REVISED

7
CHAPTER

Linear Programming Models: Graphical and Computer Methods

TEACHING SUGGESTIONS

Teaching Suggestion 7.1: *Draw Constraints for a Graphical LP Solution.*
Explain constraints of the three types (≤, =, ≥) carefully the first time you present an example. Show how to find the X_1, X_2 intercepts so a straight line can be drawn. Then provide some practice in determining which way the constraints point. This can be done by picking a few X_1, X_2 coordinates at random and indicating which direction fulfills the constraints.

Teaching Suggestion 7.2: *Feasible Region Is a Convex Polygon.*
Explain Dantzing's discovery that all feasible regions are convex (bulge outward) polygons (many-sided figures) and that the optimal solution must lie at one of the corner points. Draw both convex and concave figures to show the difference.

Teaching Suggestion 7.3: *Using the Iso-Profit Line Method.*
This method can be much more confusing than the corner point approach, but it is faster once students feel comfortable drawing the profit line. Start your first line at a profit figure you know is lower than optimal. Then draw a series of parallel lines, or run a ruler parallel, until the furthest corner point is reached. See Figures 7.6 and 7.7.

Teaching Suggestion 7.4: *QA in Action Boxes in the LP Chapters.*
There are a wealth of motivating tales of real-world LP applications in Chapters 7–9. The airline industry in particular is a major LP user.

Teaching Suggestion 7.5: *Feasible Region for the Minimization Problem.*
Students often question the open area to the right of the constraints in a minimization problem such as that in Figure 7.10. You need to explain that the solution is not unbounded in a minimization problem as it is in a maximization problem.

Teaching Suggestion 7.6: *Infeasibility.*
This problem is especially common in large LP formulations since many people will be providing input constraints to the problem. This is a real-world problem that should be expected.

Teaching Suggestion 7.7: *Alternative Optimal Solutions.*
This issue is an important one that can be explained in a positive way. Managers appreciate having choices of decisions that can be made with no penalty. Students can be made aware that alternative optimal solutions will arise again in the transportation model, assignment model, integer programming, and the chapter on network models.

Teaching Suggestion 7.8: *Importance of Sensitivity Analysis.*
Sensitivity analysis should be stressed as one of the most important LP issues. (Actually, the issue should arise for discussion with every model). Here, the issue is the source of data. When accountants tell you a profit contribution is \$8.50 per unit, is that figure accurate within 10% or within 10¢? The solution to an LP problem can change dramatically if the input parameters are not exact. Mention that sensitivity analysis also has other names, such as right-hand-side ranging, post-optimality analysis, and parametric programming.

ALTERNATIVE EXAMPLES

Alternative Example 7.1: Hal has enough clay to make 24 small vases or 6 large vases. He only has enough of a special glazing compound to glaze 16 of the small vases or 8 of the large vases. Let X_1 = the number of small vases and X_2 = the number of large vases. The smaller vases sell for \$3 each, while the larger vases would bring \$9 each.

(a) Formulate the problem.
(b) Solve graphically.

SOLUTION:

(a) Formulation

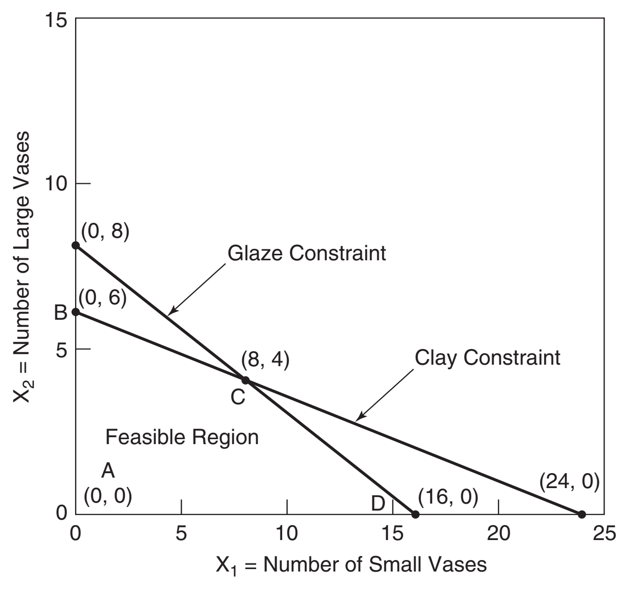
OBJECTIVE FUNCTION:

Maximize $\$3X_1 + \$9X_2$

Subject to : Clay constraint: $1X_1 + 4X_2 \leq 24$

Glaze constraint: $1X_1 + 2X_2 \leq 16$

(b) Graphical solution



Point	X_1	X_2	Income
A	0	0	$ 0
B	0	6	54
C	8	4	60*
D	16	0	48

*Optimum income of $60 will occur by making and selling 8 small vases and 4 large vases.

Draw an isoprofit line on the graph from (20, 0) to (0, 6⅔) as the $60 isoprofit line.

Alternative Example 7.2: A fabric firm has received an order for cloth specified to contain at least 45 pounds of cotton and 25 pounds of silk. The cloth can be woven out on any suitable mix of two yarns, *A* and *B*. Material *A* costs $3 per pound, and *B* costs $2 per pound. They contain the following proportions of cotton and silk (by weight):

Yarn	Cotton (%)	Silk (%)
A	30	50
B	60	10

What quantities (pounds) of *A* and *B* yarns should be used to minimize the cost of this order?

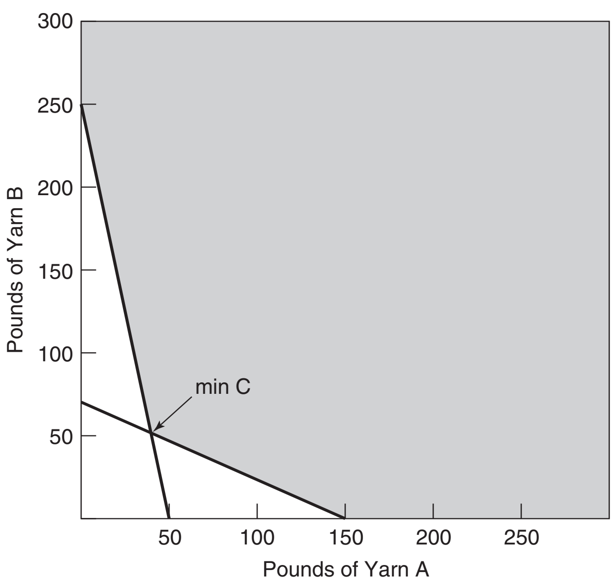
Objective function: min. $C = 3A + 2B$

Constrains: $0.30A + 0.60B \geqslant 45$ lb (cotton)

$0.50A + 0.10B \geqslant 25$ lb (silk)

Simultaneous solution of the two constraint equations reveals that $A = 39$ lb, $B = 55$ lb.

The minimum cost is $C = \$3A + \$2B = 3(39) + (2)(55) = \$227$.



SOLUTIONS TO DISCUSSION QUESTIONS AND PROBLEMS

7-1. Both minimization and maximization LP problems employ the basic approach of developing a feasible solution region by graphing each of the constraint lines. They can also both be solved by applying the corner point method. The isoprofit line method is used for maximization problems, whereas the isocost line is applied to minimization problems. Conceptually, isoprofit and isocost are the same.

The major differences between minimization and maximization problems deal with the shape of the feasible region and the direction of optimality. In minimization problems, the region must be bounded on the lower left, and the best isocost line is the one closest to the zero origin. The region may be unbounded on the top and right and yet be correctly formulated. A maximization problem must be bounded on the top and to the right. The isoprofit line yielding maximum profit is the one farthest from the zero origin.

7-2. The requirements for an LP problem are listed in Section 7.2. It is also assumed that conditions of certainty exist; that is, coefficients in the objective function and constraints are known with certainty and do not change during the period being studied. Another basic assumption that mathematically sophisticated students should be made aware of is *proportionality* in the objective function and constraints. For example, if one product uses 5 hours of a machine resource, then making 10 of that product uses 50 hours of machine time.

LP also assumes *additivity.* This means that the total of all activities equals the sum of each individual activity. For example, if the objective function is to maximize $P = 6X_1 + 4X_2$, and if $X_1 = X_2 = 1$, the profit contributions of 6 and 4 must add up to produce a sum of 10.

7-3. Each LP problem that has a feasible solution *does* have an infinite number of solutions. Only one of the points in the feasible region usually yields the *optimal* solution, but *all* of the points yield a feasible solution. If we consider the region to be continuous and accept noninteger solutions as valid, there will be an infinite number of feasible combinations of X_1 and X_2.

7-4. If a maximization problem has many constraints, then it can be very time consuming to use the corner point method to solve it. Such an approach would involve using simultaneous equations to solve for each of the feasible region's intersection points. The isoprofit line is much more effective if the problem has numerous constraints.

7-5. A problem can have alternative optimal solutions if the isoprofit or isocost line runs parallel to one of the problem's constraint lines (refer to Section 7.7 in the chapter).

7-6. This question involves the student using a little originality to develop his or her own LP constraints that fit the three conditions of (1) unboundedness, (2) infeasibility, and (3) redundancy. These conditions are discussed in Section 7.7, but each student's graphical displays should be different.

