NUMBER SYSTEM

Application of Information and Communication Technologies

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Learning Objectives

1. Distinguish among categories of numbers
2. Describe positional notation
3. Convert numbers in other bases to base 10
4. Convert base-10 numbers to numbers in other bases
5. Describe the relationship between bases 2, 8, and 16
6. Explain the importance to computing of bases that are powers of 2

Number System

- Mainly there are two types of number systems
  - Non-Positional Number System
    - Use fingers for counting. Use stones
    - Does the direction matter?
  - Positional Number System
    - Position matters

Non-positional Number System

- Use symbols such as I for 1, II for 2, III for 3, etc
- Each symbol represents the same value regardless of its position in the number
- Difficult to perform arithmetic with such a number system

Positional Number System

- Value of each digit is determined by:
  - The digit itself
  - The position of the digit in the number
  - The base of the number system

Base = total number of digits in the number system

Decimal Number system (positional system)

Name

- “decem” (Latin) => ten

Characteristics

- Ten symbols
  - 0 1 2 3 4 5 6 7 8 9
- Positional
  - $2945 \neq 2495$
  - $2945 = (2*10^3) + (9*10^2) + (4*10^1) + (5*10^0)$

Positional Notation

642 in base 10 *positional notation* is:

$$\begin{array}{lcccl} & 6 \times 10^2 = & 6 \times 100 & = 600 & \\ + & 4 \times 10^1 = & 4 \times 10 & = 40 & \\ + & 2 \times 10^0 = & 2 \times 1 & = 2 & = 642 \text{ in base 10} \end{array}$$

This number is in base 10

The power indicates the position of the number

Positional Notation

R is the base of the number

As a formula:

$$d_n * R^{n-1} + d_{n-1} * R^{n-2} + ... + d_2 * R^1 + d_1 * R^0$$

n is the number of digits in the number

d is the digit in the i^{th} position in the number

642 is $6 * 10^2 + 4 * 10 + 2 * 1$

Positional Notation

- What if 642 has a base of 13?

$$
\begin{aligned}
& 6 \times 13^2 &= \;& 6 \times 169 &= \;& 1014 \\
+\; & 4 \times 13^1 &= \;& 4 \times 13 &= \;& 52 \\
+\; & 2 \times 13^0 &= \;& 2 \times 1 &= \;& 2 \\
& & & &= \;& 1068 \text{ in base } 10
\end{aligned}
$$

- 642 in base 13 is equivalent to 1068 in base 10

Binary Number system

Name

- “binarius” (Latin) => two

Characteristics

- Two symbols
  - 0 1
- Positional
  - $1010_B \neq 1100_B$

Most (digital) computers use the binary number system

Why?

Terminology

- **Bit**: a binary digit
- **Byte**: (typically) 8 bits

Binary counting

0	0	0	0	0	0
1	1	1	1	1	1

0	10	110	1110	11110	111110
1	11	111	1111	11111	111111

Binary –Decimal Equal

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Decimal	Binary
16	10000
17	10001
18	10010
19	10011
20	10100
21	10101
22	10110
23	10111
24	11000
25	11001
26	11010
27	11011
28	11100
29	11101
30	11110
31	11111

