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BSE-1A: 25K-3077

Assignment#01

Date: 10 Sep 2025

1. $A = 2.80\,cm$ $\theta_A = 60\,cm$
$B = 1.90\,cm$ $\theta_B = -60\,cm$

$A_x = 2.80\cos(60) = 1.40$ $A_y = 1.90\sin(60) = 2.42$
$B_x = 1.90\cos(-60) = 0.95$ $B_y = 1.90\sin(-60) = -1.65$

$A = (1.40, 2.42)$ $B = (0.95, -1.65)$

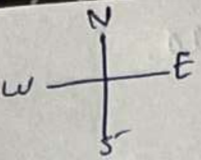
a. $A - B = (\overset{A_x}{1.40} - \overset{B_x}{0.95}), (\overset{A_y}{2.42} - (\overset{B_y}{-1.65})) = (0.45, 4.07)$
Mag $|A-B| = \sqrt{(0.45)^2 + (4.07)^2} = \sqrt{16.7} = \boxed{4.08}$
Direction $= \tan^{-1}\left(\frac{4.07}{0.45}\right) = \underline{83.69°}$

b. $A + B = (1.40 + 0.95), (2.42 + (-1.65)) = (2.35, 0.77)$
Mag $= |A+B| = \sqrt{(2.35)^2 + (0.77)^2} = \boxed{2.48}$
Direction $= \tan^{-1}\left(\frac{0.77}{2.35}\right) = \underline{18.17°}$

c. $B - A = (0.95 - 1.40), (-1.65 - 2.42) = (-0.45, -4.07)$
Mag $= |B-A| = \sqrt{(-0.45)^2 + (-4.07)^2} = 4.08$

Direction $= \tan^{-1}\left(\frac{-4.07}{-0.45}\right) = 84°$
Both Components negative
so $180 + 83.4 = \underline{263.9°}$

d. $A \times B = |A||B|\sin\theta$
$(2.80)(1.90)\sin 120 = \underline{4.60}$
Angle b/w A & B $\theta = 60 - (-60) = \underline{120°}$ so +z direction



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2.

y-axis

$V_1 = (0, -825)$

$V_2 = 1250m$ west of north $30°$ west -X

N_c 1250 Cos (30) , W_c = -1250 Sin(30)

(-625 , 1082.5)

$V_3 = 1000$ m North of east $40°$

N_c = 1000 Cos 40 , E_c = 1000 Sin 40

(766 , 642.8)

Total distence =

(-625+0+766) (-825+1082.5+642.8)

