Chebyshev's Theorem: A Clear Guide

For Dr. Faisal Bukhari's Probability and Statistics Course (Lecture 20)
English + Urdu for Exam Prep

1 Concept: Chebyshev's Theorem

1.1 English Explanation

Chebyshev's Theorem provides a way to estimate probabilities for **any distribution**, even if the shape is unknown. It states:

> The **minimum probability** that a value lies within **k standard deviations (σ)** of the mean (μ) is:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

Key points:

- Works for **non-normal** distributions.
- Estimates the probability range when distribution is **unknown**.
- Larger **k** means higher confidence that data is near the mean.

1.2 Urdu Explanation

Chebyshev's Theorem yeh kehti hai:

> Agar kisi random variable ka distribution **pata nahi**, tab bhi hum estimate kar sakte hain ke data kitne standard deviation ke andar hoga:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

Yani:

- Distribution ka shape pata na ho (normal na ho), phir bhi yeh formula kaam karta hai.
- Bara **k** matlab zyada chance ke data mean ke qareeb hoga.

2 Real-Life Application

Example: A professor says the average GPA is 2.5 with a standard deviation of 0.5 (distribution unknown). What percentage of students have a GPA between 1.5 and 3.5?

Solution:

1. Mean $\mu = 2.5$, Standard Deviation $\sigma = 0.5$
2. Range: 1.5 to 3.5
3. $k = \frac{3.5-2.5}{0.5} = 2$

4. Chebyshev's Theorem: $P(\mu - 2\sigma < X < \mu + 2\sigma) \geq 1 - \frac{1}{4} = \frac{3}{4} = 75\%$

Conclusion: At least **75%** of students have a GPA between 1.5 and 3.5.

3 Keywords to Recognize Chebyshev Questions

lightgray **Keywords in Question**	**Use Chebyshev's Theorem When...**
"At least what % lies between..."	Distribution unknown
"How much data is within __σ of mean?"	Given μ and σ, range provided
"No info about shape/distribution"	Always use Chebyshev
"P($\|X-\mu\| \geq k\sigma$)" or "outside the range"	Flip to $1 - \frac{1}{k^2}$ form
"Estimate lower bound for probability"	Distribution-free formula needed

4 Exam-Style Mini Test (Bukhari Style)

1. A variable has $\mu = 10$ and $\sigma = 2$. What is the **minimum probability** that a value lies between 6 and 14?

2. A value is more than 4 units away from mean = 5. What is the **maximum probability** this happens? $\sigma = 2$.

3. If $\mu = 40$, $\sigma = 5$, how much probability lies **outside** the range 25 to 55?

4. A random variable X has unknown distribution, $\mu = 20$, $\sigma = 4$. What can you say about the probability of X being between 12 and 28?

5. True or False: Chebyshev's Theorem only works for Normal Distributions.

4.1 Answers

1. $k = \frac{14-10}{2} = 2 \rightarrow \geq 1 - \frac{1}{4} = 0.75$ (75%)

2. $P(|X - \mu| \geq 4)$, $k = \frac{4}{2} = 2 \rightarrow \leq \frac{1}{4} = 0.25$ (25%)

3. Range = $\mu \pm 3\sigma = 40 \pm 15 = [25, 55]$, $k = 3 \rightarrow \geq 1 - \frac{1}{9} = \frac{8}{9} \approx 0.8889$. Outside: $\leq \frac{1}{9} \approx 0.1111$

4. $k = \frac{28-20}{4} = 2 \rightarrow \geq 1 - \frac{1}{4} = 0.75$ (75%)

5. **False**: Chebyshev works for **all distributions**.

5 Final Revision Tip (Cheat Sheet)

Ready for the next concept? Ask your instructor!

lightgray **What to Look For**	**What to Do**
No distribution given	Use Chebyshev
Range given around mean	Use $k = \frac{\text{Range Edge} - \mu}{\sigma}$
Need minimum probability	Use $1 - \frac{1}{k^2}$
Need maximum “outside” prob.	Use $\frac{1}{k^2}$