Applied Physics

BS Software Engineering/Information Technology

1st Semester

Lecture # 7

Presented By

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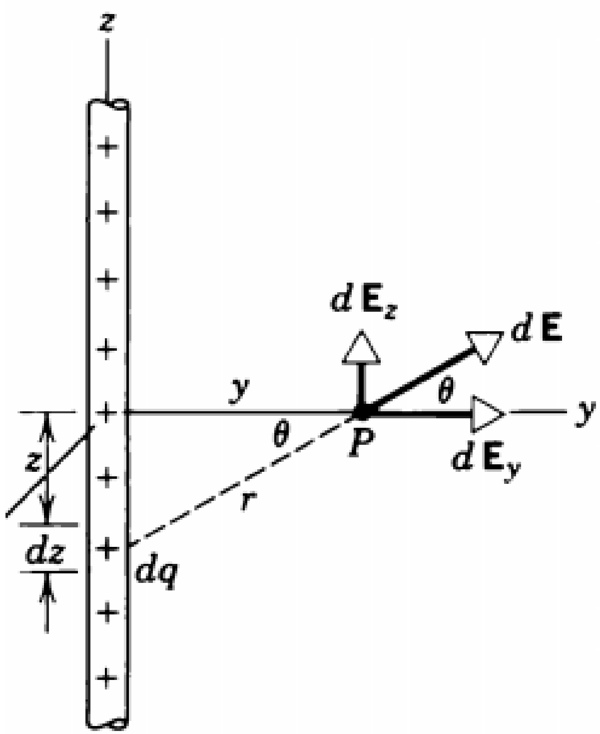
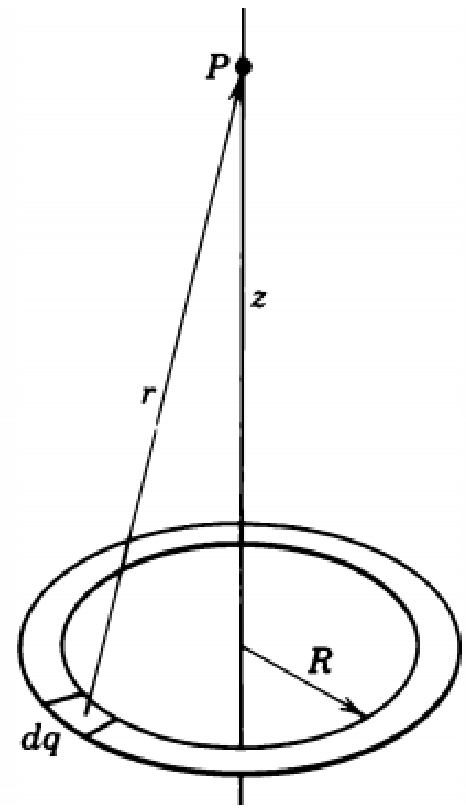
Lecture # 7

- Continuous Charge Distribution
  - Charge densities (linear, surface, volume)
- Continuous Charge Distribution
  - Uniform line of charges.
  - Uniform ring of charges.
- Chapter summery and Problems