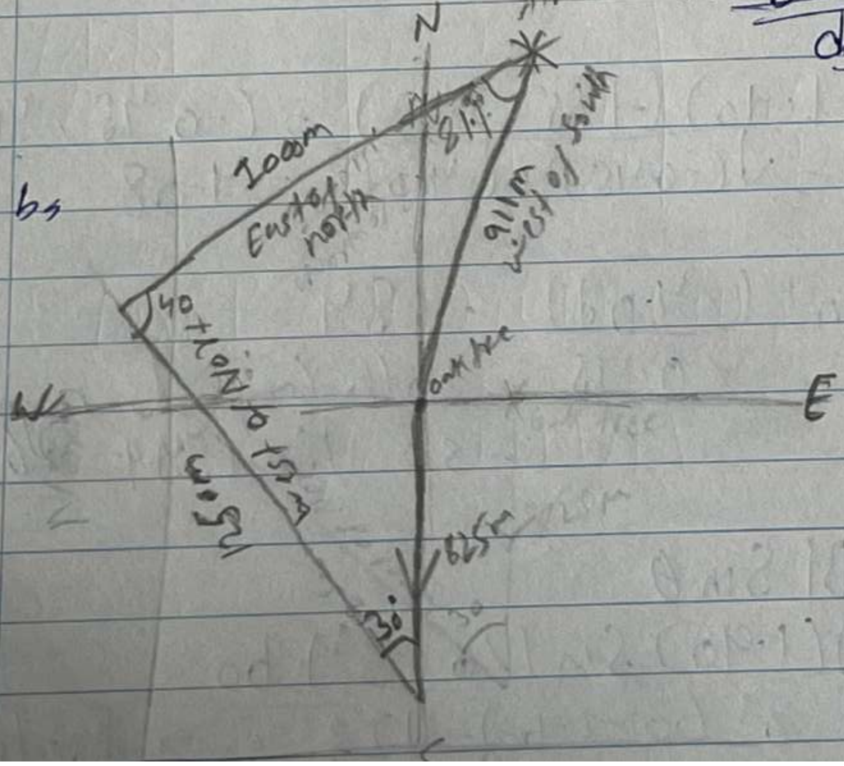
= (141.048, 900.32) meters

$Mag = \sqrt{(141.04)^2 + (900)^2} = \underline{911.3 \text{ meters}}$ total distan

$\tan^{-1}\left(\frac{900.32}{141.04}\right) = 81.10$ North east

a. To return to starting Point, travel back <u>911.3m</u> to <u>81.10°</u> to <u>South West direction</u>

b.



Not to Scale

$|A \times B| = \sqrt{(-5)^2 + (2)^2}$
$\sqrt{29}$

3. $A \times B = |A||B| \sin\theta$
$-5k + 2i = |3||3| \sin\theta$
$\sqrt{29} = 9 \sin\theta$
$\frac{\sqrt{29}}{9} = \sin\theta$
$\sin^{-1}\left(\frac{\sqrt{29}}{9}\right) = \boxed{36.79^\circ}$

4. $A = 75 = 30^\circ$
$B = 59 = 180 - 19 = 161^\circ$
$C = 25 = 180 + 52 = 232^\circ$
$D = 91 = 360 - 27 = 333^\circ$ or -27°

a. $B_x = 59 \cos 161$
$= -55.78$

B_y 58 Sin 161 =
$= 19.2$

In unit vector $= \underline{-55.7i + 19.2j}$

$C_x = 25 \cos 232$
$= -15.39$

$C_y = 25 \sin 232$
-19.70

In unit vector $= \underline{-15.39i + (-19.70)j}$

b. $A_x = 75 \cos 30$
$= 65.0$

$75 \sin 30$
$= 37.5$

$= \underline{65i + 37.5j}$

$D_x = 91 \cos(333)$
$= 81.08$

$D_y = 91 \sin(333$
$= -41.3$

$= 81.08 i + (-41.3) j$

$A+D = (65 + 81.01) i + (37.5 - 41.3) j = 146.1 i - 3.8 j$

$|A+D| = \sqrt{(146.1)^2 + (-3.8)^2} = \boxed{146.08}$

5. Cross Product is given by $|A \times B| = |A||B| \sin\theta$ where as Area of Parallelogram is giveby $= |A \times B| = |A| \cdot (|B| \sin\theta)$

Direction of Cross product is equal to the Perpendicular of the Parallelogram Plane.

$A = 3i - 2j + 4k$ & $B = -i - 4j + 2k$ find Area of triangle

$$|A \times B| = \begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ -1 & -4 & 2 \end{vmatrix} = i(-4+16) - j(6+4) + k(-12-2)$$

$12i - 10j - 14k$

$|A \times B| = \sqrt{(12)^2 + (-10)^2 + (-14)^2} = \sqrt{440} = 20.97$

Area of triangle $= \frac{1}{2} |A \times B| = \frac{20.97}{2} = \boxed{10.48 \text{ units}^2}$

6. $A \cdot B = |A||B| \cos\theta$ $|A| = |B| = X$ magnitude

$A \cdot B = x^2 \cos\theta$ $A \cdot B = \frac{1}{2} x^2$

$\frac{1}{2} \cancel{x^2} = \cancel{x^2} \cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \cos^{-1}(1/2)$