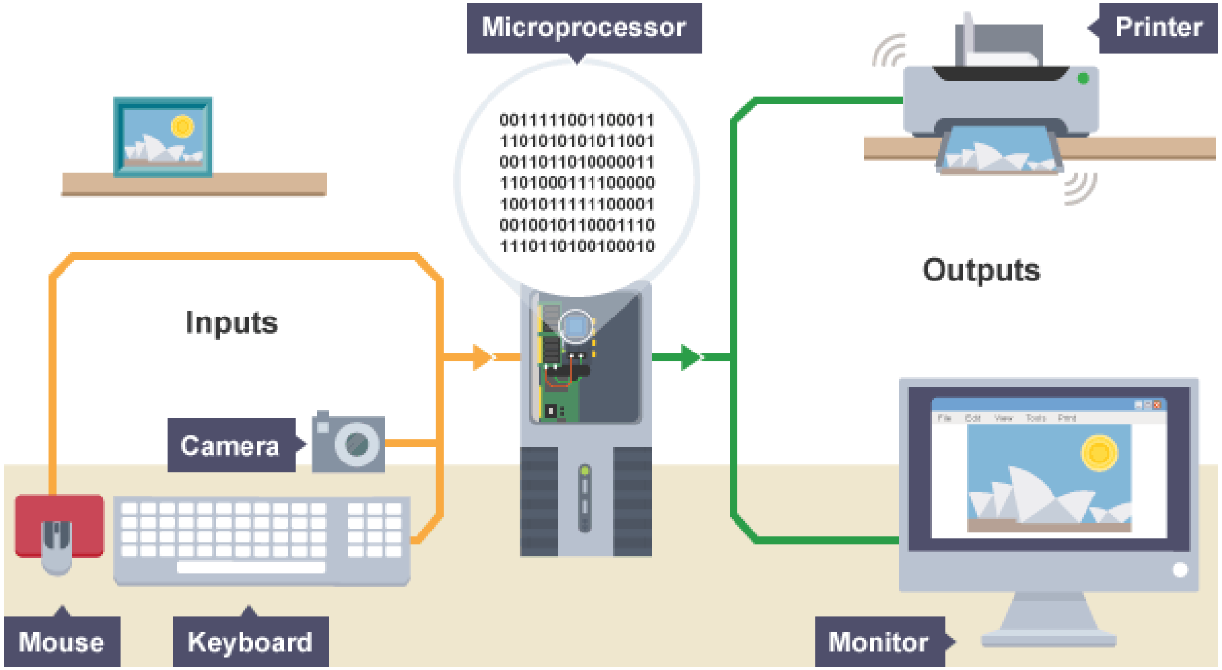
Decimal	Binary
32	100000
33	100001
34	100010
35	100011
36	100100
37	100101
38	100110
39	100111
40	101000
41	101001
42	101010
43	101011
44	101100
45	101101
46	101110
47	101111

Why Binary in Computers?

- Binary positional systems have great advantages over decimals in electronic computing
  - Circuits in computers have billions of transistors that can turn on and off
- Two reasons computer use binary
  - Two clearly distinct states that provide a safe range for reliability.
  - Least amount of necessary circuitry, which results in the least amount of space, energy consumption, and cost.

Why Binary in Computers?

- It is easy for machines to store electrical signals as ON or OFF
- Binary math is easier for computers than anything else
- Binary numbers help in digital electronic circuits using logic gates
  - Logic gates are the basic building blocks of any digital system
  - Common logic gates are AND, OR, NOT, XOR, NOR, NAND



Binary

- Recall, in decimal each position of digital is multiplied case by base ten (10)
  - $273 = 2*10^2+7*10^1+3*10^0$
  - $543.21 = 5*10^2+4*10^1+3*10+2*10^{-1}+1*10^{-2}$
  - Similarly binary equivalence is
  - $5 = 101 = 1*2^2+0*2^1+1*2^0 = 4+0+1$
  - $26 = 11010 = 1*2^4+1*2^3+0*2^2+1*2^1+0*2^0 = 16+8+0+2+0$

Binary Number

- Binary number is generally much longer than its corresponding decimal number
  - 256,058 = 111 11010 00001 11010
  - Reason: binary system has only two possibilities
  - In other words, binary digit carries less information than in decimal digit

Octal Number System

- The numbers with base 8 are called octal
  - 0, 1, 2, 3, 4, 5, 6, 7
  - There is no such thing as 8 and 9
  - We count the same way as we do for decimal, but we stop at 7

Octal-Decimal

Decimal	Octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11
10	12
11	13
12	14
13	15
14	16
15	17

Decimal	Octal
16	20
17	21
18	22
19	23
20	24
21	25
22	26
23	27
24	30
25	31
26	32
27	33
28	34
29	35
30	36
31	37

Decimal	Octal
32	40
33	41
34	42
35	43
36	44
37	45
38	46
39	47
40	50
41	51
42	52
43	53
44	54
45	55
46	56
47	57

