Probability and Statstics

Dr. Faisal Bukhari
Associate Professor
Department of Data Science
Faculty of Computing and Information Technology
University of the Punjab

Textbook

- **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

Reference books

- **Probability Demystified**, Allan G. Bluman
- **Schaum's Outline of Probability and Statistics**
- **MATLAB Primer**, Seventh Edition
- **MATLAB Demystified** by McMahon, David

References

Readings for these lecture notes:

- **Schaum's Outline of Probability, Second Edition (Schaum's Outlines)** by by Seymour Lipschutz, Marc Lipson
- **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer
- https://wordwatchtowers.wordpress.com/2009/12/21/underestimate-or-overestimate/
- Elementary Statistics, Tenth Edition, Mario F. Triola
- http://www.sjsu.edu/faculty/gerstman/

These notes contain material from the above resources.

Sampling Distribution of Means and the Central Limit Theorem

$$\overline{X} = \frac{(x_1 + x_2 + x_3 + \ldots + x_n)}{n}$$

has a normal distribution with **mean**

$$\mu_{\overline{X}} = \frac{1}{n}(\mu + \mu + \mu + \ldots + \mu) \text{ (n terms)}$$

$$= \frac{n\mu}{n}$$

$$\boldsymbol{\mu_{\overline{X}} = \mu}$$

Sampling Distribution of Means and the Central Limit Theorem

and variance

$$\boldsymbol{\sigma}^{\mathbf{2}}_{\overline{\mathbf{X}}} = \frac{1}{n^2}\left(\sigma^2 + \sigma^2 + \sigma^2 + \ \ldots + \sigma^2\right)\ (\text{n terms})$$

$$= \frac{\mathrm{n}\sigma^2}{n^2}$$

$$\boldsymbol{\sigma}^{\mathbf{2}}_{\overline{\mathbf{X}}} = \frac{\boldsymbol{\sigma}^{\mathbf{2}}}{\boldsymbol{n}}$$

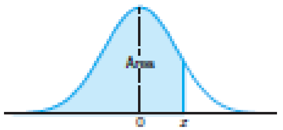


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Area under the Normal Curve [2]

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example : An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with **mean** equal to **800 hours** and a **standard deviation** of **40 hours**. Find the probability that a random sample of **16 bulbs** will have an **average life** of less than **775 hours**.

Solution : The sampling distribution of $\bar{X}$ will be approximately normal, with $\mu_{\bar{X}}$ = 800 and

$$\sigma_{\bar{X}} = \sigma / \sqrt{n}$$

$$= 40/\sqrt{16}$$

$$\sigma_{\bar{X}} = 10$$

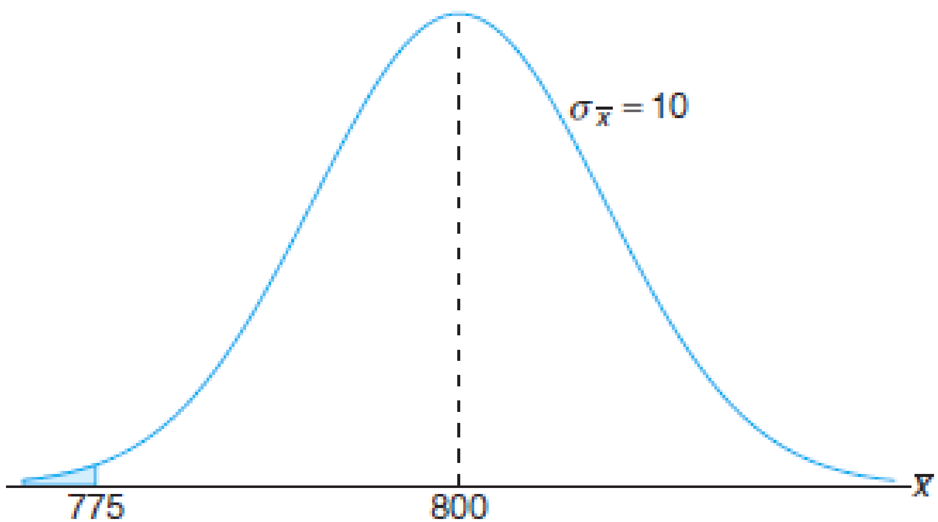
$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{775 - 800}{10}$$