7-7. The manager's statement indeed had merit if the manager understood the deterministic nature of linear programming input data. LP assumes that data pertaining to demand, supply, materials, costs, and resources are known with certainty and are constant during the time period being analyzed. If this production manager operates in a very unstable environment (for example, prices and availability of raw materials change daily, or even hourly), the model's results may be too sensitive and volatile to be trusted. The application of sensitivity analysis might be trusted. The application of sensitivity analysis might be useful to determine whether LP would still be a good approximating tool in decision making.

7-8. The objective function is not linear because it contains the product of X_1 and X_2, making it a second-degree term. The first, second, fourth, and sixth constraints are okay as is. The third and fifth constraints are nonlinear because they contain terms to the second degree and one-half degree, respectively.

7-9. For a discussion of the role and importance of sensitivity analysis in linear programming, refer to Section 7.8. It is needed especially when values of the technological coefficients and contribution rates are estimated—a common situation. When all model values are deterministic, that is, known with certainty, sensitivity analysis from the perspective of evaluating parameter accuracy may not be needed. This may be the case in a portfolio selection model in which we select from among a series of bonds whose returns and cash-in values are set for long periods.

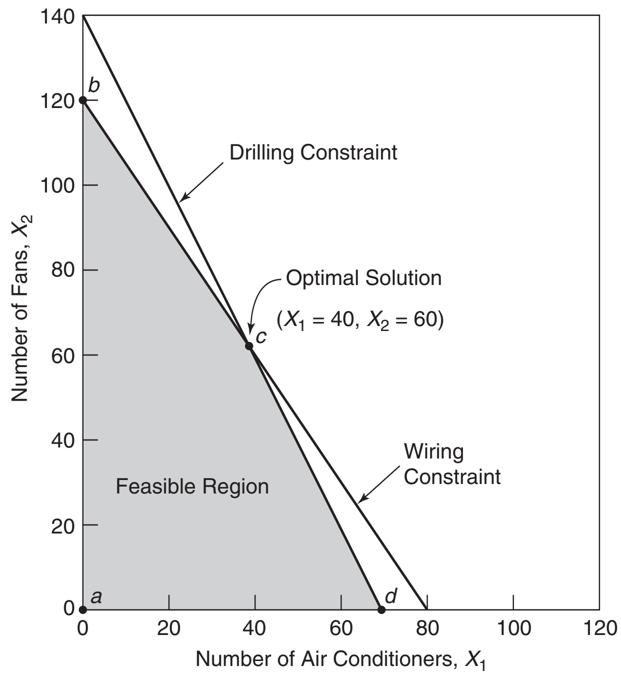
7-10. If the profit on *X* is increased from \$12 to \$15 (which is less than the upper bound), the same corner point will remain optimal. This means that the values for all variables will not change from their original values. However, total profit will increase by \$3 per unit for every unit of *X* in the original solution. If the profit is increased to \$25 (which is above the upper bound), a new corner point will be optimal. Thus, the values for *X* and *Y* may change, and the total profit will increase by at least \$13 (the amount of the increase) times the number of units of X in the original solution. The increase should normally be even more than this because the original optimal corner point is no longer optimal. Another corner point is optimal and will result in an even greater profit.

7-11. If the right-hand side of the constraint is increased from 80 to 81, the maximum total profit will increase by \$3, the amount of the dual price. If the right-hand side is increased by 10 units (to 90), the maximum possible profit will increase by 10(3) = \$30 and will be \$600 + \$30 = \$630. This \$3 increase in profit will result for each unit we increase the righthand side of the constraint until we reach 100, the upper bound. The dual price is not relevant beyond 100. Similarly, the maximum possible total profit will decrease by \$3 per unit that the right-hand side is decreased until this value goes below 75.

7-12. The student is to create his or her own data and LP formulation. (a) The meaning of the right-hand-side numbers (resources) is to be explained. (b) The meaning of the constraint coefficient (in terms of how many units of each resource that each product requires) is also to be explained. (c) The problem is to be solved graphically. (d) A simple sensitivity analysis is to be conducted by changing the contribution rate (C_j value) of the X_1 variable. For example, if C_1 was \$10 as the problem was originally formulated, the student should resolve with a \$15 value and compare solutions.

7-13. A change in a technological coefficient changes the feasible solution region. An increase means that each unit produced requires more of a scarce resource (and may lower the optimal profit). A decrease means that because of a technological advancement or other reason, less of a resource is needed to produce 1 unit. Changes in resource availability also change the feasible region shape and can increase or decrease profit.

7-14.



Let: X_1 = number of air conditioners to be produced

X_2 = number of fans to be produced

Maximize profit = $25X_1 + 15X_2$

subject to $3X_1 + 2X_2 \leq 240$ (wiring)

$2X_1 + 1X_2 \leq 140$ (drilling)

$X_1, X_2 \geq 0$

Profit at point *a* ($X_1 = 0, X_2 = 0$) = \$0

Profit at point *b* ($X_1 = 0, X_2 = 120$)

= 25(0) + (15)(120) = \$1,800

Profit at point *c* ($X_1 = 40, X_2 = 60$)

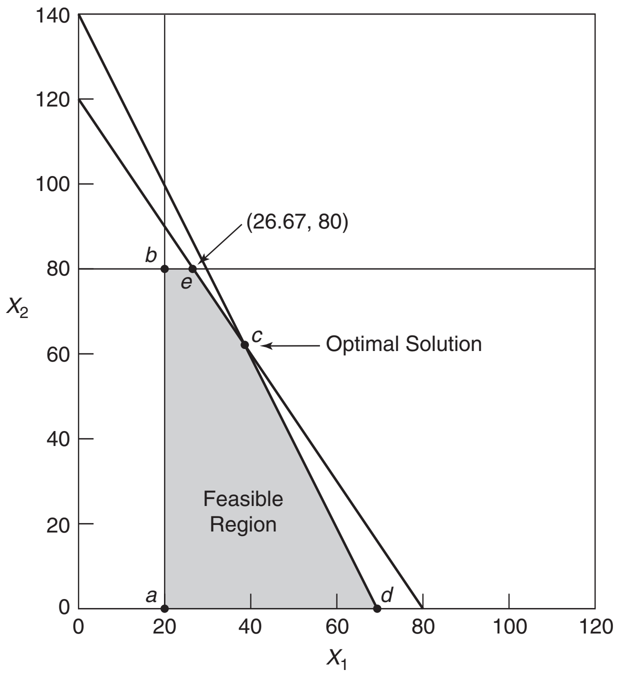
= 25(40) + (15)(60) = \$1,900

Profit at point *d* ($X_1 = 70, X_2 = 0$)

= 25(70) + (15)(0) = \$1,750

The optimal solution is to produce 40 air conditioners and 60 fans during each production period. Profit will be \$1,900.

7-15.



Maximize profit $= 25X_1 + 15X_2$

subject to $3X_1 + 2X_2 \leq 240$

$2X_1 + 1X_2 \leq 140$

$X_1 \geq 20$

$X_2 \leq 80$

$X_1, X_2 \geq 0$

Profit at point a ($X_1 = 20, X_2 = 0$)
$= 25(20) + (15)(0) = \$500$

Profit at point b ($X_1 = 20, X_2 = 80$)
$= 25(20) + (15)(80) = \$1{,}700$

Profit at point c ($X_1 = 40, X_2 = 60$)
$= 25(40) + (15)(60) = \$1{,}900$

Profit at point d ($X_1 = 70, X_2 = 0$)
$= 25(70) + (15)(0) = \$1{,}750$

Profit at point e ($X_1 = 26.67, X_2 = 80$)
$= 25(26.67) + (15)(80) = \$1{,}867$

Hence, even though the shape of the feasible region changed from Problem 7-14, the optimal solution remains the same.

7-16. Let R = number of radio ads; T = number of TV ads.

Maximize exposure $= 3{,}000R + 7{,}000T$

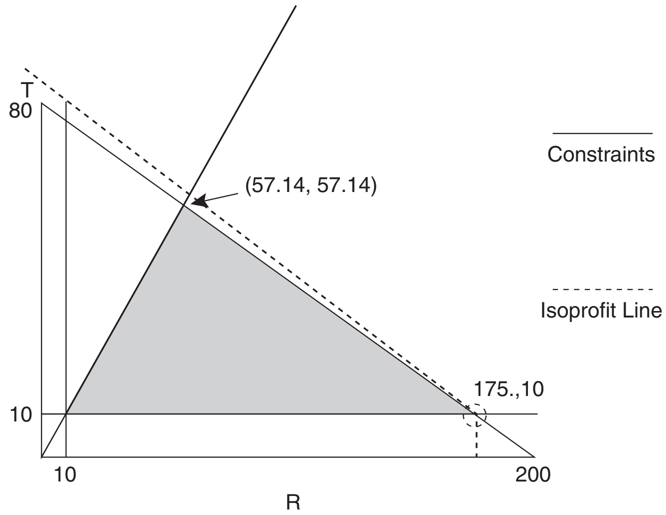
Subject to: $200R + 500T \leq 40{,}000$ (budget)

$R \geq 10$

$T \geq 10$

$R \geq T$

$R, T \geq 0$



Optimal corner point $R = 175$, $T = 10$,
Audience $= 3{,}000(175) + 7{,}000(10) = 595{,}000$ people

