Concept of Tree Diagram Representation of Sample Space

A **tree diagram** is a graphical tool used in probability theory to represent all possible outcomes in a sample space. It helps visualize the process of choosing from several different possibilities, especially when there are multiple stages or events involved. Each branch in the tree diagram represents a different possible outcome, and the paths lead to the final outcomes of an event.

Here's a detailed breakdown of the **concept of tree diagram representation of a sample space**:

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1. What is a Sample Space?

The **sample space (S)** is the set of all possible outcomes of an experiment or random trial. For example, when flipping a coin, the sample space is **{Heads, Tails}**.

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2. Structure of a Tree Diagram

A tree diagram visually represents the sample space as a series of branches. Each **node** (or point) in the diagram represents an intermediate outcome, and each **branch** represents a possible transition from one outcome to another. The **leaves** (the endpoints of the tree) represent the final outcomes.

Example: Flipping a Coin Twice

Let's say we perform an experiment where we flip a coin twice. The sample space of this experiment is:

- **{HH, HT, TH, TT}**

Tree Diagram Steps:

- **Step 1**: Start with the initial state (root node). In this case, the root represents the start of the first coin flip.
- **Step 2**: From the root, draw two branches representing the two possible outcomes of the first coin flip (Heads or Tails).
- **Step 3**: From each of these two branches, draw two more branches representing the possible outcomes of the second coin flip (Heads or Tails).
- **Step 4**: The leaves represent all possible final outcomes.

The tree diagram looks like this:

```
      (Start)
       / \
     Heads Tails
    /  \   /  \
   Heads Tails Heads Tails
```

This tree represents the four possible outcomes of flipping a coin twice: **HH, HT, TH, TT**.

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3. General Steps for Constructing a Tree Diagram:

1. **Start with the initial event** (or root node).
2. **Branch out for each possible outcome** of the first event (e.g., Heads or Tails in the coin flip).
3. For each outcome of the first event, **branch out for each possible outcome** of the second event (and so on for more events).
4. **Label each branch** with the probability of that event.
5. **Mark the leaves** with the possible outcomes.

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4. Probability in Tree Diagrams

Each branch of the tree diagram can be assigned a probability. If an event has **equally likely outcomes**, the probability of each outcome can be represented as the fraction or decimal corresponding to the likelihood.

Example: Flipping a Fair Coin Twice

- Probability of **Heads** = ½, and probability of **Tails** = ½.
- The tree diagram for flipping a coin twice will look like:

(Start)

/ \

P(Heads) P(Tails)

1/2 1/2

/ \ / \

P(Heads) P(Tails) P(Heads) P(Tails)

1/2 1/2 1/2 1/2

- The probability of getting **HH** (Heads-then-Heads) is:
  P(Heads and Heads)=1/2×1/2=1/4P(Heads \, and \, Heads) = 1/2 \times 1/2 = 1/4.
- The probability of getting **HT** (Heads-then-Tails) is:
  P(Heads and Tails)=1/2×1/2=1/4P(Heads \, and \, Tails) = 1/2 \times 1/2 = 1/4.

And so on for the other outcomes.

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5. Multiple Events and Sample Space Representation

Tree diagrams can be used for **more complex experiments** that involve multiple events. For example, rolling two dice or drawing cards from a deck.

Example: Rolling Two Dice

When rolling two dice, the sample space consists of all possible pairs of dice rolls, like **(1, 1), (1, 2), ..., (6, 6)**. The tree diagram for this experiment would have:

- **First die**: Branches for outcomes 1, 2, 3, 4, 5, 6.
- **Second die**: Each branch from the first die would have six branches representing the outcomes of the second die.

The tree diagram for this experiment would have **6 × 6 = 36 branches** and would help visualize all possible combinations.

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6. Advantages of Using Tree Diagrams

- **Visual Clarity**: Tree diagrams offer a clear, visual representation of all possible outcomes of an event or experiment.
- **Organization**: They help organize complex events and break them down into manageable steps.
- **Calculating Probabilities**: Tree diagrams make it easy to compute the probability of combined events by simply multiplying the probabilities along the branches.

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7. Example: Drawing Two Cards from a Deck Without Replacement

Let's consider drawing two cards from a deck of 52 cards, without replacement:

Step 1: Start with the root node (before any card is drawn).

- There are 52 possible outcomes for the first card.

Step 2: After drawing the first card, we have 51 cards left, so there are 51 possible outcomes for the second card.

Tree Diagram Representation:

```
        (Start)
          |
      First Draw (52 possibilities)
       /    |     ...   \
  [Card 1] [Card 2] ...   [Card 52]
```

| | |

Second Draw (51 possibilities from each branch)

| | |

[Card 1|1] [Card 1|2] ... [Card 1|52]

...

Here, the probabilities depend on the composition of the deck and the number of remaining cards.

Conclusion:

A **tree diagram** is a powerful tool in probability and statistics that helps to **visualize** the structure of sample spaces, outcomes of experiments, and compute probabilities. It is especially useful in experiments with multiple stages or events, allowing you to break down complex problems into smaller, manageable pieces.