$$= -2.5$$

$$P(\bar{X} < 775) = P(Z < -2.5)$$

$$= 0.0062.$$



```python
from scipy.stats import norm

# Given data
population_mean = 800  # Mean of the population
population_std = 40    # Standard deviation of the population
sample_size = 16       # Sample size
sample_mean = 775      # Sample mean

# Calculate the standard error of the mean (SEM)
sem = population_std / (sample_size ** 0.5)

# Calculate the z-score
z_score = (sample_mean - population_mean) / sem

# Calculate the probability using the cumulative distribution function (CDF)
probability = norm.cdf(z_score)

print(f"The probability that a random sample of 16 bulbs will have an average life of less than 775 hours is: {probability:.4f}")
```

The probability that a random sample of 16 bulbs will have an average life of less than 775 hours is: 0.0062

Example : Traveling between two campuses of a university in a city via shuttle bus takes, on average, **28 minutes** with a **standard deviation of 5** minutes. In a given week, a bus transported passengers **40 times**. What is the probability that the average transport time was more **than 30 minutes**? Assume the mean time is measured to the **nearest minute**

Solution: μ = 28, σ = 3 and n = 40

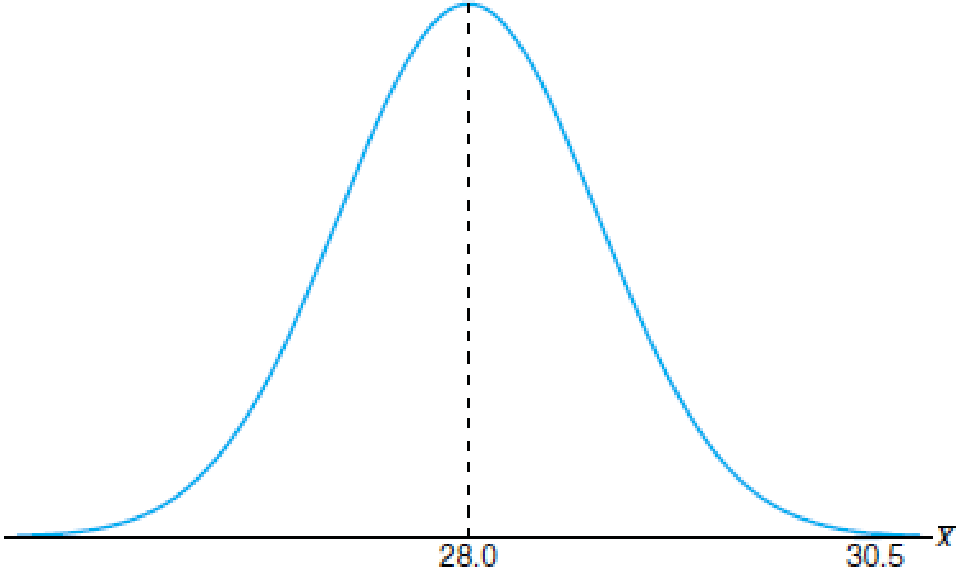
$P(\overline{X}$ > 30) =?

Since the time is measured on a continuous scale to the **nearest minute**, an $\overline{\boldsymbol{X}}$ **greater than 30** is equivalent to $\overline{\boldsymbol{x}}$ **≥ 30.5**.

$$P(\overline{X} > 30) = P\left(\frac{\overline{X}-28}{5/\sqrt{40}} \geq \frac{30.5-28}{5/\sqrt{40}}\right)$$

$$= P(Z \geq 3.16)$$

$$= 0.0008$$



```python
from scipy.stats import norm

# Given data
population_mean = 28       # Mean of the population
population_std = 5         # Standard deviation of the population
sample_size = 40           # Sample size
sample_mean = 30.5         # Sample mean

# Calculate the standard error of the mean (SEM)
sem = population_std / (sample_size ** 0.5)

# Calculate the z-score
z_score = (sample_mean - population_mean) / sem

# Calculate the probability using the complementary cumulative distribution function (1 - CDF)
probability = 1 - norm.cdf(z_score)

print(f"The probability that the average transport time was more than 30 minutes is: {probability:.4f}")
# The probability that the average transport time was more than 30 minutes is: 0.0008
```

Sampling Distribution of the Difference between Two Means

If independent samples of size n_1 and n_2 are drawn at random from two populations, discrete or continuous, with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively, then the sampling distribution of the differences of means,

$\bar{X}_1 - \bar{X}_2$, is approximately normally distributed with mean and variance given by

$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$ and $\sigma^2_{\bar{X}_1 - \bar{X}_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$

Hence,

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

is approximately a standard normal variable.