Octal Number System

- We count the same way as we do for decimals, but we stop at 7
- Means, we will have 10 after 7.
- Similarly, 20 after 17
- $(35)_{10}=(43)_8$

$(43)_8 = 4x8^1+3x8^0 = (4x8)\ 32+3 = (35)_{10}$

$(605)_8 = 6x8^2+0x8^1+5x8^0 = 6x64+0+5 = 384+5 = (389)_{10}$

Why we are discussing Octal

- The fact that there is a short cut method to convert binary number into octal
- Method: make group of 3 digits start from right to left

Binary	000	001	010	011	100	101	110	111
Octal	0	1	2	3	4	5	6	7

It saves us from calculation of lot of terms of 0's & 1's individually and respective powers of 2's for calculation. See example:

$10110101 = \underline{10}\ \underline{110}\ \underline{101} = (2\ 6\ 5)_8 = 2x8^2+6x8^1+5x8^0 = 2x64 + 6x8 + 5x1 = 128+48+5 = (181)_{10}$

$1011010111 = \underline{1}\ \underline{011}\ \underline{010}\ \underline{111} = (1327)_8 = 1x8^3+3x8^2+2x8^1+7x8^0 = 512+192+16+7 = (727)_{10}$

Hexadecimal Number System

- Hexadecimal numbers is also important in computers
- Hexadecimal has based 16 and basic symbols are
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
  - A is 10 and F is 15

Decimal-Hexadecimal

Decimal	Hexa
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Decimal	Hexa
16	10
17	11
18	12
19	13
20	14
21	15
22	16
23	17
24	18
25	19
26	1A
27	1B
28	1C
29	1D
30	1E
31	1F

Decimal	Hexa
32	20
33	21
34	22
35	23
36	24
37	25
38	26
39	27
40	28
41	29
42	2A
43	2B
44	2C
45	2D
46	2E
47	2F

Why Hexadecimal?

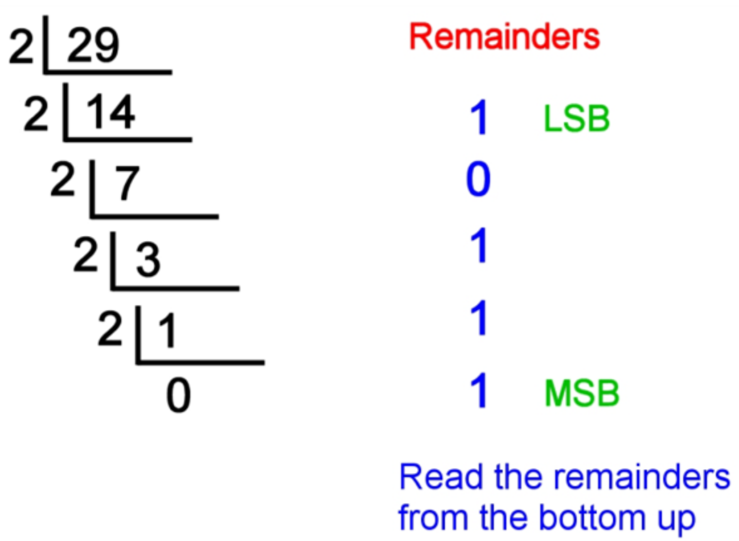
- $(43)_{16} = 4x16^1+3x16^0 = 64+3 = (67)_{10}$
- $(A5B)_{16} = Ax16^2+5x16^1+Bx16^0 = 10*256+5x16+11 = 2560+80+11 = (2651)_{10}$

- There is a shortcut method to convert binary into hexadecimal

Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Hexadec	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Decimal to Any Base

For converting a given decimal number to any base, we need to divide the number with the target base. Lets see how to convert from Decimal to Binary



29 decimal = 11101 binary

Decimal to Hexadecimal

$(423)_{10} = \quad (\ ? \ )_{16}$

Any Base to Decimal?