7-17. X_1 = number of benches produced

X_2 = number of tables produced

Maximize profit $= \$9X_1 + \$20X_2$

subject to $4X_1 + 6X_2 \leq 1{,}200$ hours

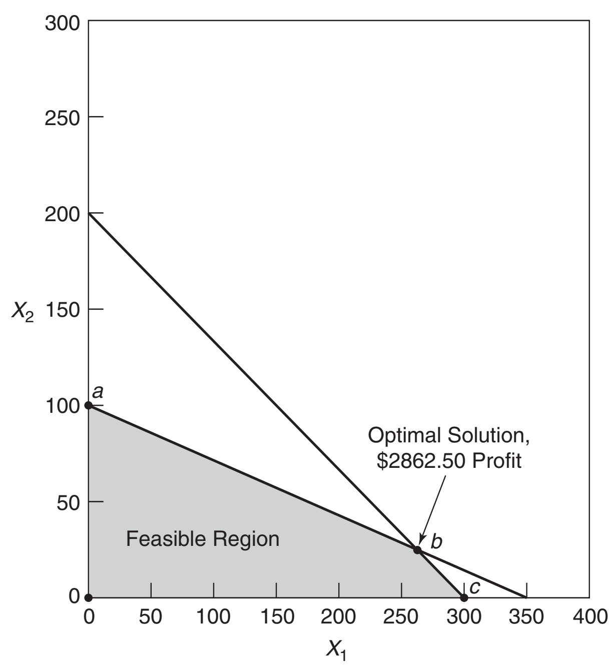
$10X_1 + 35X_2 \leq 3{,}500$ pounds

$X_1, X_2 \geq 0$

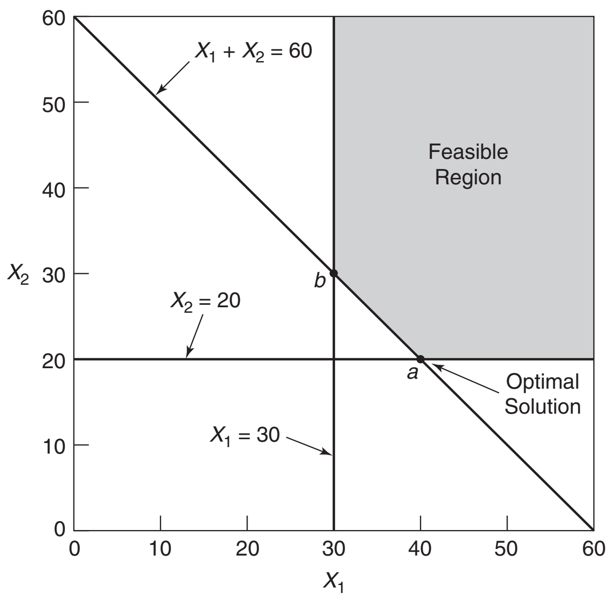
Profit at point a ($X_1 = 0, X_2 = 100$) $= \$2{,}000$

Profit at point b ($X_1 = 262.5, X_2 = 25$) $= \$2{,}862.50$

Profit at point c ($X_1 = 300, X_2 = 0$) $= \$2{,}700$



7-18.



X_1 = number of undergraduate courses

X_2 = number of graduate courses

Minimize cost = $2,500$X_1$ + $3,000$X_2$

subject to $X_1 \geq 30$

$X_2 \geqslant 20$

$X_1 + X_2 \geqslant 60$

Total cost at point *a* ($X_1 = 40$, $X_2 = 20$)

= 2,500(40) + (3,000)(20)

= $160,000

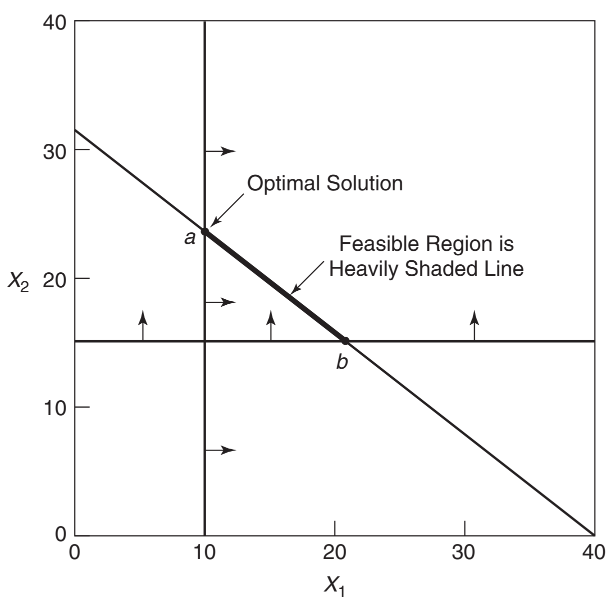
Total cost at point *b* ($X_1 = 30$, $X_2 = 30$)

= 2,500(30) + (3,000)(30)

= $165,000

Point *a* is optimal.

7-19.



X_1 = number of Alpha 4 computers

X_2 = number of Beta 5 computers

Maximize profit = $1,200$X_1$ + $1,800$X_2$

subject to $20X_1 + 25X_2 = 800$ hours
(total hours = 5 workers × 160 hours each)

$X_1 \geqslant 10$

$X_2 \geqslant 15$

Corner points: $a(X_1 = 10, X_2 = 24)$, profit = $55,200

$b(X_1 = 21\frac{1}{4}, X_2 = 15)$, profit = $52,500

Point *a* is optimal.

7-20. Let P = dollars invested in petrochemical; U = dollars invested in utility

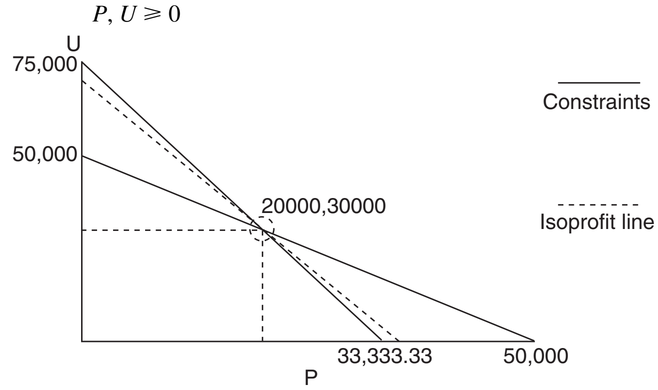
Maximize return = $0.12P + 0.06U$

Subject to:

$P + U = 50{,}000$ total investment is $50,000

$9P + 4U \leqslant 6(50{,}000)$ average risk must be less 6 [or total less than 6(50,000)]

$P, U \geqslant 0$



Corner points

P	U	Return = $0.12P + 0.06U$
0	50,000	3,000
20,000	30,000	4,200

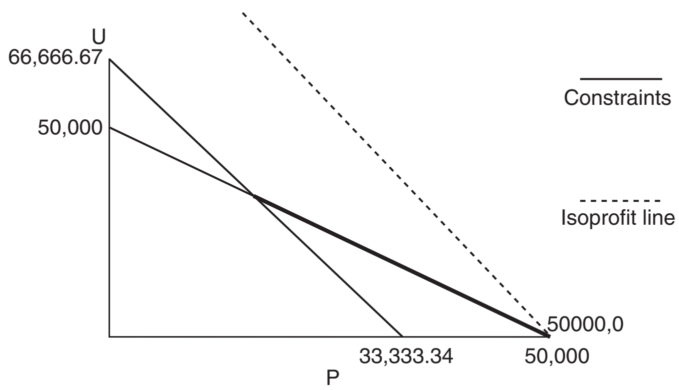
The maximum return is $4,200.
The total risk is 9(20,000) + 4(30,000) = 300,000, so average risk = 300,000/(50,000) = 6

7-21. Let P = dollars invested in petrochemical; U = dollars invested in utility

Minimize risk = $9P + 4U$

Subject to:

$P + U = 50{,}000$ total investment is $50,000



$0.12P + 0.06U \geqslant 0.08(50{,}000)$ return must be at least 8%
$P, U \geqslant 0$
Corner points

P	U	RISK = 9P + 4U
50,000	0	450,000
16,666.67	33,333.33	283,333.3

The minimum risk is 283,333.33 on \$50,000 so the average risk is 283,333.33/50,000 = 5.67. The return would be 0.12(16,666.67) + 0.06(33,333.33) = \$4,000 (or 8% of \$50,000)

7-22.

