x2 - 2 rmax}];
emin = emax/10;
ediff = (emax - emin)/20;
```



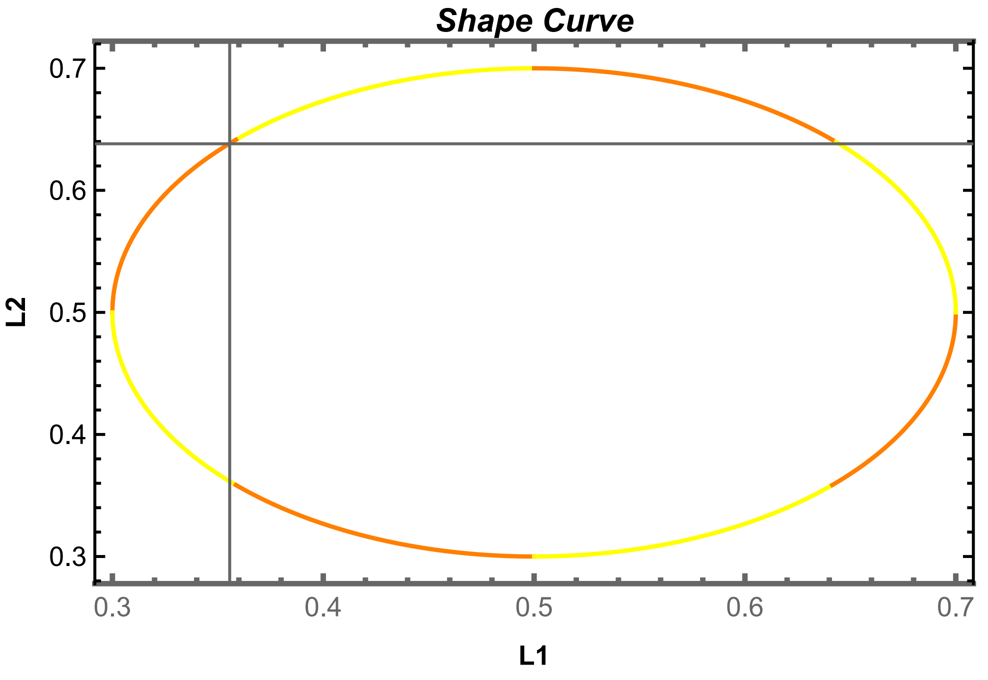



Varying e1 and e2 both
0.020
Avg Velocity 0.015
0.010
0.005
0.05
e1 0.10
0.15
0.15
0.10
e2
0.05


Varying e1 and e2 both
0.020
Avg Velocity 0.015
0.010
0.005
0.05
e1 0.10
0.15
0.15
0.10
e2
0.05


Varying (ϕ1 - ϕ2)
Average Velocity
0.05
0.00
-0.05
0
π/2
π
3π/2
2π
(ϕ1 - ϕ2)

Split Cycle Analysis

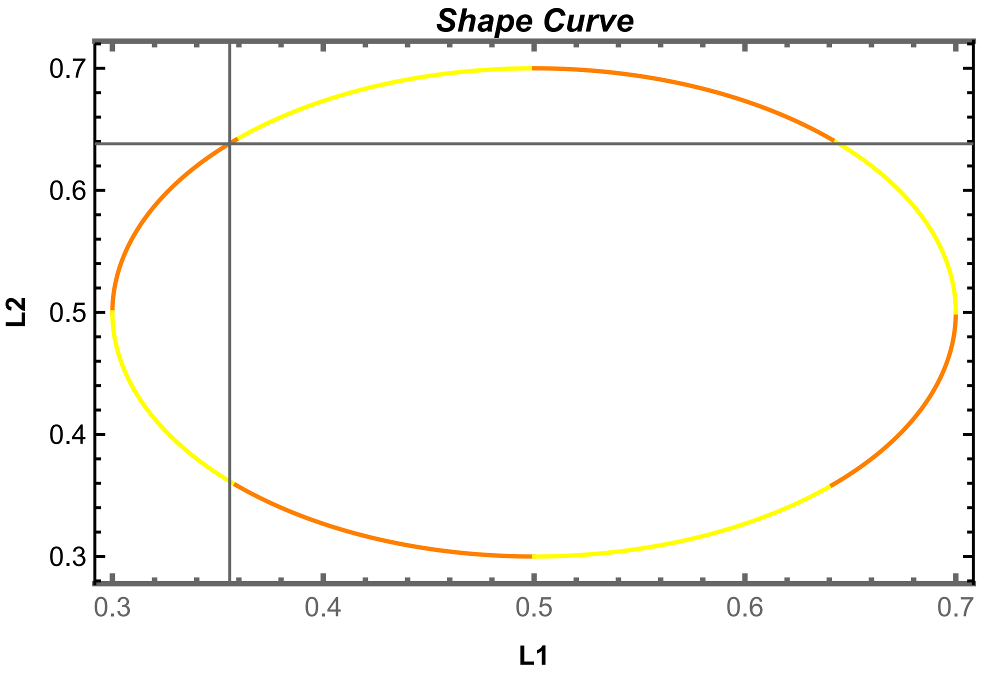



```
r1 = 0.1;
r2 = 0.1;
r3 = 0.1;
x1 = 0.5;
x2 = 0.5;
e1 = 0.2;
e2 = 0.2;