We can express any given n digit number of any base as

$$\boxed{d_n * R^{n-1} + d_{n-1} * R^{n-2} + \ldots + d_2 * R^1 + d_1 * R^0}$$

$(642)_{10} = 6 * 10^2 + 4 * 10^1 + 2 * 10^0$

$(1011)_2 = 1 * 2^3 + 0 * 2^2 + 1 * 2^1 +$
$1 * 2^0$

$(A12)_{16} = A * 16^2 + 1 * 16^1 + 2 *$
16^0

Binary to Hexa and Hexa to Binary?

Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Hexadec	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$(BC11)_{16} = \quad (\qquad\qquad\qquad\qquad)_2$

$(1110011101)_2 = \quad (\qquad\qquad)_{16}$

How many numbers can be represented using different number of bits?

1 bit 2 bits 3 bits 4 bits

Computer Arithmetic

Binary Addition

- It’s a key for binary subtraction, multiplication, division
- Four Rules

Case	A + B	Sum	Carry
1	0 + 0	0	0
2	0 + 1	1	0
3	1 + 0	1	0
4	1 + 1	0	1

Binary Addition

- Example

```
0011010 + 001100 = 00100110

        1 1                 carry
      0 0 1 1 0 1 0 = (26)10
    + 0 0 0 1 1 0 0 = (12)10
  ----------------------------

      0 1 0 0 1 1 0 = (38)10
```

Binary Addition

- Example

```
00010011  +  00111110 = 01010001

     1   1   1   1   1          carry
  0  0   0   1   0   0   1   1 = 19₁₀
+ 0  0   1   1   1   1   1   0 = 62₁₀
-----------------------------------
  0  1   0   1   0   0   0   1 = 81₁₀
```

Binary Subtraction

- It’s a key for binary subtraction, multiplication, division
- Four Rules

Case	A - B	Subtract	Carry
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Binary Subtraction

- Example

$$00100101 - 00010001 = 00010100$$

$$\begin{array}{cccccccccccl}
 & & & 0 & 1 & & & & & & & \text{borrows} \\
 & 0 & 0 & \cancel{1} & 0 & 0 & 1 & 0 & 1 & = & 37_{10} & \\
- & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & = & 17_{10} & \\
\hline
 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & = & (20)_{10} &
\end{array}$$

Binary Subtraction

- Example

00110011 - 00010110 = 00011101

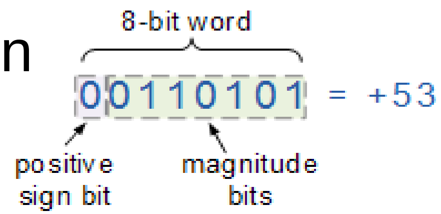
```
        1 1 1 1           borrows
    0 0 1 1 0 0 1 1 = 51_10
  - 0 0 0 1 0 1 1 0 = 22_10
-----------------------------
    0 0 0 1 1 1 0 1 = 29_10
```

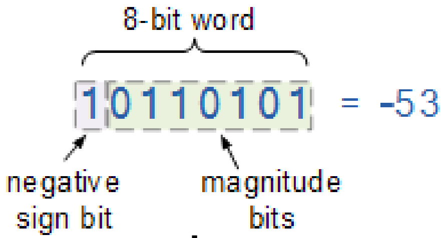
Issues in Binary Subtraction

- The classical subtraction (discussed above)
  - Works when we subtract small number from large number (27-9)
  - However, it does not work when we subtract a large number from small number
  - Alternatively, we use two methods
    - 1’s complement
    - 2’s complement

Negative Binary Number Representation

- Sign-magnitude Representation





- One’s Complement: Flip the bits to get the negative number representation

+53 = 00110101
-53 = 11001010

- Two’s Complement: Find One’s complement and then add 1 to get the negative number representation

+53 = 00110101
-53 = 11001011

1.Complement Theory

Example 1 Get 1's complement of 50

50 = 00000101

↓↓↓↓↓↓↓↓

11111010

Complement Digits

1. 1's Complement Theory

Example 2 : Subtract 1010 from 1111 using 1's complement theory. (Small negative)

$$1\,0\,1\,0 \xrightarrow{\text{1's complement}} 0\,1\,0\,1$$

$$
\begin{array}{rr}
 & 1\,1\,1\,1 \\
+ & 0\,1\,0\,1 \\
\hline
1] & 0\,1\,0\,0 \\
+ & 0\,0\,0\,1 \\
\hline
 & 0\,1\,0\,1 = (5)
\end{array}
$$

Carry "1" means the answer is positive .

