1. Arithmetic Sequence Size

(There are how many terms in this sequence?)

1.1. $\Omega(N) = N/2 \leq \lvert \{1, 2, 3, 4, 5, 6, \ldots, N\} \rvert \leq N = O(N)$ # of terms are $\Theta(N)$	for(int i=1; i<=N; i++);
1.2. $\Omega(N) = N/4 <= \lvert \{0, 2, 4, 6, 8, 10, \ldots, N\} \rvert \leq N/2 = O(N)$ # of terms are $\Theta(N)$	for(int i=0; i<=N; i+=2);
1.3. $\Omega(N) = N/4 <= \lvert \{1, 3, 5, 7, \ldots, N\} \rvert \leq N/2$ i.e $O(N)$ # of terms are $\Theta(N)$	for(int i=1; i<=N; i+=2);
1.4. $\Omega(N) = N/6 <= \lvert \{1, 4, 7, 10, \ldots, N\} \rvert \leq N/3$ i.e. $O(N)$ # of terms are $\Theta(N)$	for(int i=1; i<=N; i+=3);
1.5. $\Omega(N) = \lvert \{1, 1+k, 1+2k, 1+3k, 1+4k, 1+5k, \ldots, N\} \rvert \leq N/k$ i.e. $O(N)$ if k is a constant	for(int i=1; i<=N; i+=k);
1.6. $\Omega(N/\log N) = \lvert \{1, 1+\log N, 1+2\log N, 1+3\log N, 1+4\log N, 1+5\log N, \ldots, N\} \rvert \leq N/\log N$ i.e. $O(N/\log N)$	K = log N; for(int i=1; i<=N; i+=k);
1.7. $\Omega(\sqrt{N}) = \lvert \{1, 1+\sqrt{N}, 1+2\sqrt{N}, 1+3\sqrt{N}, 1+4\sqrt{N}, 1+5\sqrt{N}, \ldots, N\} \rvert \leq N/\sqrt{N}$ $= O(\sqrt{N})$ i.e. $O(\sqrt{N})$	K = $\sqrt{N}$; for(int i=1; i<=N; i+=k);
(int i=1; i<=N; i+=10); N/10 times Similarly for(int i=1; i<=N; i+=20); N/20 times for(int i=1; i<=N; i+= $\sqrt{N}$); $N/\sqrt{N} = \sqrt{N}$ ====> $N = \sqrt{N}.\sqrt{N}$	

2. Arithmetic Series and relatives

Applications of 1+2+3+4+...+N = $\frac{N(N+1)}{2}$ If you don't remember this formula.

Proof

Upper Bound 1+2+3+4+5+6+ ....(N-3)+ (N-2)+ (N-1)+ N/2 <= N + N + N + ... + N replacing every value by the highest term
<= N x N = N^2 = O(N^2)

Lower Bound ~~1+2+3+4+5+6+...+ N/2~~+ (N/2+1)+(N/2+2) ....(N-3)+ (N-2)+ (N-1)+ N ignoring the first half
>= (N/2+1)+(N/2+2) + ... +(N-3)+ (N-2)+ (N-1)+ N and now replacing with the smallest value
>= N/2 + N/2 + N/2 + N/2 + ... + N/2 + N/2 + N/2 = N/2 x N/2 = ¼ N^2 = $\Omega(N^2)$
f(N) = Θ(N^2)

2.1 $\Omega(T2) \leq 1+2+3+4+5+6+ \ldots T-3+ T-2+ T-1+ T \leq O(T^2)$ ====> $\Theta(T2)$	
2.2 $\Omega(N^2) \leq 1+2+3+4+5+6+ + \ldots + N/2+N/2+1+ \ldots +N-3+ N-2+ N-1+ N \leq O(N^2)$ ===> $\Theta(N^2)$ $\Theta(N)$ (1)1+(1,2)2+(1,2,3)3+(1,2,3,4)4+......+(1,2,3,4.....,N)N $\Theta(N^2)$	for(int i=1; i<=N; i++) for(int j=1; j<=i; j++)
2.3 $\Omega(N^2) \leq 1+2+3+4+5+6+ \ldots (N/2-3)+ (N/2-2)+ (N/2-1)+ N/2 \leq O(N^2)$ ====> $\Theta(N^2)$ $\Theta(N)$ (1)1+(1,2)2+(1,2,3)3+(1,2,3,4)4+......+(1,2,3,4.....,N)N $\Theta(N^2)$	for(int i=1; i<=N; i+=2) for(int j=1; j<=i; j++)
2.4 $\Omega(N^2) \leq 1+2+3+4+5+6+ \ldots (N/3-3)+ (N/3-2)+ (N/3-1)+ N/3 \leq O(N^2)$ ====> $\Theta(N^2)$ $\Theta(N)$ (1)1+(1,2)2+(1,2,3)3+(1,2,3,4)4+......+(1,2,3,4.....,N/3)N/3 $\Theta(N^2)$	for(int i=1; i<=N; i+=3) for(int j=1; j<=i; j++)
2.5 $\Omega(N) \leq 1+2+3+4+5+6+ \ldots +\sqrt{N}$ <= $O((\sqrt{N})^2)$ $\leq O(N)$	

