Probability and Statstics

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Textbooks

- **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

- **Elementary Statistics: Picturing the World,** 6th Edition, Ron Larson and Betsy Farber

- **Elementary Statistics**, 13th Edition, Mario F. Triola

Reference books

- **Probability Demystified**, Allan G. Bluman
- **Schaum's Outline of Probability and Statistics**
- **MATLAB Primer**, Seventh Edition
- **MATLAB Demystified** by McMahon, David

Reference

Readings for these lecture notes:

MATLAB Demystified, David McMahon

MATLAB® Primer, Seventh Edition, Timothy A. Davis Kermit Sigmon

Elementary Statistics PICTURING THE WORLD by **Ron Larson and Betsy Farber**

https://www.blackjackinfo.com/knowledge-base/blackjack-theory-and-math/a-question-for-the-statistics-experts/

http://math.stackexchange.com/questions/598808/if-you-roll-a-fair-six-sided-die-twice-whats-the-probability-that-you-get-the

These notes contain material from the above resources.

Relational operators

The relational operators in MATLAB are:

- < less than
- > greater than
- <= less than or equal
- >= greater than or equal
- == equal
- ~= not equal

Note that = is used in an assignment statement whereas == is a relational operator.

Logical operators:

Relational operators may be connected by **logical operators**:

- & and
- | or
- ~ not
- && short-circuit and
- || short-circuit or

fix()

fix(X) rounds the elements of X to the nearest integers towards zero.

Examples of fix()

```
>> fix(5.5)

ans =
    5

>> fix(5.9)

ans =
    5
```

Example: ***Intervals*** on the real line, defined below, appear very often in mathematics. Here a and b are real numbers *with* $a < b$.

Open interval from a to b = $(a,b) = \{x : a < x < b\}$

Closed interval from a to b = $[a,b] = \{x : a \leq x \leq b\}$

Open-closed interval from a to b = $(a,b] = \{x : a < x \leq b\}$

Closed-open interval from a to b = $[a,b) = \{x : a \leq x < b\}$

The **open-closed** and **closed-open** intervals are also called ***half*-open**

rand()

rand(): returns an n-by-n matrix containing pseudorandom values drawn from the standard uniform distribution on the **open interval** (0,1).

Example 1 of rand()

```
>> n = rand(1,10)
```

n =

Columns 1 through 3

0.1576 0.9706 0.9572

Columns 4 through 6

0.4854 0.8003 0.1419

Columns 7 through 9

0.4218 0.9157 0.7922

Column 10

0.9595

Example 2 of rand()

```
>> n = fix(10*rand(1,10))

n =

  Columns 1 through 6

     8     9     1     9     6     0

  Columns 7 through 10

     2     5     9     9
```

Simulation

- A **simulation** is the use of a **mathematical or physical model** to **reproduce the conditions** of a situation or process. Collecting data often involves the **use of computers**.
- Simulations allow you to study situations that are **impractical or even dangerous** to create in real life, and often they save time and money.
- For instance, **automobile manufacturers** use **simulations with dummies** to study the effects of **crashes on humans**.

cumsum (Cumulative Sum)

```
% Original data
x = [1 2 3 4 5];

% Cumulative sum
y = cumsum(x);

% Display result

disp('Original Vector:');
disp(x);
disp('Cumulative Sum:');
disp(y);
```

What is randi in MATLAB?

randi stands for **"random integer"** and is a built-in MATLAB function used to generate random integers from a uniform discrete distribution

r = **randi**([imin, imax], m, n)

- **imin, imax:** Range of integers (inclusive)
- **m, n:** Size of the matrix you want (rows × columns)
- **r:** Output matrix filled with random integers

Single die roll

x = **randi**([1,6])

Gives one random number between 1 and 6 (like rolling a die).

Simulate 10 die rolls

x = **randi**([1,6], 1, 10)

Gives a 1 × 10 row vector with random die rolls.

Generate a 3×3 matrix of random integers between 10 and 20

```
A = randi([10, 20], 3, 3)
```

Objective

- To demonstrate the Law of Large Numbers using a MATLAB simulation.

- We simulate tossing a fair **coin multiple times (e.g., 10000 times)** and observe how **the empirical probability of getting heads converges to the theoretical probability** (0.5).