Electric field due to continuous charge distribution





A continuous distribution of charge is described by its charge density

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

For linear charge distribution

Linear charge density relation: $\lambda = \frac{dq}{dl}$

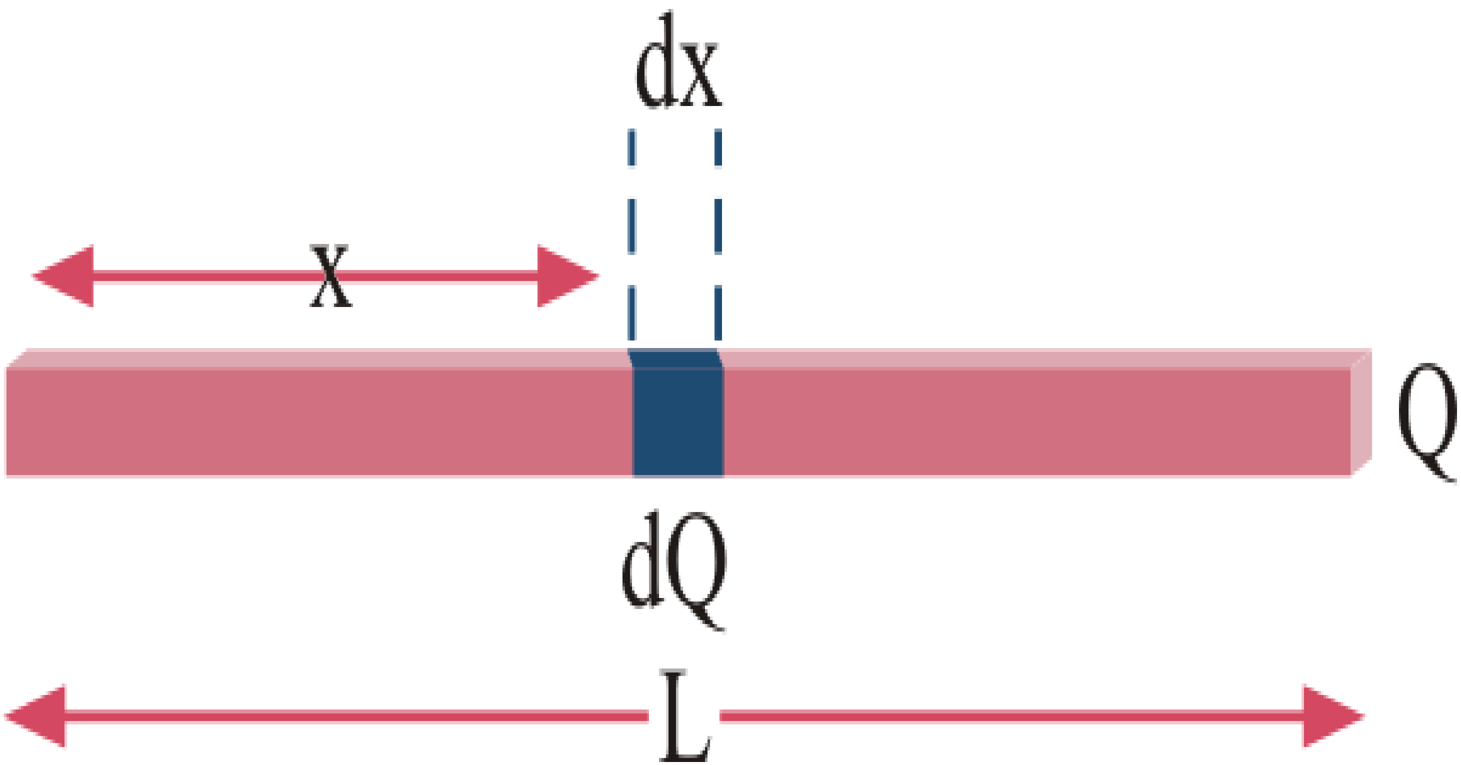
For uniform surface charge

Surface charge density relation: $\sigma = \frac{dq}{ds}$

For uniform volume charge

Volume charge density relation: $\rho = \frac{dq}{dV}$

Linear charge density relation: $\lambda = \frac{dQ}{dx}$



Steps to find out electric field **E** of ring of charges at point P

- An *Element*'s field
- Finding resultant **E** by

a) Cancelling components

b) Adding components

- Integrating

The field set up by a continuous charge distribution can be computed by dividing the distribution into infinitesimal elements *dq*. Each element of charge establishes a field $d\boldsymbol{E}$ at a point F, and the resultant field at P is then found from the superposition principle by adding (that is, integrating) the field contributions due to all the charge elements, or

$$\boldsymbol{E} = \int d\boldsymbol{E} \text{...........(i}$$

In calculating the electric field of a continuous charge distribution, the general strategy is to choose an arbitrary element of charge *dq*, find the electric field ***dE*** at the observation point P, and then integrate over the distribution using Eq. 1 to find the total field E. In many cases, the charge element dq is treated as a point charge and gives a contribution to the field ***dE*** of magnitude

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2},$$

where r is the distance from the charge element *dq* to the point P. In other cases, we can simplify calculations by choosing *dq* to be an element in the form of a charge distribution that gives a known field ***dE.***

Uniform Line of Charges

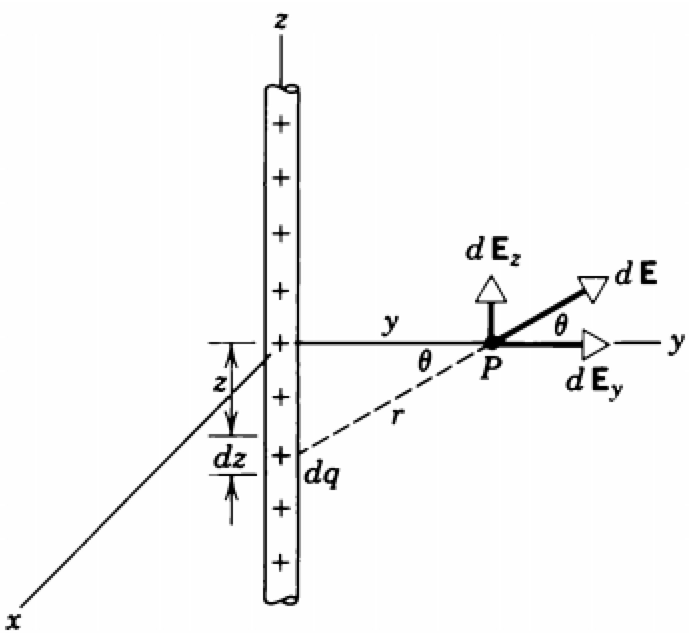
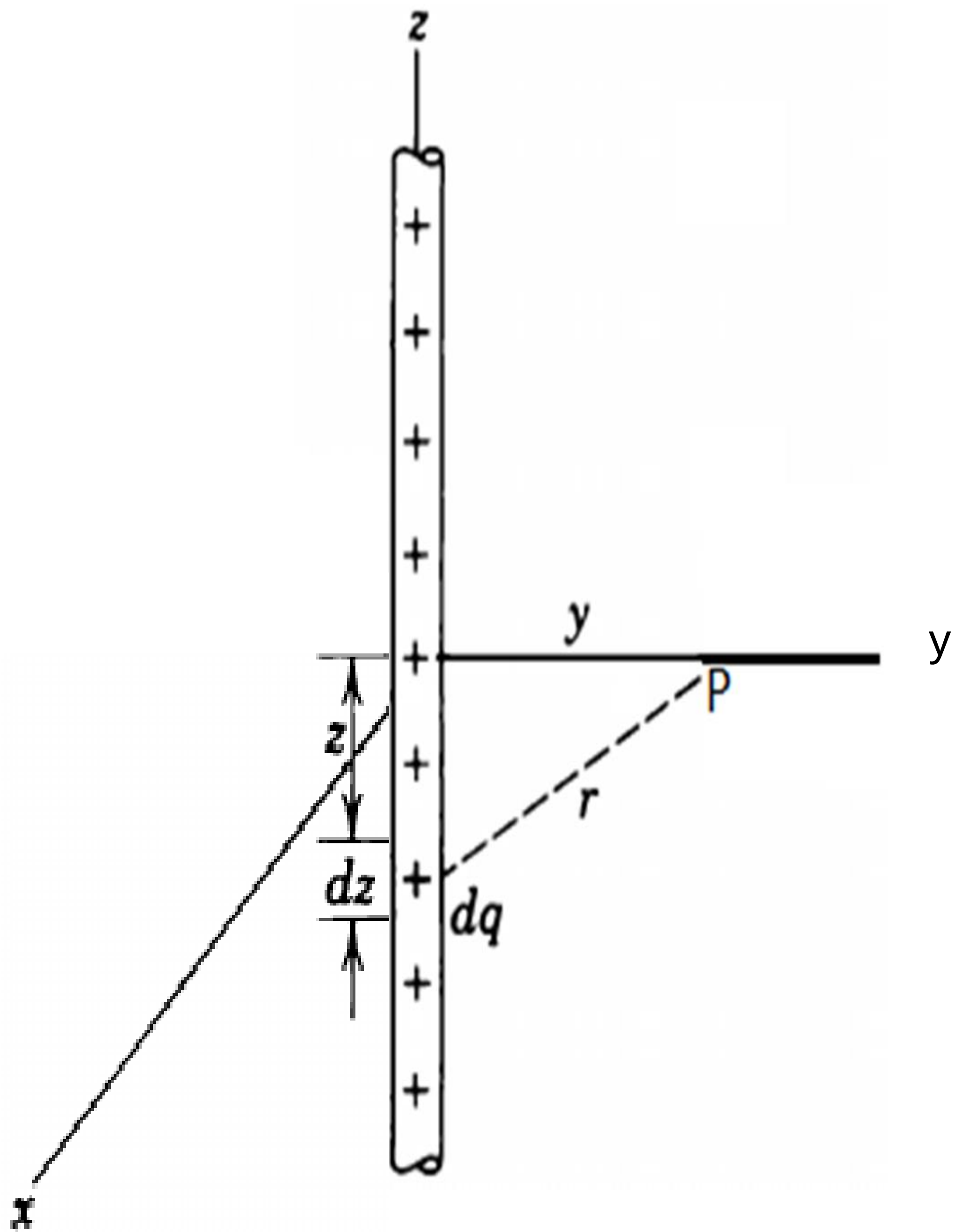


