

21230006

M. Umar Farooq

Question# 01

Answer the following questions about the function whose derivative is $f'(x) = x^3 + 5x^2 + 6x$

- (i) What are the critical points of f ?
- (ii) On what intervals is f increasing or decreasing?
- (iii) Inflection points and intervals for which function is concave up and

and concave down?

(iv) At what points, if any, does f assume local maximum and minimum values?

$$f'(x) = x^3 + 5x^2 + 6x$$

→ Critical point:

$$f'(x) = 0$$

$$0 = x^3 + 5x^2 + 6x$$

$$= x^3 + 3x^2 + 2x^2 + 6x$$

$$= x^2(x+3) + 2x(x+3)$$

$$0 = (x^2 + 2x)(x+3)$$

$$x^2 + 2x = 0$$

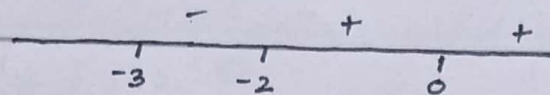
$$x + 3 = 0$$

$$x(x+2) = 0$$

$$x = -3$$

$$x = 0 \mid x = -2$$

ii, Increasing/decreasing



$$f(-1) = (-1)^3 + 5(-1)^2 + 6(-1) = -1 + 5 - 6 = -7 + 5 = -2$$

$$f(-2.5) = (-2.5)^3 + 5(-2.5)^2 + 6(-2.5) = -15.625 + 31.25 - 15 = 0.625$$

$$f(-4) = (-4)^3 + 5(-4)^2 + 6(-4) = -64 + 80 - 24 = -8$$

$$f(1) = (1)^3 + 5(1)^2 + 6(1)$$

$$= 1 + 5 + 6$$

$$= 12$$

increasing
decreasing

$$(-2, 0) \cup (0, \infty)$$

$$(-\infty, -3) \cup (-3, -2)$$

(iii) Concave/up and down.

$$f''(x) = 3x^2 + 10x + 6$$

$$f''(x) = 0$$

$$0 = 3x^2 + 10x + 6$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{100 - 4(3)(6)}}{2(3)}$$

$$= \frac{-10 \pm \sqrt{28}}{6}$$

$$= \frac{-10 \pm 2\sqrt{7}}{6}$$

$$x = \frac{-10 + 2\sqrt{7}}{6}$$

$$x = \frac{-10 - 2\sqrt{7}}{6}$$

$$x = -0.7$$

$$x = -2.54$$

$$f''(1) = 3(-1)^2 + 10(1) + 6 = -1$$

$$f''(3) = 3(3)^2 + 10(3) + 6 = 63$$

$$f''(1) = 3(1)^2 + 10(1) + 6$$

Concave up

$$(-\infty, -2.54) \cup (-0.7, \infty)$$

Concave down

$$(-2.54, -0.7)$$

iv)

maxima/minima

maxima = $\left(-\frac{10}{3}, \frac{662}{27}\right)$ local maxima

minima = $(0, 6)$ local minima