Sampling Distribution of the Difference between Two Means

- If both n_1 and n_2 are greater than or equal to **30**, the normal approximation for the distribution of $\bar{X}_1 - \bar{X}_2$ is very good when the underlying distributions are **not too far away** from normal.
- However, even when n_1 and n_2 are **less than 30**, the **normal approximation** is reasonably good e**xcept** when the populations are decidedly **nonnormal**. Of course, if both populations are normal, then

  $\bar{X}_1 - \bar{X}_2$ has a normal distribution no matter what the sizes of n_1 and n_2 are.

Example Paint Drying Time: Two independent experiments are run in which two different types of paint are compared. **Eighteen** specimens are painted using type *A*, and the drying time, in hours, is recorded for each. The same is done with type *B*. The **population standard deviations** are both known to be **1.0**.

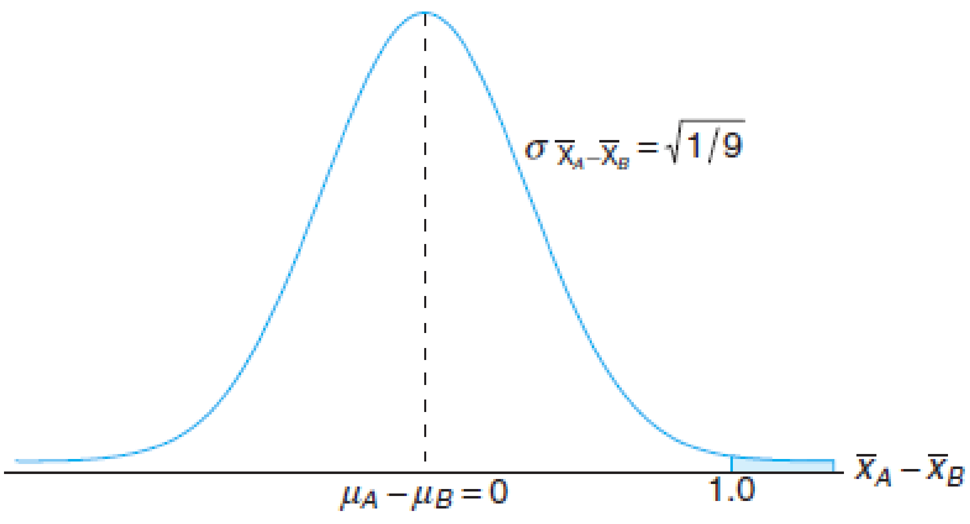
Assuming that the **mean drying time is equal** for the two types of paint, find $P(\bar{X}_A - \bar{X}_B > 1.0)$, where $\bar{X}_A$ and $\bar{X}_B$ are average drying times for samples of size $n_A = n_B = 18$.

Solution : $P(\overline{X}_A - \overline{X}_B > 1.0) = ?$

$$\mu_{\overline{X}_A - \overline{X}_B} = \mu_A - \mu_B = 0$$

and $$\sigma^2_{\overline{X}_A - \overline{X}_B} = \frac{\sigma^2_A}{n_A} + \frac{\sigma^2_B}{n_B}$$

$$\sigma^2_{\overline{X}_1 - \overline{X}_2} = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$$



$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\Rightarrow Z = \frac{1 - 0}{\sqrt{\frac{1}{9}}} = 3$$

$$P(Z > 3.0) = 1 - P(Z < 3.0)$$

$$= 1 - 0.9987$$

$$= 0.0013.$$

```python
from scipy.stats import norm
# Given data
sigma_A = 1.0          # Population standard deviation for type A
sigma_B = 1.0          # Population standard deviation for type B
n_A = 18               # Sample size for type A
n_B = 18               # Sample size for type B

# Calculate the standard error of the mean (SEM)
sem = ((sigma_A ** 2) / n_A + (sigma_B ** 2) / n_B) ** 0.5

# Calculate the z-score assuming equal means (mu_A - mu_B = 0)
z_score = 1.0 / sem

# Calculate the probability using the complementary cumulative
distribution function (1 - CDF)
probability = 1 - norm.cdf(z_score)
print(f"The probability that (mu_A - mu_B) > 1.0 is:
{probability:.4f}")
# The probability that (mu_A - mu_B) > 1.0 is: 0.0013
```

Interval Estimation

- There are many situations in which it is preferable to determine an interval within which we would expect to find the value of the parameter. Such an interval is called an **interval estimate**.