Simulation of Law of Large Numbers

```
clc;
clear all;
% Number of tosses
N = 10000;

% Simulate N tosses: 1 = Heads, 0 = Tails

coin_tosses = randi([0 1], 1, N);

% Cumulative mean (empirical probability of Heads)
empirical_prob_heads = cumsum(coin_tosses) ./ (1:N);
```

Simulation of Law of Large Numbers

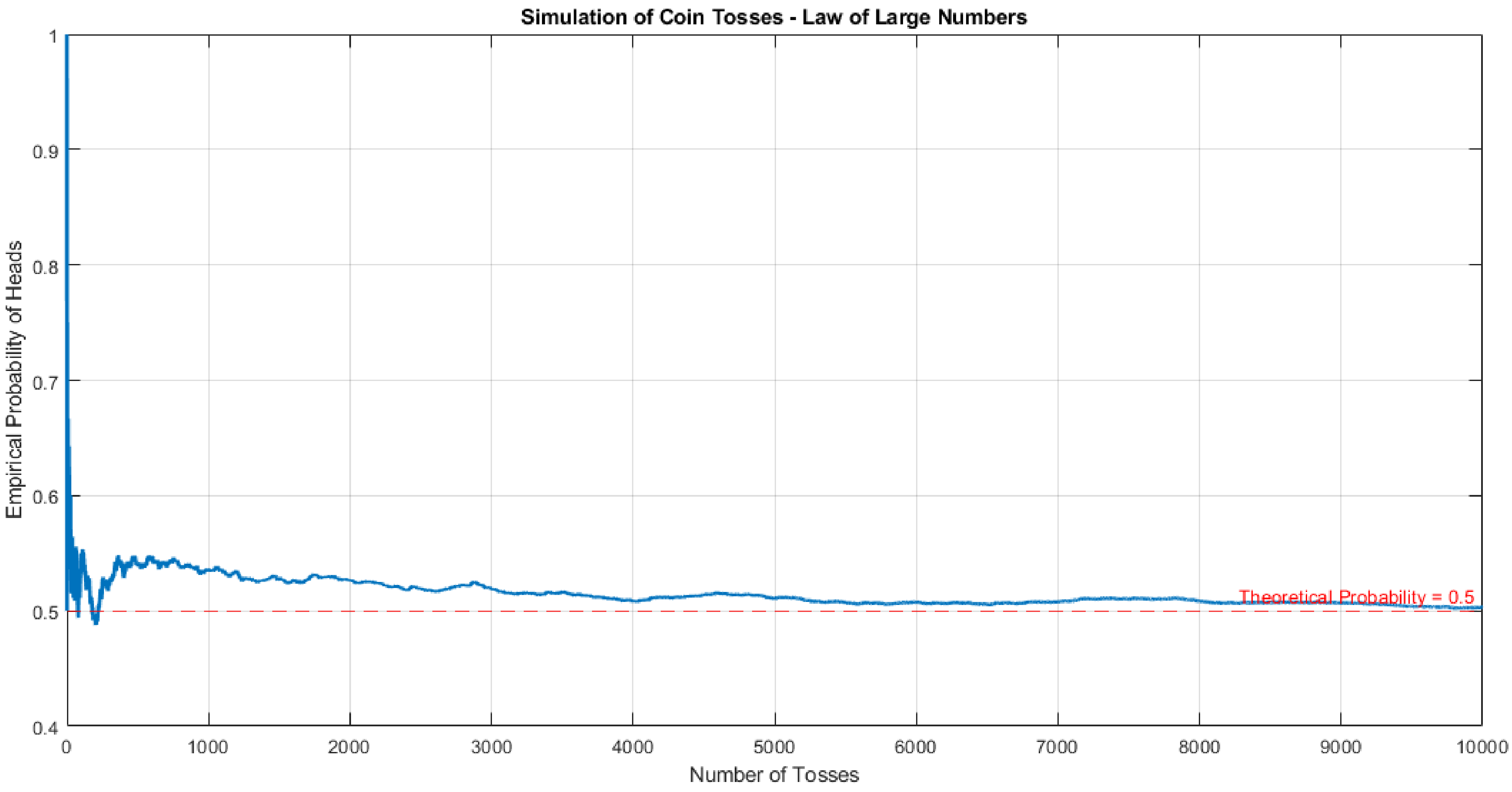
% Plotting the empirical probability vs number of tosses

```
figure;

plot(1:N, empirical_prob_heads, 'LineWidth', 2);
```

