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Exercise 4.4

Question 3

$x_1 + 5x_2 = 7$ ①
$x_1 - 2x_2 - 2$ ⑪ $(x_1, x_2) = ?$

Subtracting eq ⑪ from eq ①

$$x_1 + 5x_2 = 7$$
$$x_1 - 2x_2 = -2$$
$$- \quad + \quad +$$

$$7x_2 = 9$$
$$x_2 = 9/7$$

putting value $x_2 = 9/7$ in eq ⑪

$$x_1 - 2(9/7) = -2$$
$$x_1 = -2 + 18/7$$
$$x_1 = 4/7$$

$$(x_1, x_2) = (4/7, 9/7)$$

Question 6

$$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 1 & 6 \end{bmatrix} \sim \quad r_4 \leftarrow -3r_3 + r_4$$

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Multiply row 3 by -3 and add it on row 4

$$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -5 & 15 \end{bmatrix} \sim \quad R_4 \leftarrow \tfrac{1}{5} R_4$$

Multiply row 4 by 1/5

$$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

Question 18

$$\begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \begin{bmatrix} 1 & -2 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix} \end{array}$$

$$x_1 - 2x_2 + 3x_4 = -2 \quad (I)$$
$$x_2 - 4x_4 = 7 \quad (II)$$
$$x_3 = 6 \quad (III)$$
$$x_4 = -3 \quad (IV)$$

Put $x_4 = -3$ in eq (II)

$$x_2 - 4(-3) = 7$$
$$x_2 + 12 = 7$$
$$x_2 = -5$$

put $x_2 = -5$ and $x_4 = -3$ in eq (I)

$$x_1 - 2(-5) + 3(-3) = -2$$
$$x_1 + 10 - 9 = -2$$

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$x_1 = -2 \div 1$

$x_1 = -3$

$x_1 = 17 \quad x_2 = 5 \quad x_3 = 6 \quad x_4 = -8$

Question 12

$x_1 - 3x_2 + 4x_3 = -4$

$3x_1 - 7x_2 + 7x_3 = -8$

$-4x_1 + 6x_2 - x_3 = 7$

$$\left[\begin{array}{cccc} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{array}\right] \quad \begin{array}{l} \\ r_2 \leftarrow -3r_1 + r_2 \\ r_3 \leftarrow 4r_1 + r_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 15 & -9 \end{array}\right] \quad r_3 \leftarrow 3r_2 + r_3$$

$$\left[\begin{array}{cccc} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & 0 & 0 & -3 \end{array}\right] \leftarrow$$

the last row indicates $0 = -3$ this system is inconsistent and has no Solution Set

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$-3r_2 + r_4 = r_4$

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Question 16

$$x_1 \qquad -2x_4 = -3$$
$$2x_2 + 2x_3 \qquad = 0$$
$$x_3 + 3x_4 = 1$$
$$-2x_1 + 3x_2 + 2x_3 + x_4 = 5$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \quad r_4 \leftarrow 2r_1 + r_4$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix} \quad r_2 \leftarrow r_2/2 \quad r_4 \leftarrow 3r_2 + r_4$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{bmatrix} \quad r_4 \leftarrow r_3 + r_4$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system is consistent and due to x_4 being free variable has infinite number of solution

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Question 19

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \sim$$

$r_2 \leftarrow -3r_1 + r_2$

$$\begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$$

For system to be consistent

$$6 - 3h \neq 0$$

and $6 - 3h = 0$ when

$$h = 2$$

So for the system to be consistent

$$h \in R \mid h \neq 2$$

Question 25

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix} \sim$$

$r_3 \leftarrow 2r_1 + r_3$

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{bmatrix} \sim$$

$r_3 \leftarrow r_2 + r_3$

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{bmatrix}$$

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For the system to be consistent

$$K_4 2gth = 0$$

Question no 33

$$4T_1 - T_2 - T_4 = 30$$
$$-T_1 + 4T_2 - T_3 = 60$$
$$-T_2 + 4T_3 - T_4 = 70$$
$$-T_1 - T_3 + 4T_4 = 40$$

These are system of linear equations.

Question no 34

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \sim r_1 \leftrightarrow r_4$$

$$\begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \sim \begin{matrix} r_2 \leftarrow r_1 + r_2 \\ r_4 \leftarrow 4r_1 + r_4 \end{matrix}$$

$$\begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & 4 & 100 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix} \sim \begin{matrix} r_1 \leftarrow -1 \times r_1 \\ r_2 \leftarrow 1/4\, r_2 \\ r_3 \leftarrow r_2 + r_3 \\ r_4 \leftarrow r_2 + r_4 \end{matrix}$$