- An interval estimate of a population parameter $\boldsymbol{\theta}$ is an interval of the form $\widehat{\boldsymbol{\theta}}_L < \boldsymbol{\theta} < \widehat{\boldsymbol{\theta}_U}$, where $\widehat{\boldsymbol{\theta}}_L$ and $\widehat{\boldsymbol{\theta}_U}$ depend on the value of the **statistic** $\widehat{\boldsymbol{\theta}}$ for a particular sample and also on the sampling distribution of $\widehat{\boldsymbol{\theta}}$.

Single Sample: Estimating the Mean

- The sampling distribution of $\overline{X}$ is centered at μ, and in most applications the variance is smaller than that of any other estimators of μ.

- Thus, the sample mean $\overline{x}$ will be used as a point estimate for the population mean μ. Recall that $\boldsymbol{\sigma^2_{\overline{X}} = \sigma^2/n}$, so a large sample will yield a value of $\overline{x}$ that comes from a sampling distribution with a small variance. Hence, $\boldsymbol{\overline{x}}$ is likely to be a very accurate estimate of $\boldsymbol{\mu}$ when ***n* is large**.

Type I Error vs. Type II error

- Rejection of the **null hypothesis** when **it is true** is called a **type I error**.

- Nonrejection of the null hypothesis when it is false is called a **type II error**.

- The probability of committing a **type I error**, also called the **level of significance**, is denoted by the **Greek letter α**

Type I Error (False Positive)

Definition:

A Type I error occurs when we **reject a null hypothesis that is actually true**.

In other words, we detect an effect or difference that doesn't actually exist.

Example:

In a medical trial, concluding a drug works when it actually does not. This could lead to approving an ineffective drug.

Real-life analogy:

Declaring someone guilty when they are actually innocent.

Type II Error (False Negative)

Definition:

A Type II error occurs when we fail to reject a null hypothesis that is false. In other words, we miss detecting an effect or difference that is present.

Example:

In a drug trial, concluding the drug has no effect when it actually works. This could prevent a useful drug from being approved.

Real-life analogy:

Declaring someone innocent when they are actually guilty.

Summary of Errors

	True State (H_0 is True)	True State (H_0 is False)
Reject H_0	Type I Error (α)	Correct Decision
Fail to Reject H_0	Correct Decision	Type II Error (β)

Medical Testing and Security Systems

Type I Error (False Positive)

- **Medical Testing:**
- **Scenario:** A diagnostic test for a disease.
- **Example:** A patient is diagnosed with a disease, but they do not have it. The test falsely detects the condition.

- **Security Systems:**
- **Scenario:** Airport security screening for prohibited items.
- **Example:** A security system flags a harmless item as a threat, leading to unnecessary checks and delays.

Spam Email Filters and Criminal Justice

Type I Error (False Positive)

- **Spam Email Filters:**
- **Scenario:** Email filters detecting spam.
- **Example:** A legitimate email is marked as spam and sent to the junk folder, potentially missing important messages.

- **Criminal Justice:**
- **Scenario:** A legal trial.
- **Example:** A court convicts an innocent person of a crime they did not commit, a false positive.

COVID-19 Testing and Machine Learning Models

Type I Error (False Positive)

- **COVID-19 Testing:**
- **Scenario:** Testing for COVID-19 infection.
- **Example:** A test result comes back positive, but the person does not have COVID-19, causing unnecessary quarantine.

- **Machine Learning Models:**
- **Scenario:** Classifying images as cats or dogs.
- **Example:** A machine learning model classifies a picture of a dog as a cat, a false positive.

Fire Alarm and Pregnancy Test

Type I Error (False Positive)

- **Fire Alarm System:**
- **Scenario:** A fire alarm detects smoke.
- **Example:** The alarm goes off due to burnt toast, even though there is no real fire, a false signal.