====> Θ(N) Θ($N^{1/2}$) (1)1+(1,2)2+$_{(1,2,3)}$3+(1,2,3,4)4+......+(1,2,3,4....,$N^{1/2}$)$N^{1/2}$ Θ(N^2)	for(int i=1; i<=$N^{1/2}$; i+=1) for(int j=1; j<=i; j++)

2.6 $\Omega((log\ N)^2) \leq$ 1+2+3+4+5+6+ .... +$log\ N$ <= $O((log\ N)^2) \leq \Theta(\log^2 N)$
(1)1+(1,2)2+$_{(1,2,3)}$3+(1,2,3,4)4+......+(1,2,4,8...., N)log N = Θ($\log^2$ N) ===>
(1)1+(1,2)2+$_{(1,2,3)}$3+(1,2,3,4)4+......+(1,2,4,8...., N)log N
	for(int i=1; i<=N; i*=2) Θ(log N)
	for(int j=1; j<=i; j*=2)
	<u>Example 2</u>
	for(int i=1; i<=log N; i++)
	for(j=1; j<=i; j++) ; ___

2.7 $\Omega(N^4) \leq$ 1+2+3+4+5+6+ .... + N^2 $\leq O(N^4)$ ====> $\Theta(N^4)$
(1)1+(1,2)2+$_{(1,2,3)}$3+(1,2,3,4)4+......+(1,2,3,4....,N^2)N^2 = Θ(N^4) ===>
	for(int i=1; i<=N*N; i=1)
	Θ(N^2)
	for(int j=1; j<=i; j++)

2.8 $\Omega(N^6) \leq$ 1+2+3+4+5+6+ .... + $N^3 \leq O(N^6)$ Θ(N^3) (1)1+(1,2)2+$_{(1,2,3)}$3+(1,2,3,4)4+......+(1,2,3,4....,N^3)N^3 Θ(N^6)	for(int i=1; i<=N*N*N; i=1) for(int j=1; j<=i; j++)
2.9 $\Omega(N^{2k}) \leq$ 1+2+3+4+5+6+ .... +N^k <= $O(N^k x N^k)$	
2.10 $\Omega(N^3) \leq 1^2+2^2+3^2+4^2+5^2+6^2+ \ldots + N^2 \leq O(N^3)$ (1)1+(1,2,3,4)2+$_{(1,2,3,\ldots,9)}$9+(1,2,3,4...,16)16+......+(1,2,3,4....,N^2)N^2 = Θ(N^3)	for(int i=1; i<=N; i=1) Θ(N) for(int j=1; j<=i*i; j++)
2.11 $\Omega(N^4) \leq 1+2^3+3^3+4^3+5^3+6^3+ \ldots + N^3 \leq O(N^4)$ Θ(N) (1)1+(1,2,3,...,8)8+$_{(1,2,3,\ldots,27)}$27+(1,2,3,4...,64)64+......+(1,2,3,4....,N^3)N^3 Θ(N^4)	for(int i=1; i<=N; i=1) for(int j=1; j<=i*i*i; j++)
2.12 $\Omega(N^{k+1}) \leq 1^k+2^k+3^k+4^k+5^k+6^k+ \ldots +N^k$ <= $O(N^{k+1})$	

3. Some Examples

$\sqrt{N} * \sqrt{N} = N$ for(int i=1; i*i<=N; i++) Sum++; O($\sqrt{N}$) for(int i=1; i*i<=N*N; i++) Sum++; O(N)
for(int i=1; i*i*i<=N*N; i++) Sum++; O($N^{2/3}$) for(int i=1; i*i*i<=N; i++) Sum++; O($N^{1/3}$)