

# All Calc 2 Integration Techniques

(50 integrals for calc 2 students)

Video: <https://youtu.be/XOUwldufY9Y>

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## I. Know your Derivatives

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$$(Q1.) \int \sec^2 x \, dx$$

$$(Q2.) \int \frac{1}{x} \, dx$$

$$(Q3.) \int \frac{1}{\sqrt{1-x^2}} \, dx$$

$$(Q4.) \int \sec x \tan x \, dx$$

$$(Q5.) \int \frac{1}{1+x^2} \, dx$$

$$(Q6.) \int \cos x \, dx$$

$$(Q7.) \int \sin x \, dx$$

$$(Q8.) \int e^x \, dx$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(b^x) = b^x \ln b$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

## II. Reverse Power Rule

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$$(Q9.) \int \sqrt{x}(x+4) \, dx$$

$$(Q10.) \int \frac{1+x^6}{x^2} \, dx$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$$

## III. U sub

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$$(Q11.) \int 4x^3 \sec^2(x^4) \, dx$$

$$(Q12.) \int \frac{x^3}{1+x^4} \, dx$$

$$(Q13.) \int \frac{x}{1+x^4} \, dx$$

$$(Q14.) \int \frac{1}{1+\sqrt{x}} \, dx$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

$$\int f'(g(x))g'(x) \, dx = f(g(x)) + C$$

$$\int f(ax+b) \, dx = \frac{1}{a} \int f(u) \, du$$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

## IV. Know the famous ones (part 1. famous first step)

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$$(Q15.) \int \tan x \, dx$$

$$(Q16.) \int \sec x \, dx$$

$$(Q17.) \int \frac{1}{x^3+x} \, dx$$

## V. Say NO to these integral additions

~~$$(Q18.) \int \sin^3 x \, dx = \frac{1}{4} \sin^4 x + C$$~~

~~$$(Q19.) \int \frac{1}{1+\sqrt{x}} \, dx = \ln|1+\sqrt{x}| + C$$~~

~~$$(Q20.) \int e^{x^2} \, dx = \frac{1}{2x} e^{x^2} + C$$~~

~~$$(Q21.) \int \tan^{-1} x \, dx = \frac{1}{1+x^2} + C$$~~

~~$$(Q22.) \int x^2 \sin x \, dx = -\frac{1}{3} x^3 \cos x + C$$~~

$$\int x^3 \, dx = \frac{1}{4} x^4 + C \quad \int \sin^3 x \cos x \, dx = \frac{1}{4} \sin^4 x + C$$

$$\int \frac{1}{1+x} \, dx = \ln|1+x| + C \quad \int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int e^{2x} \, dx = \frac{1}{2} e^{2x} + C \quad \int f(ax+b) \, dx = \frac{1}{a} \int f(u) \, du$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\int (x^2 + \sin x) \, dx = \int x^2 \, dx + \int \sin x \, dx = \frac{1}{3} x^3 - \cos x + C$$

## VI. Know the famous ones (part2. non-elementary integrals)

$$\int e^{x^2} \, dx \quad \int e^{-x^2} \, dx \quad \int \frac{\sin x}{x} \, dx \quad \int \frac{\cos x}{x} \, dx$$

$$\int \frac{e^x}{x} \, dx \quad \int \frac{1}{\ln x} \, dx \quad \int \sin(x^2) \, dx \quad \int \cos(x^2) \, dx$$