Figure A uniform line of charge of great length. The element of length dz gives a contribution $d\mathbf{E}$ to the electric field at point P, whose distance y from the line is small compared with the length of the line.


z
y
y
P
z
r
dz
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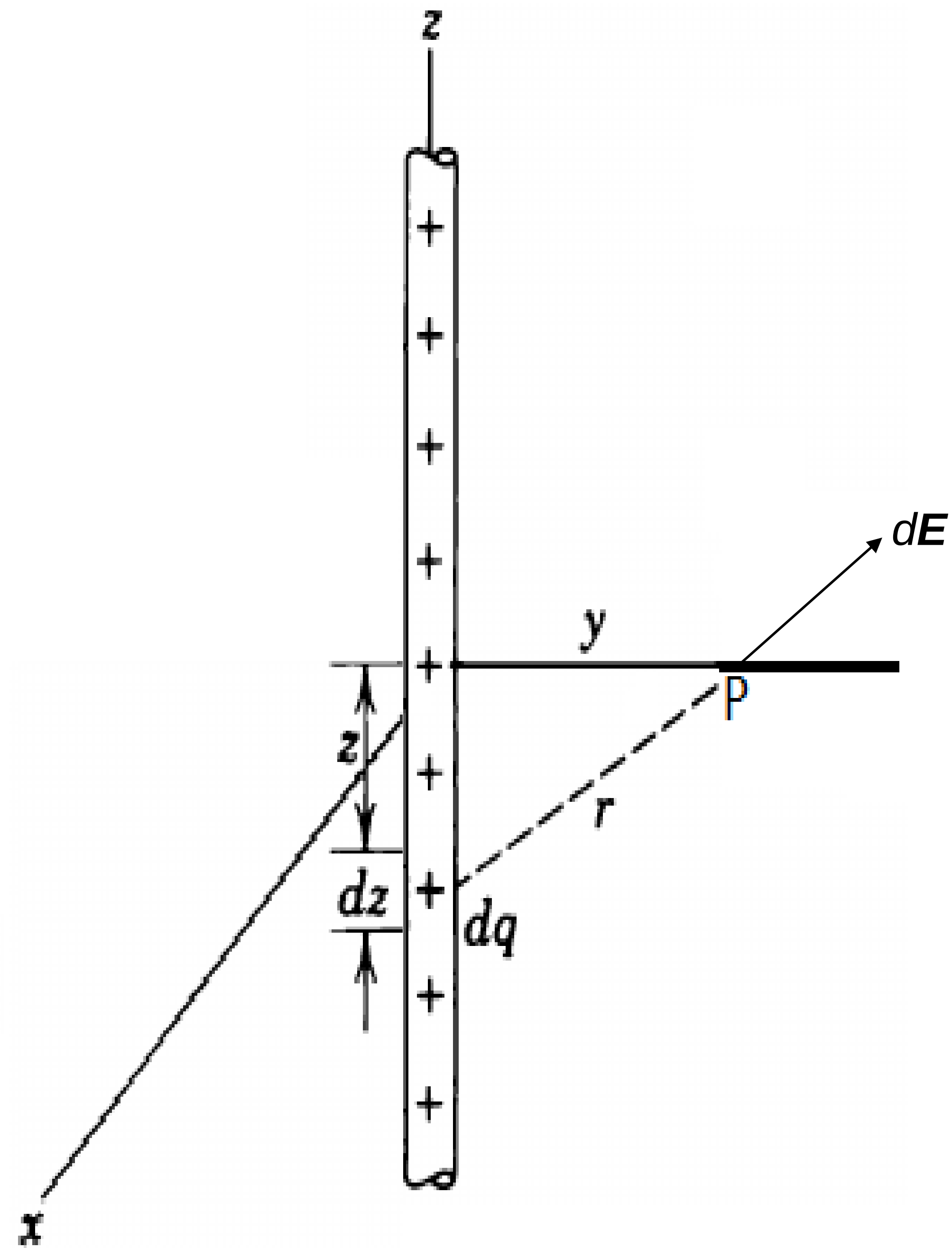
x