1. 1's Complement Theory

Example 3 : Subtract 1010 from 1000 using 1's complement theory. (Large negative)

$$1\,0\,1\,0 \xrightarrow{\text{1's complement}} 0\,1\,0\,1$$

$$\begin{array}{rr} & 1\,0\,0\,0 \\ + & 0\,1\,0\,1 \\ \hline 0] & 1\,1\,0\,1 \end{array}$$

Carry "0" means the answer is negative and available in 1's complement form.

$$1\,1\,0\,1 \longrightarrow 0\,0\,1\,0 = (2)$$

2’s Complement Arithmetic

1. How to get 2’s complement form
2. Arithmetic operation using 2’s complement theory

2's Complement Arithmetic (How to get 2's complement form..?)

Example 1

$$\left.\begin{array}{r} 5 = 00000101 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 11111010 \end{array}\right\} \text{Complement Digits}$$

$$\left.\begin{array}{r} 11111010 \\ +1 \\ \hline -5 = 11111011 \end{array}\right\} \text{Add 1}$$

Example 2

$$\left.\begin{array}{r} -13 = 11110011 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 00001100 \end{array}\right\} \text{Complement Digits}$$

$$\left.\begin{array}{r} 00001100 \\ +1 \\ \hline 13 = 00001101 \end{array}\right\} \text{Add 1}$$

2's Complement Arithmetic
(Method)

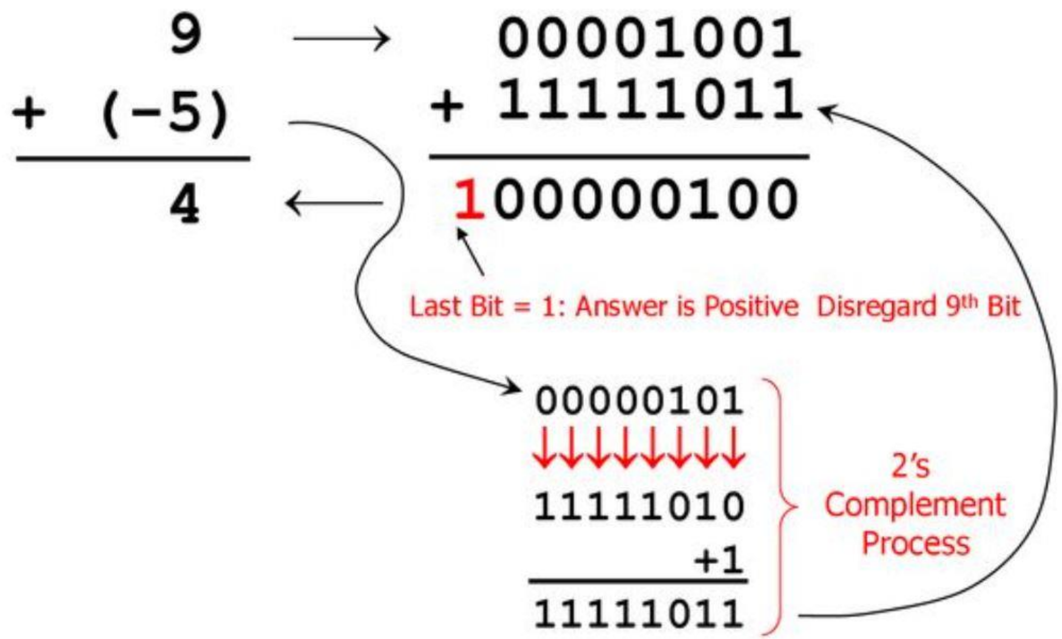
1. Read both the operands
2. Negative operand(s) (if any) is converted into 2's complement form
3. Add both the numbers
4. If carry is generated (i.e. =1) then the resultant number is positive and in original form
5. If carry is not generated(when we have negative operand) then the carry is assumed =0.
6. If carry is not generated (with both the operand positive) then the answer is Positive.
7. Carry zero means the resultant number is negative and in a 2's complement form.
8. Convert the 2's complement form into the original form.