$\boxed{\theta = 60^\circ}$

7. $A = (30, -20)$
$B = (60, 80)$
$C = (-10, -10)$
$D = (40, -30)$
$E = (-70, 60)$

A to B: 1/2 distance $= \frac{60+30}{2}, \frac{80+(-20)}{2}$

New Points $= (45, 30)$

Np to C 1/3 dis $= \frac{-10-45}{3}, \frac{-10-30}{3}$

$N = -18.3, -13.3$

New points $= 45 - 18.3, 30 - 13.3$
$= (26.7, 16.7)$

NP = New points

NewPato D = 1/4 distance $= \frac{40-26.7}{4}, \frac{-30-16.7}{4}$
$= 3.32, -11.67$

New points $= 26.7 + 3.3, 16.7 - 11.7$ ≈ 11.7
$= (30, 5)$

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NP to E : 1/5 distance = $\frac{-70-30}{5}$, $\frac{60-5}{5}$

= -20 , 11

Final points = (30-20) , (5+11)

$\boxed{(10, 16)}$

↳ Treasure is dig at these points ≥

After Rearranging Order

A = (60, 80)

B = (30, -20)

C = (-10, -10)

D = (40, -30)

E = (-70, 60)

A to B, 1/2 distance = $\frac{30-60}{2}$, $\frac{-20+(80)}{2}$

= -15 , -50

NP = (60-15) , 80 - 50)

= (45, 30)

NP to C : 1/3 distance

= $\frac{-10-45}{3}$, $\frac{-10-30}{3}$ = -18.3 , -13.3

Final Result = a, Coordinates of treasure are (10, 16)

b, As we see, we get same points again after rearranging which shows that points not depend on order b/c point is average of All 5 trees -

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8. a is +ve in X direction = ai
b is +ve in Y direction = bj

a. If $d>0$, a/d, Same as ai +ve $\hat{x}$ direction
b. If $d<0$, a/d, opposite of ai -ve $\hat{x}$ direction

c. Magnitude = $\vec{a}\cdot\vec{b} = (ai)\cdot(bj) = ab(i\cdot j) = ab(0) = 0$
magnitude is 0

d. Magnitude of $a\cdot b/d$
$a\cdot b = 0$
$0/d = 0$ ∴ Magnitude is Zero Ans

e. Direction of $\vec{a}\times\vec{b}$
$= (ai)\times(bj) = ab(i\times j) = abk$
↳ +ve Z direction.

f. Direction of $\vec{b}\times\vec{a}$
$(bj)\times(ai) = ab(j\times i) = ab(-k) = -abk$
So -ve Z-direction

g. Magnitude of vector product $(a\times b)$
$|a\times b| = |abk| = ab$ $a>0$ & $b>0$

h. Magnitude of vector product $(b\times a)$
$|b\times a| = |-abk| = ab$

i. Magnitude of $\vec{a}\times\vec{b}/d$ $= \frac{(abk)}{d} = \left(\frac{ab}{d}\right)k$
$\frac{|ab|}{d} = \frac{ab}{|d|}$

j Direction of $a \times b / d$ if d is positive
if $d > 0$

$$\frac{a \times b}{d} = \left(\frac{ab}{d}\right)k$$ which is +ve z direction

If d is negative, direction would be -ve z direction

9. $H = 3.0\,m$
$L = 4.3\,m$
$W = 3.7\,m$

a. Magnitude $= \sqrt{(3)^2+(4.3)^2+(3.7)^2} = 6.41\,m$

b. No, Length is not smaller than displacement b/c displacement is shortest distance from Starting to ending point

c. Yes, it might be greater, if fly move by making different corners.

d. Yes, it might be same, if fly move from corner to corner in straight direction.

e. In unit vector: $4.3\hat{i} + 3.7\hat{j} + 3.0\hat{k}$

f. If we unfold the box, differently each time

i. $(3.0+3.7) \times 4.3 = 6.70 \times 4.3$ diagonally
ii. $(3.0+4.3) \times 3.7 = 7.3 \times 3.7$ diagonally.
iii. $(3.7+4.3) \times 3 = 8.0 \times 3.0$ diagonally

i. $\sqrt{(6.7)^2+(4.3)^2} = 7.95\,m$
ii. $\sqrt{(7.3)^2+(3.7)^2} = 8.18\,m$
iii. $\sqrt{(8)^2+(3)^2} = 8.53\,m$

So shortest distance is $7.95\,m$.