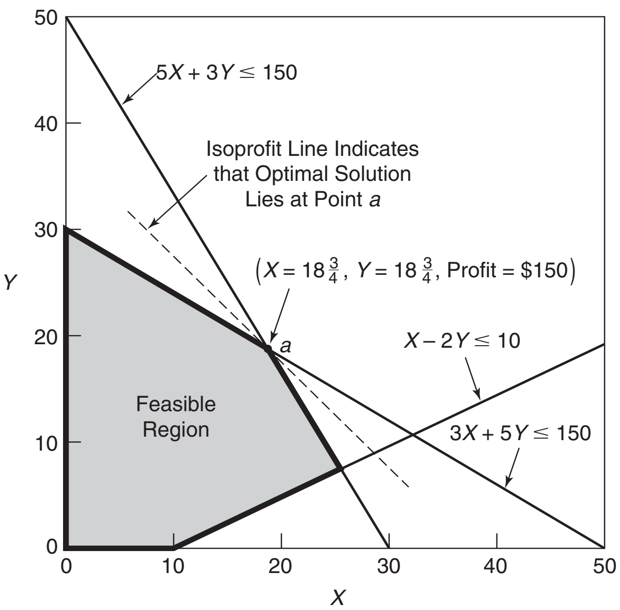
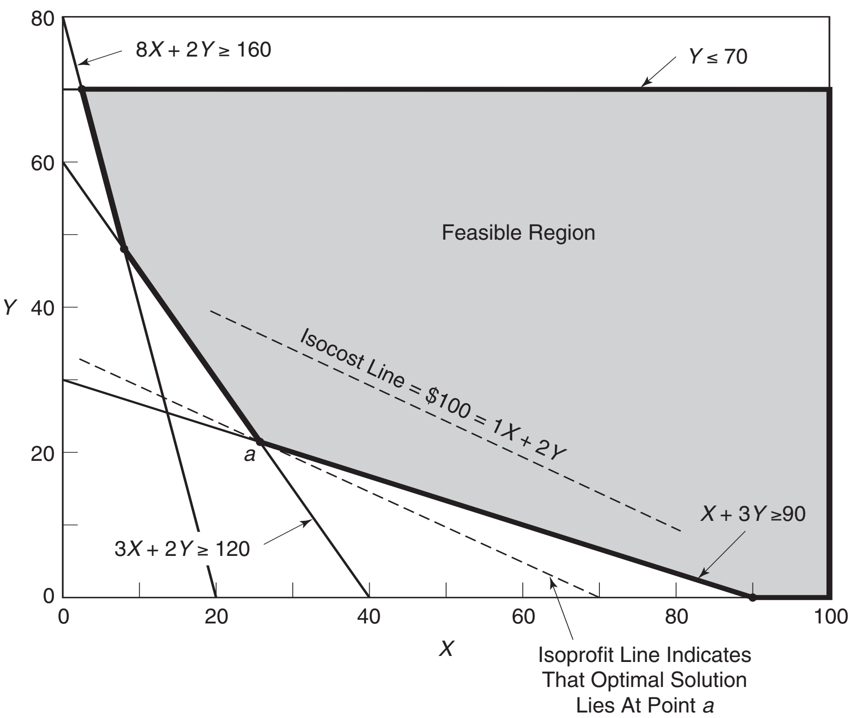


Figure for Problem 7-23.



Note that this problem has one constraint with a negative sign. This may cause the beginning student some confusion in plotting the line.

7-23. Point a lies at intersection of constraints (see figure below):

$$3X + 2Y = 120$$
$$X + 3Y = 90$$

Multiply the second equation by -3 and add it to the first (the method of simultaneous equations):

$$3X + 2Y = 120$$
$$\underline{-3X - 9Y = -270}$$
$$-7Y = -150 \Rightarrow Y = 21.43 \quad \text{and} \quad X = 25.71$$

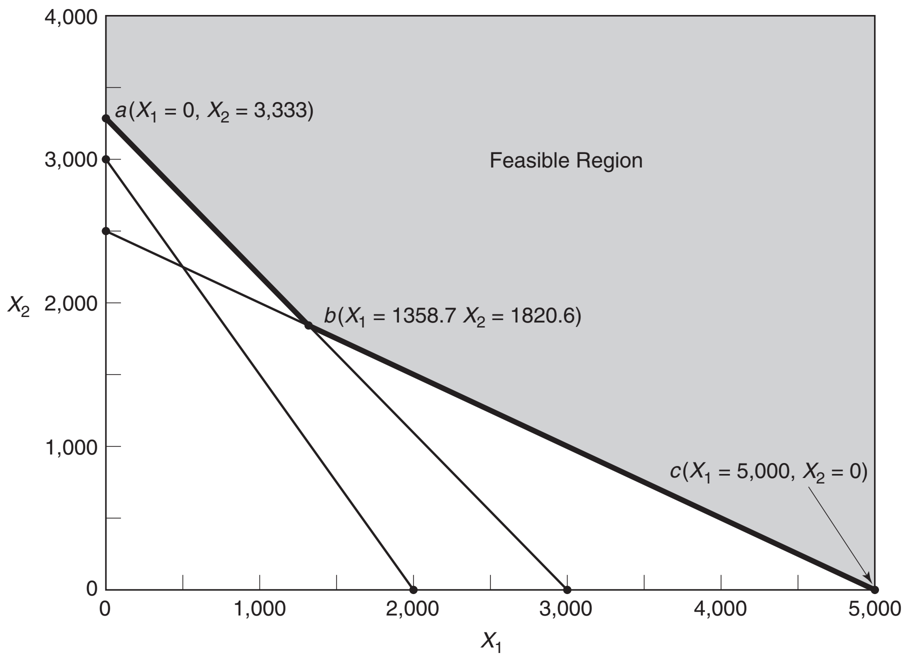
$$\text{Cost} = \$1X + \$2Y = \$1(25.71) + (\$2)(21.43)$$
$$= \$68.57$$

7-24. X_1 = \$ invested in Louisiana Gas and Power
X_2 = \$ invested in Trimex Insulation Co.
Minimize total investment = $X_1 + X_2$

subject to
$$\$0.36X_1 + \$0.24X_2 \geqslant \$720$$
$$\$1.67X_1 + \$1.50X_2 \geqslant \$5{,}000$$
$$0.04X_1 + 0.08X_2 \geqslant \$200$$

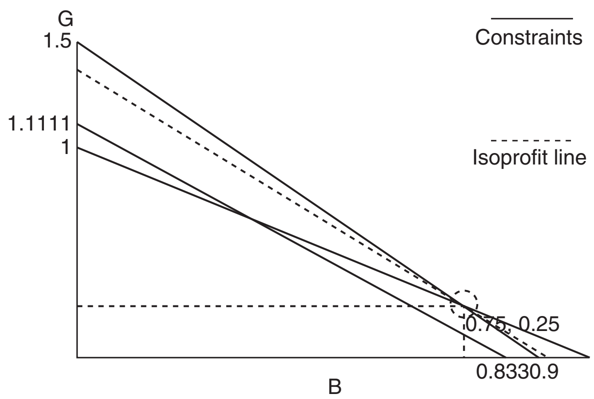
Investment at a is \$3,333.
Investment at b is \$3,179. ← optimal solution
Investment at c is \$5,000.
Short-term growth is \$926.09.
Intermediate-term growth is \$5,000.
Dividends are \$200.
See graph.

Figure for Problem 7-24.



7-25. Let B = pounds of beef in each pound of dog food
G = pounds of grain in each pound of dog food
Minimize cost = $0.90B + 0.60G$
Subject to:

$B + G = 1$	the total weight should be 1 pound
$10B + 6G \geq 9$	at least 9 units of Vitamin 1
$12B + 9G \geq 10$	at least 10 units of Vitamin 2
$B, G \geq 0$	



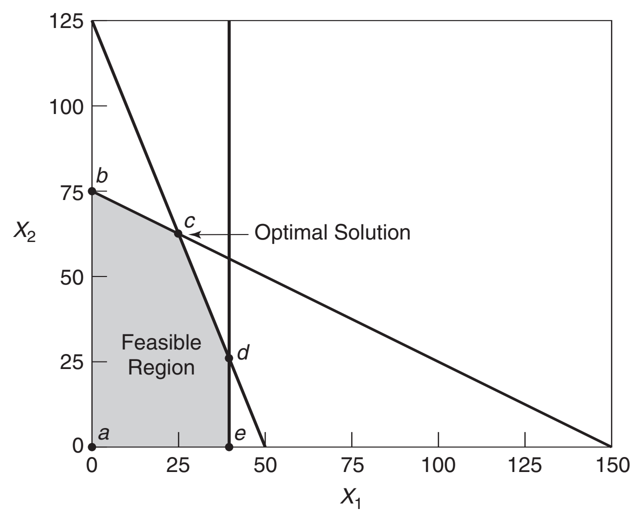
The feasible corner points are (0.75, 0.25) and (1,0). The minimum cost solution
B = 0.75 pounds of beef, G = 0.25 pounds of grain, cost = \$0.825,
Vitamin 1 content = 10(0.75) + 6(0.25) = 9
Vitamin 2 content = 12(0.75) + 9(0.25) = 11.25

7-26. Let X_1 = number of barrels of pruned olives
X_2 = number of barrels of regular olives
Maximize profit = $\$20X_1 + \$30X_2$
subject to $5X_1 + 2X_2 \leq 250$ (labor hours)
$1X_1 + 2X_2 \leq 150$ (acres)
$X_1 \leq 40$ (barrels)
$X_1, X_2 \geq 0$

a. Corner point $a = (X_1 = 0, X_2 = 0)$, profit = 0
Corner point $b = (X_1 = 0, X_2 = 75)$, profit = \$2,250
Corner point $c = (X_1 = 25, X_2 = 62\frac{1}{2})$, profit = \$2,375 ← optimal profit
Corner point $d = (X_1 = 40, X_2 = 25)$, profit = \$1,550
Corner point $e = (X_1 = 40, X_2 = 0)$, profit = \$800

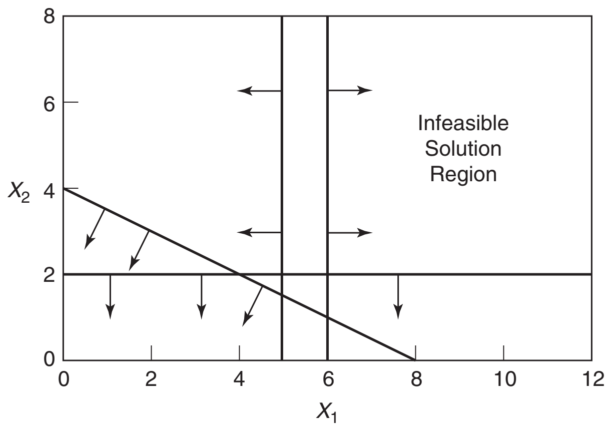
b. Produce 25 barrels of pruned olives and 62½ barrels of regular olives.

c. Devote 25 acres to pruning process and 125 acres to regular process.