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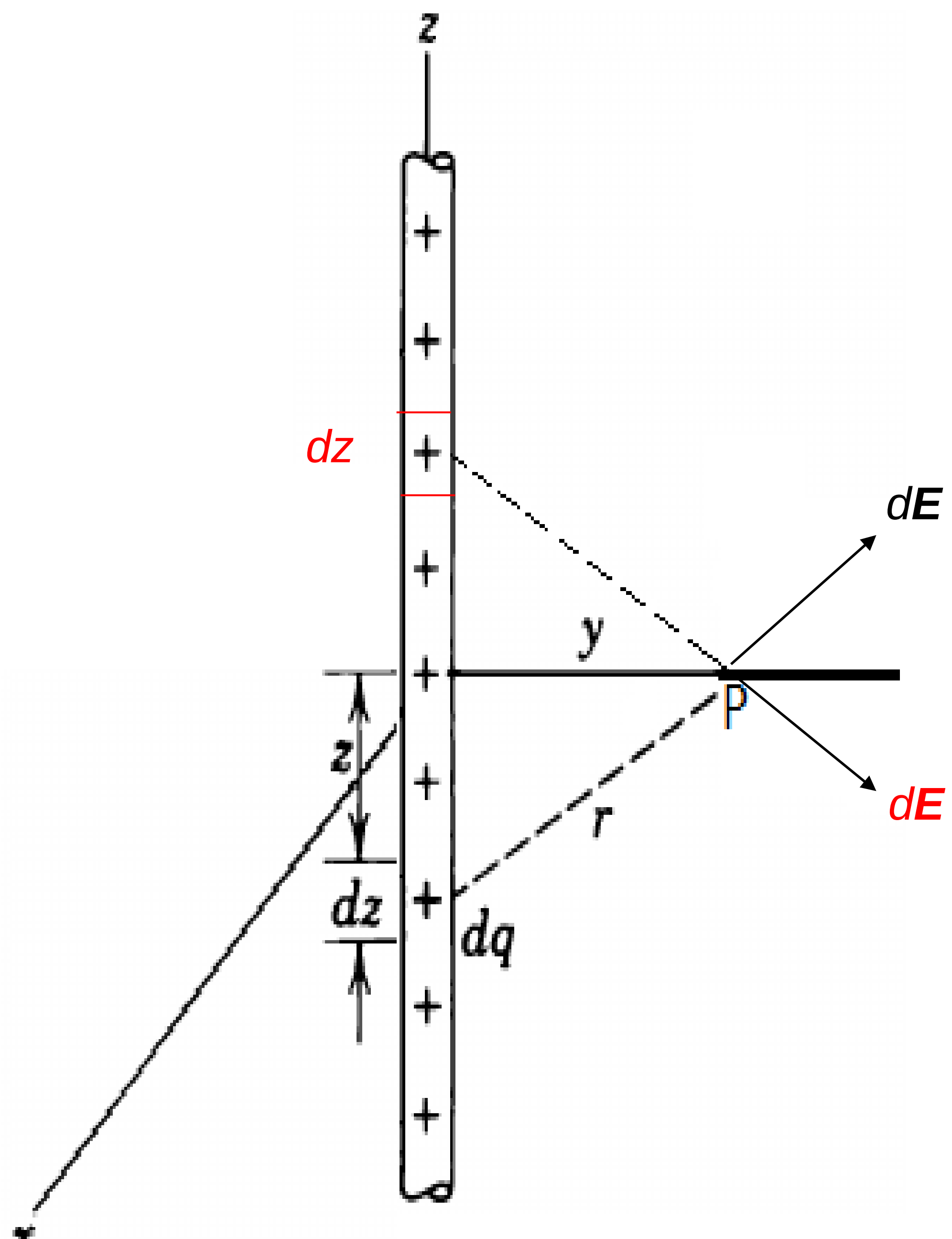