- **Pregnancy Test:**
- **Scenario:** A home pregnancy test.
- **Example:** A test indicates pregnancy, but the woman is not pregnant, leading to confusion and emotional distress.

Scientific Research and Face Recognition Technology

- **Scientific Research:**
- **Scenario:** A study testing the effect of a drug.
- **Example:** Researchers conclude that a drug is effective, but in reality, the drug has no real effect.

- **Face Recognition Technology:**
- **Scenario:** A facial recognition system for security.
- **Example:** The system identifies an innocent person as a suspect, leading to wrongful detainment.

What is a Type II Error?

A Type II error, or false negative, occurs when a test fails to detect an effect or relationship that actually exists. This means concluding there is no effect when there is one.

Medical, Spam, and Product Quality Examples

Type II error, or false negative

1. Medical Testing:

- A test fails to detect cancer in a patient who has it.
- Consequences: Delayed treatment.

2. Spam Detection:

- A spam email is classified as legitimate.
- Consequences: Harmful messages.

3. Product Quality Testing:

- A faulty product passes quality control.
- Consequences: Defective products reach customers.

COVID-19, Jury Trial, and Statistical Testing

Type II error, or false negative

4. COVID-19 Testing:

- An infected person tests negative.
- Consequences: Continued spread of virus.

5. Jury Trial:

- A guilty defendant is acquitted.
- Consequences: Justice is not served.

6. Statistical Testing:

- A test fails to detect a true relationship.
- Consequences: Incorrect research findings.

Summary

- Type II errors occur in various contexts when a test or system fails to detect an existing issue. Their consequences can range from minor inconveniences to significant societal impacts.

Single Sample: Estimating the Mean

According to the **Central Limit Theorem**, we can expect the sampling distribution of $\bar{X}$ to be approximately normally distributed with mean $\mu_{\bar{X}} = \mu$ *and* $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

$P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$, *where* $Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

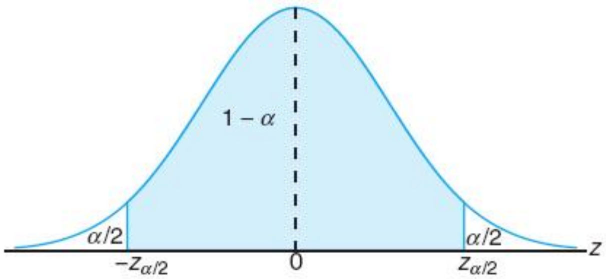
$$\Longrightarrow P\left(-z_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = 1 - \alpha$$

$$\Longrightarrow P\left(- z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \bar{x} - \mu < z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$\Longrightarrow P\left(-\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < -\mu < -\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$\Longrightarrow P\left(\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} > \mu > \bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$\Longrightarrow P\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$



Confidence Interval on μ, σ^2 Known [1]

If $\overline{x}$ is the mean of a random sample of size n from a population with known variance σ^2, a $100(1 - \alpha)\%$ confidence interval for μ is given by

$$\overline{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \overline{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Confidence Interval on μ, σ^2 Known [2]

Example 6.11: The **average** zinc concentration recovered from a sample of measurements taken in **36** different locations in a river is found to be **2.6** grams per milliliter. Find the **95% and 99%** confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is **0.3** gram per milliliter.

Area under the Normal Curve [1]

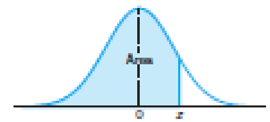


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Area under the Normal Curve [2]

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Solution:

The point estimate of μ is $\bar{x} = 2.6$. The z-value leaving an area of 0.025 to the right, and therefore an area of 0.975 to the left, is $z_{0.025} = 1.96$ (Table A.3). Hence, the 95% confidence interval is

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\alpha/2 = 0.05/2 = 0.0250$

$z_{\alpha/2} = z_{0.0250} = 1.96$ $\quad\because$ **$1 - \alpha/2 = 1 - 0.250 = 0.9750$**

95% confidence interval for μ is

$$2.6 - (1.96)\left(\frac{0.3}{\sqrt{36}}\right) < \mu < 2.6 + (1.96)\left(\frac{0.3}{\sqrt{36}}\right)$$