2's Complement Arithmetic
(Method)

1. Read both the operands
2. Negative operand (if any) is converted into 2's complement form
3. Add both the numbers (2's complement of negative operand with the other one).
4. If carry is generated (i.e. =1) then the resultant number is positive and in original form
5. If carry is not generated(when we have negative operand) then the carry is assumed =0.
6. Carry zero means the resultant number is negative and in a 2's complement form.
7. Convert the 2's complement form into the original form.

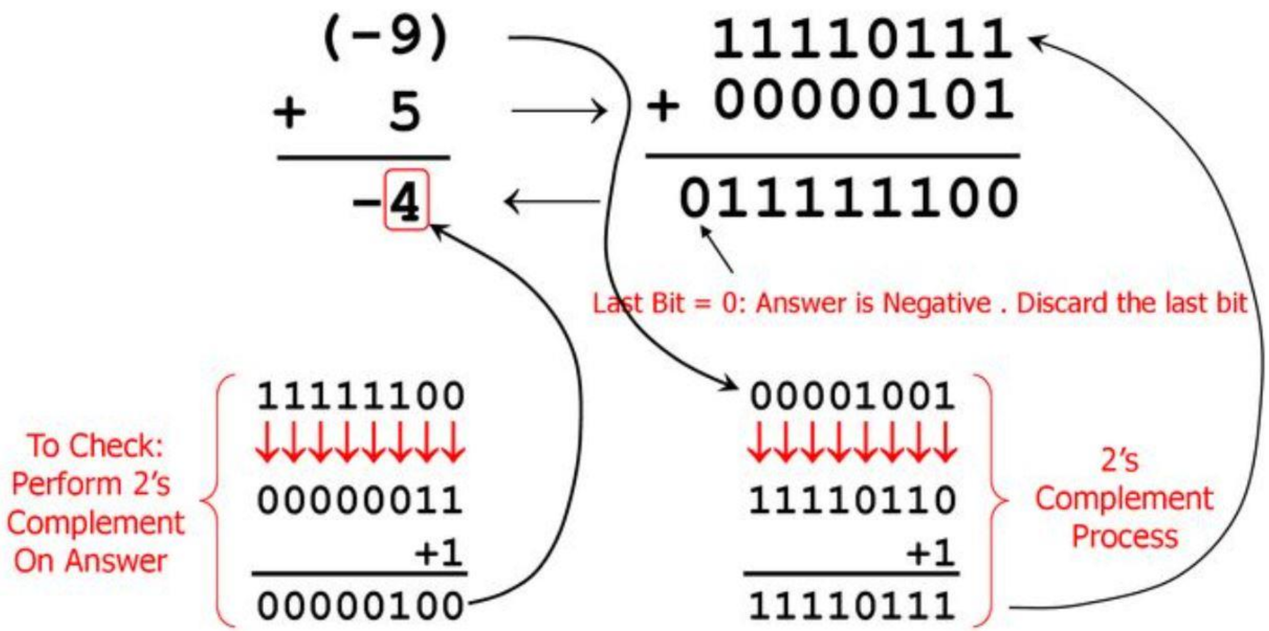
POS + NEG → POS Answer

Take the 2’s complement of the negative number and use regular binary 8-bit addition.



POS + NEG → NEG Answer

Take the 2's complement of the negative number and use regular 8-bit binary addition.