16 $F_4 = -(A+B+C)$ (fourth force that oppose all other force

A: 100 N at 30° +ve x-axis.

$A_x = 100 \cos 30 = 86.6$

$A_y = 100 \sin 30 = 50$

B: 80 N at 120° x-axis

$B_x = 80 \cos 120 = -40$

$B_y = 80 \sin 120 = 69.3$

C = 40 N at 233 +x-axis

$C_x = 40 \cos 233 = -24.1$

$C_y = 40 \sin 233 = -31.9$

Sum of All forces = $R_x = 86.6 + (-40) + (-24.1) = 22.5$

$R_y = 50 + 69.3 + (-31.9) = 87.4$

$R = (22.5, 87.4)$

So fourth force that oppose all three force will be $-R = (-22.5, -87.4)$

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fourth force

$|F_4|$ = Magnitude $= \sqrt{(-22.5)^2 + (-87.4)^2} = \boxed{90.28\ N}$

Direction $-\theta = \tan^{-1}\left(\frac{-87.4}{-22.5}\right) = 75.4$

In 3^{rd} Quadrant $= 180 + 75.4 = \boxed{255.4^\circ}$ +ve x axis

1] Every Edge = 1m

Q1 So OQ = Pass from origin (0,0,0) to (1,1,1)

$OQ = 1\hat{i} + 1\hat{j} + 1\hat{k}$

half of the face

OR = Move towards the Center (1, 0.5, 0.5)

so $OR = 1\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$

Q3 Cosine of angle b/w OQ & OR

$OQ \cdot OR = (1i + 1j + 1k) \cdot (1i + \frac{1}{2}j + \frac{1}{2}k)$

$= 1 + \frac{1}{2} + \frac{1}{2} = 2$

$|OR|$ Mag $= \sqrt{(1)^2 + (0.5)^2 + (0.5)^2} = \frac{\sqrt{3}}{\sqrt{2}}$

Thus $\cos = \frac{OQ \cdot OR}{|OQ||OR|} = \frac{2}{(\sqrt{3})(\sqrt{3}/2)} = \frac{2\sqrt{2}}{3}$

$|OQ| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \boxed{19.49^\circ}$

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12) $d_1 = 4i + 5j$
$d_2 = -3i + 4j$

a) $d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 4 & 5 & 0 \\ -3 & 4 & 0 \end{vmatrix} = i(0) - j(0) + k(16 + 15) = 31k$

b) $d_1 \cdot d_2 = (4i + 5j) \cdot (-3i + 4j)$
$= -12 + 20 = 8$

c) $(d_1 + d_2) \cdot d_2 = 4i + (-3i) + 5j + 4j$
$(1i + 9j) \cdot (-3i + 4j)$
$-3 + 36 = 33$

13) $A = 4i + 3j \quad B = -13i + 7j$

$\vec{A} + \vec{B} = (4i - 13i) + (3j + 7j)$
$-9i + 10j$

Magnitude $= \sqrt{(-9)^2 + (10)^2} = \sqrt{181} = 13.45m$

direction $\theta = \tan^{-1}\left(\frac{10}{-9}\right) = -47.9 \simeq \boxed{-48°}$

~~13~~ 14. $(2i-3j+4k)\times A = 4i+3j-k)$

$B\times A = C$

so C must be perpendicular to B & A

& their dot product must be equal to Zero:

$(2i-3j+4k)\cdot(4i+3j-k) = 8-9-4 = \boxed{-5 \neq 0}$

$B\cdot C \neq 0$ so ~~no~~ any 'A' vector holds for $\boxed{B\times A = C}$

15. Suppose two vectors: X & Y

Sum is perpendicular to difference

$(x+y)\cdot(x-y) = 0$

$x\cdot x - \cancel{x\cdot y} + \cancel{y\cdot x} - y\cdot y = 0$

$x\cdot x - y\cdot y = 0$

$x\cdot x = y\cdot y$

$|x|^2 = |y|^2$

Take sqroot

$|X| = |y|$

Both vectors have equal magnitudes.