τ1 = 0;
τ2 = 0.125;
τ3 = 0.25;
τ4 = 0.375;
τ5 = 0.5;
τ6 = 0.625;
τ7 = 0.75;
τ8 = 0.875;
τ9 = 1;
```

```
0.00265616

0.00962921

0.0106845

0.0106845

0.00962921

0.00265616

-0.00295805

-0.00295805
```

Observation:
The most contracted states (3 and 4) contribute the most to the net displacement

Split Cycle Analysis

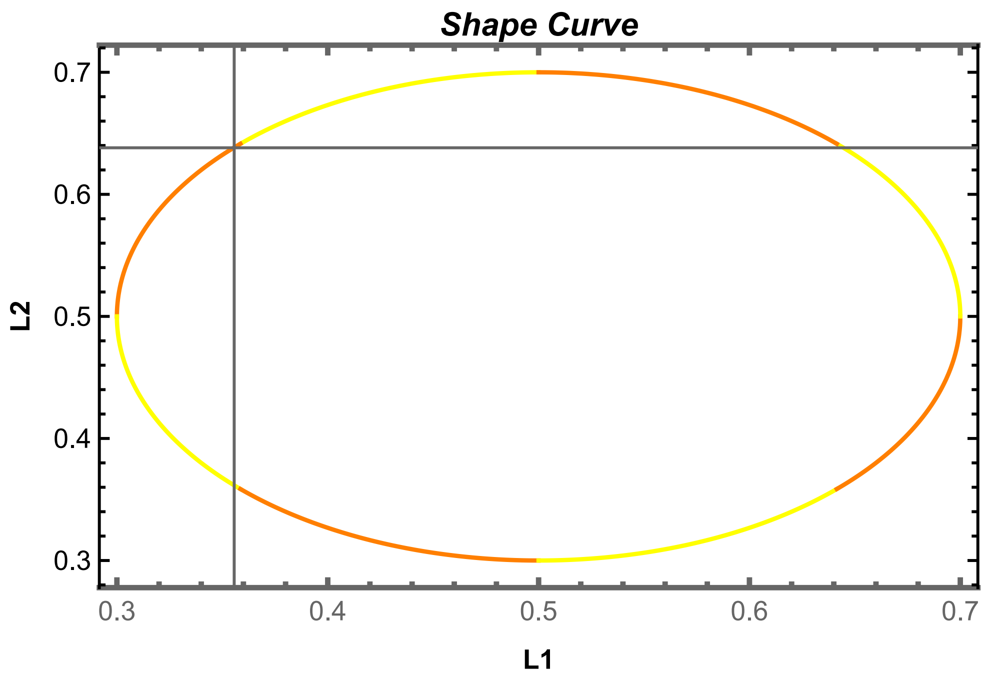



```
r1 = 0.15;
r2 = 0.075;
r3 = 0.15;
x2 = 0.6;
e1 = 0.2;
e2 = 0.2;

τ1 = 0;
τ2 = 0.125;
τ3 = 0.25;
τ4 = 0.375;
τ5 = 0.5;
τ6 = 0.625;
τ7 = 0.75;
τ8 = 0.875;
τ9 = 1;
```

```
-0.00712219

0.0115125

0.0375062

0.0312563

0.0116737

-0.00858383

-0.0213024

-0.020612
```

Observation:
The most contracted states (3 and 4) contribute the most to the net displacement. On bringing these spheres closer together, the contribution has increased.

Split Cycle Analysis

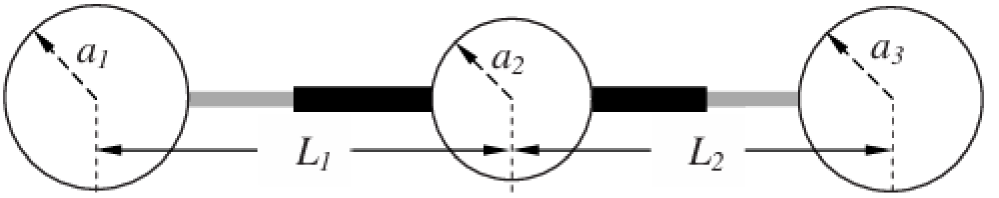



```
r1 = 0.01;
r2 = 0.01;
r3 = 0.01;
x2 = 0.8;
e1 = 0.2;
e2 = 0.2;

τ1 = 0;
τ2 = 0.125;
τ3 = 0.25;
τ4 = 0.375;
τ5 = 0.5;
τ6 = 0.625;
τ7 = 0.75;
τ8 = 0.875;
τ9 = 1;
```

```
0.00355007

0.0413403

0.0329297

0.000210655

-0.000793675

-0.000782087

-0.000661924

0.000103378
```

Observation:
In cases of sheer asymmetry in x1 and x2, the contracted state theory fails simply because now the states where the average rate of change of the link is higher, will contribute most to the displacement (2 and 3) while the other link is at the most contracted.

3 Spheres, 2 Active Link with Oseen Interactions



Inferences:

1. We get a net displacement due to Oseen interactions and time-reversal asymmetry.
2. The internal mechanics of the cycle show that certain splits carry more contribution based on their average rate of change of link length as well as the contraction of the link in that split. This explains the net displacement.
3. The orientation of the swimmer does not matter, only the deformation mechanism matters.
4. The expression given in the literature approximates the actual behavior closely.
5. Maximum net displacement occurs at a phase difference of pi/2
6. More asymmetry in the resting lengths of the links leads to more displacement (keeping e1 and e2 constant)