$2.47 < \mu < 2.73.$

$\alpha/2 = 0.01/2 = 0.005$

$\because\ 1 - \alpha/2 = 1 - 0.005 = 0.9950$

$z_{\alpha/2} = z_{0.005} = 2.575$

99% confidence interval for μ is

$2.6 - (2.575)\left(\frac{0.3}{\sqrt{36}}\right) < \mu < 2.6 + (2.575)\left(\frac{0.3}{\sqrt{36}}\right)$

$2.6 – 0.13 < \mu < 2.6 + 0.13$

$2.47 < \mu < 2.73.$

```python
import scipy.stats as stats

# Given data
sample_mean = 2.6                # Sample mean
population_std = 0.3             # Population standard
deviation
sample_size = 36                 # Sample size
confidence_levels = [0.95, 0.99]  # Confidence levels

# Calculate the standard error of the mean (SEM)
sem = population_std / (sample_size ** 0.5)

# Calculate the confidence intervals for the given
confidence levels
confidence_intervals = []
for confidence_level in confidence_levels:
    # Calculate the Z-score for the given confidence level
    z_score = stats.norm.ppf((1 + confidence_level) / 2)

    # Calculate the margin of error
    margin_of_error = z_score * sem
```

```python
# for loop continue from the previous slide
# Calculate the confidence interval
    lower_limit = sample_mean - margin_of_error
    upper_limit = sample_mean + margin_of_error

    confidence_intervals.append((lower_limit, upper_limit))

# Print the results
for confidence_level in confidence_levels:
# Calculate the Z-score for the given confidence level
    z_score = stats.norm.ppf((1 + confidence_level) / 2)

#95% Confidence Interval: (2.5020, 2.6980)
#99% Confidence Interval: (2.4712, 2.7288)
```

Error In Estimating μ By $\bar{x}$ [1]

- The 100(1−α)% confidence interval provides an estimate of the accuracy of our point estimate. If μ is actually the center value of the interval, then $\bar{x}$ estimates μ without error.

- Most of the time, however, **$\bar{x}$ will not be exactly equal to μ** and the point estimate will **be in error**. The size of this error will be the absolute value of the difference between μ and $\bar{x}$, and we can be 100(1 − α)% confident that this **difference will not exceed $z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$**

Error In Estimating μ By $\overline{x}$ [2]

Theorem 9.1: If $\overline{\mathrm{x}}$ is used as an estimate of μ, we can be 100(1 − α)% confident that the error will not exceed $z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Area under the Normal Curve [1]

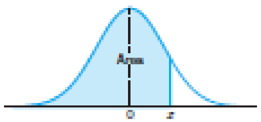


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Area under the Normal Curve [2]

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

$z_{\alpha/2} = z_{0.0250} = 1.96$ $\because 1 - \alpha/2 = 1 - 0.250 = 0.9750$

$z_{\alpha/2} = z_{0.005} = 2.575$ $\because 1 - \alpha/2 = 1 – 0.005 = 0.9950$

- We are **95% confident** that the sample mean $\overline{x}$ = 2.6 differs from the true mean μ by an amount less than **(1.96)(0.3)/√36 = 0.1** and **99% confident** that the difference is less than **(2.575)(0.3)/√36 = 0.13**.
- Frequently, we wish to know how **large a sample** is necessary to ensure that the error in estimating μ will be less than a specified amount ***e***.
- By Theorem 9.1, we must choose *n* such that

$$z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = e$$

Theorem 9.2: If $\overline{x}$ is used as an estimate of μ, we can be $100(1 - \alpha)\%$ confident that the error will not exceed a specified amount e when the sample size is

$$\boldsymbol{n = \left(\frac{\sigma z_{\alpha/2}}{e}\right)^{2}}$$

When solving for the sample size, n, we round all fractional values up to the next whole number. By adhering to this principle, we can be sure that our degree of confidence never falls below $100(1 - \alpha)\%$.

Example: How **large a sample** is required if we want to be 95% confident that our estimate of μ in the previous Example is off by **less than 0.05**?

Solution:

The population standard deviation is σ = 0.3. Then, by Theorem 9.2

n = {(1.96)(0.3) /0.05}^{2} = 138.3.

Therefore, we can be 95% confident that a random sample of size **139** will provide an estimate $\overline{x}$ differing from μ by an amount less than 0.05.