Statistical Inference: A Clear Guide

For Dr. Faisal Bukhari's Probability and Statistics Course (Lecture 21)
English + Urdu for Exam Prep

1 Topic 1: Population vs Sample

1.1 English Explanation

- **Population**: All the data you are concerned with (e.g., all students in PU).
- **Sample**: A part of the population (e.g., 30 students from PU).
- In real life, we **can't test everyone**, so we use samples to **estimate** population characteristics.

1.2 Urdu Explanation

- **Population**: Wo **poora group** jis par research ho rahi ho (jaise PU ke sab students).
- **Sample**: Us group ka **chhota hissa** (jaise 30 random students).
- Har kisi se data lena mushkil hota hai, isliye **sample** se estimate karte hain.

2 Topic 2: Statistic vs Parameter

lightgray **Term**	**Meaning (English)**	**Urdu**
Parameter	Value from population (e.g., μ, σ)	Asal value jo poori population ka hissa ho
Statistic	Value from sample (e.g., $\bar{x}$, s)	Sample se nikaala gaya number

Real Use: You don't know the population mean μ. You take a sample, calculate $\bar{x}$, and use it to estimate μ.

3 Topic 3: Point Estimate vs Interval Estimate

3.1 Point Estimate

- A single value (e.g., $\bar{x}$) used to estimate a parameter (e.g., μ).
- Example: "We estimate the average height is **5.6 feet**."

3.2 Interval Estimate

- A range (e.g., $[5.4, 5.8]$) with a confidence level.
- Example: "We are **95% confident** that average height is between 5.4 and 5.8."

3.3 Urdu Explanation

- **Point Estimate**: Ek number (jaise $\bar{x} = 5.6$).
- **Interval Estimate**: Range (5.4 se 5.8 tak) jisme asal value hone ka chance ho.

4 Topic 4: Confidence Interval Formula (for μ when σ known)

$$\text{CI} = \bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- σ = population standard deviation (known)
- n = sample size
- $Z_{\alpha/2}$ = z-critical value (from z-table)

lightgray **Confidence Level**	α	**z-value**
90%	0.10	1.645
95%	0.05	1.96
99%	0.01	2.575

5 Keywords for Confidence Interval Questions

lightgray **Clue Words**	**What to Do**
"Estimate with 95% confidence"	Use CI formula
"Margin of error"	Use $E = Z \cdot \frac{\sigma}{\sqrt{n}}$
"Interval in which parameter lies"	Use CI
"Find z for 90%, 95%, 99%"	Use z-table
"Give upper/lower bound"	CI upper/lower

6 Exam-Style Questions (Bukhari Style)

1. A sample of 40 values has mean = 20, $\sigma = 4$. Find the 95% confidence interval.
2. If $n = 25$, $\bar{x} = 75$, $\sigma = 10$, find the margin of error at 99% confidence.
3. What z-value corresponds to 90% confidence?
4. Interpret: "We are 95% confident that μ lies between 58 and 62." Is this interval likely to include μ?
5. Population mean weight is unknown. You sample 100 people, mean weight is 70 kg, $\sigma = 5$. Find the CI.

6.1 Answers

1. $\text{CI} = 20 \pm 1.96 \cdot \frac{4}{\sqrt{40}} = 20 \pm 1.24 \Rightarrow [18.76, 21.24]$
2. $E = 2.575 \cdot \frac{10}{\sqrt{25}} = 2.575 \cdot 2 = 5.15$
3. $Z = 1.645$
4. Yes, the interval is designed to capture μ with 95% certainty.

5. CI $= 70 \pm 1.96 \cdot \frac{5}{\sqrt{100}} = 70 \pm 0.98 \Rightarrow [69.02, 70.98]$

7 Fast Cheat Sheet (Revision Aid)

lightgray **Concept**	**Formula / Shortcut**
Margin of Error (E)	$Z \cdot \frac{\sigma}{\sqrt{n}}$
CI (when σ known)	$\bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}$
Z for 95%	1.96
Z for 90%	1.645
Z for 99%	2.575

Ready for Hypothesis Testing or more Confidence Interval practice? Ask your instructor!