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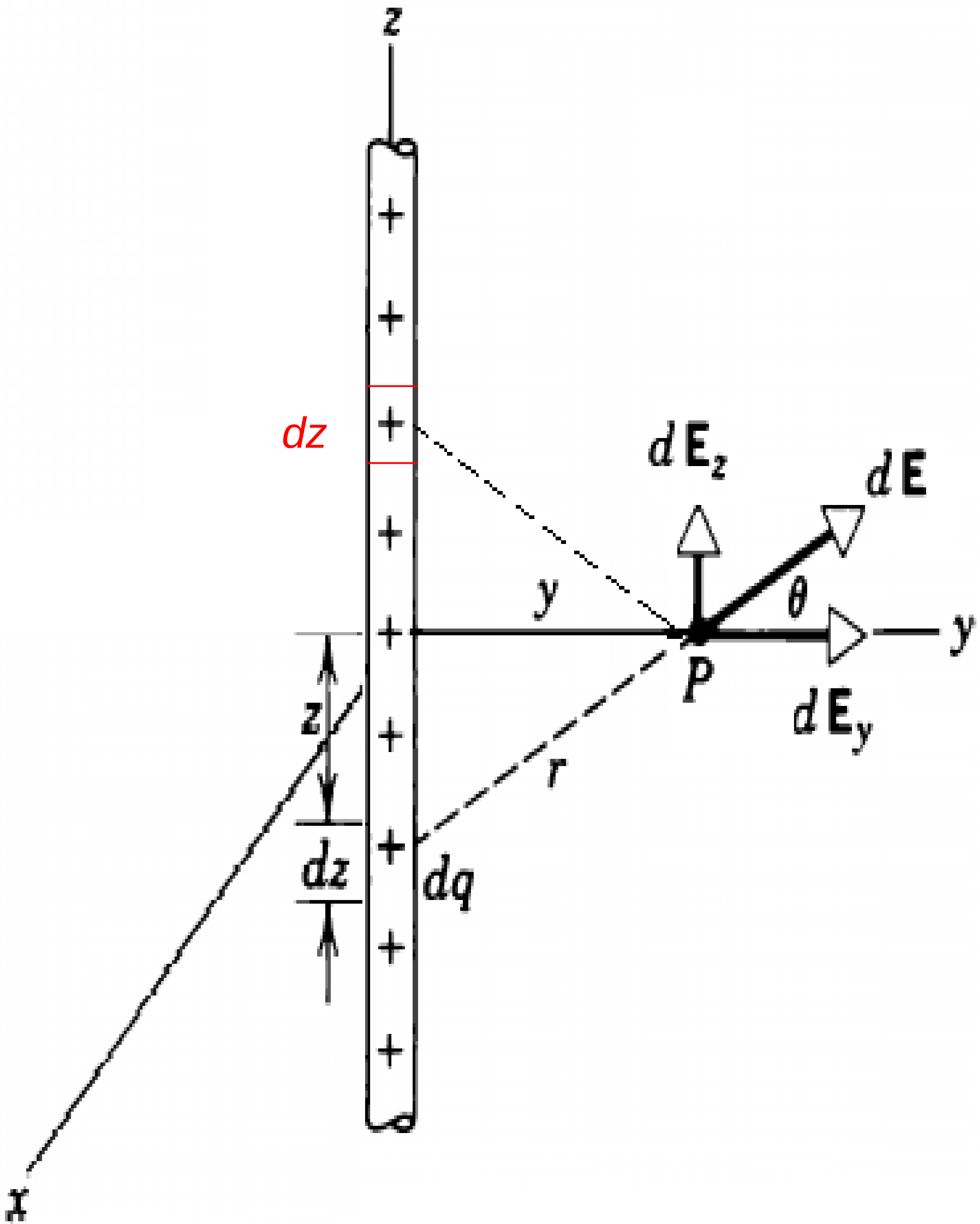
z
dz
$d\mathbf{E}_z$
$d\mathbf{E}$
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dq
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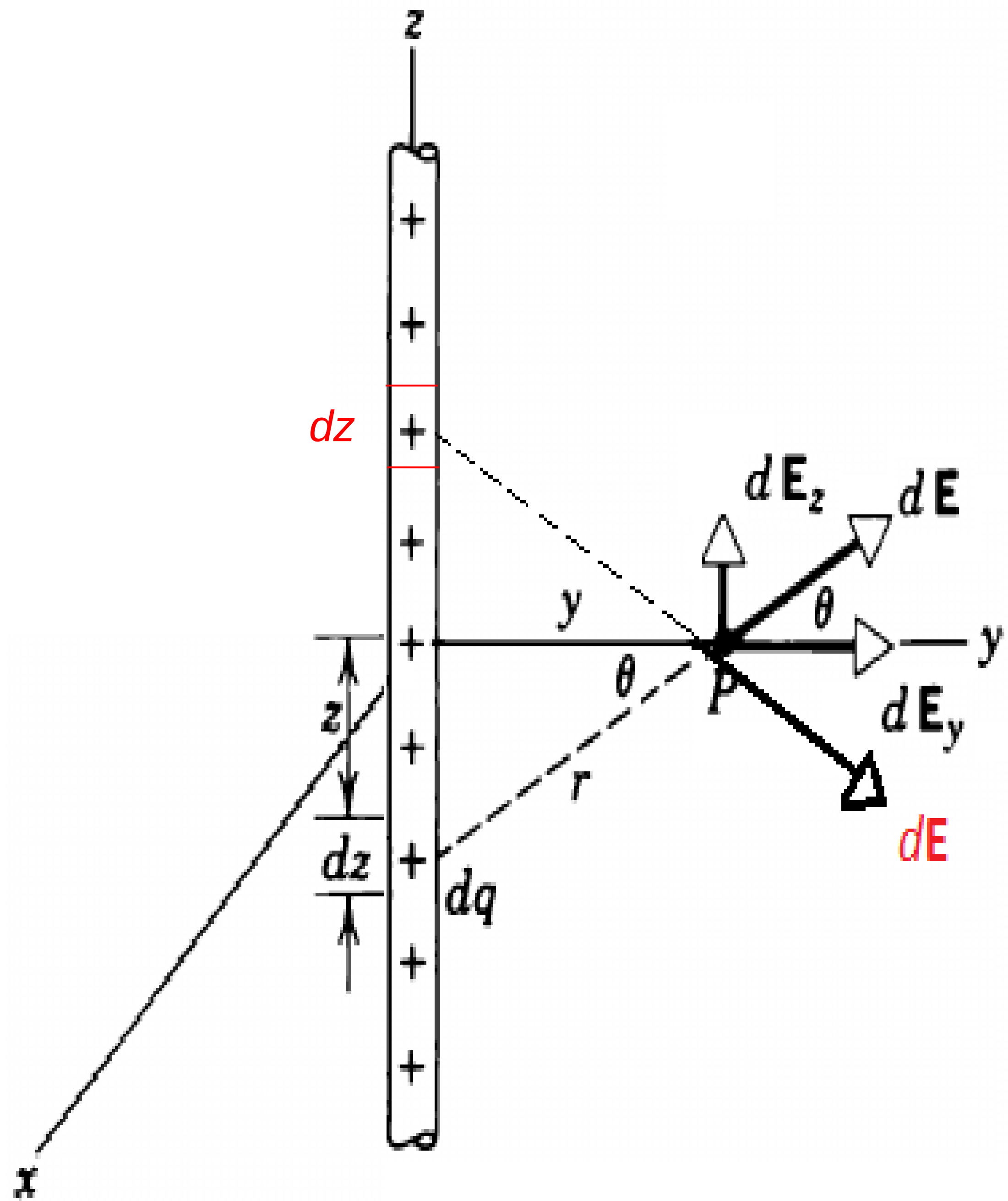
z
dz
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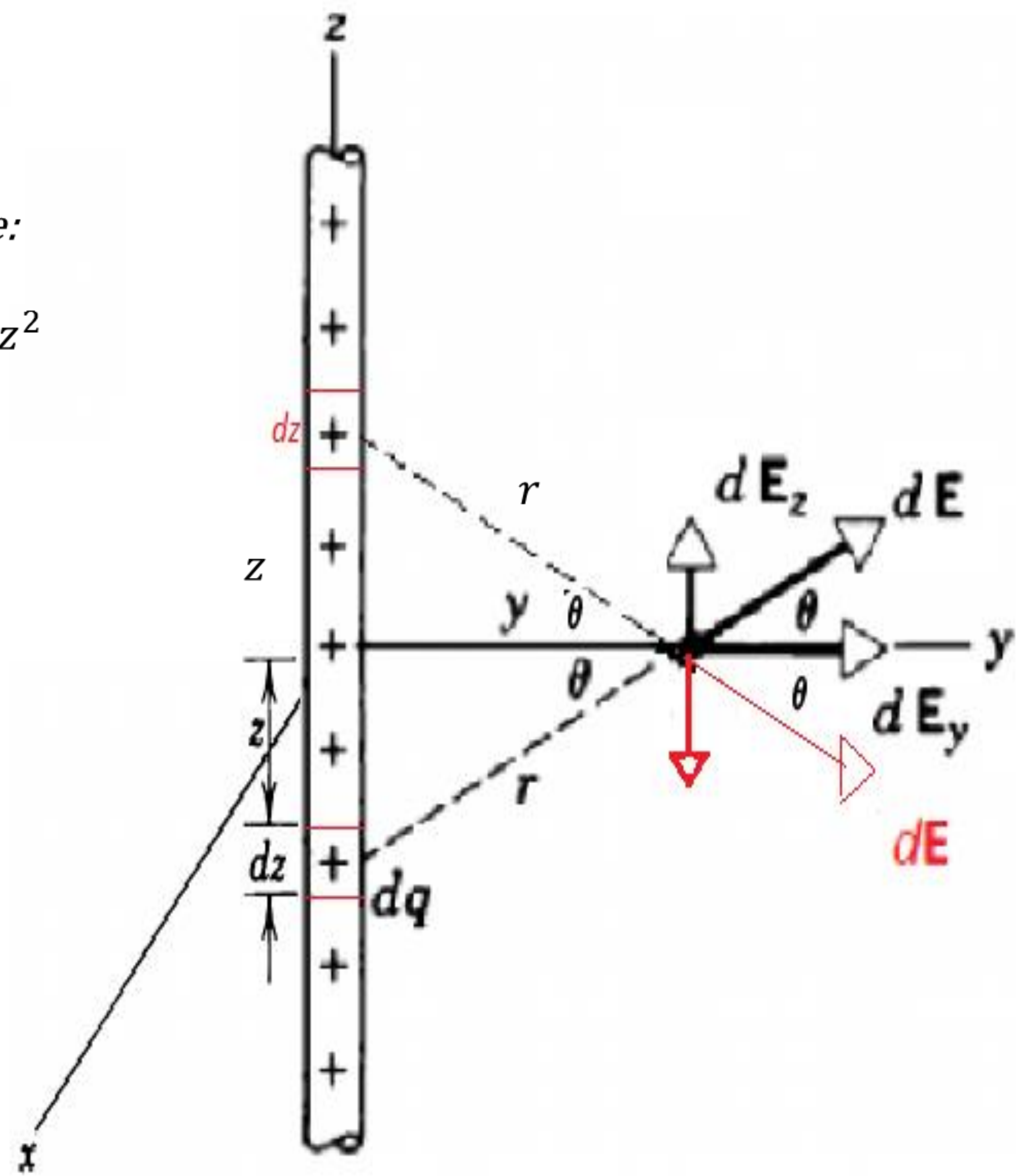
θ
P
dE_y
z
r
dE
dz
dq
x