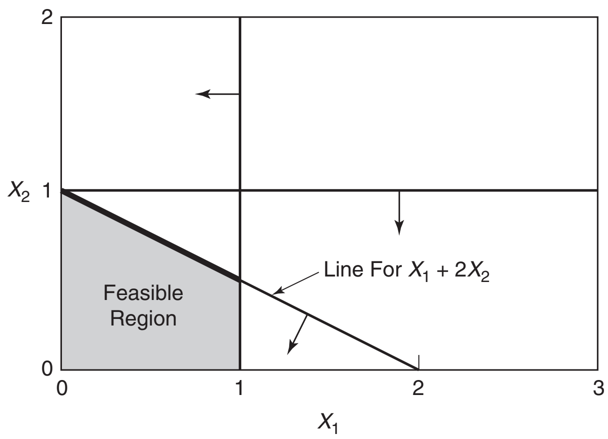


7-27.

Formulation 1:

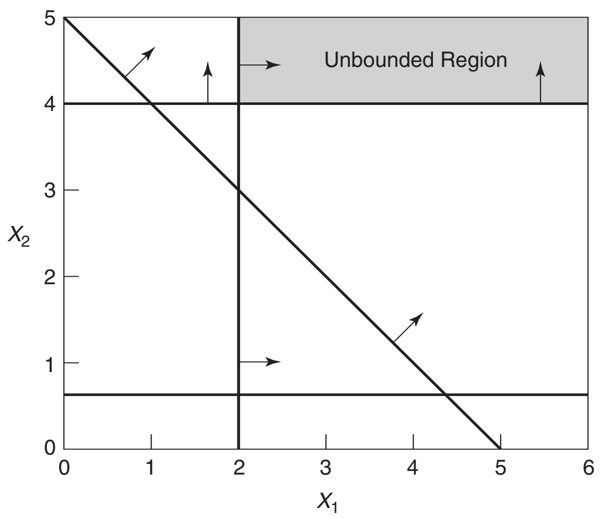


Formulation 2:

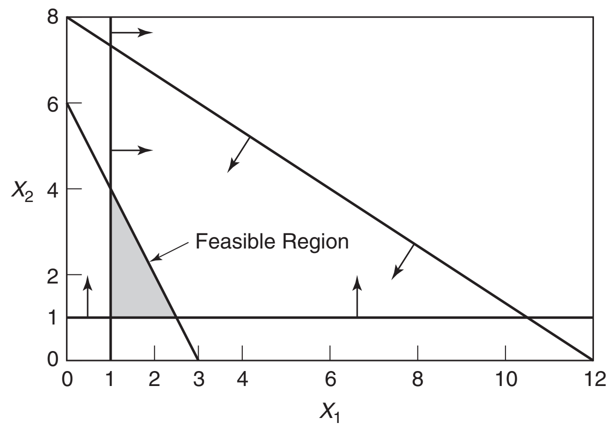


While formulation 2 is correct, it is a special case. $X_1 + 2X_2 = 2$ line—this is also the same slope as the isoprofit line $X_1 + 2X_2$ and hence there will be more than one optimal solution. As a matter of fact, every point along the heavy line will provide an "alternate optimum."

Formulation 3:

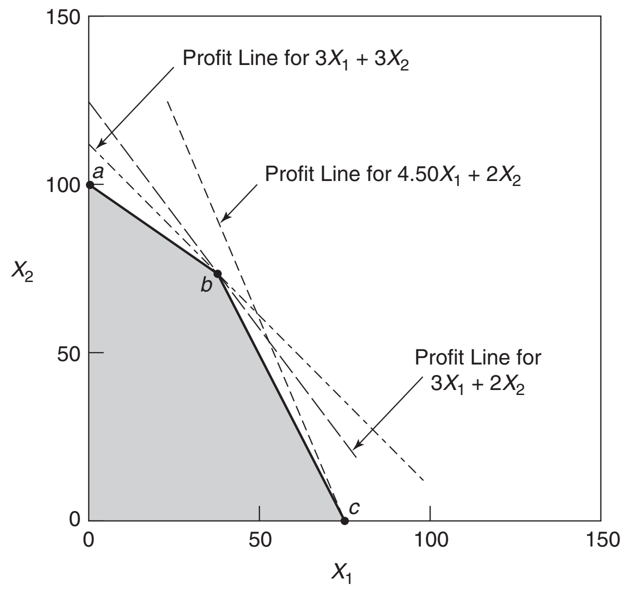


Formulation 4:

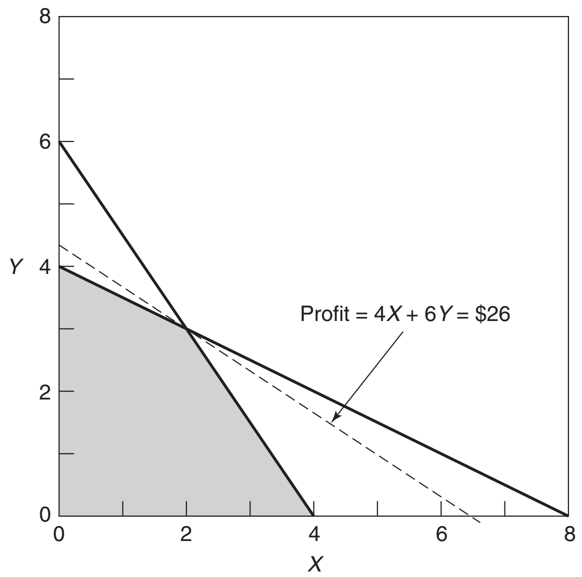


Formulation 4 appears to be proper as is. Note that the constraint $4X_1 + 6X_2 \leqq 48$ is redundant.

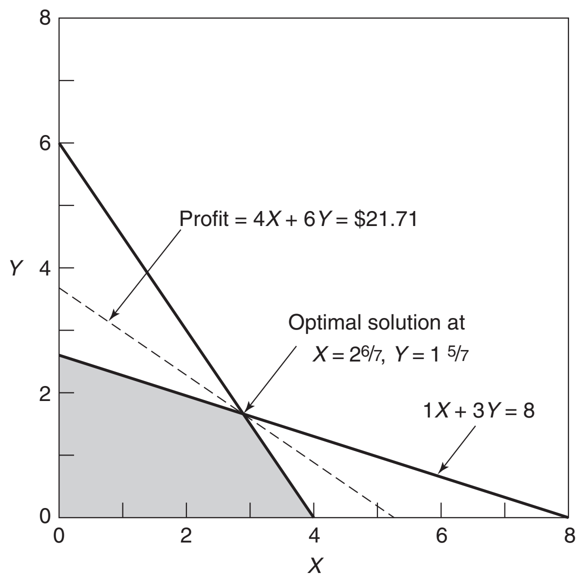
7-28. Using the isoprofit line or corner point method, we see that point *b* (where $X = 37.5$ and $Y = 75$) is optimal if the profit $= \$3X + \$2Y$. If the profit changes to \$4.50 per unit of *X*, the optimal solution shifts to point *c*. If the objective function becomes $P = \$3X + \$3Y$, the corner point *b* remains optimal.



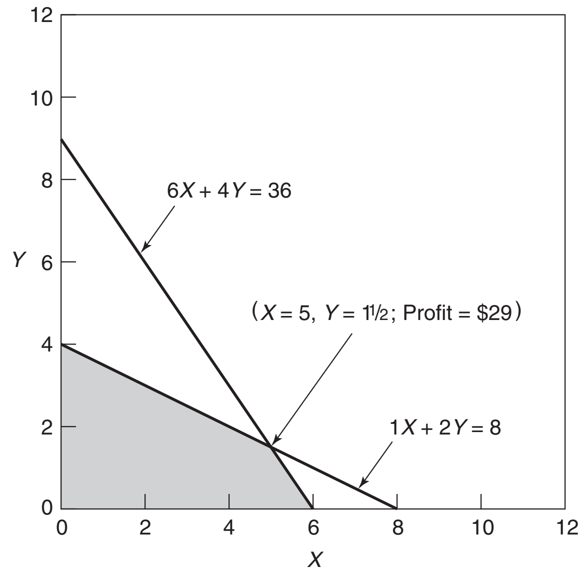
7-29. The optimal solution of \$26 profit lies at the point $X = 2$, $Y = 3$.



If the first constraint is altered to $1X + 3Y \leq 8$, the feasible region and optimal solution shift considerably, as shown in the next column.



7-30.



Using the corner point method, we determine that the optimal solution mix under the new constraint yields a \$29 profit, or an increase of \$3 over the *\$26* profit calculated. Thus, the firm should not add the hours because the cost is more than \$3.