% yline create a horizontal line

```
yline(0.5, 'r--', 'Theoretical Probability = 0.5');

xlabel('Number of Tosses');
```

```
ylabel('Empirical Probability of Heads');

title('Simulation of Coin Tosses - Law of Large Numbers');

grid on;
```


Simulation of Coin Tosses - Law of Large Numbers
Empirical Probability of Heads
Number of Tosses
Theoretical Probability = 0.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

Simulation of coin tosses [1]

Question: Simulate the outcomes of 1000 biased coin tosses with **p[Head] = 0.3**

Solution:

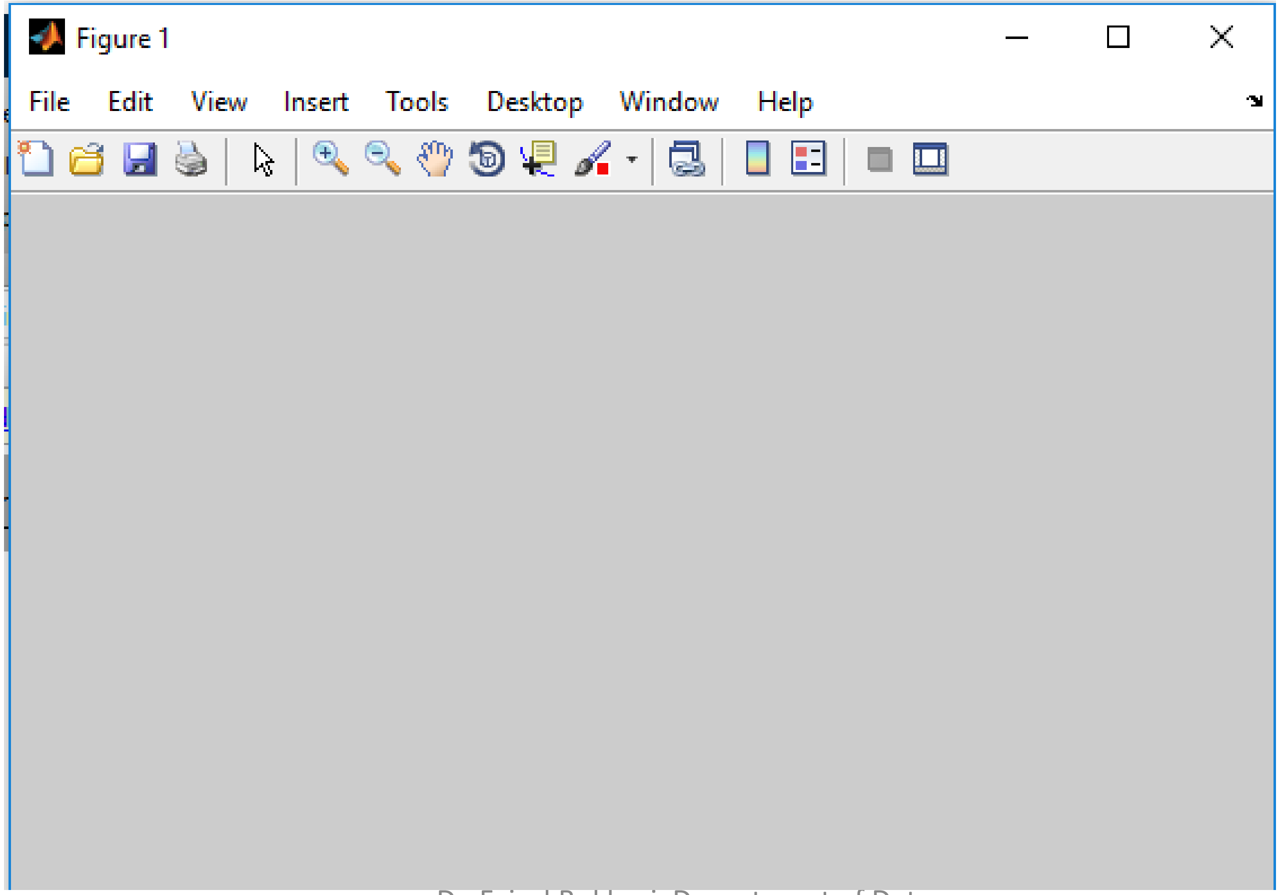
```
randomNumber = rand(1000,1);

headsOutOf1000 = randomNumber <= 0.3;

totalNumberOfHeads =
sum(headsOutOf1000);

probabilityOfHeads = totalNumberOfHeads
/1000;
```