From Figure:

$r^2 = y^2 + z^2$

$\cos\theta = y / r$



Consider a section of an infinite line of charge whose linear charge density has the constant value λ. Let us find the field **E** at a distance y from the line.

The magnitude of the field contribution ***dE*** due to charge element *dq* will be:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The vector ***dE*** has y components and z components. Now we can use symmetry argument to simplify the problem. Here Ey component will add up but *E*z must be zero because for every charge element at positive z, there is a corresponding element at negative *z,* such that z components of their field cancel each other at point P. Thus **E** entirely points at y direction.

$$dE_y = dE\cos\theta$$

$$E_y = \int dE_y$$

We are left with the y component only. Because the contribution of E_y from the top and bottom halves of the rods are equal, we can write:

$$E = E_y = 2\int_0^{\infty} \cos\theta \; dE$$

Putting values of $\cos\theta$ and dE,

$$E = 2\int_0^{\infty} \frac{y}{r}\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

By Pythagoras theorem, from figure $y^2 + z^2 = r^2$

And linear charge density relation: $\lambda = \frac{dq}{dz}$

$$E = 2\int_0^{\infty} \frac{y\lambda}{4\pi\varepsilon_0}\frac{dz}{(y^2+z^2)^{3/2}}$$

$$E = \frac{y\lambda}{2\pi\varepsilon_0}\int_0^{\infty}\frac{dz}{(y^2+z^2)^{3/2}} \text{.................ii}$$

Putting trigonometric function over here, from fig:

$$\tan\theta = \frac{z}{y}$$

$$z = y\tan\theta$$

Differentiating on both sides

$$dz = y\ \sec^2\theta\, d\theta$$

Putting values of z and dz in eq ii, we get:

$$E = \frac{y\lambda}{2\pi\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{y\ sec^2\theta d\theta}{(y^2 + y^2\ \tan^2\theta)^{\frac{3}{2}}}$$

$$= \frac{y\lambda}{2\pi\varepsilon_0}\int_0^{\pi/2} \frac{y\ sec^2\theta d\theta}{(y^2\ )^{3/2}(1+\tan^2\theta)^{3/2}}$$

$$= \frac{y\lambda}{2\pi\varepsilon_0}\int_0^{\pi/2} \frac{y\ sec^2\theta d\theta}{y^3(sec^2\theta)^{3/2}}$$

$$= \frac{y^2\lambda}{y^3 2\pi\varepsilon_0}\int_0^{\pi/2} \frac{sec^2\theta d\theta}{\sec^3\theta}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\int_0^{\pi/2} \cos\theta\ d\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}$$