7-31. a. The corner points and profits are

$X = 0$, $Y = 0$, profit $= 0$

$X = 60$, $Y = 0$, profit $= 300$

$X = 30$, $Y = 60$, profit $= 510 \leftarrow$ Optimal solution

$X = 0$, $Y = 80$, profit $= 480$

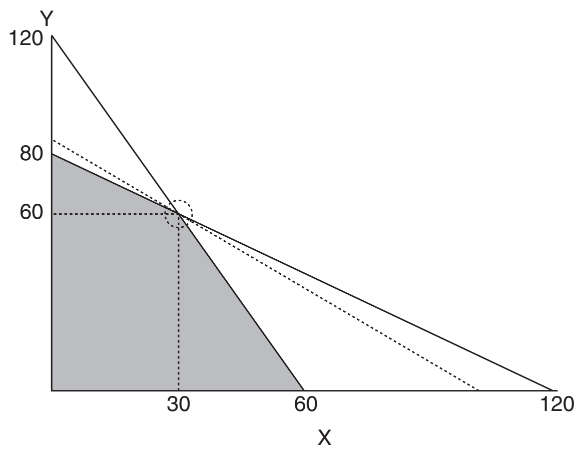
b. If profit $= 8X + 6Y$, the optimal solution is at the same corner point but profit increases.

$X = 0$, $Y = 0$, profit $= 0$

$X = 60$, $Y = 0$, profit $= 480$

$X = 30$, $Y = 60$, profit $= 600 \leftarrow$ Optimal solution

$X = 0$, $Y = 80$, profit $= 480$



c. If profit = $3X + 6Y$, a new corner point is optimal.

$X = 0$, $Y = 0$, profit = 0

$X = 60$, $Y = 0$, profit = 180

$X = 30$, $Y = 60$, profit = 450

$X = 0$, $Y = 80$, profit = 480 ← Optimal solution

7-32. The corner points change and the new optimal solution is X = 40, Y = 40, and profit = 440. The corner points are

X = 0, Y = 0, profit = 0

X = 60, Y = 0, profit = 300

X = 40, Y = 40, profit = 440 ← Optimal solution

X = 0, Y = 60, profit = 360

7-33. a. It could increase by 7 (for an upper limit of 12) or decrease by 1 (for a lower limit of 4).

b. Profit would increase by the dual value of 0.75.

c. Profit would increase by 10 times the dual price or 10(0.75) = $7.50.

7-34. a. 25 units of product 1 and 0 units of product 2.

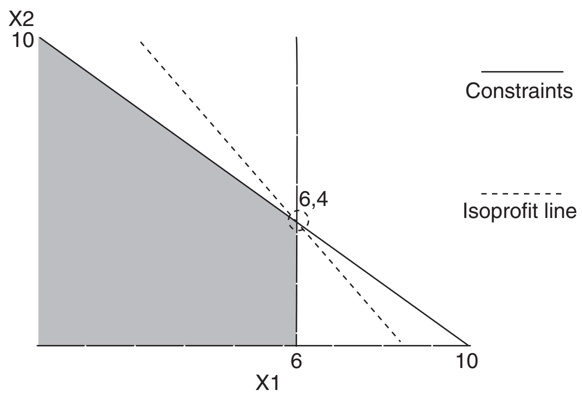
b. All of resource 3 is being used (there is no slack for constraint 3). A total of 25 units of resource 1 is being used since there were 45 units available and there are 20 units of slack. A total of 75 units of product 2 being used since there were 87 units available and there are 12 units of slack.

c. The dual price for constraint 1 is 0, for constraint 2 is 0, and for constraint 3 is 25.

d. You should try to obtain resource 3 because the dual price is 25. This means profit will increase by 25 for each unit of resource 3 that we obtain. Therefore, we should pay up to $25 for this.

e. If management decided to produce one more unit of product 2 (currently 0 units are being produced), the total profit would decrease by 5 (the amount of the reduced cost).

7-35.



a. The feasible corner points and their profits are:

Feasible corner points	Profit = 8X1 + 5X2
(0,0)	0
(6,0)	48
(6,4)	68
(0,10)	50

The optimal solution is X1 = 6, X2 = 4, profit = $68.

b. The feasible corner points and their profits are:

Feasible corner points	Profit = 8X1 + 5X2
(0,0)	0
(6,0)	48
(6,5)	73
(0,11)	55

The new optimal solution is X1 = 6, X2 = 5, profit = $73. Profit increased $5, so this is the dual price for constraint 1.

c. The feasible corner points and their profits are:

Feasible corner points	Profit = 8X1 + 5X2
(0,0)	0
(6,0)	48
(0,6)	30

As a result of this change, the feasible region got smaller. Profit decreased by $20. The right-hand side decreased by 4 units, and the profit decreased by 4 x dual price.

d. The feasible corner points and their profits are:

Feasible corner points	Profit = 8X1 + 5X2
(0,0)	0
(5,0)	40
(0,5)	25

As a result of this change, the feasible region got smaller. Profit decreased by $28. Although there was a 5-unit change in the right-hand side of constraint 1, the dual price found in part b is not valid when the right-hand side of this constraint goes below 6 (which is a 4-unit decrease).

e. The computer output indicates that the dual price for constraint 1 is $5, but this is valid up to a lower bound of 6. Once the right-hand side goes lower than this, the dual price is no longer relevant.

g. When the right-hand side goes beyond the limits, a new corner point becomes optimal so the dual price is no longer relevant.

7-36. Let: X_1 = number of coconuts carried

X_2 = number of skins carried

Maximize profit = $60X_1 + 300X_2$ (in rupees)

subject to $5X_1 + 15X_2 \leq 300$ pounds

$\frac{1}{8}X_1 + 1X_2 \leq 15$ cubic feet

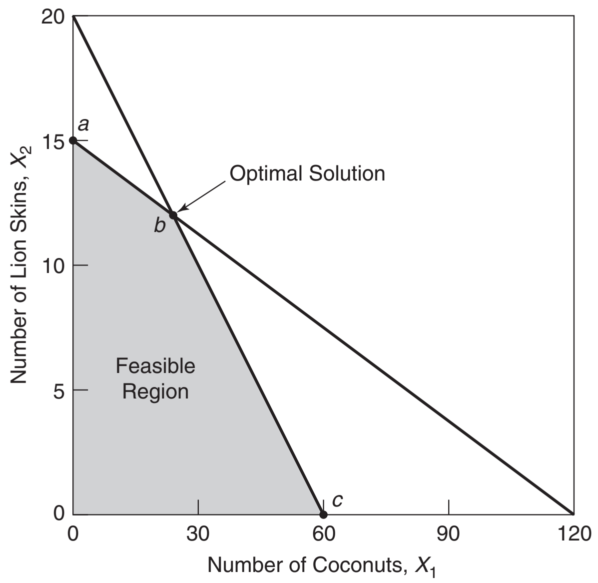
$X_1, X_2 \geq 0$

At point a: ($X_1 = 0, X_2 = 15$), P = 4,500 rupees

At point b: ($X_1 = 24, X_2 = 12$), P = 1,440 + 3,600 = 5,040 rupees

At point c: ($X_1 = 60, X_2 = 0$), P = 3,600 rupees

The three princes should carry 24 coconuts and 12 lions' skins. This will produce a wealth of 5,040 rupees.



7-37. a. \$120,000 in money market fund; \$80,000 in stock fund; total risk = 1,560,000

b. Total return = \$14,000. Rate of return = 14,000/200,000 = 0.07

c. The investments would not change since 14 is less than the upper bound for this coefficient. The total risk would increase.

d. The total risk would worsen by 2 (the dual value) per additional dollar.

e. No. The amount invested in the money market fund is greater than \$50,000 for the original solution.

7-38. a. \$40,000 in money market fund; \$160,000 in stock fund; total return = 18,000

b. Total risk = 12(160,000) + 5(40,000) = 2,120,000. Average risk = 2,120,000/200,000 = 10.6.

c. No. The change is above the lower bound.

d. Dual value = 0.10 = 10%

e. Total return would change by (dual price)(change in RHS) = (−0.05)(10,000) = −500.

7-39. a. Let: X_1 = number of pounds of stock X purchased per cow each month

X_2 = number of pounds of stock Y purchased per cow each month

X_3 = number of pounds of stock Z purchased per cow each month

Four pounds of ingredient Z per cow can be transformed to:

4 pounds × (16 oz/lb) = 64 oz per cow

5 pounds = 80 oz

1 pound = 16 oz

8 pounds = 128 oz

$3X_1 + 2X_2 + 4X_3 \geqslant 64$ (ingredient A requirement)

$2X_1 + 3X_2 + 1X_3 \geqslant 80$ (ingredient B requirement)

$1X_1 + 0X_2 + 2X_3 \geqslant 16$ (ingredient C requirement)

$6X_1 + 8X_2 + 4X_3 \geqslant 128$ (ingredient D requirement)

$X_3 \leq 5$ (stock Z limitation)

Minimize cost = $\$2X_1 + \$4X_2 + \$2.50X_3$

b. Cost = \$80

X_1 = 40 lbs. of X

X_2 = 0 lbs. of Y

X_3 = 0 lbs. of Z

7-40. Let: X_1 = number units of XJ201 produced

X_2 = number units of XM897 produced

X_3 = number units of TR29 produced

X_4 = number units of BR788 produced

Maximize profit = $9X_1 + 12X_2 + 15X_3 + 11X_4$

subject to

$0.5X_1 + 1.5X_2 + 1.5X_3 + 1X_4 \leq 15{,}000$ (hours of wiring time available)

$0.3X_1 + 1X_2 + 2X_3 + 3X_4 \leq 17{,}000$ (hours of drilling time available)

$0.2X_1 + 4X_2 + 1X_3 + 2X_4 \leq 26{,}000$ (hours of assembly time available)

$0.5X_1 + 1X_2 + 0.5X_3 + 0.5X_4 \leq 12{,}000$ (hours of inspection time)

$X_1 \geqslant 150$ (units of XJ201)

$X_2 \geqslant 100$ (units of XM897)