figure

figure: opens up a new figure window

Example of figure command

>> figure



hold on vs. hold off

hold on: holds current plot

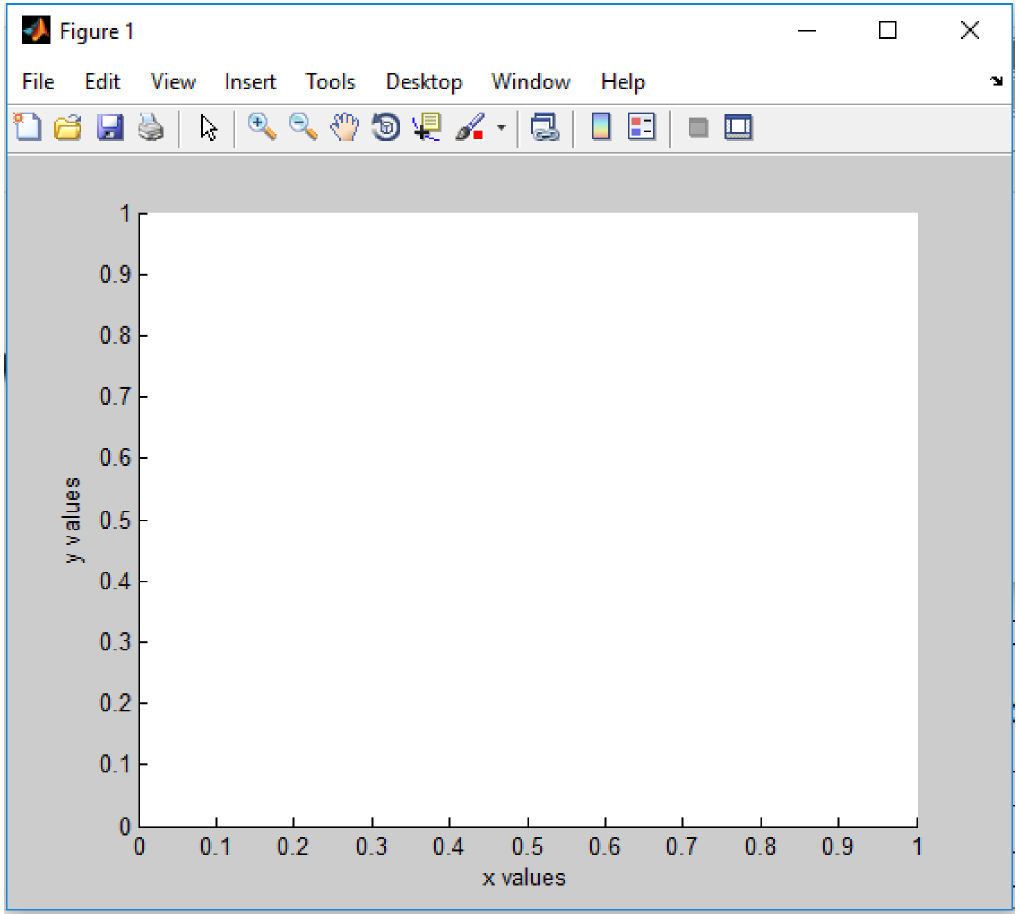
hold off: releases current plot

xlabel vs. ylabel

xlabel: Labels the x-axis

ylabel: Labels the y-axis

```
>> figure
>> hold on
>> xlabel('x values')
>> ylabel('y values')
```