Uniform Ring of Charges

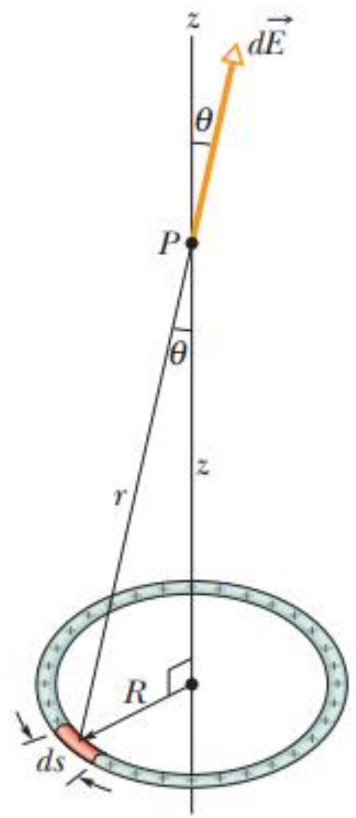
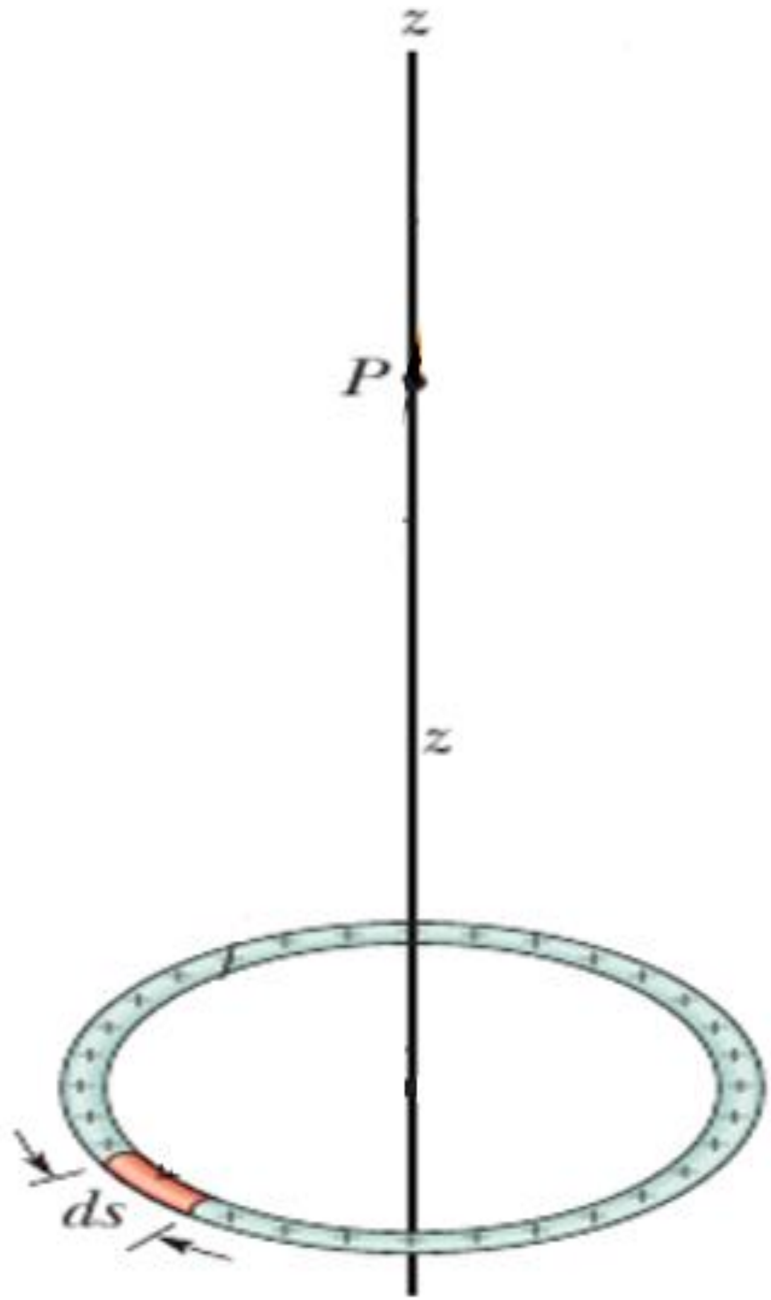
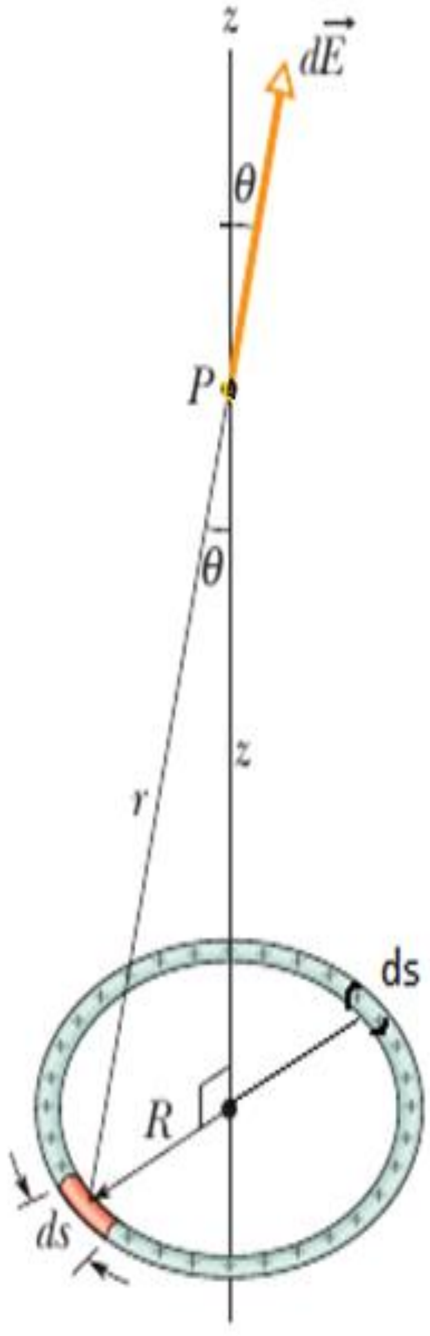
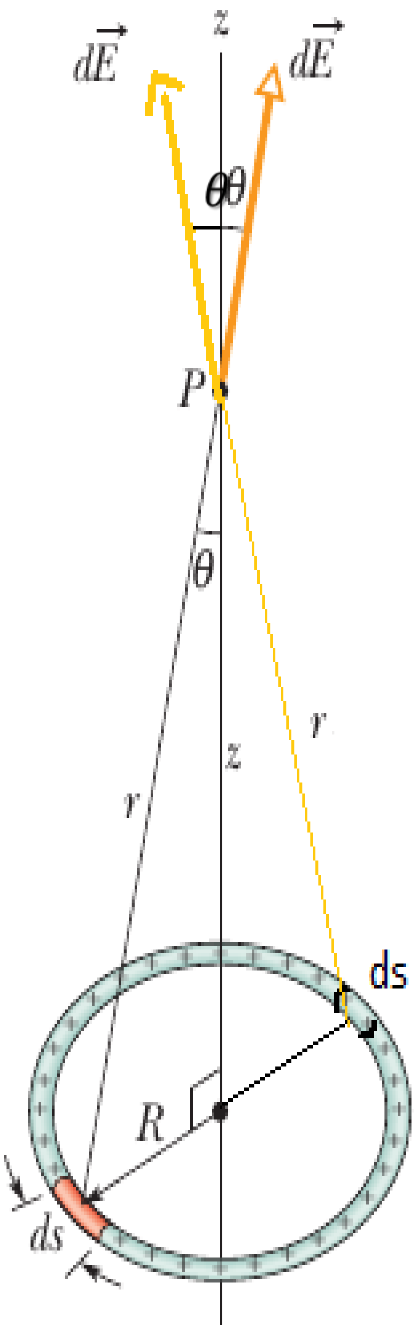