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$$\begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 4 & 0 & 95 \\ 0 & 0 & -4 & 16 & 240 \end{bmatrix} \quad r_4 \leftarrow r_3 + r_4$$

$$\begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 4 & 0 & 95 \\ 0 & 0 & 0 & 16 & 385 \end{bmatrix} \quad \begin{matrix} r_4 \leftarrow r_4/16 \\ r_2 \leftarrow -r_4 + r_2 \\ r_1 = 4r_4 + r_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 900/16 \\ 0 & 1 & 0 & 0 & 25 - 385/16 \\ 0 & 0 & 4 & 0 & 95 \\ 0 & 0 & 0 & 1 & 385/16 \end{bmatrix} \quad r_3 \leftarrow r_3/4$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 225/4 \\ 0 & 1 & 0 & 0 & 15/16 \\ 0 & 0 & 1 & 0 & 95/4 \\ 0 & 0 & 0 & 1 & 385/16 \end{bmatrix} \quad r_1 \leftarrow -r_3 + r_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 130/4 \\ 0 & 1 & 0 & 0 & 15/16 \\ 0 & 0 & 1 & 0 & 95/4 \\ 0 & 0 & 0 & 1 & 385/16 \end{bmatrix}$$

S.S $\begin{cases} x_1 = 130/4 \\ x_2 = 15/16 \\ x_3 = 95/4 \\ x_4 = 385/16 \end{cases}$

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Exercise 1.2.

Question 4

Pivot position

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \sim \begin{matrix} r_2 \leftarrow -3r_1 + r_2 \\ r_3 \leftarrow -5r_1 + r_3 \end{matrix}$$

Pivot column

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix} \sim r_3 \leftarrow 2r_2 + r_3$$

pivot

Pivot column

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & -10 \end{bmatrix} \sim \begin{matrix} r_2 \leftarrow -\frac{1}{4} r_2 \\ r_3 \leftarrow r_3/10 \end{matrix}$$

Pivot column

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \sim \begin{matrix} r_2 \leftarrow 3r_3 + r_2 \\ r_1 \leftarrow 7r_3 + r_1 \end{matrix}$$

Pivot position

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \sim r_1 \leftarrow -3r_2 + r_1$$

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$$\begin{bmatrix} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{1} & 2 & 0 \\ 0 & 0 & 0 & \boxed{-4} \end{bmatrix}$$ 3 pivots

Question no 6

3 × 2 non zero matrix's possible echelon form

■ used for pivots * used for any non-zero value

i) $\begin{bmatrix} ■ & * \\ 0 & ■ \\ 0 & 0 \end{bmatrix}$ 2 pivots

ii) $\begin{bmatrix} ■ & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ 1 pivot

Question no 11

$$\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix} \sim \begin{matrix} r_2 \leftarrow 3r_1 + r_2 \\ r_3 \leftarrow 2r_1 + r_3 \end{matrix}$$

$$\begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim r_1 \leftarrow \frac{1}{3} r_1$$

$$\begin{bmatrix} 1 & -4/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 4/3 x_2 + 2/3 x_3 = 0$

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S.S $\begin{cases} x_1 = +4/3\,x_2 - 2/3\,x_3 \\ x_2 \text{ is free variable} \\ x_3 = \text{free variable.} \end{cases}$

Question no 15

a) $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & 0 \end{bmatrix}$

System is consistent with a unique solution.

b) $\begin{bmatrix} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 & \blacksquare \end{bmatrix}$

System is inconsistent

Question no 20

$x_1 + 3x_2 = 2$

$3x_2 + hx_2 = k$

$\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix} \sim$ $r_2 \leftarrow -3r_1 + r_2$

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -9+h & -6+k \end{bmatrix}$

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i) No solution

$h = 9$ and $K \neq 6$

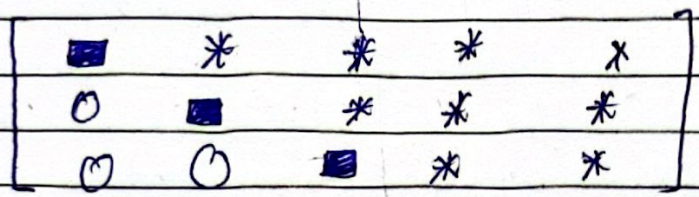
ii) One Solution

$h \neq 9$ and $K \in R$

iii) Many Solutions

$h = 9$ and $K = 6$

Question no 23



↘ example Matrix

System will be consistent as there are only 3 rows and there are 3 pivot columns so each row contains at least one non zero element (augmented column excluded).

Question no 33

$a_0 + a_1(1) + a_2(1)^2 = 12$

$a_0 + a_1(2) + a_2(2)^2 = 15$

$a_0 + a_1(3) + a_2(3)^2 = 16$

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$$\begin{bmatrix} -1 & 1 & 1 & 12 \\ 1 & 2 & 4 & 15 \\ 1 & 3 & 9 & 16 \end{bmatrix} \sim \quad r_2 \leftarrow -r_1 + r_2, \quad r_3 \leftarrow -r_1 + r_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & 8 & 4 \end{bmatrix} \sim \quad r_3 \leftarrow -2r_2 + r_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 2 & -2 \end{bmatrix} \sim \quad r_3 \leftarrow \frac{1}{2} r_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \quad r_2 \leftarrow -3r_3 + r_2, \quad r_1 \leftarrow -r_3 + r_1$$

$$\begin{bmatrix} 1 & 1 & 0 & 13 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \quad r_1 \leftarrow -r_2 + r_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$a_0 = 7$
$a_1 = 6$
$a_2 = -1$

Interpolating Polynomial :-

$$P(t) = a_0 + a_1 t + a_2 t^2$$
$$= 7 + 6t - t^2$$