Confidence Intervals (σ Known): A Clear Guide

For Dr. Faisal Bukhari's Probability and Statistics Course
English + Urdu with Exam-Style Questions

1 What is a Confidence Interval?

1.1 English Explanation

A **Confidence Interval (CI)** is a **range** that likely contains the **true population mean** (μ). Instead of a single number (e.g., $\bar{x} = 65$), we say:

> "We are 95% confident that μ is between 63 and 67."

This means:

- The population mean **might or might not** be in the range.
- If we repeat the experiment 100 times, **95 out of 100 intervals** will include μ.

Formula (when σ is known):

$$\text{CI} = \bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

1.2 Urdu Explanation

Confidence Interval ka matlab:

> "Hum itne percent confident hain ke asal mean (μ) iss range ke andar hoga."

Agar 95% CI hai:

- Yani 100 baar experiment karein, to **95 baar** jo range milegi, usme μ hoga.
- Yeh 100% guarantee nahi, sirf confidence hai.

Formula:

$$\text{CI} = \bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}$$

2 Keywords (Exam Clues)

lightgray **Statement Includes...**	**Use This**
"Estimate mean with 90%/95% confidence"	CI formula with Z-value
"Margin of error is..."	$Z \cdot \frac{\sigma}{\sqrt{n}}$
"Range in which μ lies..."	Interpretation logic
"Find how large sample should be"	Sample size formula
"We want the error to be less than..."	Use Theorem 9.2

3 Dr. Bukhari-Style Twisted Questions

Below are 10 logic-based, confusing word problems designed to mimic Dr. Faisal Bukhari's exam style.

1. A steel factory is adjusting its cutting machines. The management suspects the average length of steel rods should be 150 cm, but recent samples show deviation. From a batch of 64 rods, the sample mean is 148.5 cm. Past records show $\sigma = 4$ cm. **What range should the manager consider to be 99% confident about the true mean length of rods?**

2. A researcher claims the average reaction time of drivers is less than 0.6 seconds. From a random sample of 49 drivers, the mean is 0.58 seconds. Assume $\sigma = 0.07$. **With 95% confidence, can the researcher defend this claim?**

3. A survey of 100 farmers revealed an average monthly income of Rs. 18,000 with $\sigma = 2,500$. The policy team wants to ensure that any estimation is accurate within Rs. 400 with 95% confidence. **Is this sample size enough?**

4. A bottle-filling machine claims to fill exactly 500 ml per bottle. A sample of 36 bottles shows a mean of 497.8 ml, $\sigma = 6$ ml. **Find the 95% confidence interval. Should the machine be adjusted?**

5. A sample of 25 batteries has a mean life of 80 hours. Past data shows $\sigma = 5$ hours. You want to be 99% confident that the average is within ± 1.5 hours. **Is your current sample size enough?**

6. A hospital administrator collects data on patient wait times. From a sample of 81 patients, mean wait time is 23 minutes, $\sigma = 6$ minutes. **Find a 90% confidence interval for the average patient wait time.**

7. An engineer checks the strength of metal sheets. He wants to estimate average strength (in psi) within 3 units of error at 95% confidence, $\sigma = 15$. **What is the minimum sample size required?**

8. An experiment on drug performance gives $\bar{x} = 72.3$ from 16 patients, with $\sigma = 3.8$. The scientist wants a 90% confidence interval. **What is the maximum possible deviation from the true mean (i.e., margin of error)?**

9. A quality inspector claims the mean length of packets is exactly 20 cm. A random sample of 100 packets has $\bar{x} = 19.7$ cm, $\sigma = 1.5$ cm. **Construct a 99% CI. Should the claim be rejected?**

10. To test whether a factory's output has changed, an auditor takes a sample of 49 outputs with average 230 units/day and $\sigma = 14$. He constructs a 95% CI and finds it is [226.08, 233.92]. **Can the auditor say the output has changed from the previous mean of 235 units/day?**

3.1 Answers

1. $Z = 2.575$, $E = 2.575 \cdot \frac{4}{\sqrt{64}} = 1.2875$, CI $= 148.5 \pm 1.2875 \Rightarrow [147.21, 149.79]$

2. $Z = 1.96$, $E = 1.96 \cdot \frac{0.07}{\sqrt{49}} = 0.0196$, CI $= 0.58 \pm 0.0196 \Rightarrow [0.5604, 0.5996]$. 0.6 is outside, so claim is **supported**.

3. $Z = 1.96$, $E = 1.96 \cdot \frac{2500}{\sqrt{100}} = 490$. Error too high, so **not enough**.

4. $Z = 1.96$, $E = 1.96 \cdot \frac{6}{\sqrt{36}} = 1.96$, CI $= 497.8 \pm 1.96 \Rightarrow [495.84, 499.76]$. 500 is outside, so **adjustment needed**.

5. $Z = 2.575$, $E = 2.575 \cdot \frac{5}{\sqrt{25}} = 2.575$. Error > 1.5, so **not enough**.

6. $Z = 1.645$, $E = 1.645 \cdot \frac{6}{\sqrt{81}} = 1.097$, CI $= 23 \pm 1.097 \Rightarrow [21.90, 24.10]$

7. $n = \left(\frac{1.96 \cdot 15}{3}\right)^2 = 96.04 \Rightarrow$ **n = 97**

8. $Z = 1.645$, $E = 1.645 \cdot \frac{3.8}{\sqrt{16}} = 1.563$. **Margin of error = 1.563.**

9. $Z = 2.575$, $E = 2.575 \cdot \frac{1.5}{\sqrt{100}} = 0.386$, CI $= 19.7 \pm 0.386 \Rightarrow [19.314, 20.086]$. 20 is within, so claim is **accepted**.

10. 235 is **outside** the interval [226.08, 233.92], so output has **changed**.

4 Final Cheat Sheet (Revision Aid)

lightgray **Topic**	**Key Formula / Rule**
Confidence Interval (σ known)	$\bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}$
Margin of Error (E)	$Z \cdot \frac{\sigma}{\sqrt{n}}$
Sample Size for max error e	$n = \left(\frac{Z \cdot \sigma}{e}\right)^2$
Higher Confidence	Wider CI
Bigger Sample Size	Narrower CI
Z-values to remember	1.645 (90%), 1.96 (95%), 2.575 (99%)
CI Interpretation	μ **likely** lies within this range

Want to practice more or move to Confidence Intervals for Proportions or σ Unknown? Ask your instructor!