Verify the logic using following combinations:

1: (10) –(01)
2: (10) –(02)
3: (10) –(05)
4: (10) –(08)
5: (10) –(09)
6: (10) –(10)
7**: (210) –(08)**
8: (120) –(55)
9: (52) –(18)

2's Complement Arithmetic (Examples)

A+B	A	B	2's of B	Addition	Ans
A=10 B=-1	1 0 1 0	0 0 0 1	1 1 1 0 0 0 0 1 1 1 1 1	1 0 1 0 1 1 1 1 CY =1 So ans is +ve 1 1 0 0 1	+9
B=-2		0 0 1 0	1 1 0 1 0 0 0 1 1 1 1 0	1 0 1 0 1 1 1 0 CY =1 So ans is +ve 1 1 0 0 0	+8
B=-5		0 1 0 1	1 0 1 0 0 0 0 1 1 0 1 1	1 0 1 0 1 0 1 1 1 0 1 0 1 CY =1 So ans is +ve	+5
B=-8		1 0 0 0	0 1 1 1 0 0 0 1 1 0 0 0	1 0 1 0 1 0 0 0 1 0 0 1 0 CY =1 So ans is +ve	+2
B=-9		1 0 0 1	0 1 1 0 0 0 0 1 0 1 1 1	1 0 1 0 0 1 1 1 1 0 0 0 1 CY =1 So ans is +ve	+1
B=-10		1 0 1 0	0 1 0 1 0 0 0 1 0 1 1 0	1 0 1 0 0 1 1 0 1 0 0 0 0 CY =1 So ans is +ve	+0

Example: Perform 2's complement subtraction on 210-08

201= 1 1 0 1 0 0 1 0

8= 0 0 0 0 1 0 0 0 2's complement of 8 is = 1 1 1 1 1 0 0 0

Add both the numbers:

```
  1 1 0 1 0 0 1 0
 +1 1 1 1 1 0 0 0
1 1 1 0 0 1 0 1 0
```

Carry = 1 means and is positive +202

2's Complement Arithmetic (Examples on varying number of bits)

Example: Perform 2's complement arithmetic for (30)-(50) using

1: 6-bit number system
2: 8-bit number system

Example: Perform 2's complement arithmetic for (30)-(50) using:

1: 6-bit number system

(30)= 0 1 1 1 1 0
(-50)= 1 1 0 0 1 0 2's complement 0 0 1 1 0 1
0 0 0 0 0 1

0 0 1 1 1 0

Add both the numbers

0 1 1 1 1 0
0 0 1 1 1 0

0 1 0 1 1 0 0

Carry =0 means number is negative and in 2's compl form

0 1 0 0 1 1
0 0 0 0 0 1

0 1 0 1 0 0 = -20

2: 8-bit number system

(30)= 0 0 0 1 1 1 1 0
(-50)= 0 0 1 1 0 0 1 0 2's complement 1 1 0 0 1 1 0 1
0 0 0 0 0 0 0 1

1 1 0 0 1 1 1 0

Add both the numbers

0 0 0 1 1 1 1 0
1 1 0 0 1 1 1 0

0 1 1 1 0 1 1 0 0

Carry =0 means number is negative and in 2's compl form

0 0 0 1 0 0 1 1
0 0 0 0 0 0 0 1

0 0 0 1 0 1 0 0 = -20

(r-1)'s and r's complement

Example of decimal number system

So r becomes 10's complement

And (r-1) becomes 9's complement

How to get (r-1)'s complement of decimal number...?

How do we get (r-1)'s complement ...?

(r-1)'s i.e. 9's complement of decimal number can be obtained by ((10^n - 1)-number) where n represents the number of digits in given number.

Example 1: Find 9's complement of $(1234)_{10}$

9's complement = (10^4-1)-1234 = 9999-1234 = 8765

How do we get (r)'s complement ...?

r's i.e. 10's complement can be obtained by (10^n – number) where n represents the number of digits in given number.

Example 2: Find 10's complement = 10^4-1234 = 8766.

We will move to subtraction using them. Remember always the number to be subtracted (negative number) is converted to 9's or 10's complement.