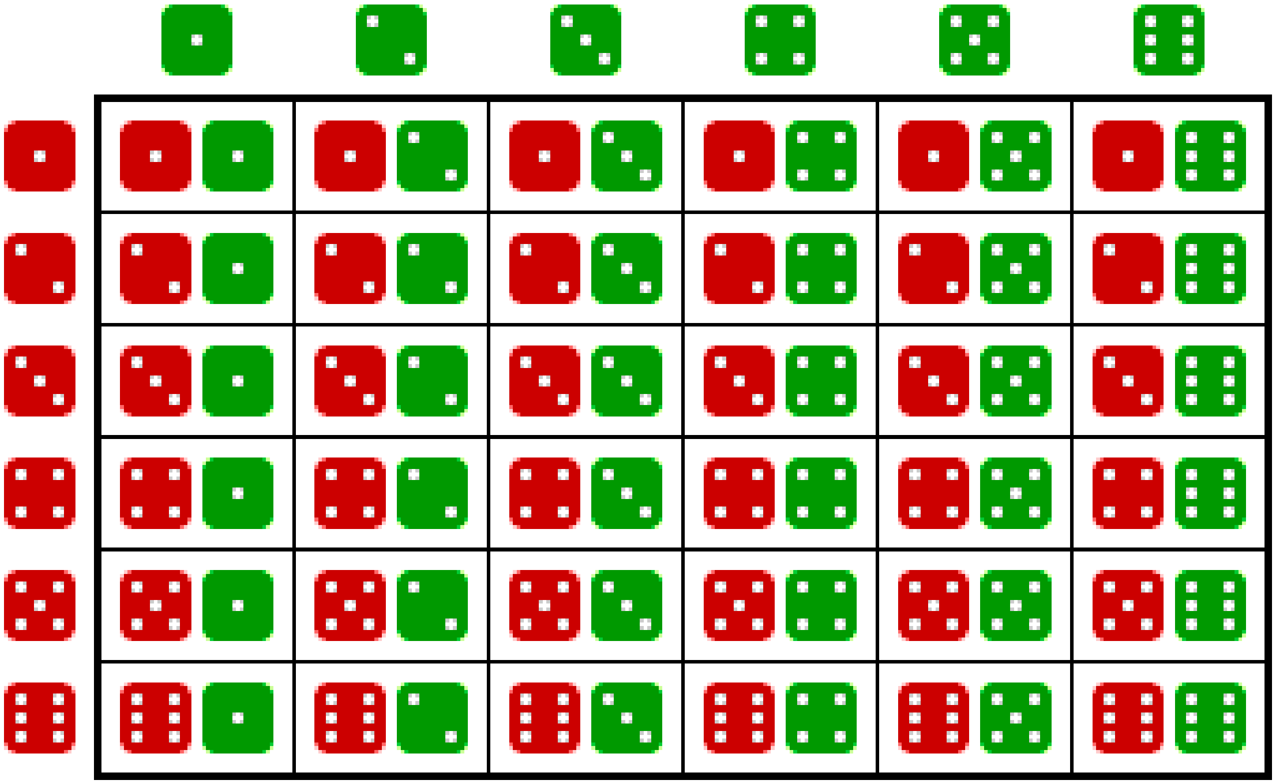


bar()

bar: Bar graph.

bar(X,Y) draws the columns of the M-by-N matrix Y as M groups of N vertical bars. The vector X must not have duplicate values.

Simulation of the sum of two fair dice [1]



Simulation of the sum of two fair dice [2]

Simulate the sum of outcome of two dice (e.g., number of times 2, 3, 4, …, or 12 appears), when two are rolled 10, 000 times.