$$\int x^x \, dx \quad \int \sqrt{1+x^3} \, dx$$

## VII. Integration by Parts

$$(Q23.) \int x \cos(x^2) \, dx$$

$$(Q24.) \int x \cos x \, dx$$

$$(Q25.) \int x^3 \ln x \, dx$$

$$(Q22^*) \int x^2 \sin x \, dx$$

$$(Q25^*) \int x^3 \ln x \, dx$$

$$(Q26.) \int e^x \sin(2x) \, dx$$

$$(Q21^*) \int \tan^{-1} x \, dx$$

$$(Q27.) \int \frac{\ln x}{\sqrt{x}} \, dx$$

$$(Q28.) \int x^2 e^{3x} \, dx$$

$$(Q29.) \int x \sec x \tan x \, dx$$

$$d(uv) = u \, dv + v \, du$$

$$\int u \, dv = uv - \int v \, du$$

## VIII. Use Trig Identities

$$(Q30.) \int \sin^2 x \cos x \, dx$$

$$(Q31.) \int \sin^2 x \, dx$$

$$(Q18^*) \int \sin^3 x \, dx$$

$$(Q32.) \int \sec^4 x \, dx$$

$$(Q33.) \int \sec^4 x \tan x \, dx$$

$$(Q34.) \int \tan^3 x \, dx$$

$$(Q35.) \int \sec^3 x \, dx$$

$$\int \left( \begin{array}{c} \text{an expression} \\ \text{in terms of } \sin x \end{array} \right) \cos x \, dx \quad \int \left( \begin{array}{c} \text{an expression} \\ \text{in terms of } \cos x \end{array} \right) \sin x \, dx$$

$$\int \left( \begin{array}{c} \text{an expression} \\ \text{in terms of } \tan x \end{array} \right) \sec^2 x \, dx \quad \int \left( \begin{array}{c} \text{an expression} \\ \text{in terms of } \sec x \end{array} \right) \sec x \tan x \, dx$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$\sec^2 \theta = \tan^2 \theta + 1$$

$$\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha - \beta) + \sin(\alpha + \beta))$$

$$\sin \alpha \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

## IX. Trig Sub

$$(Q36.) \int \sqrt{x^2 - 6x + 9} \, dx$$

$$(Q37.) \int \sqrt{x^2 + 9} \, dx$$

$$(Q38.) \int \frac{1}{x\sqrt{x^2 - 4}} \, dx$$

$$(Q39.) \int \sqrt{1 - x^2} \, dx$$

$$(Q40.) \int \frac{1}{(25 + x^2)^{\frac{3}{2}}} \, dx$$

$$(Q41.) \int \frac{x}{(25 + x^2)^{\frac{3}{2}}} \, dx$$

$$(Q42.) \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$$

you see	you let	you use
$\sqrt{x^2 + a^2}$	$x = a \tan \theta$	$\tan^2 \theta + 1 = \sec^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$\sec^2 \theta - 1 = \tan^2 \theta$
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$1 - \sin^2 \theta = \cos^2 \theta$

## X. Partial Fractions

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$$(Q43.) \int \frac{x^3}{x^2+9} dx$$

$$(Q44.) \int \frac{8x-17}{x^2-5x+4} dx$$

$$(Q45.) \int \frac{4x^2-9x+2}{(x+3)(x^2+4)} dx$$

$$(Q46.) \int \frac{1}{x^2+6x+13} dx$$

$$(Q47.) \int \frac{2x-5}{x^3+x^2} dx$$

$$(Q48.) \int \frac{2x^2+8x+5}{x^2+4x+13} dx$$

$$(Q49.) \int \frac{6x^2+31x+45}{x^3+6x^2+9x} dx$$

$$(Q50.) \int \frac{1}{x^2-a^2} dx$$

$$\int \frac{1}{(ax+b)^n} dx = \frac{1}{a(1-n)}(ax+b)^{1-n} + C, \quad n \neq 1$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int \frac{x}{ax^2+b} dx = \frac{1}{2a} \ln|ax^2+b| + C$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

### linear factors

$$\frac{8x-17}{x^2-5x+4} = \frac{A}{x-1} + \frac{B}{x-4}$$

### Irreducible Quadratic Factors

$$\frac{4x^2-9x+2}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

### Repeated Factors

$$\frac{2x-5}{x^2(x+1)} = \frac{Ax+B}{x^2} + \frac{C}{x+1} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\frac{2x-5}{x^3(x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+1} + \frac{E}{(x+1)^2}$$

$$\frac{6x^2+31x+45}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$\frac{2x-5}{(x+1)(x^2+4)^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$