$X_3 \geqslant 300$ (units of TR29)

$X_4 \geqslant 400$ (units of BR788)

7-41. Let

S_{N1} = number of standard racquets produced in current month on normal time

S_{O1} = number of standard racquets produced in current month on overtime

S_{N2} = number of standard racquets produced in next month on normal time

S_{O2} = number of standard racquets produced in next month on overtime

P_{N1} = number of professional racquets produced in current month on normal time

P_{O1} = number of professional racquets produced in current month on overtime
P_{N2} = number of professional racquets produced in next month on normal time
P_{O2} = number of professional racquets produced in next month on overtime
I_S = number of standard racquets left in inventory at end of current month
I_P = number of professional racquets left in inventory at end of current month

Minimize cost = $40S_{N1} + 50S_{O1} + 44S_{N2} + 55S_{O2} + 60P_{N1} + 70P_{O1} + 66P_{N2} + 77P_{O2} + 2_{IS} + 2_{IP}$

Subject to:

$I_S = S_{N1} + S_{O1} - 180$	Standard racquets remaining is number produced less demand
$I_P = P_{N1} + P_{O1} - 90$	Professional racquets remaining is number produced less demand
$S_{N2} + S_{O2} + I_S \geqslant 200$	Demand for standard racquets next month
$P_{N2} + P_{O2} + I_P \geqslant 120$	Demand for professional racquets next month
$S_{N1} + P_{N1} \leqslant 230$	Capacity in current month on normal time
$S_{O1} + P_{O1} \leqslant 80$	Capacity in current month on overtime
$S_{N2} + P_{N2} \leqslant 230$	Capacity next month on normal time
$S_{O2} + P_{O2} \leqslant 80$	Capacity next month on overtime

All variables $\geqslant 0$

7-42. a. Let: X_1 = number of MCA regular modems made and sold in November

X_2 = number of MCA intelligent modems made and sold in November

Data needed for variable costs and contribution margin (refer to the table on the bottom of this page):

Hours needed to produce each modem:

$$\text{MCA regular} = \frac{5{,}000 \text{ hours}}{9{,}000 \text{ modems}} = 0.555 \text{ hour/modem}$$

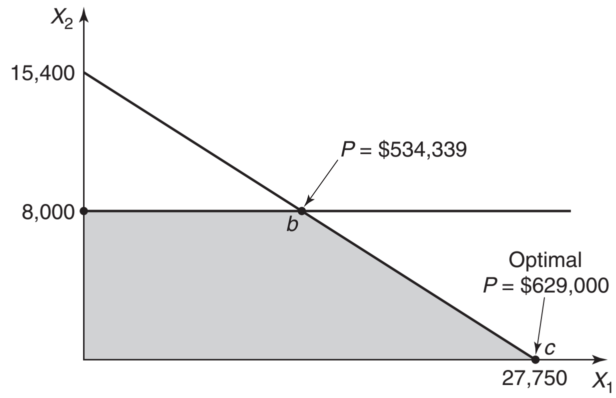
$$\text{MCA intelligent} = \frac{10{,}400 \text{ hours}}{10{,}400 \text{ modems}} = 1.0 \text{ hour/modem}$$

Maximize profit = $\$22.67X_1 + \$29.01X_2$

subject to $0.555X_1 + 1.0X_2 \leq 15{,}400$ (direct labor hours)

$X_2 \leq 8{,}000$ (intelligent modems)

b.



c. The optimal solution suggests making all MCA regular modems. Students should discuss the implications of shipping no MCA intelligent modems.

7-43. Minimize cost = $12X_1 + 9X_2 + 11X_3 + 4X_4$

subject to

$$X_1 + X_2 + X_3 + X_4 = 50$$
$$X_4 \geq 7.5$$
$$X_1 + X_2 \geq 22.5$$
$$X_2 + X_3 \leq 15.0$$

Solution:

X_1 = 7.5 pounds of C-30
X_2 = 15 pounds of C-92
X_3 = 0 pounds of D-21
X_4 = 27.5 pounds of E-11

Cost = \$3.35.

7-44. Let A_1 = gallons of crude A used in Regular
A_2 = gallons of crude A used in Premium
A_3 = gallons of crude A used in Super
B_1 = gallons of crude B used in Regular
B_2 = gallons of crude B used in Premium
B_3 = gallons of crude B used in Super

Minimize cost = $0.42A_1 + 0.42A_2 + 0.42A_3 + 0.47B_1 + 0.47B_2 + 0.47B_3$

Subject to

$0.40A_1 + 0.52B_1 \geq 0.41(A_1 + B_1)$

Table for Problem 7-42(a)

	MCA Regular Modem		MCA Intelligent Modem	
	Total	**Per Unit**	**Total**	**Per Unit**
Net sales	$424,000	$47.11	$613,000	$58.94
Variable costs[a]				
Direct labor	60,000	6.67	76,800	7.38
Indirect labor	9,000	1.00	11,520	1.11
Materials	90,000	10.00	128,000	12.31
General expenses	30,000	3.33	35,000	3.37
Sales commissions	31,000	3.44	60,000	5.76
Total variable costs	$220,000	$24.44	$311,320	$29.93
Contribution margin	$204,000	$22.67	$301,680	$29.01

[a]Depreciation, fixed general expense, and advertising are excluded from the calculations.

$0.40A_2 + 0.52B_2 \geq 0.44(A_2 + B_2)$

$0.40A_3 + 0.52B_3 \geq 0.48(A_3 + B_3)$

$A_1 + B_1 \geq 20{,}000$

$A_2 + B_2 \geq 15{,}000$

$A_3 + B_3 \geq 10{,}000$

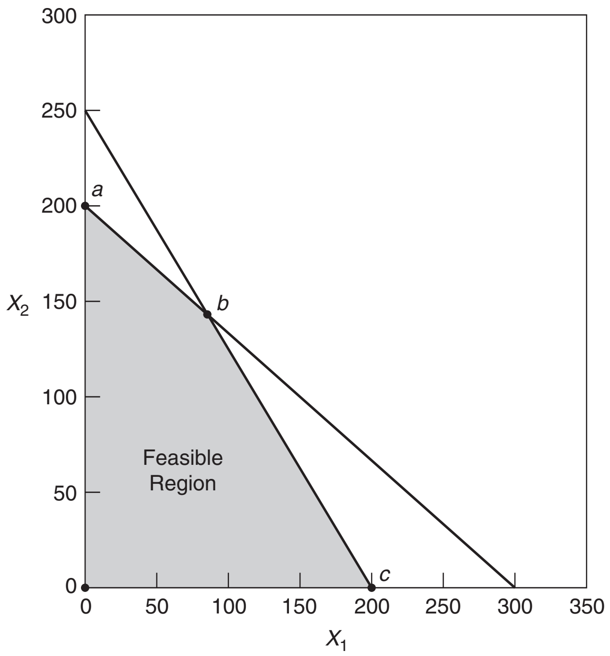
$A_1, A_2, A_3, B_1, B_2, B_3 \geq 0$

The solution is

A_1 = 18,333.33 gallons of crude A used in Regular; A_2 = 10,000 gallons of crude A used in Premium; A_3 = 3,333.33 gallons of crude A used in Super; B_1 = 1.666.67 gallons of crude B used in Regular, B_2 = 5,000 gallons of crude B used in Premium ; B_3 = 6,666.67 gallons of crude B used in Super; total cost = $19,566.67.

SOLUTIONS TO INTERNET HOMEWORK PROBLEMS

7-45.



X_1 = number of model *A* tubs produced

X_2 = number of model *B* tubs produced

Maximize profit = $90X_1 + 70X_2$

subject to $125X_1 + 100X_2 \leqslant 25{,}000$ (steel)

$20X_1 + 30X_2 \leqslant 6{,}000$ (zinc)

$X_1, X_2 \geqslant 0$

Profit at point *a* ($X_1 = 0, X_2 = 200$) = $14,000

Profit at point *b* ($X_1 = 85.71, X_2 = 142.86$) = $17,714.10

Profit at point *c* ($X_1 = 200, X_2 = 0$) = $18,000 ← optimal solution

7-46. Let: X_1 = number of pounds of compost in each bag

X_2 = number of pounds of sewage waste in each bag

Minimize cost = $5X_1 + 4X_2$ (in cents)

subject to $X_1 + X_2 \geqslant 60$ (pounds per bag)

$X_1 \geq 30$ (pounds compost per bag)

$X_2 \leqslant 40$ (pounds sewage per bag)

Corner point *a:*

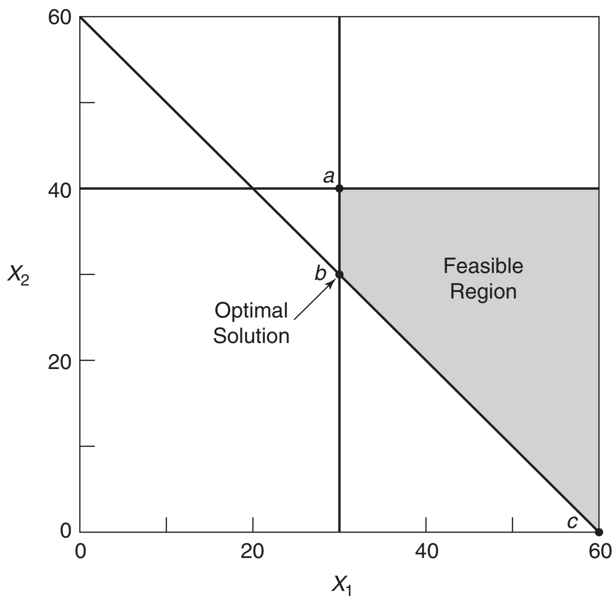
($X_1 = 30, X_2 = 40$) ⇒ cost = 5(30) + (4)(40) = $3.10