9's Complement Arithmetic

Example 1: Get 9's complement of following numbers $(19)_{10}$, $(146)_{10}$, $(4397)_{10}$

99	999	9999
-19	-146	-4397
80	853	5602

9's Complement Arithmetic

Example 1: Perform subtraction using 9's complement on the data set given below.

a: 18-06, b: 39-23 c: 34-49 d: 49-84

06 is negative number,
We will get 9's complement of 06

```
  9 9
- 0 6
-----
  9 3
```

We will ADD 93 and 18

```
  9 3
+ 1 8
-----
1 1 1
```

A carry is generated means answer is +ve
ADD the carry back to the sum "11".

```
  1 1
+   1
-----
  1 2
```

Try for (018) – (006)

9's Complement Arithmetic

Example 2: Perform 9's complement subtraction on the data set given below.
a: 18-06, b: 39-23 c: 34-49 d: 49-84

23 is negative number,
We will get 9's complement of 23

$$\begin{array}{r} 9\,9 \\ -\ 2\,3 \\ \hline 7\,6 \end{array}$$

We will ADD 76 and 39

$$\begin{array}{r} 7\,6 \\ +\ 3\,9 \\ \hline 1\ 1\,5 \end{array}$$

A carry is generated .
ADD the carry back to the sum "15".

$$\begin{array}{r} 1\ 5 \\ +\ \ 1 \\ \hline 1\ 6 \end{array}$$

Try for 039-023

9’s Complement Arithmetic

Example 3: Perform 9’s complement subtraction on the data set given below.
a: 18-06, b: 39-23 c: 34-49 d: 49-84

49 is negative number,
We will get 9’s complement of 49

```
  9 9
- 4 9
-----
  5 0
```

We will ADD 50 and 34

```
   5 0
 + 3 4
 -----
 0 8 4
```

A carry is **not** generated that means answer is Negative.
Get 9’s complement of 84

```
  9 9
_ 8 4
-----
  1 5
```

Answer is -15

Try for 034-049

9’s Complement Arithmetic

Example 4: Perform 9’s complement subtraction on the data set given below.

a: 18-06, b: 39-23 c: 34-49 d: 49-84

84 is negative number,
We will get 9’s complement of 84

```
  9 9
- 8 4
-----
  1 5
```

We will ADD 15 and 49

```
  4 9
+ 1 5
-----
0 6 4
```

A carry is **not** generated that means answer is Negative.
Get 9’s complement of 64

```
  9 9
_ 6 4
-----
  3 5
```

Answer is -35

Try for 049-084

10’s Complement Arithmetic

Example 1: Perform 10’s complement subtraction on the data set given below.

a: 24-09, b: 69-32 c: 265-347

09 is negative number,
We will get 10’s complement of 09

$$\begin{array}{r} 99 \\ -\ 09 \\ \hline 90 \\ +\ 01 \\ \hline 91 \end{array}$$

We will ADD 91 and 24

$$\begin{array}{r} 24 \\ +\ 91 \\ \hline \mathbf{1}15 \end{array}$$

A carry is generated that means answer is Positive.
Answer is +15

Try for 024-009

10's Complement Arithmetic

Example 2: Perform 10's complement subtraction on the data set given below.

a: 24-09, b: 69-32 c: 265-347

32 is negative number,
We will get 10's complement of 32

$$\begin{array}{r} 99 \\ -\ 32 \\ \hline 67 \\ +\ 01 \\ \hline 68 \end{array}$$

We will ADD 68 and 69

$$\begin{array}{r} 68 \\ +\ 69 \\ \hline \mathbf{1}37 \end{array}$$

A carry is generated that means answer is Positive.
Answer is +37

Try for 069-032

10's Complement Arithmetic

Example 3: Perform 10's complement subtraction on the data set given below.

a: 24-09, b: 69-32 c: 265-347

347 is negative number,
We will get 10's complement of 347

```
  9 9 9
- 3 4 7
-------
  6 5 2
+   0 1
-------
  6 5 3
```

We will ADD 653 and 265

```
  653
+ 265
-----
0 918
```

A carry is not generated that means answer is Negative and in 10's complement form.

Get 10's complement form of 918

```
 9 9 9
-9 1 8
------
 0 8 1
+0 0 1
------
 0 8 2
```

Ans = -82

Try for 0265-0347