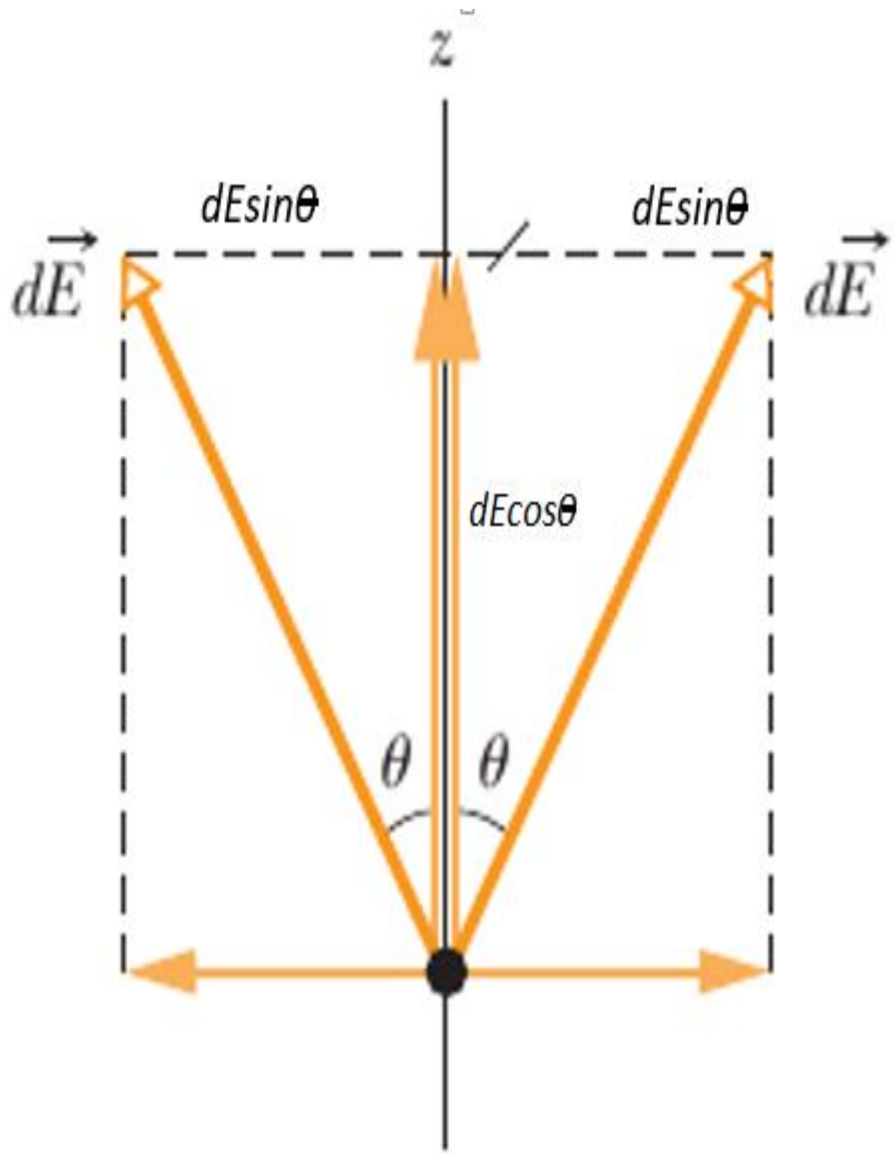
Figure A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point *P*.


z
P
z
ds


z
$d\vec{E}$
θ
P
θ
z
r
ds
R
ds



z
$d\vec{E}$
$d\vec{E}$
$\theta\theta$
P
θ
r
z
r
ds
R
ds


z
dEsinθ
dEsinθ
$\vec{dE}$
$\vec{dE}$
dEcosθ
θ
θ

Consider a differential element of the ring of length ds located at an arbitrary position on the ring in figure. It contains an element of charge given by Eq

$$dq = \lambda \, ds$$

This element sets up a differential field *dE* at point P.

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, ds}{r^2} = \frac{\lambda\, ds}{4\pi\epsilon_0(z^2 + R^2)}$$

…………………(i

Consider the charge element on the opposite side of the ring. It too contributes the field magnitude $d\boldsymbol{E}$ but the field vector leans at angle θ in the opposite direction from the vector from our first charge element, as indicated in the side view of Figure.

Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its symmetric partner on the opposite side of the ring.

To find the resultant field at P we must add up, vectorially, all the field contributions ***dE*** made by the differential elements of the ring.

All the remaining components are in the positive direction of the z axis, so we can just add them up as scalars. Thus we can already tell the direction of the net electric field at P, directly away from the ring.

The vector addition becomes a scalar addition of components parallel to the axis. The z component of d**E** is dEcosθ. From Figure we see that:

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}} \qquad \text{.....................(ii}$$

If we multiply eq(i) and eq(ii) gives us the parallel field component from each charge element:

$$dE_z = dE \cos\theta = \frac{z\lambda\, ds}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}}$$

To add the various contributions, we need add only the lengths of the elements, because all other quantities in above equation have the same value for all charge elements. Thus:

$$E_z = \int dE \cos\theta = \frac{z\lambda}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \int ds$$
$$= \frac{z\lambda(2\pi R)}{4\pi\epsilon_0(z^2 + R^2)^{3/2}},$$

In which the integral is simply 2πR, the circumference of the ring. But $\lambda(2\pi R)$ is q, the total charge on the ring, so that we can write above eq as:

$$E_z = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}$$

This expression is for charged ring.

Review & Summary

Electric Field To explain the electrostatic force between two charges, we assume that each charge sets up an electric field in the space around it. The force acting on each charge is then due to the electric field set up at its location by the other charge.

Definition of Electric Field The *electric field* $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

Electric Field Lines *Electric field lines* provide a means for visualizing the direction and magnitude of electric fields. The electric field vector at any point is tangent to a field line through that point. The density of field lines in any region is proportional to the magnitude of the electric field in that region. Field lines originate on positive charges and terminate on negative charges.

Field Due to a Point Charge The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

Field Due to a Continuous Charge Distribution The electric field due to a *continuous charge distribution* is found by treating charge elements as point charges and then summing, via integration, the electric field vectors produced by all the charge elements to find the net vector.

A Point Charge in an Electric Field

In the preceding four modules we worked at the first of our two tasks: given a charge distribution, to find the electric field it produces in the surrounding space. Here we begin the second task: to determine what happens to a charged particle when it is in an electric field set up by other stationary or slowly moving charges.

What happens is that an electrostatic force acts on the particle, as given by

$$\vec{F} = q\vec{E}, \tag{22-28}$$

in which q is the charge of the particle (including its sign) and $\vec{E}$ is the electric field that other charges have produced at the location of the particle. (The field is *not* the field set up by the particle itself; to distinguish the two fields, the field acting on the particle in Eq. 22-28 is often called the *external field.* A charged particle or object is not affected by its own electric field.) Equation 22-28 tells us

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

Dipole in an Electric Field When an electric dipole of dipole moment $\vec{p}$ is placed in an electric field $\vec{E}$, the field exerts a torque $\vec{\tau}$ on the dipole:

$$\vec{\tau} = \vec{p} \times \vec{E}. \tag{22-34}$$

The dipole has a potential energy U associated with its orientation in the field:

$$U = -\vec{p} \cdot \vec{E}. \tag{22-38}$$

This potential energy is defined to be zero when $\vec{p}$ is perpendicular to $\vec{E}$; it is least ($U = -pE$) when $\vec{p}$ is aligned with $\vec{E}$ and greatest ($U = pE$) when $\vec{p}$ is directed opposite $\vec{E}$.

Quiz # 1

First two chapters

Wednesday 3rd February 2021.