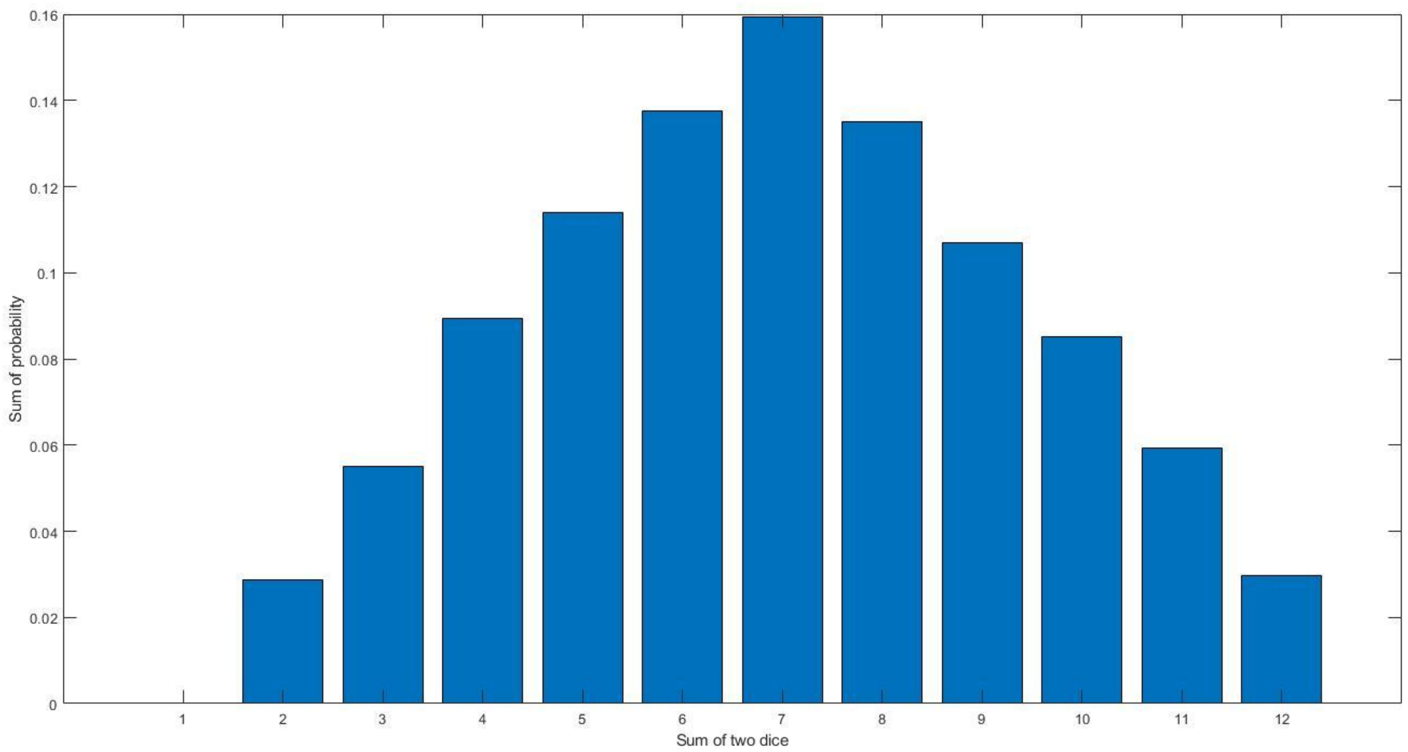
		White Die					
		1	2	3	4	5	6
Red Die	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
	6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

```
% create die 1
Die1 = floor(6 * rand(10000, 1) + 1 );
% create die 2
Die2 = floor(6 * rand(10000, 1) + 1);
% sum of 2 dice
SumOfDice = Die1 + Die2;
% check if sum is 2
D2 = SumOfDice == 2;
% compute probability of 2
probD2 = sum(D2)/10000;
D3 = SumOfDice == 3;
probD3 = sum(D3) /10000;
```

```
D4 = SumOfDice == 4;

probD4 = sum(D4) /10000;

D5 = SumOfDice == 5;

probD5 = sum(D5) /10000;

D6 = SumOfDice == 6;

probD6 = sum(D6) /10000;

D7 = SumOfDice == 7;

probD7 = sum(D7) /10000;
```

```
D8 = SumOfDice == 8;

probD8 = sum(D8) /10000;

D9 = SumOfDice == 9;

probD9 = sum(D9) /10000;

D10 = SumOfDice == 10;

probD10 = sum(D10) /10000;

D11 = SumOfDice == 11;

probD11 = sum(D11) /10000;
```

```
D12 = SumOfDice == 12;

probD12 = sum(D12) /10000;

probD1 = 0;
p = [probD1 , probD2, probD3, probD4,
  probD5,     probD6, probD7,  probD8,
  probD9, probD10, probD11, probD12 ]';

bar(p)
hold on
xlabel('Sum of two dice')
ylabel('Probability')
```



Sum of probability
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
9
10
11
12
Sum of two dice

Understanding histcounts in MATLAB

histcounts is a MATLAB function used to compute the frequency distribution of data into bins.