Corner point *b:*

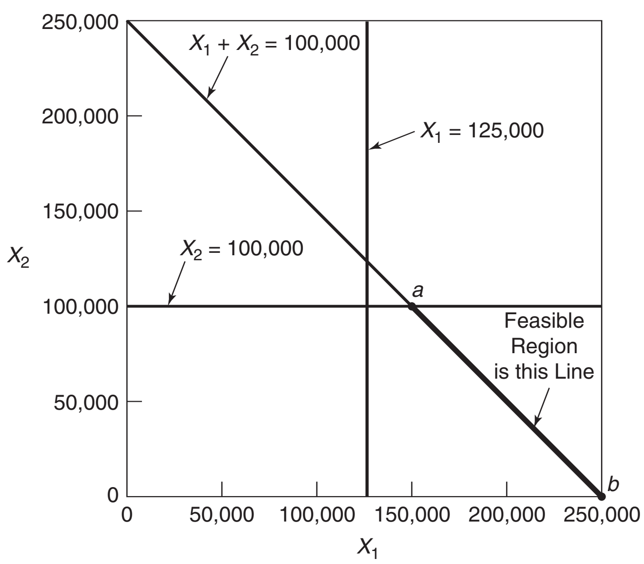
($X_1 = 30, X_2 = 30$) ⇒ cost = 5(30) + (4)(30) = $2.70

Corner point *c:*

($X_1 = 60, X_2 = 0$) ⇒ cost = 5(60) + (4)(0) = $3.00



7-47.



X_1 = \$ invested in Treasury notes

X_2 = \$ invested in bonds

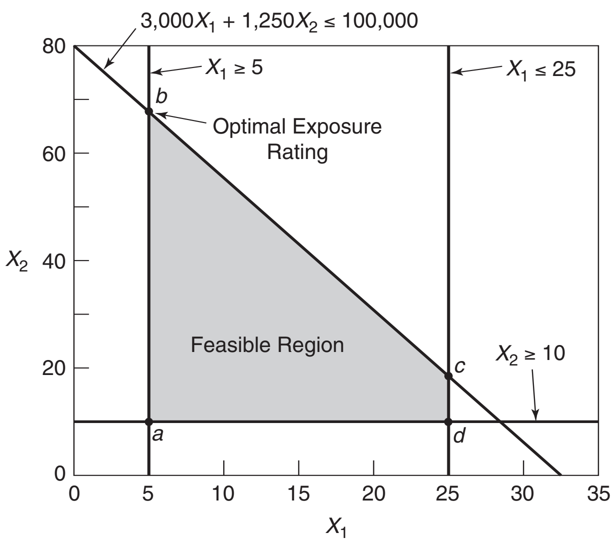
Maximize ROI $= 0.08X_1 + 0.09X_2$

$$X_1 \geq \$125{,}000$$
$$X_2 \leq \$100{,}000$$
$$X_1 + X_2 = \$250{,}000$$
$$X_1, X_2 \geq 0$$

Point a ($X_1 = 150{,}000$, $X_2 = 100{,}000$), ROI = \$21,000 ← optimal solution

Point b ($X_1 = 250{,}000$, $X_2 = 0$), ROI = \$20,000

7-48.



Let: X_1 = number of TV spots

X_2 = number of newspaper ads

Maximize exposures $= 35{,}000X_1 + 20{,}000X_2$

subject to $3000X_1 + 1{,}250X_2 \leq \$100{,}000$

$$X_1 \geq 5$$
$$X_1 \leq 25$$
$$X_2 \geq 10$$

Point a ($X_1 = 5$, $X_2 = 10$), exposure = 375,000

Point b ($X_1 = 5$, $X_2 = 68$), exposure = 175,000
+ 1,360,000
= 1,535,000
(optimal)

Point c ($X_1 = 25$, $X_2 = 20$), exposure = 875,000
+ 400,000
= 1,275,000

Point d ($X_1 = 25$, $X_2 = 10$), exposure = 875,000
+ 200,000
= 1,075,000

7-49. Maximize $Z = [220 - (0.45)(220) - 44 - 20]X_1 + [175 - (0.40)(175) - 30 - 20]X_2 = 57X_1 + 55X_2$

Constraints:

$X_1 + X_2 \leq 390$ production limit

$2.5X_1 + 2.4X_2 \leq 960$ labor hours

Corner points:

$X_1 = 384$, $X_2 = 0$, profit = \$21,888

$X_1 = 0$, $X_2 = 390$, profit = \$21,450

$X_1 = 240$, $X_2 = 150$, profit = \$21,930

Students should point out that those three options are so close in profit that production desires and sensitivity of the RHS and cost coefficient are important issues. This is a good lead-in to the discussion of sensitivity analysis. As a matter of reference, the right-hand side ranging for the first constraint is a production limit from 384 to 400 units. For the second constraint, the hours may range only from 936 to 975 without affecting the solution.

The objective function coefficients, similarly, are very sensitive. The \$57 for X_1 may increase by 29 cents or decrease by \$2. The \$55 for X_2 may increase by \$2 or decrease by 28 cents.

SOLUTION TO MEXICANA WIRE WORKS CASE

1. Maximize P = 34 W75C + 30 W33C + 60 W5X + 25 W7X subject to:

1 W75C ≤ 1,400
1 W33C ≤ 250
1 W5X ≤ 1,510
1 W7X ≤ 1,116
1 W75C + 2 W33C + 0 W5X + 1 W7X ≤ 4,000
1 W75C + 1 W33C + 4 W5X + 1 W7X ≤ 4,200
1 W75C + 3 W33C + 0 W5X + 0 W7X ≤ 2,000
1 W75C + 0 W33C + 3 W5X + 2 W7X ≤ 2,300
1 W75C ≥ 150
1 W7X ≥ 600

Solution: Produce:

1,100 units of W75C—backorder 300 units
250 units of W33C—backorder 0 units
0 units of W5X—backorder 1,510 units
600 units of W7X—backorder 516 units

Maximized profit will be \$59,900. By addressing quality problems listed earlier, we could increase our capacity by up to 3% reducing our backorder level.

2. Bringing in temporary workers in the Drawing Department would not help. Drawing is not a binding constraint. However, if these former employees could do rework, we could reduce our rework inventory and fill some of our backorders thereby increasing profits. We have about a third of a month's output in rework inventory. Expediting the rework process would also free up valuable cash.

3. The plant layout is not optimum. When we install the new equipment, an opportunity for improving the layout could arise. Exchanging the locations for packaging and extrusion would create a better flow of our main product. Also, as we improve our quality and reduce our rework inventory, we could capture some of the space now used for rework storage and processing and put it to productive use.

Our machine utilization of 63% is quite low. Most manufacturers strive for at least an 85% machine utilization. If we could determine the cause(s) of this poor utilization, we might find a key to a dramatic increase in capacity.

INTERNET CASE STUDY: AGRI-CHEM CORPORATION

This case demonstrates an interesting use of linear programming in a production setting.

Let X_1 = ammonia
X_2 = ammonium phosphate
X_3 = ammonium nitrate
X_4 = urea
X_5 = hydrofluoric acid
X_6 = chlorine
X_7 = caustic soda
X_8 = vinyl chloride monomer

Objective function:

$$\text{Maximize Profit} = 80X_1 + 120X_2 + 140X_3 + 140X_4 + 90X_5 + 70X_6 + 60X_7 + 90X_8$$

Subject to the following constraints:

$X_1 \leq 1{,}200$ $\quad X_5 \leq 560$
$X_2 \leq 540$ $\quad X_6 \leq 1{,}200$
$X_3 \leq 490$ $\quad X_7 \leq 1{,}280$
$X_4 \leq 160$ $\quad X_8 \leq 840$

Current natural gas usage = 85,680 cu. ft. $\times$ 10^3/day
20 percent curtailment = 68,554 cu. ft. $\times$ 10^3/day
Hence, the ninth constraint is:

$$8X_1 + 10X_2 + 12X_3 + 12X_4 + 7X_5 + 18X_6 + 20X_7 + 14X_8 \leq 68{,}544$$

The following is the production schedule (tons/day);

$X_1 = 1{,}200$ $\quad X_5 = 560$
$X_2 = 540$ $\quad X_6 = 1{,}200$
$X_3 = 490$ $\quad X_7 = 423.2$
$X_4 = 160$ $\quad X_8 = 840$

Objective function value = \$487,192

Because of the natural gas curtailment, the caustic soda production is reduced from 1280 tons/day to 425 tons/day.

For a 40 percent natural gas curtailment, the ninth constraint is:

$$8X_1 + 10X_2 + 12X_3 + 12X_4 + 7X_5 + 18X_6 + 20X_7 + 14X_8 \leq 51{,}408$$

The optimal solution results in the following production schedule:

$X_1 = 1200$ $\quad X_5 = 560$
$X_2 = 540$ $\quad X_6 = 718{,}2$
$X_3 = 490$ $\quad X_7 = 0$
$X_4 = 160$ $\quad X_8 = 840$

Objective function value: \$428,075.6

The caustic soda production is eliminated completely and the chlorine production is reduced from 1,200 to 720 tons/day.