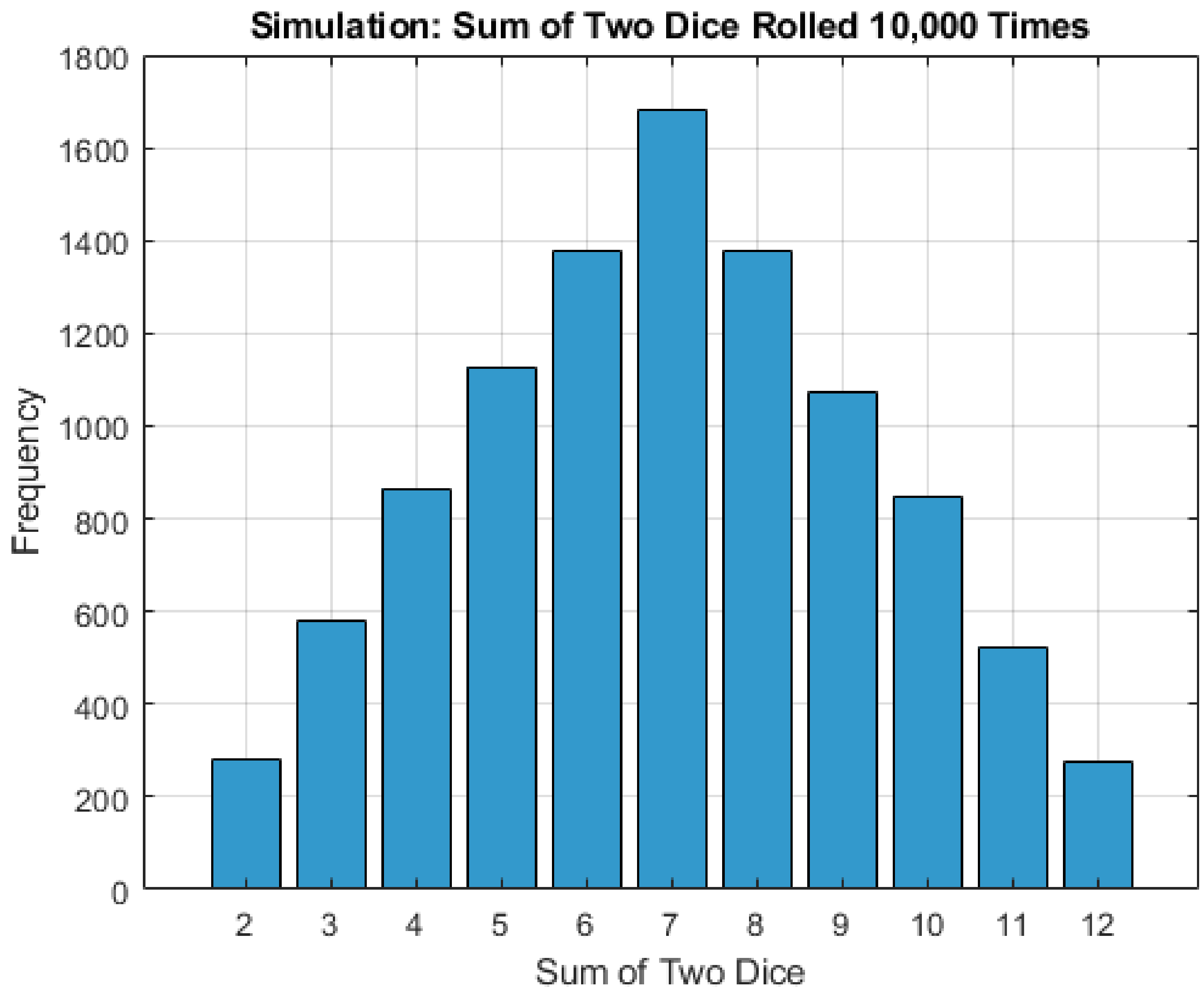
counts **= histcounts(data, edges)**

- **data:** The input array (e.g., sum of dice rolls).
- **edges:** The bin edges that define intervals for grouping values.
- **counts:** An array representing the number of occurrences in each bin.

Optimized Code

```
clc;
clear;

% Number of rolls
N = 10000;

% Simulate rolling two dice
die1 = randi([1, 6], 1, N);
die2 = randi([1, 6], 1, N);

% Compute sum of outcomes
sum_dice = die1 + die2;
```

```
% Count occurrences of each sum (2 to 12)
outcome_values = 2:12;
outcome_counts = histcounts(sum_dice, [1.5:1:12.5]);

% Plot the results
figure;
bar(outcome_values, outcome_counts, 'FaceColor', [0.2 0.6 0.8]);
xlabel('Sum of Two Dice');
ylabel('Frequency');

title('Simulation: Sum of Two Dice Rolled 10,000 Times');
grid on;
```


Simulation: Sum of Two Dice Rolled 10,000 Times
1800
1600
1400
1200
1000
800
600
400
200
0
Frequency
2
3
4
5
6
7
8
9
10
11
12
Sum of Two Dice